Online documentation. Part I:
Built-in electron distributions

As has been said in the main text, although the algorithm itself is valid for a general case, the analytical built-in electron distribution functions $G(E,\mu)$ have the factorized form

$$G(E,\mu) = u(E)g(\mu), \tag{1}$$

that is, they can be written as a product of the energy (or momentum) distribution function $u(E)$ and the angular distribution function $g(\mu)$. Currently, to cover a representatively broad range of possibilities needed for practical applications including contribution from thermal and nonthermal electrons, nine types of energy distributions and five types of angular distributions are implemented. The index of the energy distribution is specified by the parameter `ParmIn[17]` in a call to the gyrosynchrotron code (see online documentation, Part II), and the index of the angular distribution is specified by the parameter `ParmIn[19]`; any combination of the energy and angular distribution is possible. However, in the library `libGS_Std_HomSrc_CEH` the selected anisotropy is applied throughout any assumed distribution including the Maxwellian component if present, while in the library `libGS_Std_HomSrc_C` the Maxwellian component (in THM, TNT, TNP, and TNG distributions) is always adopted isotropic. The distribution parameters (whose number and meaning depend on the type of distribution) are specified by different elements of the array `ParmIn`. These parameters will be described below.

For the factorized electron distribution (1), the functions $u(E)$ and $g(\mu)$ can be normalized independently. We assume that the distribution functions satisfy the following normalization conditions

$$2\pi \int\limits_{E_{\min}}^{E_{\max}} u(E)\,\mathrm{d}E = n_\mathrm{e}, \qquad \int\limits_{-1}^{1} g(\mu)\,\mathrm{d}\mu = 1, \tag{2}$$

where n_e is the number density of electrons having the energy between $E_{\min}$ and $E_{\max}$.

Sample emission spectra in Figures 1–9 were calculated for the following source parameters:

- *visible source area* $S = 10^{20}$ *cm*^{2}*;*
- *source depth* $L = 10^{10}$ *cm;*
- *magnetic field* $B = 180$ *G;*
- *thermal plasma density* $n_0 = 3\times 10^9$ cm^{-3}, *unless different is specified (for the thermal, thermal/nonthermal, and kappa distributions);*
- *thermal plasma temperature* $T_0 = 3\times 10^7$ *K, unless different is specified (for the thermal, thermal/nonthermal, and kappa distributions);*
- *viewing angle* $\theta = 45^\circ$, *unless different is specified (for the anisotropic pitch-angle distributions).*

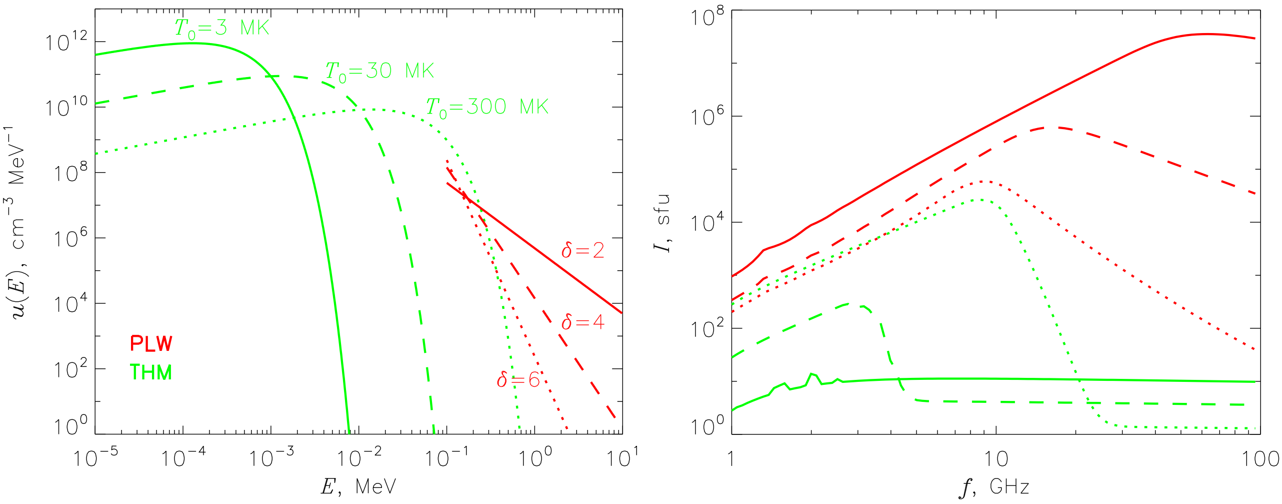

Figure 1: Thermal electron distribution (for $n_0 = 3 \times 10^9$ cm^{-3} and different electron temperatures) and single power-law electron distribution over kinetic energy (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and different power-law indices δ). For the thermal distributions, the emission spectra at high frequencies are dominated by free-free emission.

Distributions over energy

Sample emission spectra for different energy distributions (in Figures 1–7) were calculated under the assumption that the pitch-angle distribution is isotropic (ISO).

Thermal distribution (THM; index 2)

Relativistic thermal distribution is given by the expression

$$u(\gamma)\,\mathrm{d}\gamma = \frac{n_0}{2\pi}\frac{\gamma\sqrt{\gamma^2-1}}{\theta K_2(1/\theta)}\exp\left(-\frac{\gamma}{\theta}\right)\,\mathrm{d}\gamma, \tag{3}$$

where n_0 is the number density of the thermal electrons, γ is the Lorentz-factor, $\theta = k_\mathrm{B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, k_B is the Boltzmann constant, and K_2 is the MacDonald function of the second order.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[2]` $= T_0$ [K];
- `ParmIn[11]` $= n_0$ [cm^{-3}];
- `ParmIn[17]` $= 2$.

Note that the above parameters, T_0 and n_0, are also used beyond the case of thermal distribution. For other energy distributions, n_0 and T_0 are considered as the background plasma density and temperature, respectively. They are used to calculate the dispersion parameters of the electromagnetic waves (n_0) and the free-free contribution (both n_0 and T_0). However, the gyrosynchrotron contribution of the thermal electrons is calculated only if one explicitly chooses the thermal distribution as the "main" electron distribution by setting `ParmIn[17]` $= 2$ (or, more broadly, the class of thermal/nonthermal distributions, see below); otherwise, this contribution is neglected in GS computation.

Examples of the thermal distributions and the corresponding gyrosynchrotron emission spectra are shown in Figure 1.

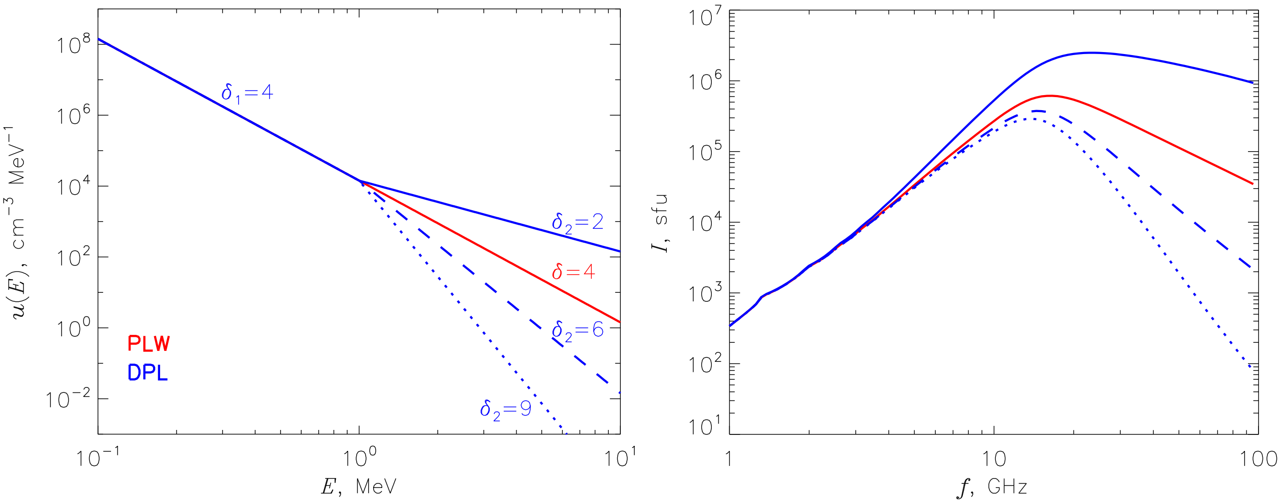

Figure 2: Double power-law electron distribution (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{break} = 1$ MeV, $E_\mathrm{max} = 10$ MeV, $\delta_1 = 4$, and different high-energy power-law indices δ_2). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

Single power-law distribution over kinetic energy (PLW; index 3)

Power-law distributions of the nonthermal electrons over kinetic energy $E = mc^2(\gamma - 1)$ are widely used for interpretation of solar radio and hard X-ray emissions. These distributions are given by the expression

$$u(E)\,\mathrm{d}E = AE^{-\delta}\,\mathrm{d}E \quad \text{for} \quad E_\mathrm{min} < E < E_\mathrm{max}, \tag{4}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_\mathrm{b}}{2\pi}\frac{\delta - 1}{E_\mathrm{min}^{1-\delta} - E_\mathrm{max}^{1-\delta}}, \tag{5}$$

where n_b is the number density of the nonthermal electrons. The logarithmic normalization for $\delta = 1$ is not implemented, however, one can arbitrarily approach this case taking δ very close but slightly different from 1.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[6]` $= E_\mathrm{min}$ [MeV];
- `ParmIn[7]` $= E_\mathrm{max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[12]` $= n_\mathrm{b}$ [cm^{-3}];
- `ParmIn[17]` $= 3$.

Examples of the single power-law distributions and the corresponding gyrosynchrotron emission spectra are shown in Figure 1.

Double power-law distribution over energy (DPL; index 4)

In this case the electron spectrum consists of two parts (high-energy and low-energy), where both the high-energy and low-energy parts are described by power laws, but with different indices. These distributions (double power-law or broken power-law) can be described by the following expression:

$$u(E)\,\mathrm{d}E = \mathrm{d}E \begin{cases} A_1 E^{-\delta_1}, & \text{for } E_{\min} < E \le E_{\text{break}}, \\ A_2 E^{-\delta_2}, & \text{for } E_{\text{break}} \le E < E_{\max}, \end{cases} \tag{6}$$

and 0 outside the range from $E_{\min}$ to $E_{\max}$. In the above expression, $A_1 E_{\text{break}}^{-\delta_1} = A_2 E_{\text{break}}^{-\delta_2}$ (to make the function continuous), $\delta_1 \neq 1$, and $\delta_2 \neq 1$. In the library `libGS_Std_HomSrc_CEH` the normalization factor is given by

$$A_1^{-1} = \frac{2\pi}{n_{\mathrm{b}}} \left(\frac{E_{\min}^{1-\delta_1} - E_{\text{break}}^{1-\delta_1}}{\delta_1 - 1} + E_{\text{break}}^{\delta_2-\delta_1} \frac{E_{\text{break}}^{1-\delta_2} - E_{\max}^{1-\delta_2}}{\delta_2 - 1} \right), \tag{7}$$

i.e., n_{b} is the number density of nonthermal electrons between $E_{\min}$ and $E_{\max}$, and A_2 is found using the above continuity condition. In the library `libGS_Std_HomSrc_C`, normalization (5) is instead used for the purpose of easier comparison between the DPL and PLW results.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[6]` $= E_{\min}$ [MeV];
- `ParmIn[7]` $= E_{\max}$ [MeV];
- `ParmIn[8]` $= E_{\text{break}}$ [MeV];
- `ParmIn[9]` $= \delta_1$;
- `ParmIn[10]` $= \delta_2$;
- `ParmIn[12]` $= n_{\mathrm{b}}$ [cm^{-3}];
- `ParmIn[17]` $= 4$.

Examples of the double power-law distributions and the corresponding gyrosynchrotron emission spectra are shown in Figure 2.

Thermal/nonthermal distribution over energy (TNT; index 5)

This distribution ensures a smooth transition from nonthermal to the thermal distribution at low energies by means of expression

$$u(E)\,\mathrm{d}E = \mathrm{d}E \begin{cases} u_{\text{THM}}(E), & \text{for } E \le E_{\text{cr}}, \\ A E^{-\delta}, & \text{for } E_{\text{cr}} \le E < E_{\max}, \end{cases} \tag{8}$$

and 0 for $E > E_{\max}$. In the above expression, $u_{\text{THM}}(E)$ is the thermal distribution function (3), $A = u_{\text{THM}}(E_{\text{cr}}) E_{\text{cr}}^{-\delta}$ to make the function continuous, the matching point E_{cr} satisfies the condition $E_{\text{cr}} < E_{\max}$, and $\delta > 1$. In our codes, the matching point E_{cr} is defined as the energy corresponding to the momentum p_{cr}

$$p_{\text{cr}}^2 = \frac{p_{\text{THM}}^2}{\varepsilon}, \tag{9}$$

where p_{THM} is the mean thermal momentum corresponding to the energy $k_{\mathrm{B}} T_0$, and the parameter ε specifies location of the turning point (the distribution becomes purely thermal when $\varepsilon < p_{\text{cr}}^2 / p^2(E_{\max})$).

For small ε, number density of the nonthermal electrons (with $E > E_{\text{cr}}$) is much less than that of the thermal electrons. Therefore we assume that the normalization condition remains approximately the same as for the thermal distribution (3), and the total electron number density $n_{\mathrm{e}} \simeq n_0$. However, it should be noted that actually the total electron density slightly exceeds n_0.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

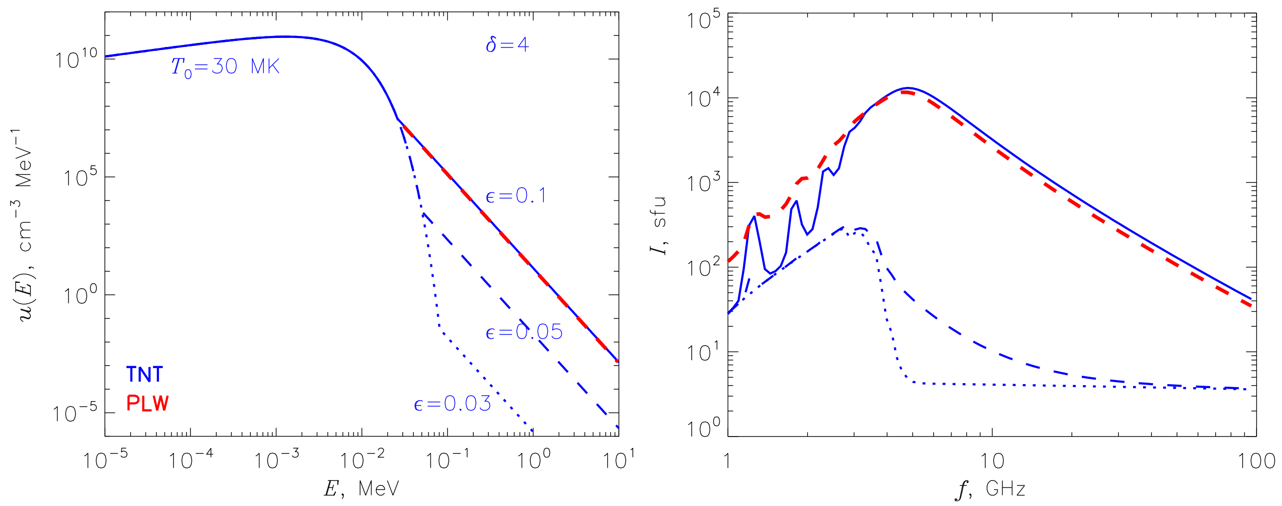

Figure 3: Thermal/nonthermal electron distribution over kinetic energy (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\delta = 4$, and different matching parameters ε). For $\varepsilon = 0.05$ and $\varepsilon = 0.03$, the emission spectra at high frequencies are dominated by free-free emission. For $\varepsilon = 0.03$, the emission spectrum (shown by dotted line) is nearly the same as for the purely thermal distribution with $T_0 = 3 \times 10^7$ K (shown by green dashed line in Figure 1), because the contribution of nonthermal particles is negligible in this case. Red dashed line represents the nonthermal "tail" of the thermal/nonthermal distribution; for $\varepsilon = 0.1$, this "tail" behaves as the single power-law distribution with $n_\mathrm{b} = 10^6$ cm^{-3}, $E_\mathrm{min} = 0.03$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$.

- `ParmIn[2]` $= T_0$ [K];
- `ParmIn[3]` $= \varepsilon$;
- `ParmIn[7]` $= E_\mathrm{max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[11]` $= n_0$ [cm^{-3}];
- `ParmIn[17]` $= 5$.

Number density of the nonthermal electrons n_b is not specified explicitly, while it is calculated consistently using n_0, T_0, ε, δ, and E_max.

Examples of the thermal/nonthermal distributions over energy and the corresponding gyrosynchrotron emission spectra are shown in Figure 3.

Kappa distribution (KAP; index 6)

Another way of describing the smooth transition from the thermal distribution to a nonthermal tail is a so-called Kappa distribution, which is widely used to quantify particle distributions in the interplanetary plasma. It is convenient to express the Kappa distribution in terms of the Lorentz-factor γ:

$$u(\gamma)\,\mathrm{d}\gamma = A\frac{\gamma\sqrt{\gamma^2-1}}{\theta^{3/2}\left[1+\dfrac{\gamma-1}{(\varkappa-3/2)\theta}\right]^{\varkappa+1}}\,\mathrm{d}\gamma \quad \text{for} \quad E < E_\mathrm{max}, \tag{10}$$

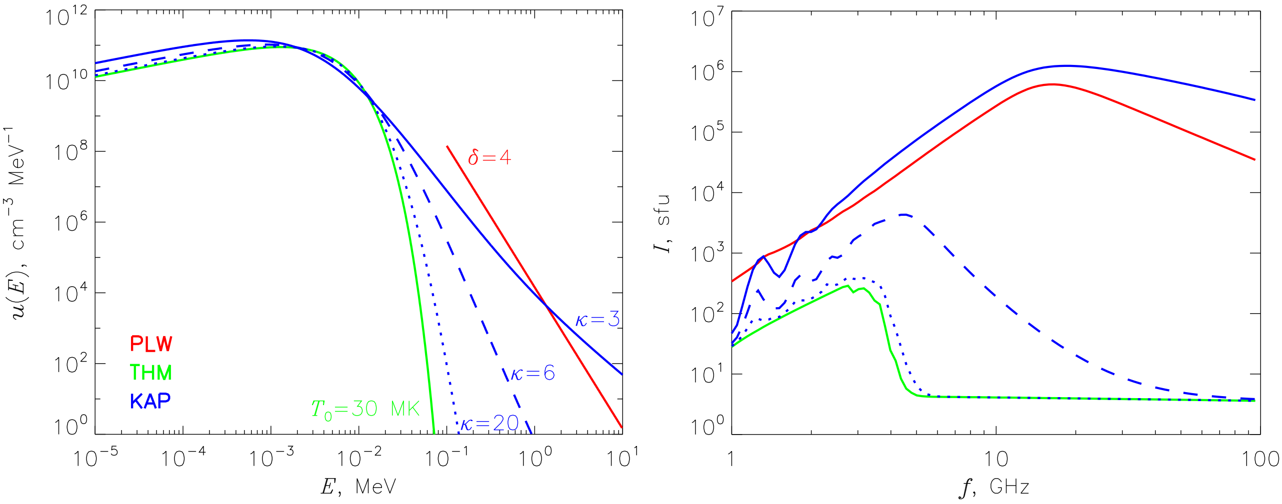

Figure 4: Kappa distribution (for $n_\mathrm{e} = 3\times10^9$ cm^{-3}, $T_0 = 3\times10^7$ K, and different values of the parameter $\varkappa$). Thermal distribution (for the same particle number density and temperature) and single power-law distribution (for $n_\mathrm{b} = 3\times10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and $\delta = 4$) are given for reference. For the thermal distribution and kappa distributions with $\varkappa = 6$ and $\varkappa = 20$, the emission spectra at high frequencies are dominated by free-free emission.

and 0 otherwise. In the above expression, $\theta = k_\mathrm{B}T_0/(mc^2)$ is the normalized thermal energy for the temperature T_0, and $\varkappa$ is the distribution parameter. The normalization factor A is calculated numerically by using normalization condition (2). Kappa distribution becomes purely thermal distribution when $\varkappa \to \infty$.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[2]` $= T_0$ [K];
- `ParmIn[4]` $= \varkappa$;
- `ParmIn[7]` $= E_\mathrm{max}$ [MeV];
- `ParmIn[11]` $= n_\mathrm{e}$ [cm^{-3}];
- `ParmIn[17]` $= 6$.

Note that for kappa distribution, there is no unique demarkation between the thermal and nonthermal particles, so the total number density is specified.

Examples of the kappa distributions and the corresponding gyrosynchrotron emission spectra are shown in Figure 4.

Power-law distribution over momentum (PLP; index 7)

Power-law distribution of the nonthermal electrons over the absolute value of momentum is given by the expression

$$u(p)\,\mathrm{d}p = Ap^{-\delta}\,\mathrm{d}p \quad \text{for} \quad p_\mathrm{min} < p < p_\mathrm{max}, \tag{11}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_\mathrm{b}}{2\pi}\frac{\delta-3}{p_\mathrm{min}^{3-\delta} - p_\mathrm{max}^{3-\delta}}, \tag{12}$$

where n_b is the number density of nonthermal electrons, $p_\mathrm{min} = p(E_\mathrm{min})$, and $p_\mathrm{max} = p(E_\mathrm{max})$, the case of $\delta = 3$ is not implemented.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

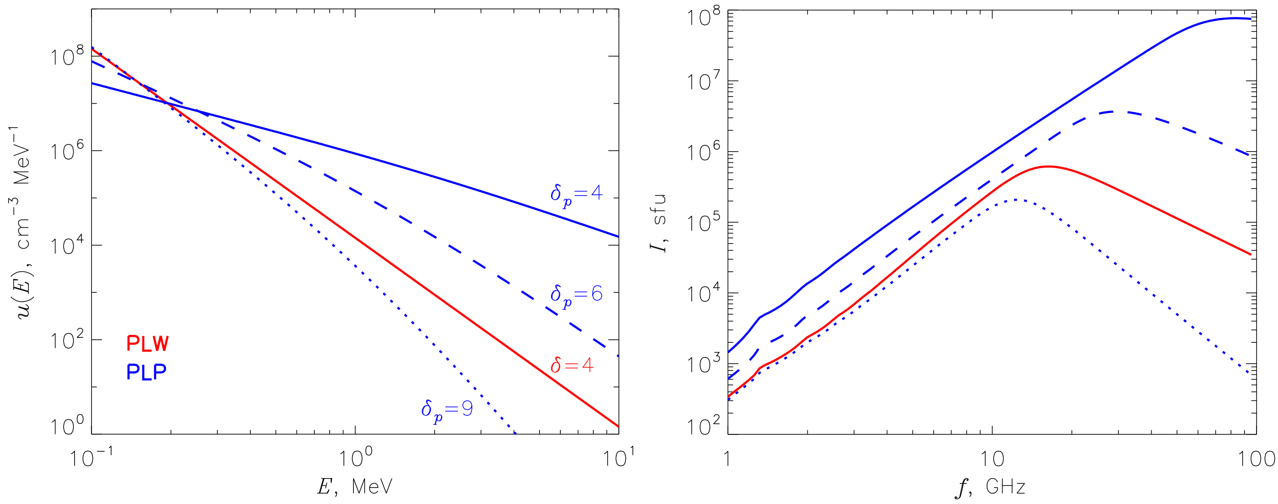

Figure 5: Power-law electron distribution over momentum (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and different power-law indices δ_p). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

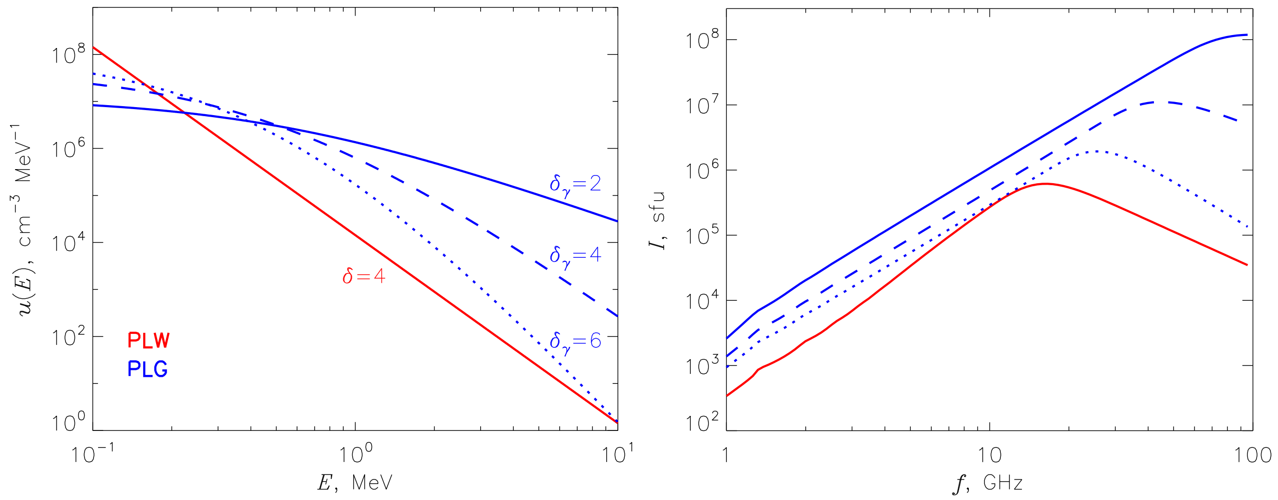

Figure 6: Power-law electron distribution over Lorentz factor (for $n_\mathrm{b} = 3 \times 10^7$ cm^{-3}, $E_\mathrm{min} = 0.1$ MeV, $E_\mathrm{max} = 10$ MeV, and different power-law indices δ_γ). Single power-law distribution (for the same particle number density and $\delta = 4$) is given for reference.

- `ParmIn[6]` $= E_\mathrm{min}$ [MeV];
- `ParmIn[7]` $= E_\mathrm{max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[12]` $= n_\mathrm{b}$ [cm^{-3}];
- `ParmIn[17]` $= 7$.

Note that this distribution is not a power-law when expressed via the electron energy. And, vice versa, power-law distribution over energy becomes non-power-law when expressed via the electron momentum.

Examples of the power-law distributions over momentum and the corresponding gyrosynchrotron emission spectra are shown in Figure 5.

Power-law distribution over Lorentz factor (PLG; index 8)

Power-law distribution of the nonthermal electrons over Lorentz factor is given by the expression

$$u(\gamma)\,\mathrm{d}\gamma = A\gamma^{-\delta}\,\mathrm{d}\gamma \quad \text{for} \quad \gamma_{\min} < \gamma < \gamma_{\max}, \tag{13}$$

and 0 otherwise. The normalization constant A equals

$$A = \frac{n_\mathrm{b}}{2\pi}\frac{\delta - 1}{\gamma_{\min}^{1-\delta} - \gamma_{\max}^{1-\delta}}, \tag{14}$$

where n_b is the number density of nonthermal electrons, $\gamma_{\min} = \gamma(E_{\min})$, and $\gamma_{\max} = \gamma(E_{\max})$, the case of $\delta = 1$ is not implemented.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[6]` $= E_{\min}$ [MeV];
- `ParmIn[7]` $= E_{\max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[12]` $= n_\mathrm{b}$ [cm^{-3}];
- `ParmIn[17]` $= 8$.

Again, this distribution is different from the power-law distributions over energy or momentum.

Examples of the power-law distributions over Lorentz factor and the corresponding gyrosynchrotron emission spectra are shown in Figure 6.

Thermal/nonthermal distribution over momentum (TNP; index 9)

This distribution is similar to the thermal/nonthermal distribution over energy (index 5) with the only difference that the nonthermal part (at $E > E_\mathrm{cr}$) is described by the power-law distribution over the absolute value of momentum, that is

$$u(p)\,\mathrm{d}p = \mathrm{d}p \begin{cases} u_\mathrm{THM}(p), & \text{for} \quad p < p_\mathrm{cr}, \\ Ap^{-\delta}, & \text{for} \quad p_\mathrm{cr} \le p < p_{\max}, \end{cases} \tag{15}$$

and 0 for $p > p_{\max}$. In the above expression, $u_\mathrm{THM}(p)$ is the thermal distribution function (3) expressed via momentum, p_cr is given by Eq. (9), $p_{\max} = p(E_{\max})$, and location of the matching point and the matching conditions are the same as for the TNT distribution.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[2]` $= T_0$ [K];
- `ParmIn[3]` $= \varepsilon$;
- `ParmIn[7]` $= E_{\max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[11]` $= n_0$ [cm^{-3}];
- `ParmIn[17]` $= 9$.

An example of the thermal/nonthermal distribution over momentum and the corresponding gyrosynchrotron emission spectrum are shown in Figure 7.

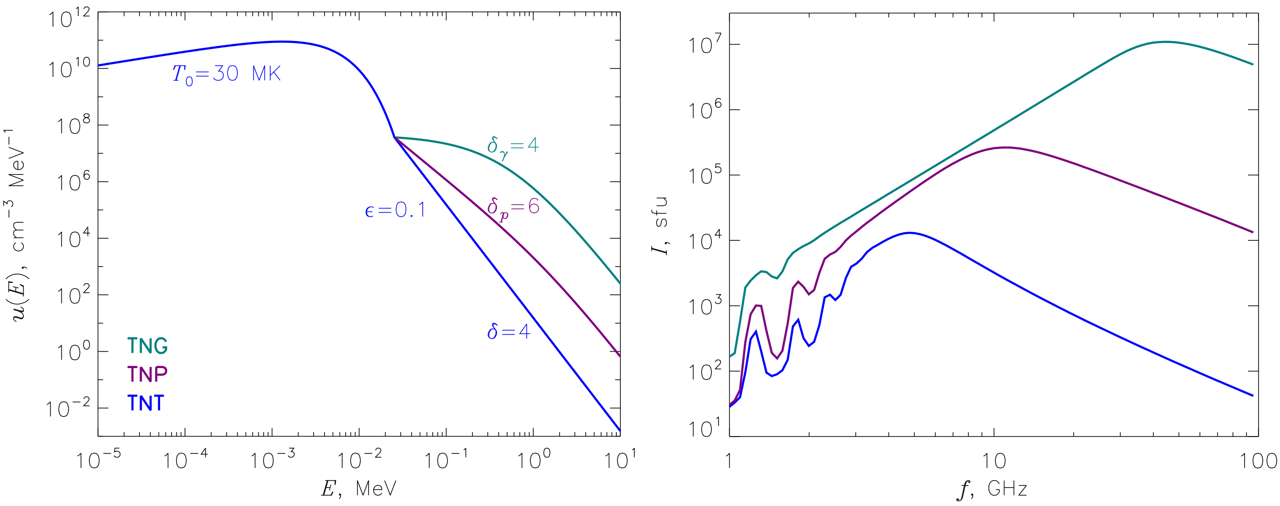

Figure 7: Different thermal/nonthermal electron distributions (for $n_0 = 3 \times 10^9$ cm^{-3}, $T_0 = 3 \times 10^7$ K, $\varepsilon = 0.1$). All the distributions have different numbers of fast electrons above $E_{\rm cr}$.

Thermal/nonthermal distribution over Lorentz factor (TNG; index 10)

This distribution is similar to the thermal/nonthermal distribution over energy (index 5) with the only difference that the nonthermal part (at $E > E_{\rm cr}$) is described by the power-law distribution over the Lorentz factor, that is

$$u(\gamma)\,\mathrm{d}\gamma = \mathrm{d}\gamma \left\{ \begin{array}{ll} u_{\rm THM}(\gamma), & \text{for} \quad \gamma < \gamma_{\rm cr}, \\ A\gamma^{-\delta}, & \text{for} \quad \gamma_{\rm cr} \le \gamma < \gamma_{\rm max}, \end{array} \right. \tag{16}$$

and 0 for $\gamma > \gamma_{\rm max}$. In the above expression, $u_{\rm THM}(\gamma)$ is the thermal distribution function (3) expressed via Lorentz factor, $\gamma_{\rm cr} = \gamma(p_{\rm cr})$, $\gamma_{\rm max} = \gamma(E_{\rm max})$, and location of the matching point and the matching conditions are the same as for the TNT distribution with index 5.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[2]` $= T_0$ [K];
- `ParmIn[3]` $= \varepsilon$;
- `ParmIn[7]` $= E_{\rm max}$ [MeV];
- `ParmIn[9]` $= \delta$;
- `ParmIn[11]` $= n_0$ [cm^{-3}];
- `ParmIn[17]` $= 10$.

An example of the thermal/nonthermal distribution over Lorentz factor and the corresponding gyrosynchrotron emission spectrum are shown in Figure 7.

If the energy distribution index differs from the above values (2-10) then the single power-law distribution over energy (index 3) will be used.

Distributions over pitch-angle

Sample emission spectra for the different pitch-angle distributions (in Figures 8–9) were calculated under the assumption that the energy distribution is a single power-law (PLW) with $n_\mathrm{b} = 3 \times 10^7$ *cm*$^{-3}$, $E_\mathrm{min} = 0.1$ *MeV,* $E_\mathrm{max} = 10$ *MeV, and* $\delta = 4$.

Isotropic distribution (ISO; index 1 or 0)

In this case, the electron distribution does not depend on pitch-angle, that is

$$g(\mu) = \mathrm{const} = \frac{1}{2}. \tag{17}$$

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[19]` = 1.

For the library ***`libGS_Std_HomSrc_CEH`***: the use of indices 0 and 1 yields equivalent results, given all other parameters are identical. *For the library* ***`libGS_Std_HomSrc_C`***: the use of index 1 yields the result computed according to the new continuous code, while the use of index 0 activates the original (less accurate but the fastest) Petrosian-Klein code.

Exponential loss-cone distribution (ELC; index 2)

Symmetric loss-cone distribution with exponential boundary is given by the expression

$$g(\mu) = A \begin{cases} 1, & \text{for } \; |\mu| < \mu_c, \\ \exp\left(-\dfrac{|\mu| - \mu_c}{\Delta\mu}\right), & \text{for } \; |\mu| \geq \mu_c, \end{cases} \tag{18}$$

where $\mu_c = \cos\alpha_c > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_c + \Delta\mu - \Delta\mu \exp\left(\frac{\mu_c - 1}{\Delta\mu}\right)\right]. \tag{19}$$

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[19]` = 2;
- `ParmIn[20]` = α_c [degrees];
- `ParmIn[22]` = $\Delta\mu$.

An example of the exponential loss-cone distribution and the corresponding gyrosynchrotron emission spectrum are shown in Figure 8.

Gaussian loss-cone distribution (GLC; index 3)

Symmetric loss-cone distribution with gaussian boundary is given by the expression

$$g(\mu) = A \begin{cases} 1, & \text{for } \; |\mu| < \mu_c, \\ \exp\left[-\dfrac{(|\mu| - \mu_c)^2}{\Delta\mu^2}\right], & \text{for } \; |\mu| \geq \mu_c, \end{cases} \tag{20}$$

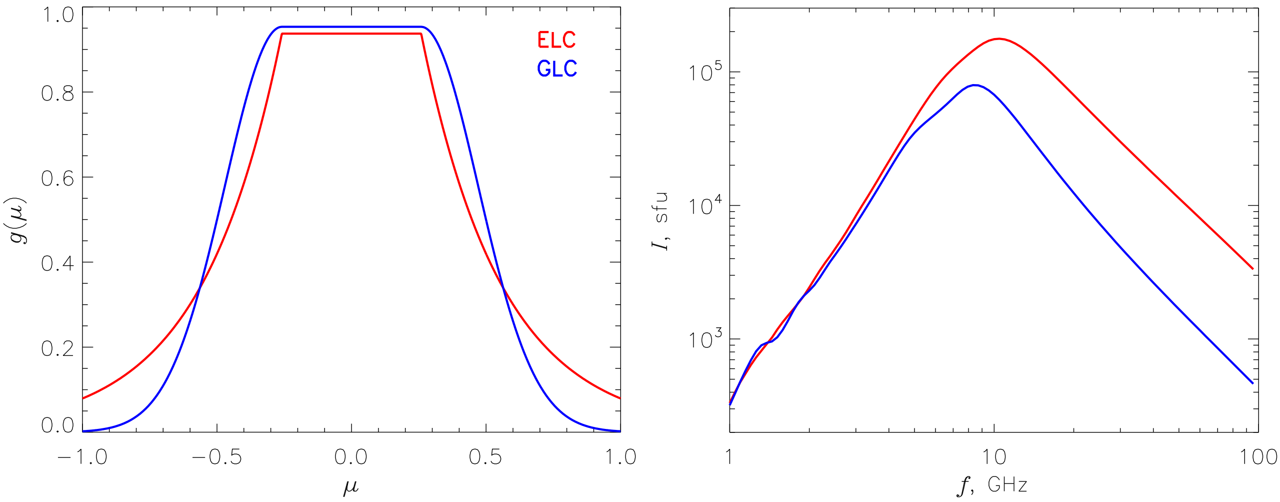

Figure 8: Exponential and gaussian loss-cone distributions (for $\alpha_c = 75°$ and $\Delta\mu = 0.3$). The emission spectra are calculated for the propagation angle $\theta = 30°$.

where $\mu_c = \cos\alpha_c > 0$ is the loss-cone boundary, and the parameter $\Delta\mu$ determines the sharpness of the loss-cone boundary. The normalization factor A is given by

$$A^{-1} = 2\left[\mu_c + \frac{\sqrt{\pi}}{2}\Delta\mu\,\mathrm{erf}\left(\frac{1-\mu_c}{\Delta\mu}\right)\right], \tag{21}$$

where erf is the error function.

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[19]` $= 3$;
- `ParmIn[20]` $= \alpha_c$ [degrees];
- `ParmIn[22]` $= \Delta\mu$.

An example of the gaussian loss-cone distribution and the corresponding gyrosynchrotron emission spectrum are shown in Figure 8.

Gaussian distribution (GAU; index 4)

Gaussian distribution is given by the expression

$$g(\mu) = A\exp\left[-\frac{(\mu-\mu_0)^2}{\Delta\mu^2}\right], \tag{22}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and $\Delta\mu$ is the beam angular width. The above expression represents the beam along the field line for $\mu_0 = \pm 1$, the transverse beam for $\mu_0 = 0$, and an oblique beam (or a hollow-beam) otherwise. The normalization factor A is given by

$$A^{-1} = \frac{\sqrt{\pi}}{2}\Delta\mu\left[\mathrm{erf}\left(\frac{1-\mu_0}{\Delta\mu}\right) + \mathrm{erf}\left(\frac{1+\mu_0}{\Delta\mu}\right)\right]. \tag{23}$$

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[19]` $= 4$;
- `ParmIn[21]` $= \alpha_0$ [degrees];

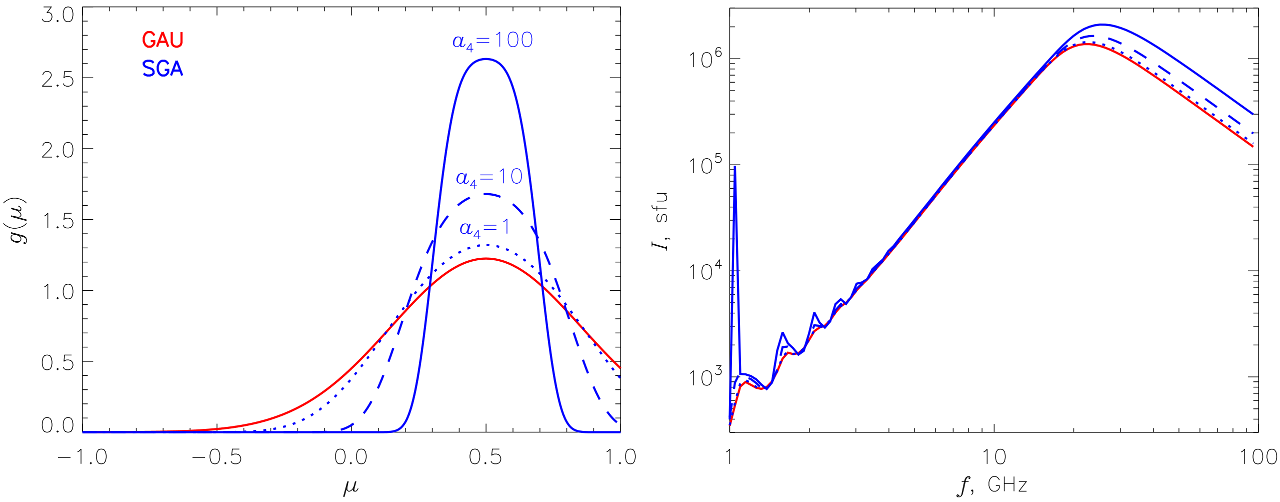

Figure 9: Gaussian distribution (for $\alpha_0 = 60°$ and $\Delta\mu = 0.5$) and supergaussian distribution (for the same α_0 and $\Delta\mu$, and different values of the parameter a_4). The emission spectra are calculated for the propagation angle $\theta = 60°$.

- `ParmIn[22]` $= \Delta\mu$.

If $\alpha_0 = \pi/2$, this distribution coincides with GLC distribution with $\alpha_c = \pi/2$, otherwise they are different from each other.

An example of the gaussian distribution and the corresponding gyrosynchrotron emission spectrum are shown in Figure 9.

"Supergaussian" distribution (SGA; index 5)

This distribution is very similar to the GAU distribution near its maximum (μ_0) but decreases more rapidly at some angular distance from μ_0. Such a shape is achieved by adding a term with fourth degree of $(\mu - \mu_0)$ to the argument of exponent in (22), that is

$$g(\mu) = A \exp\left[-\frac{(\mu-\mu_0)^2 + a_4(\mu-\mu_0)^4}{\Delta\mu^2}\right], \tag{24}$$

where $\mu_0 = \cos\alpha_0$ is the beam direction, and the beam angular width and shape near the maximum are determined by the parameters $\Delta\mu$ and a_4. The normalization factor A is calculated numerically by using normalization condition (2).

In our gyrosynchrotron codes, the parameters of this distribution are specified as:

- `ParmIn[19]` $= 5$;
- `ParmIn[21]` $= \alpha_0$ [degrees];
- `ParmIn[22]` $= \Delta\mu$;
- `ParmIn[23]` $= a_4$.

Examples of the "supergaussian" distribution and the corresponding gyrosynchrotron emission spectra are shown in Figure 9.

If the angular distribution index differs from the above values (0–5) then the isotropic distribution (index 1) will be used; however, values above 100 should be avoided as they are reserved for future use and testing.