Study of Doppler reflectometry capability to determine the perpendicular velocity and the k-spectrum of the density fluctuations using a 2D full-wave code

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Abstract

The capability of Doppler reflectometry to determine the perpendicular velocity and the perpendicular wave-number spectrum of the density fluctuations is studied with a two-dimensional full-wave code in X mode. The code is applied to study the influence of the probing beam and plasma characteristics on Doppler measurements. The results show that Gaussian beams can provide accurate perpendicular velocity measurements for broad ranges of antenna tilt angles, turbulence levels, turbulence spectral widths and cut-off layer curvatures. The capability to determine the perpendicular wave-number spectrum of the density fluctuations is studied under different turbulence levels and antenna tilt angles. We consider an optimum case: a Gaussian beam with a relatively large beam waist and a plasma slab. The results show a linear dependence between the scattered power and the turbulence level for turbulence levels up to 2%; At higher turbulence levels this relationship becomes non linear. The numerical results also show that the reflectometer response depends on the probed wave-number. Such dependence, though weak, should be taken into account in the evaluation of the wave-number spectra with Doppler reflectometry.

1. Introduction

Doppler reflectometry allows the determination of the perpendicular velocity, \( v_\perp \), and the perpendicular wave-number spectrum, \( k_\perp \)-spectrum, of the density fluctuations [1-4]. A deliberated antenna tilt angle \( \theta \) with respect to the cut-off layer makes the reflectometer sensitive to density perturbations with perpendicular wave-number \( k_\perp = 2k_0 \sin \theta \). If the antenna pattern and the antenna tilt angle are properly chosen the antenna selects preferentially the diffraction order -1\(^{st}\) whereas the 0\(^{th}\) order is partially or even completely suppressed. The spectral resolution of the Doppler reflectometer \( \Delta k / k_\perp \) determines its capability to separate both diffraction orders and therefore to achieve quantitative results. The perpendicular velocity of the density fluctuations can be obtained from the Doppler shift \( f_D \) of -1\(^{st}\) diffraction order and the returned signal intensity contains information about the turbulence level. Thus, the \( k_\perp \)-spectrum can be measured by changing the antenna tilt angle, i.e. by changing the probed wave-number of the density fluctuations.

From the simulated spectra it is possible to estimate the Doppler shifted structure characteristics: the Doppler peak amplitude \( A_p \), the Doppler shifted peak frequency \( f_p \),
and the spectral width $\Delta f$. These magnitudes are obtained by fitting the non-symmetric part of the simulated spectra with a Gaussian function $A_p \exp\left(-\left(f - f_p\right)^2 / \Delta f^2\right)$.

Many parameters may affect the Doppler measurements: beam size and beam curvature, antenna tilt angle, cut-off layer curvature, turbulence level, wave-number spectrum of the turbulence, etc. The influence of these parameters in determining the perpendicular velocity of density fluctuations was already reported in [5] and it is briefly summarized in section 2. New results studying the capability of Doppler reflectometry to determine the perpendicular velocity and the $k_\perp$-spectrum in a large range of turbulence levels and antenna tilt angles are shown in sections 3 and 4.

2. Effect of the cut-off and wave-front curvatures

The spectral resolution of a Doppler reflectometer depends on the probing beam size, $w_0$, the cut-off layer curvature, $R_{\text{plasma}}$, and the wave front curvatures, $R_{\text{beam}}$. Such dependence can be estimated as [1]:

$$\frac{\Delta k}{k_\perp} = \frac{\sqrt{2}}{w_0 k_0 \sin \theta} \left[1 + (w_0^2 k_0 / \rho)^2 \right]^{1/2}$$

with $\rho = R_{\text{beam}} R_{\text{plasma}} / (R_{\text{beam}} + R_{\text{plasma}})$.

Figure 1a shows the estimated spectral resolution (1) as a function of the normalized beam waist for a plasma with high curvature in red ($R_{\text{plasma}} / \lambda_0 \approx 30$), and for a plasma slab in blue. The antenna tilt angle is $\theta = 18^\circ$ and the probing frequency is 40 GHz. Two different wave front curvatures are considered: a negligible wave front curvature (beam waist located at the cut-off layer), and a non-negligible one (beam waist located 6 cm in front of the cut-off layer).

![Figure 1a](image1a.png)

**Figure 1.** (a) Estimation of the spectral resolution given by (1) as a function of the normalized beam waist for two plasma geometries and two wave front curvatures. (b) Spectral resolution of the simulated spectra as a function of the beam waist for both plasma geometries. The black star is the spectral resolution obtained with a plasma slab and a standard horn.
The figure shows that there is an optimum beam size (best spectral resolution) in the case of high curvature plasmas (red line) while in a plasma slab the resolution improves as the beam size increases (blue line). When the beam divergence is considered (blue and red dots) the spectral resolution gets worse for narrow beam waists but the same qualitative behaviour is observed for both cut-off layer curvatures.

Figure 1b shows the 2D full-wave results for comparison with the previous estimates shown in figure 1a. It displays the normalized spectral width of the simulated spectra, $\Delta f / f_D$, for both plasma curvatures and non-negligible wave front curvature. In both cases the trend of $\Delta f / f_D$ follows that of the estimated $\Delta k / k_\perp$. In the slab geometry $\Delta f / f_D$ decreases as the beam size increases (blue dots), while an optimum beam size (minimum $\Delta f / f_D$) is found for high curvature plasmas (red dots). Figure 1b also shows the normalized spectral width of the simulated spectra obtained with a standard horn and a plasma slab (black star). In this case, the beam divergence is much higher than the one with Gaussian beams and therefore the spectral resolution is very poor.

In the cases shown in figure 1b obtained with Gaussian beams the obtained Doppler frequency agrees with the expected Doppler shift within 5%. However, if standard horns are used the error in the determination of the perpendicular velocity is, in general, much higher than 5%. The accuracy in the Doppler shift measurements depends on both, the spectral resolution and the shape of the wave-number spectrum region probed by the reflectometer [5,6]. The ratio between the spectral width of the turbulence and the wave-number probed by the reflectometer $k_w / k_\perp$ is the relevant parameter. Depending on this parameter the wave-number range probed by the reflectometer, $k_\perp \pm \Delta k$, may lie within the nearly-flat region of the spectrum or within the region where the amplitude changes (see figure 2).

![Figure 2. Input k-spectrum of the density fluctuations.](image)

The effect of the spectral width of the turbulence is studied changing the ratio $k_w / k_\perp$ from 0.3 to 2, for standard gain horns and a plasma slab and for Gaussian beams with optimum spot size and a high curvature cut-off layer. The 2D full-wave results are shown in figure 3. In these simulations the spectral width of the turbulence is scanned...
and the wave-number probed by the reflectometer is fixed (θ = 18°) except for two cases where the probed wave-number is different (θ = 30°). As shown in the figure, the frequency shift of the signals obtained with standard gain horns and a plasma slab agrees with the expected Doppler shift only if $k_w/k_\perp > 1$. The error in the estimation of the Doppler frequency reaches 20% in the range $k_w/k_\perp : 0.6 - 1$ and further increases for lower $k_w/k_\perp$ values. The underestimation of the rotation velocity can be explained in terms of the different amplitude of high and low wave-numbers within the range probed by the reflectometer, $k_\perp \pm \Delta k$. The response of the diagnostic is lower for high wave-numbers because in this range the spectrum of density fluctuations has a low intensity and, therefore, the amplitude of the Doppler structure drops in the high frequency region.

Gaussian beams with optimum spot size provide good Doppler frequency values in the range $k_w/k_\perp > 0.6$ even in plasmas with high curvature. The error is large only if the probed wave-number is too high and/or the spectral width is too narrow ($k_w/k_\perp < 0.5$).

![Graph showing normalized Doppler shifted peak frequency as a function of the normalized spectral width of the turbulence.](image)

**Figure 3.** Normalized Doppler shifted peak frequency as a function of the normalized spectral width of the turbulence.

### 3. Turbulence level and antenna tilt angle scan

In this section we study the effects of the turbulence level and the antenna tilt angle in determining the perpendicular velocity of the density fluctuations. To avoid the effect of the cut-off layer curvature, we consider a plasma slab and, for the sake of simplicity, a linear density profile. The beam waist size is $w_0 = 6\lambda_0$ and it is located $12\lambda_0$ in front of the cut-off layer (7\lambda_0 vacuum + 5\lambda_0 plasma) Thus, the wave front curvature can be neglected in the cut-off layer. The antenna tilt angle is varied from 8° up to 30° and the turbulence level ranges from 0.5% up to 10%. The input k-spectrum is essentially flat within the wave-number range probed by the reflectometer as shown in figure 4.
Figure 5 shows the power spectra of the simulated signals for the antenna tilt angle and turbulence level scans. The results are shown for three different antenna tilt angles, $\theta = 18^\circ$ (a), $\theta = 22^\circ$ (b) and $\theta = 30^\circ$ (c), and four turbulence levels $\delta n_e/n_e = 0.5, 2.0, 4.8$ and $10\%$ from top to bottom. The frequency axis is normalized to the expected Doppler shift at each particular value of the antenna tilt angle. The spectra show a clear Doppler shifted peak located close to the expected Doppler shift $f_D$ irrespective of the turbulence level and the antenna tilt angle.

The results of the Gaussian fits of the spectra are shown in figure 6. Figure 6a shows the Doppler shifted peak frequency, $f_p$, of the simulated spectra normalized to the expected Doppler frequency, $f_D$, obtained for $\theta = 8^\circ$, $14^\circ$, $18^\circ$, $22^\circ$, $30^\circ$ and four different rms values of the turbulence (0.5, 2.0, 4.8, and $10\%$). The important result is that over the range of turbulence levels and antenna tilt angles studied, the estimated perpendicular velocity of density fluctuations agrees with the input velocity value within a $5\%$.
Figure 6b shows the spectral width $\Delta f$ of the simulated spectra normalized with the expected Doppler shift, $f_D$, as a function of the turbulence level. The rms value of the turbulence increases from $\delta n_e/n_e = 0.5\%$ up to $\delta n_e/n_e = 10\%$. The results are represented for four tilt angles: $\theta = 8^\circ$, $14^\circ$, $22^\circ$ and $30^\circ$. This figure shows that the spectral width of the turbulence is influenced by the turbulence amplitude as expected from the non-linear theory of Doppler reflectometry [7]. An increasing Doppler peak broadening is found when the turbulence level becomes larger. However, the spectral resolution of the system is good enough and the estimated perpendicular velocity agrees with the input velocity value with moderate errors even for a relatively large turbulence level, as high as 10%.

4. Capability to determine the $k_z$-spectrum

In this section we study the relationship between the scattered power, the turbulence level and the antenna tilt angle. Such relationships determine the capability of Doppler reflectometry to measure the perpendicular wave-number spectrum of the density fluctuations.

To study the reflectometer response for different turbulence levels we fix the antenna tilt angle and change the turbulence level from 0.1% up to 10%. Figure 7 shows the Doppler peak amplitude of the simulated power spectra as a function of the rms value of the turbulence for three different antenna tilt angles: $\theta = 8^\circ$, $\theta = 18^\circ$, and $\theta = 30^\circ$. This result shows a linear dependence between the Doppler peak amplitude and the turbulence level if the rms value of the turbulence $\delta n_e/n_e$ is below 2-3%. Above this value, the dependence is not longer linear and the Doppler peak amplitude becomes almost constant when the turbulence level $\delta n_e/n_e$ is larger than 7%. The saturation at high turbulence levels is due to multiple scattering of the microwave beam from the density fluctuations close to the cut-off layer.
Figure 7 also shows that the reflectometer response depends on the antenna tilt angle, i.e. on the probed wave-number. Such dependence is summarized in figure 8. It shows the Doppler peak amplitude of the simulated power spectra as a function of the normalized wave-number probed by the reflectometer. The results are shown for four values of the turbulence level $\delta n / n_0 = 0.5, 2, 4.8$ and 10 %. The solid line represents a power law $A_p \propto (k_\perp / k_0)^{-0.9}$. The Doppler peak amplitude depends not only on the rms value of the turbulence but also on the probed wave-number. The Doppler peak amplitude decreases as $1 / k_\perp^{0.9}$ as the perpendicular wave-number probed by the reflectometer increases and this dependence appears to be almost independent of the turbulence level.

Similar angle dependence was found in [8] for a single wave-number density fluctuation which satisfies the selectivity condition $k_\perp = 2 k_0 \sin \theta$. Such dependence is used in [8] to calibrate the numerical results obtained with a flat input k-spectrum and
different antenna tilt angles. Once the numerical results are calibrated the obtained k-spectrum reproduces the input one qualitatively.

However, more simulations are needed to identify the parameters which could modify the observed $1/k_{⊥}^{0.9}$ decrease of the scattered power when the antenna tilt angle increases.

Conclusions

Numerical results show that Gaussian beams with optimum spot size can be used to determine the perpendicular velocity of the density fluctuations with high accuracy even in plasmas with high-curvature cut-off layers. Gaussian beams with low beam divergence improve the spectral resolution and minimize the error in the determination of the perpendicular velocity in a wide range of turbulence spectral widths. On the contrary, antenna systems with high beam divergence should be avoided even in plasmas with very low-curvature cut-off layers.

The results show that optimized Gaussian beams provide accurate velocity values for all the turbulence levels and antenna tilt angles studied. The spectral resolution of the system is influenced by the turbulence amplitude. However, even at high turbulence levels, it is good enough to separate both diffraction orders and achieve quantitative information.

The relationship between the scattered power and the turbulence level is only linear if the turbulence level is below 2-3%. At higher turbulence levels, the relationship becomes non linear and the scattered power becomes almost constant if the turbulence level is larger than 7%. Thus, care must be taken in estimating the density fluctuation level from the measured spectra.

The numerical results show that the reflectometer response depends on the antenna tilt angle ($1/k_{⊥}^{0.9}$) and such dependence appears to be independent of the turbulence level.

References

[8] C. Lechte, E. Holzhauer, G. Conway, and U. Stroth. (This proceedings)