Doppler reflectometry on ASDEX Upgrade: Foundations and latest results

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1. Introduction

Over the last few years microwave Doppler reflectometry has developed into an important diagnostic technique with systems either routinely operating or under active development on a growing number of fusion devices. The drive behind Doppler reflectometry is its range of applications, particularly its potential to provide high spatial and temporal resolved measurements of the radial electric field, \( E_r \) and its fluctuations. In addition to new applications there has been steady progress in developing the theoretical foundations and in refining the measurement technique itself. In this paper we review the diagnostic fundamentals, discuss recent developments and identify some of the outstanding issues. As illustrations we use example data and simulations from the ASDEX Upgrade (AUG) Doppler reflectometer system.

2. Doppler reflectometry technique

By deliberately tilting a standard fluctuation reflectometer to the plasma cutoff (non-normal incidence angle) a hybrid diagnostic is created with the wavenumber sensitivity of coherent scattering and the radial localization of microwave reflectometry. As the microwave beam propagates into the plasma it is refracted due to the non-normal gradient, as shown schematically in fig. 1, until it is reflected at the cutoff where the refractive index squared \( N^2 \) is a minimum. For a plane cutoff layer this occurs at \( N^2 = \sin^2 \theta_o \) where \( \theta_o \) is the incident angle at the plasma boundary [1]. At the turning point the beam is propagating parallel to the cutoff layer and is sensitive to \( N^2 \) fluctuations - i.e. density fluctuations - which scatter the beam. If the fluctuation spectrum contains wavenumbers satisfying the Bragg condition \( m k = 2 k_o \sin \theta \) (for the monostatic antenna configuration shown in fig. 1) where \( m \) is the Bragg order, then some of the beam will be scattered back to the reflectometer antenna. If the fluctuations are moving with velocity \( u \) then the received signal is Doppler frequency shifted by \( 2 \pi f_D = \vec{u} \cdot \vec{k} = u_\perp k_\perp + u_\parallel k_\parallel + u_r k_r \).

Figure 1: Schematic of measurement technique.
ally, the parallel and radial components to the Doppler shift can be neglected so that the Doppler frequency is directly proportional to the velocity of the fluctuations moving in the plasma $E \times B$ frame, $u_\perp = v_{E \times B} + v_{ph}$. If the fluctuation phase velocity $v_{ph}$ is known, or is negligible, then the radial electric field $E_r$ can be extracted [2,3,4]. The diagnostic can be extended by adding a second beam with a slight frequency difference to measure the instantaneous $E_r$ shear and turbulence correlation properties [5] (independent of $v_{ph}$). Since $\tilde{E}_r \rightarrow \hat{v}_{E \times B} \rightarrow \tilde{f}_D$ this allows direct access to long wavelength $E_r$ oscillations such as GAMs and zonal flows [6]. Here, the turbulence is used as a tracer simply to generate the Doppler signal, however, since the intensity of the Doppler peak is determined by the turbulence amplitude at the probed $k_\perp$ the turbulence spectrum can be measured by varying the tilt angle [4]. Further, when $v_{E \times B} \lesssim v_{ph}$ then the turbulence phase velocity behaviour can be studied [7].

3. Analysis technique

Fig. 2 shows an edge $u_\perp$ profile for the $-2T/+0.8\,MA$ ELMy H-mode shot #18676 with 13 MW of NBI, together with example Doppler spectra from the AUG dual channel V-band Doppler system ($50 - 75\,GHz$) using fixed poloidally tilted antenna pairs on the tokamak low-field-side in either O or X-mode polarization [2]. Adjacent antennae launch and collect the scattered signal which is down-converted using a heterodyne receiver with in-phase and quadrature detection. The complex amplitude signals ($I + iQ = Ae^{i\phi}$) are sampled at $20\,MHz$ for upto $7$ seconds. The Doppler frequency is extracted by fitting Gaussians to the asymmetric component of the complex amplitude spectra. A radial profile can be measured in $50 - 200\,ms$ by stepping the microwave probe frequency staircase fashion. The profile repetition period is set by the desired radial resolution (number of steps, typically $>20$) and the Doppler resolution (step length: $3 - 20\,ms$). Each measurement position and $k_\perp = 2Nk_\alpha$ are obtained using the TORBEAM beam tracing code [8] with a fitted density profile using DCN interferometry, Thomson scattering, Lithium beam and FM profile reflectometry data, and equilibrium reconstructions from the CLISTE code. In the tokamak scrape-off-layer $u_\perp$ always flows in the ion drift direction (i.e. $+E_r$) but reverses across the separatrix due to the pedestal pressure gradient and poloidal fluid velocity to form a deep negative $E_r$ well [2,3]. Fig. 2 shows that during an ELM the $E_r$ well collapses in concert with the density profile. To calculate $E_r$ required various assumptions: (a) Negligible radial and parallel contributions to the
Doppler shift, (b) ray tracing gives the correct $k_\perp$, (c) negligible turbulence phase velocity, (d) sufficient turbulence amplitude and (e) appropriate antenna characteristics and plasma geometry. The following sections address the applicability of these requirements.

3. Parallel and radial wavenumber

Broad-band density turbulence generally has a symmetric radial $k_r$ spectrum and $u_r$ velocity distribution - i.e. balanced inward and outward movement - which will broaden the reflectometer spectrum without contributing to the Doppler shift. Radially propagating disturbances, such as density ‘blobs’ and filaments expected in the edge/SOL region, may of course still generate Doppler shifts (cf. original phase runaway problem). Distinguishing filamentary radial movement from pure transverse rotation may require coherence techniques and additional normal incidence reflectometry. Conversely, the parallel component is asymmetric, but fortunately small. Tokamak turbulence is generally drift-wave like in nature with parallel wavenumbers scaling inversely with the field line connection length, i.e. $k_\parallel \approx 1/qR$. For medium sized tokamaks such as AUG with a major radius of $R \sim 2$ m and safety factors $q > 1$ this gives $k_\parallel < 0.005 \text{ cm}^{-1}$, which is 3 to 4 orders smaller than typical probed $k_\perp$ values of $5 - 10 \text{ cm}^{-1}$. To contribute significantly the parallel velocity would need to be $u_\parallel > 10^3 \times u_\perp$, which is not met even in strongly rotating NBI driven discharges with core toroidal fluid velocities up to 100 kms$^{-1}$. Hence, for most discharge conditions the radial and parallel components can, to first order, be neglected.

4. Perpendicular wavenumber

As the microwave beam propagates into the plasma its wavenumber decreases with the refractive index $k_i^2 = N^2 k_0^2$ as it is refracted [9]. This is illustrated in fig. 3 which shows contours of the wave electric field from a 2D FDTD full-wave code [5] for a 65 GHz X-mode parallel beam approaching a slab density gradient at $\theta = 28^\circ$. The dashed line is the cutoff which, from geometric optics for a slab geometry [1], occurs at $N^2 = \sin^2 \theta$. The contours show the enhancement of the electric field intensity at the cutoff creating an interac-

Figure 3: Electric field contours from 2D full-wave code and 3D beam tracing plot for a 65 GHz X-mode beam incident on a slab density at $-28^\circ$. 
tion zone of width $\Delta r$ and $\Delta k$. The Bragg backscatter condition is thus $k_\perp = 2k_i$ where $k_i$ is the projection of the local wavenumber at the cutoff layer into the perpendicular plane. Overlaid in fig. 3 is the central ray and beam extent from the TORBEAM paraxial WKB beam tracing code, which uses a complex eikonal solution of the wave equation to retain diffraction effects and computes the beam propagation within the cold plasma approximation [8]. The Gaussian beam has a 300 cm focal length and a 3.2 cm initial radius to match the full-wave simulation. The remarkable agreement between the results, not only with the central ray tracking the field maxima, but the corresponding behaviour of the 3 dB beam width, validates (at least for these conditions) beam tracing - despite the WKB approximation - as an appropriate tool for estimating $k_i$, and the width of the interaction zone, i.e. radial and $k$ error bars, from the beam width (cf. weighting function results of Bulanin [10]). The main advantage is that a beam can be traced with a single run using real tokamak equilibria and density profiles in a few seconds compared to full-wave simulations with approximated geometries requiring several hours of computation.

5. Turbulence phase velocity

The wave scattering essentially occurs from turbulence moving in the plasma $E \times B$ (guiding centre) frame, i.e. the velocity measured is $u_\perp = v_{E \times B} + v_{ph}$ (which can be shown from theoretical considerations, e.g. diamagnetic cancelation [11]). If the turbulence phase velocity $v_{ph}$ is known, or small $v_{E \times B} \gg v_{ph}$, then the $E \times B$ velocity, and hence the radial electric field $E_r$, can be extracted directly from the Doppler shift with good accuracy. To assess the magnitude of $v_{ph}$, a series of numerical simulations using linear and non-linear gyrokinetic turbulence codes were performed for real tokamak conditions. Two regions of particular interest, shown in fig. 4; the H-mode edge gradient region and the region inside the pedestal; were modelled for the typical $T_e \sim 2.4 T, I = 1.0 MA$ Deuterium improved H-mode AUG discharge #17222. Fig. 5 shows the computed growth rate $\gamma$ and real frequency $\omega_r$ (in units of $c_s/R$) against $k_\theta \rho_s$ for the edge and core from the GS2 gyrokinetic code in linear mode (the input parameters are shown in the figures). For the edge case GS2 predicts phase velocities much smaller than the diamagnetic drift velocity, $v^* = -\rho_s c_s/L_n$, expected for the dominant toroidal electron drift wave (EDW) turbulence. For $k_\theta \rho_s < 0.6$ the phase velocity is closer to $v_{ph} \approx +3 \rho_s c_s/R$, approximately 500 ms$^{-1}$ for the parameters indicated in the figure. A second mode is dominant at higher $k_\theta \rho_s$ with a smaller phase velocity, which can change sign depending on the magnetic shear. The low shear case $\hat{s} = 3.5$ simulates the local flattening of the $q$ profile due to edge currents predicted by the
CLISTE code. Edge turbulence is expected to be more non-linear than the core, so corresponding simulations have been performed with the non-linear gyro-kinetic GENE code. The results are preliminary, but similar behaviour is observed. Ion-temperature gradient

\[
\frac{R}{L_{Ti,e}} = 50, \frac{R}{L_n} = 25, n_e = 2 \times 10^{19}, T_e = 700, q = 3.7, s = 3.5 \text{ & } 5, \alpha = 2 \text{ (EM) col.}
\]

\[
\frac{R}{L_{Ti}} = \frac{R}{L_{Te}} = \frac{R}{L_n} = 8, n_e = 4 \times 10^{19}, T_e = 2000, T_e/T_i = 1, q = 2, s = 2, \alpha = 0.5 \text{ (EM) col.}
\]

\[
v_{ph} = 3(c_s \rho_s/R) = 523 \text{ m/s}
\]

\[
v_{ph} = -c_s \rho_s/L_n = v_{cs} = -4.3 \text{ km/s}
\]

\[
v_{ph} = 300 \text{ m/s}
\]

\[
v_{ph} = -24(c_s \rho_s/R) = -11.9 \text{ km/s}
\]

\[
v_{ph} = 6(c_s \rho_s/R) = 1.5 \text{ km/s}
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**Figure 5:** GS2 gyrokinetic (linear) simulations for (l) H-mode edge and (r) core conditions.

(ITG) or trapped electron mode (TEM) turbulence is expected to be more dominant in the shallower core gradient region. Linear GK simulations indicate that \( v_{ph} \sim O(3)\rho_s c_s/R \) (c.f. [12]), for pure ITG or pure TEM. However, for the more realistic tokamak conditions with \( \eta_i \approx 1 \) the GS2 code gives \( v_{ph} \) of the order of a few hundred m/s for \( k_{\perp} \rho_s \ll 0.6 \), above which an ETG branch appears with a very high phase velocity - larger than the measured \( u_{\perp} \). In the edge with probed \( k_{\perp} \rho_s \sim 0.8 \) the simulations indicate that \( v_{ph} \) maybe negligible for EDW, ITG and TEM, but in the core with \( k_{\perp} \rho_s \sim 2.5 \) ETG may not be ignored if present. Thus, it is vital to know the nature of the turbulence.

6. Turbulence amplitude

An important factor is the turbulence amplitude at the probed wavenumber. Reflectometer simulations [13] and measurements [2] show that at low fluctuation amplitudes the \( m = -1 \) spectral peak will be lost in the wings of a dominant \( m = 0 \) peak or beneath the diagnostic noise floor. Turbulence k-spectrum measurements and non-linear numerical simulations from the GEMR gyro-fluid code, shown in fig.6, indicate that for ITG turbulence the spectrum peaks around \( k_{\perp} \rho_s \sim 0.3 \) with a spectral index around \( n = -3 \). Since \( \rho_s = c_s/\omega_{ci} \) depends on the machine parameters this defines the practical range of probable \( k_{\perp} \), i.e. the antenna tilt angles. However, the Bragg condition \( k_{\perp} = mk = 2k_o \sin \theta \) means higher \( m \) orders will also appear at the same Doppler frequency for any fixed tilt. Fig. 7 shows the maximum reflected power vs normalized sinusoidal perturbation wavelength \( \Lambda/\lambda_o \) at fixed tilt \( \theta = 10^\circ \) and spot size \( w/\lambda_o = 5 \) for increasing perturbation amplitude \( h/\lambda_o \) (from
physical optics simulation [12]). The dominant peak is the \( m = -1 \) Bragg order, but with rising amplitude the \( m = -1 \) peak saturates and the \( m = -2 \) peak becomes stronger. This implies that if the main \( m = -1 \) order is weakened due to strong roll-off in the turbulence \( k \) spectrum (high spectral index) then the second order signal due to stronger turbulence at smaller \( k \) maybe become significant. Where the turbulence is being used purely as a tracer this is a positive effect which extends operational range in the tilt angle. But, if the aim is to measure the \( k \)-spectrum then this could create ambiguity in the probed \( k \) and distortion in the measured spectrum.

7. Resolution

For flat cutoff layers the diagnostic \( k \) resolution scales inversely with the beam radius \( \Delta k \propto 2\sqrt{2/w} \), and a localization \( \Delta r \) scaling with the Airy lobe width and the decay length of the evanescent electric field pattern around the cutoff, but cutoff layer curvature and beam divergence increase the spread in the probed \( k_i \) and hence reduce the localization and resolution. The optimal antenna is a low divergence Gaussian beam with large diameter and a phase front matched to cutoff curvature. However, even well focused horn antennas have side-lobes, often as high as \( -15 \) dB, at typically around \( \pm 15^\circ \) or so. If the main lobe signal is poor, for example weak turbulence, probing the core or high \( k_{\perp} \), then additional Doppler peaks may enter via the side-lobes. Fig. 8 shows ray traces for the AUG O-mode antenna at 70 GHz for shot \#18053 tilted at \( 34^\circ \) with \( \pm 11^\circ \) side-lobes (dashed). The corresponding spectrum (insert) shows a large \( f_D \) peak from the main \( k_{\perp} = 8.8 \) cm\(^{-1} \) ray and a second \( f_D \sim 0 \) peak (often negative frequency) consistent with a stronger amplitude but lower \( k_{\perp} \) probed by the upper side-lobe. If the peaks are well separated...
by a large $u_\perp$ then strong side-lobes might be used to advantage by allowing simultaneous multiple $k_\perp$ probing.

8. Discussion

The growing number of Doppler reflectometer installations and applications can only help to improve the methodology and accuracy of the technique. The analysis technique described in section 3 can be applied almost routinely to AUG data. However, the analysis chain is far from being automated. While the preparation of fitted density profiles (often time consuming), equilibria generation and beam-tracing can be independent, the extraction of $f_D$ from difficult spectral data still requires human judgement and interaction. To date, the $E_r$ measurements are impressive, but further diagnostic validation via comparative measurements with other diagnostics is a high priority. Other outstanding issues include, further modelling of the instrument response [14], particularly full-wave simulations of the $k$-spectrum response, the susceptibility to higher Bragg orders and potential to distort the measured $k$-spectrum. The effect of $k$-spectrum roll-off on the measured $f_D$ is also of concern [13,15], as well as turbulence amplitude thresholds and non-linear effects. Finally, more non-linear turbulence simulations are required, particularly concerning the ETG amplitude and phase velocity.

9. References