45mins

Magnetic Reconnection Rate & X-line Spread in Collisionless Plasmas

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Collaborators: M. Hesse, Tak Chu. Li, Fan Guo, Xiaocan Li, W. Daughton, P. Cassak,
M. Shay, Takuma Nakamura, Hui Li, Shan Wang, Li-Jen Chen et al. & MMS team

Outlines

- ★ Theory of Reconnection Rate; Past —Present
- ★ X-line Orientation & Spread
- ★ Magnetospheric Multiscale Mission (MMS)
  & Solar Observations of Reconnection Rates
- ★ Summary & Future (Unsolved questions)

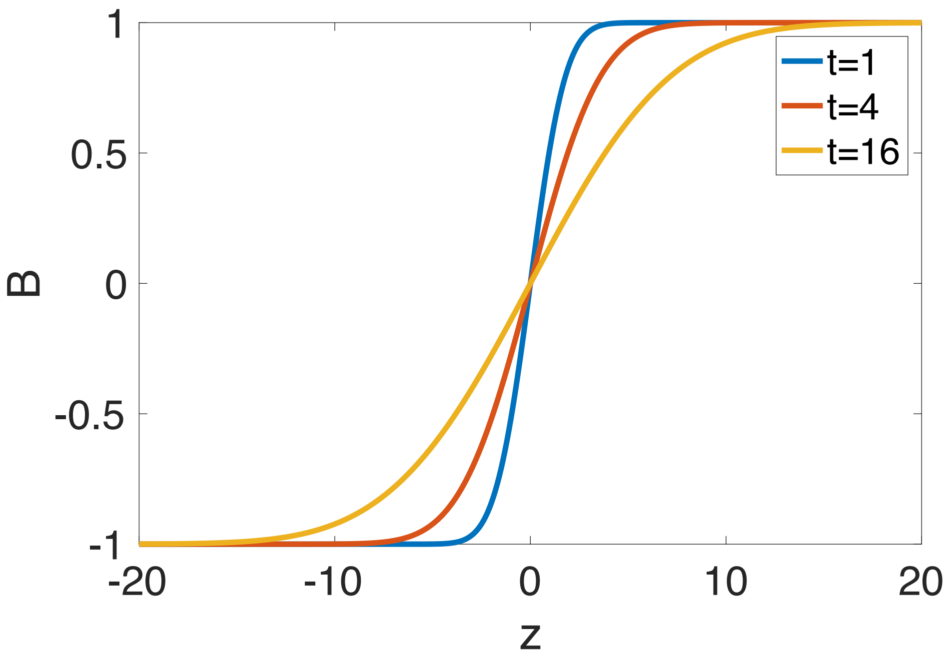
Theory of Reconnection Rate
Past — Present

Magnetic Diffusion (<1953) (i.e., Dungey, 1953)

Induction eqn:

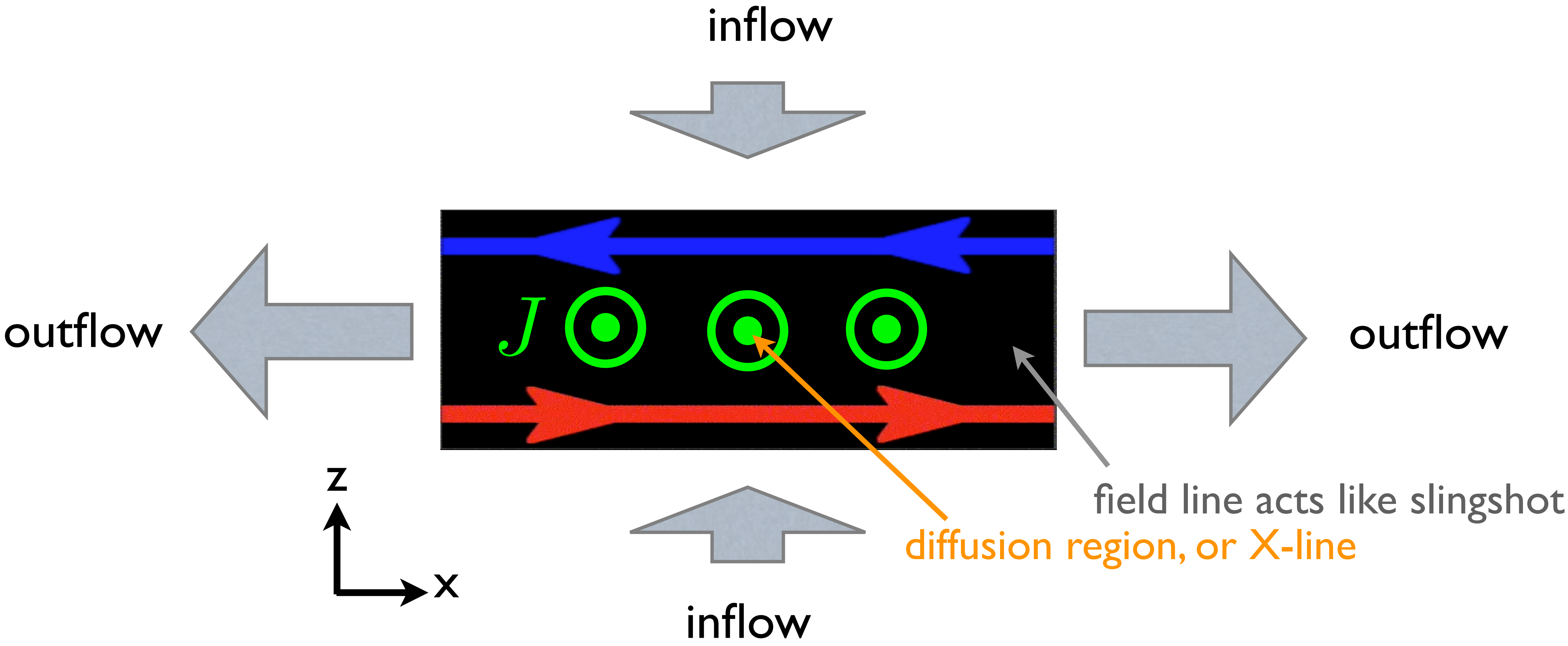
$$\frac{\partial \mathbf{B}}{\partial t} = \cancel{\nabla \times (\mathbf{V} \times \mathbf{B})} + \eta \nabla^2 \mathbf{B}$$

$$\rightarrow B = B_0 \mathrm{erf}\left[z/\sqrt{4\eta t}\right]$$



- Becomes broader & broader over time…. No steady state…
- Too slow to explain the dissipation of magnetic energy in flares.

Magnetic Reconnection

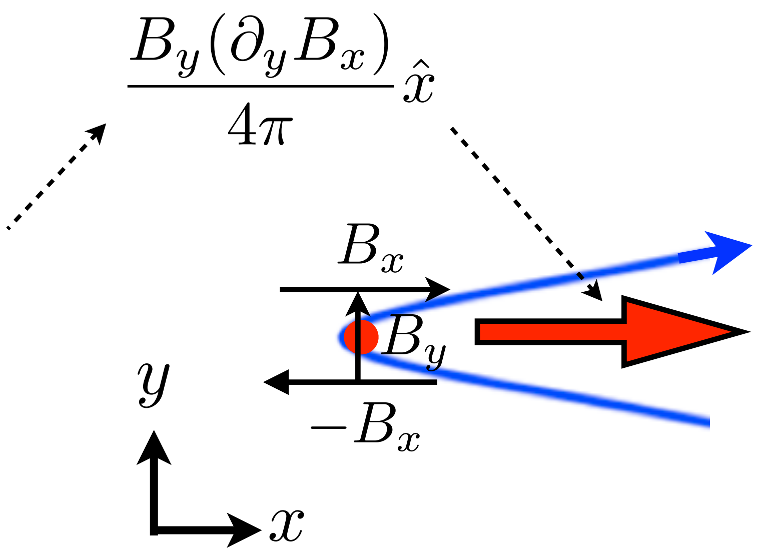


1. Inflow brings in magnetic flux (*frozen-in*)
2. Field lines break & reconnect (*frozen-in* is violated *!!*)
3. Reconnected field line shoots out plasma (*frozen-in*)
4. Pressure drop sucks in plasma inflow
1. Inflow brings in magnetic flux (*frozen-in*)
2. .....
3. ...

A self-driven process!!!

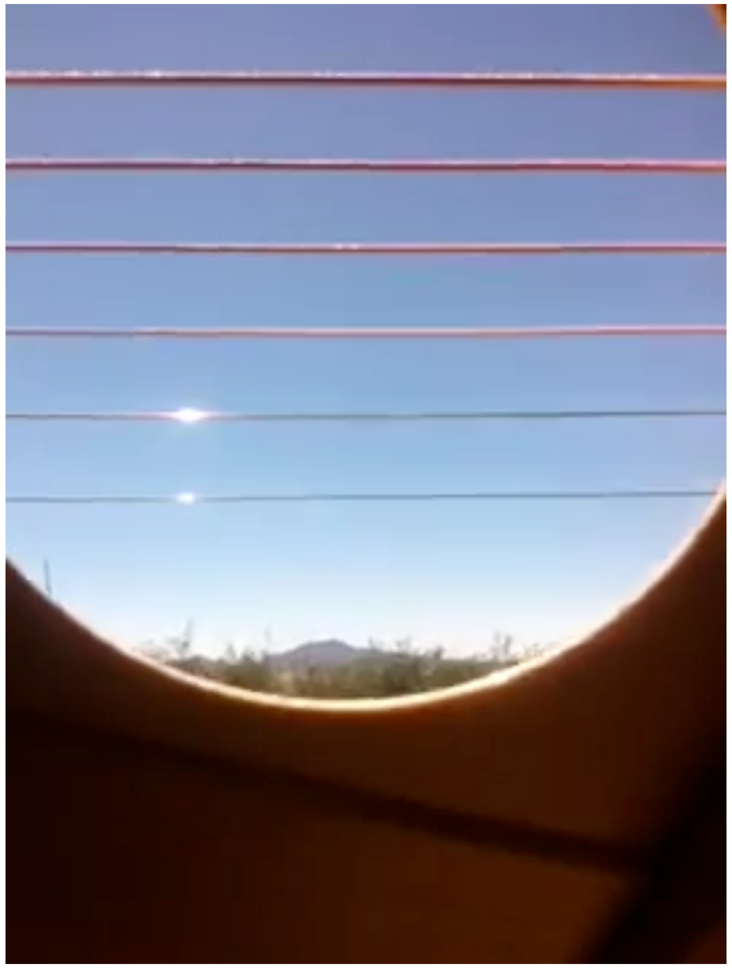
Magnetic tension & Alfvén waves

magnetic tension: $\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi}$



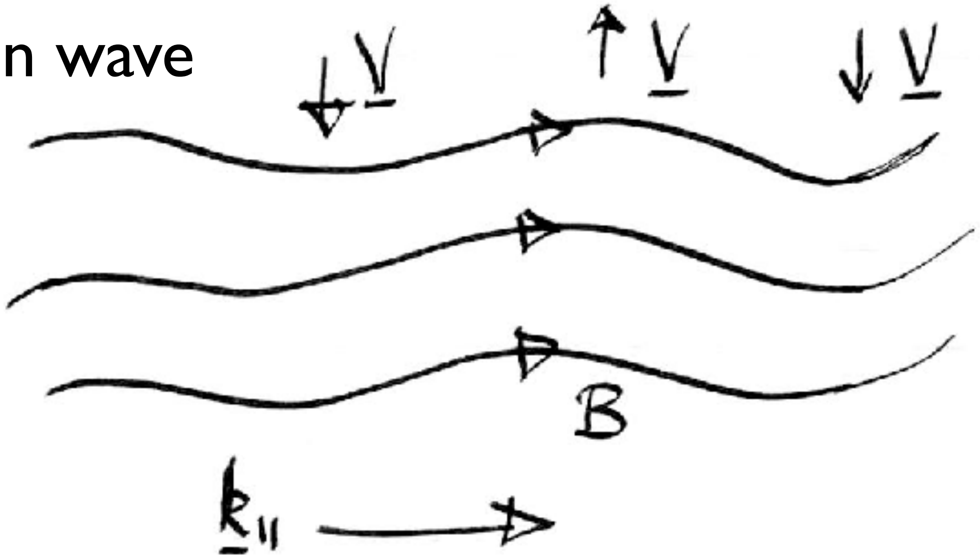
plasma inertia: $nm_i\frac{d\mathbf{V}}{dt}$

vibration of guitar strings



(Youtube: iphone 4 inside a guitar oscillation! VERY COOL!)

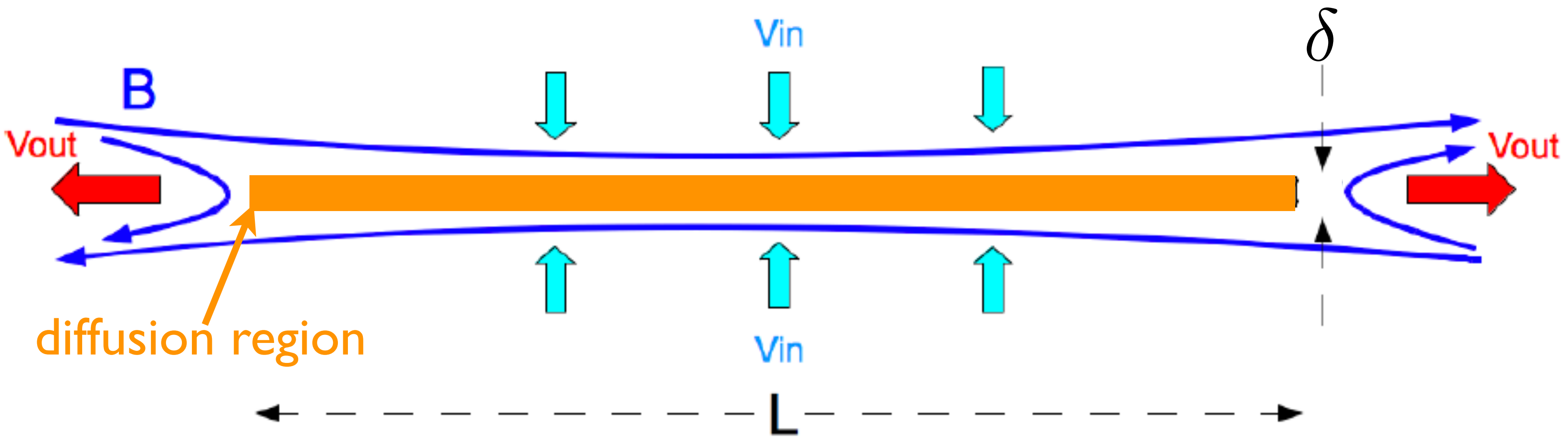
Alfvén wave



Alfvén speed

$$\rightarrow V_A \sim \frac{B}{\sqrt{4\pi nm_i}}$$

Sweet-Parker solution (1957)



mass conservation: $\nabla \cdot (n\mathbf{V}) \simeq 0$ $\quad \rightarrow V_{in} L \simeq V_{out} \delta$

momentum eq.: $\dfrac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} \simeq n m_i \mathbf{V} \cdot \nabla \mathbf{V}$ $\quad \rightarrow V_{out} \simeq \dfrac{B}{\sqrt{4\pi n m_i}} = V_A$!

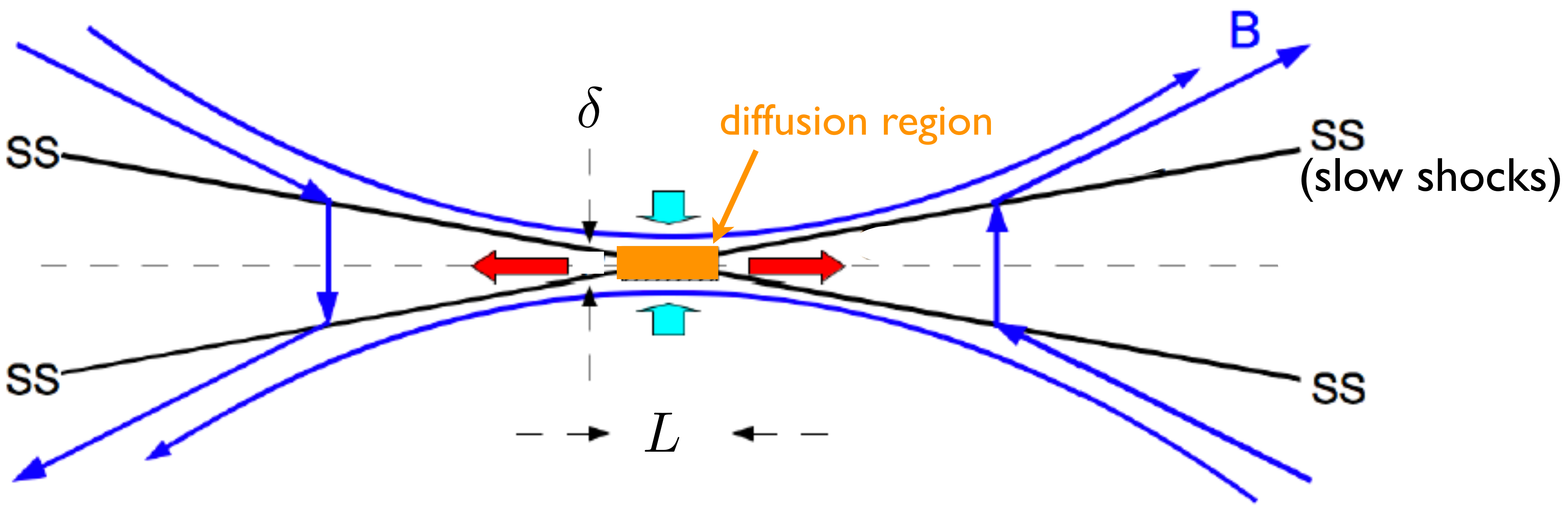
tension inertia

normalized reconnection rate $\rightarrow$ $\boxed{R \equiv \dfrac{V_{in}}{V_A} \sim \dfrac{\delta}{L}}$

- However, this model has a small δ/L , the rate is also too small to explain the time-scales in solar flare. (Parker 1963)

- To explain the flares, it requires R~ 0.1. (Parker 1973)

Petschek solution (1964)

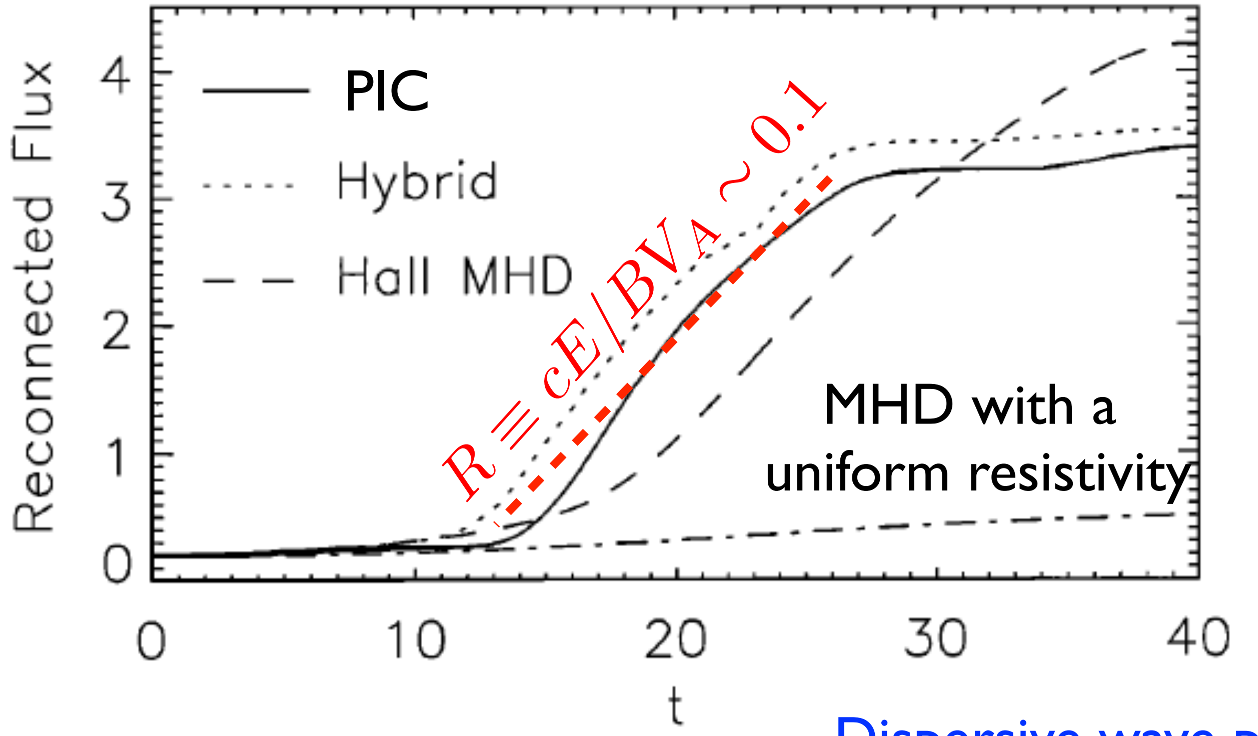


Reconnection rate is much larger if $R \sim \frac{\delta}{L} \uparrow$

- However, this is not a self-consistent solution. (Sato & Hayashi, 79; Biskamp, 86)

*aspect ratio = aspect ratio of the diffusion region

GEM Reconnection Challenge (2001)



(Birn+, 2001)

Dispersive wave picture

(Sonnerup 79; Shay+ 98; Rogers+ 01; Drake+ 08)

Ohm's Law in plasmas:

$$\mathbf{E} + \overset{\text{convection}}{\frac{\mathbf{v}_i \times \mathbf{B}}{c}} = \overset{\text{resis.}}{\eta \mathbf{J}} + \overset{\text{Hall term}}{\frac{1}{nec}\mathbf{J} \times \mathbf{B}} - \overset{\text{pressure}}{\frac{1}{ne}\nabla \cdot \mathbf{P}_e} + \overset{\text{inertia}}{\frac{m_e}{e^2}\frac{d\mathbf{J}/n}{dt}}$$

	resis.	Hall term	pressure	inertia	
MHD:	✓				slow
Hall MHD:	✓	✓			↑ fast with R~0.1 ↓
Hybrid:	✓	✓		✓	
PIC:		✓	✓	✓	

Standing Dispersive Wave Picture

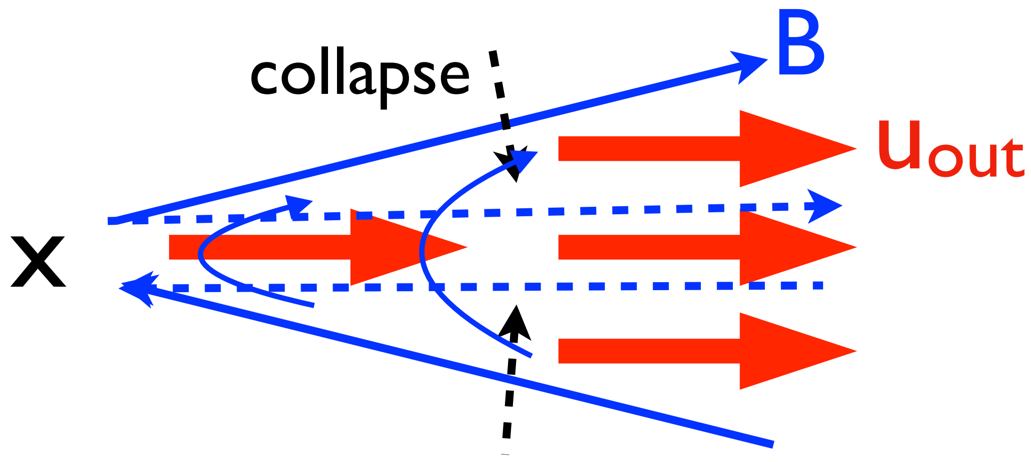
— outflow is driven by magnetic tension force

Without the Hall term…

Alfvén wave

$$\omega \propto k$$

$$\rightarrow u_{out} \sim \omega/k \sim constant$$



— collapses back to a long Sweet-Parker layer

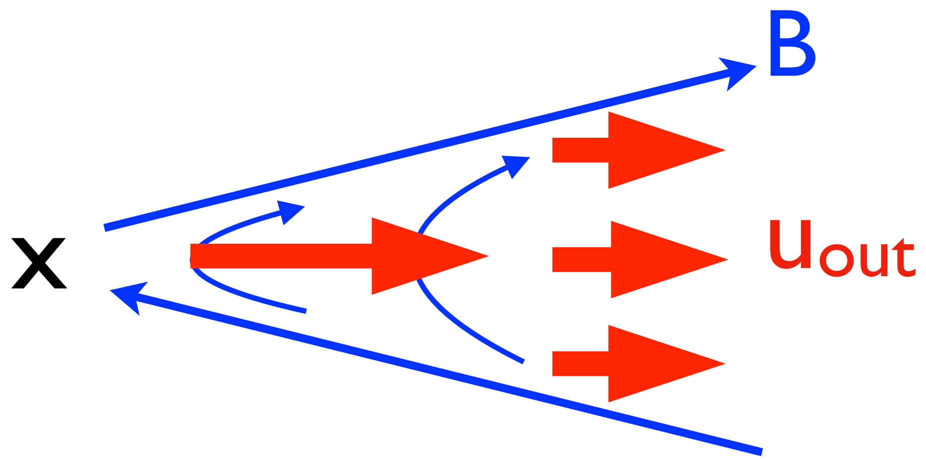
With the Hall term…

Whistler wave $(b_g = 0)$

Kinetic Alfvén wave $(b_g \neq 0)$

$$\omega \propto k^2$$

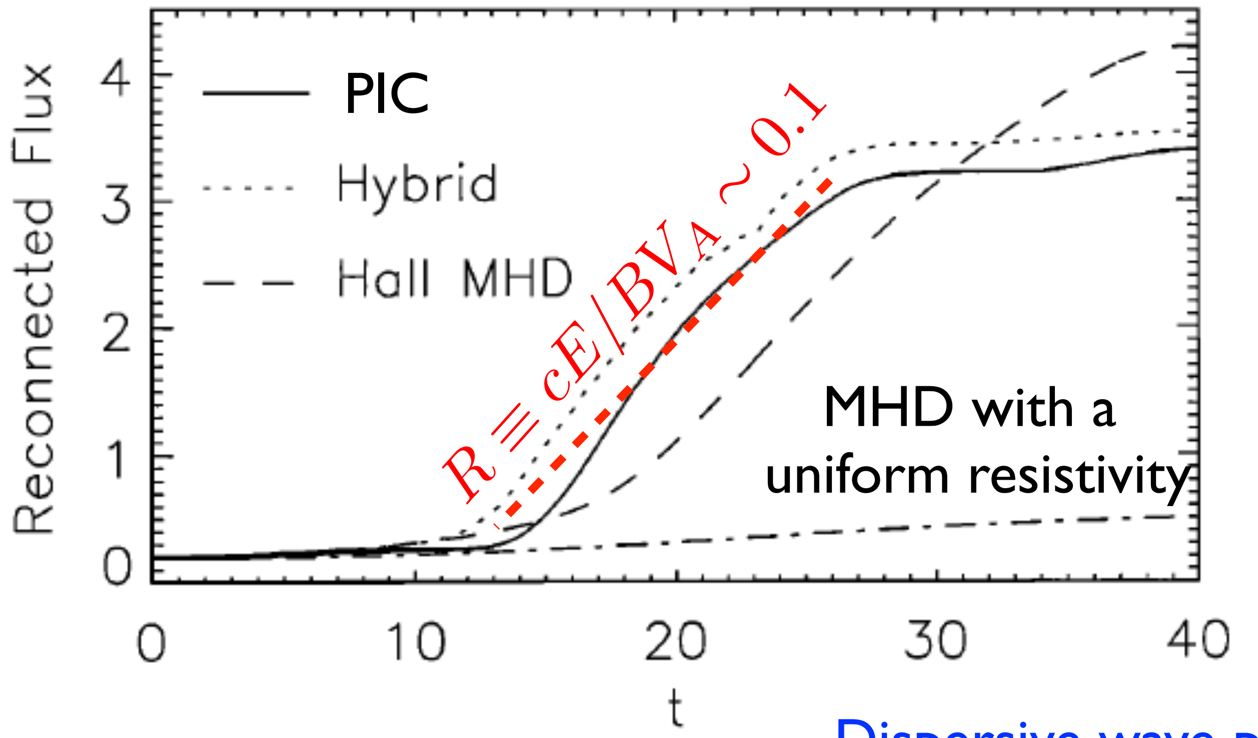
$$\rightarrow u_{out} \sim \omega/k \propto k$$



— stays opened!

- This seems to explain the difference of reconnections in resistive-MHD vs. Two-fluid/Hybrid/PIC models. (Birn+ 2001, Rogers+ 2001, Shay+ 1998, Mandt+ 1994)

GEM Reconnection Challenge (2001)



(Birn+ 2001)

Dispersive wave picture

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Ohm's Law in plasmas:

$$\mathbf{E}+\overset{\text{convection}}{\frac{\mathbf{v}_i\times\mathbf{B}}{c}}=\overset{\text{resis.}}{\eta\mathbf{J}}+\overset{\text{Hall term}}{\frac{1}{nec}\mathbf{J}\times\mathbf{B}}-\overset{\text{pressure}}{\frac{1}{ne}\nabla\cdot\mathbf{P}_e}+\overset{\text{inertia}}{\frac{m_e}{e^2}\frac{d\mathbf{J}/n}{dt}}$$

	convection	resis.	Hall term	pressure	inertia	
MHD:		✓				slow
Hall MHD:		✓	✓			↑
Hybrid:		✓	✓		✓	fast with R~0.1
PIC:			✓	✓	✓	↓

However, electron-positron (PIC): ✓ ✓

(Bessho & Bhattacharjee, 05; Daughton+ 07; Swisdak+ 08; Liu+ 09)

strong guide field limit (PIC): ✓ ✓

(Liu+14; TenBarge+14; Stainer+15; Cassak+15)

also fast with R~0.1

Q1: Why is the fast rate R~ 0.1?

Q2: What is the localization mech.?

To be solved.

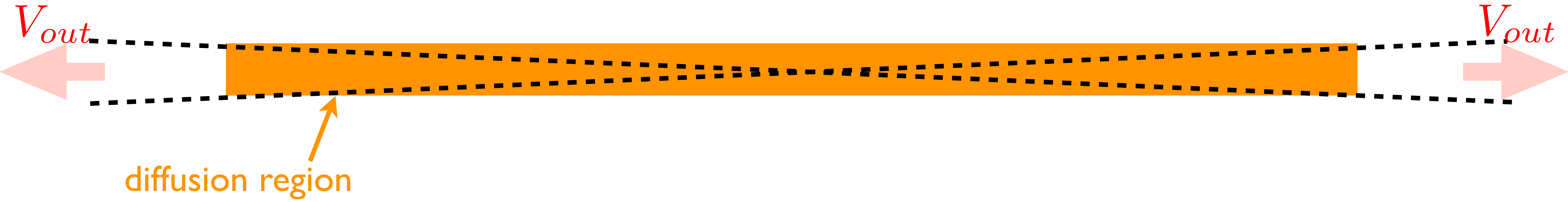
Q1: How to explain the fast reconnection rate value of order 0.1 in different systems?

-- including PIC, hybrid, Hall-MHD, MHD with a localized resistivity, even MHD with numerical resistivity only…etc

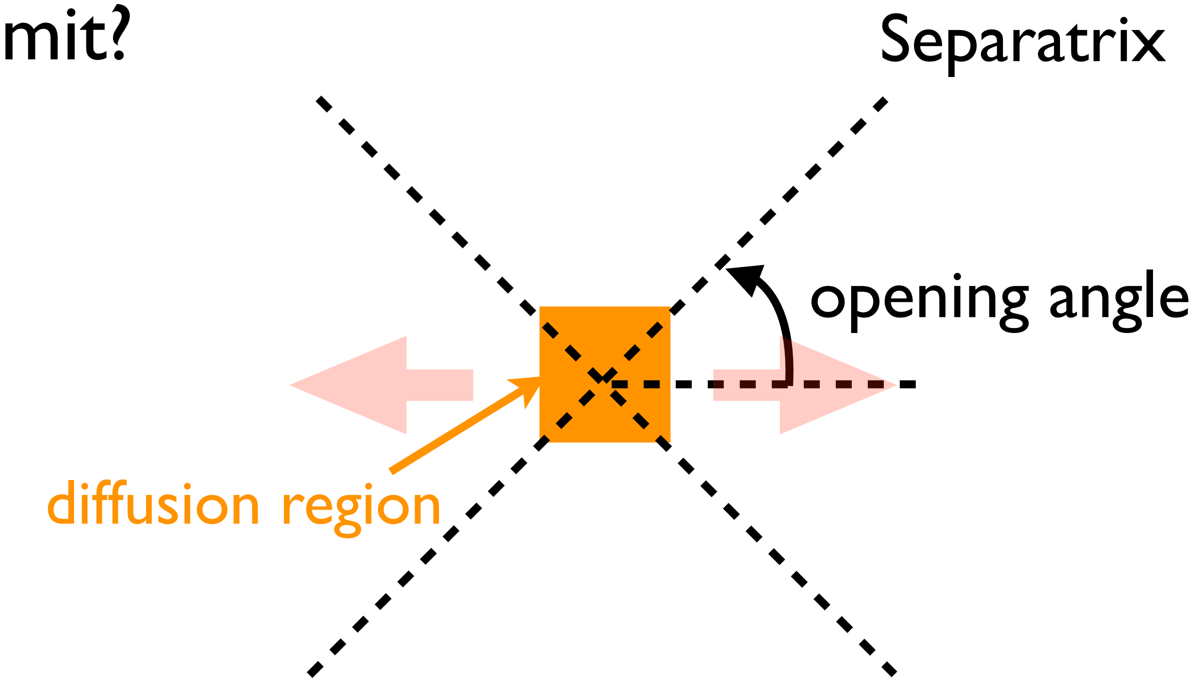
*clue: can not be the diffusion-scale physics!

Two extreme limits…

1. In the small δ/L limit, $R \sim \delta/L \rightarrow 0$



2. How about the large δ/L limit?



It turns out that when $\delta/L \rightarrow 1$, $R \rightarrow 0$!

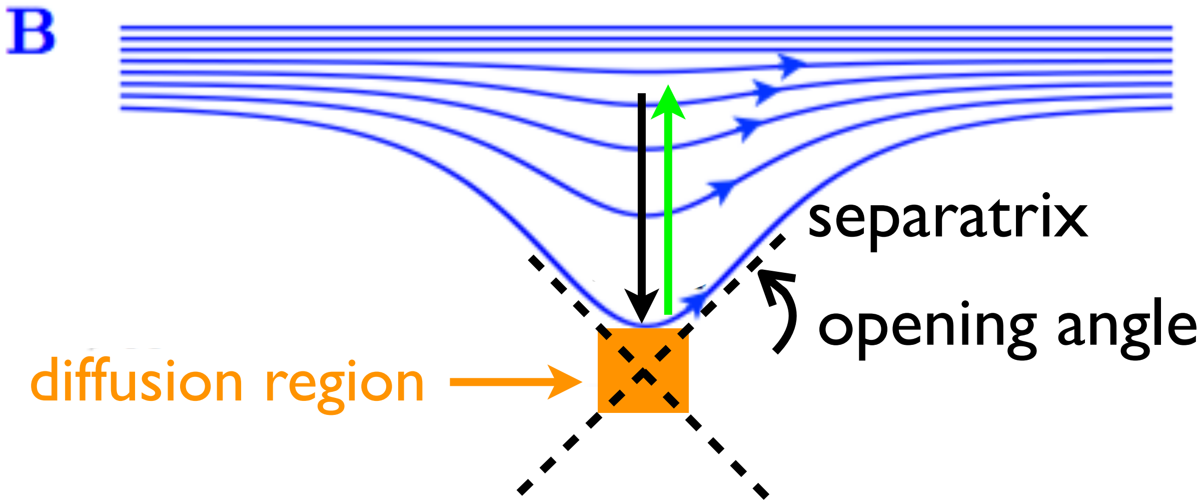
✦ There must be a maximum in between these two limits~

The Key: Geometry & Force balance!

In the large diffusion region aspect ratio, δ/L , limit .....

$$\overset{\text{tension}}{\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi}} \simeq \overset{\text{magnetic pressure}}{\frac{\nabla(B^2)}{8\pi}} + \overset{\text{inertia}}{nm_i\mathbf{V}\cdot\nabla\mathbf{V}}$$

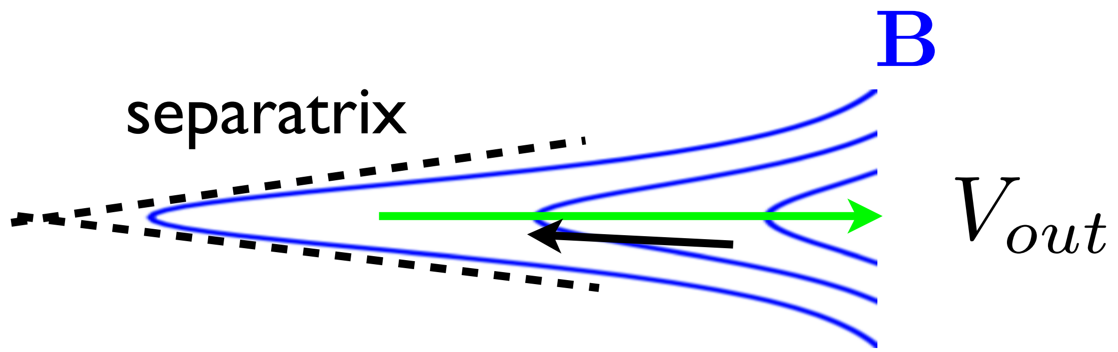
@ inflow region



$\rightarrow$ reduction of the reconnecting field!!!

$\rightarrow R\downarrow$

@ outflow region



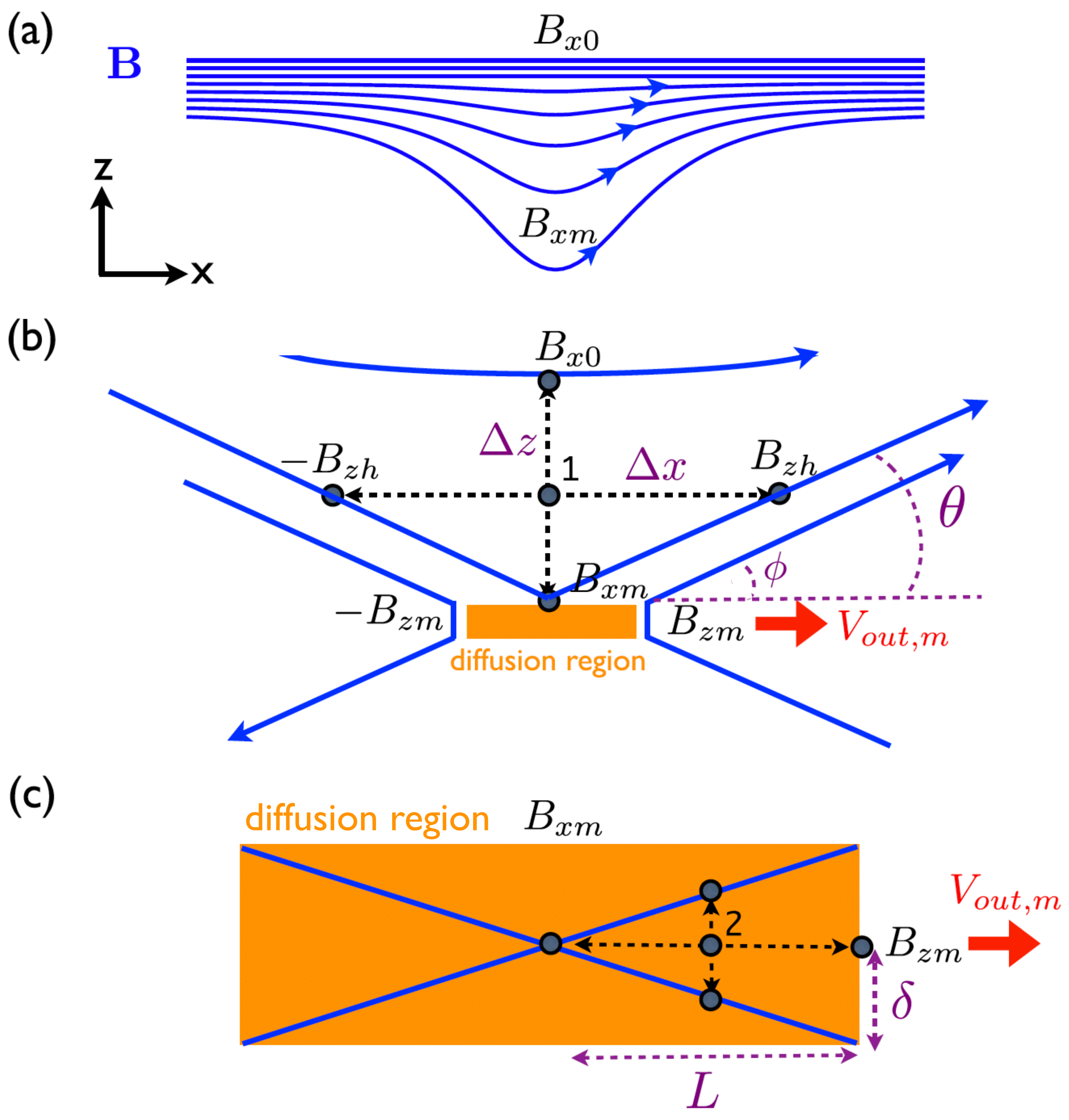
$\rightarrow$ reduction of the outflow speed!!!

$\rightarrow R\downarrow$

- Constraints imposed at the inflow & outflow regions (upper) bound the rate!

Back-of-the-envelope calculation…

(Liu+ PRL 2017)



step1: Introduce the scale-separation~

step2: analyze the force-balance at point 1

$\rightarrow B_{zm}(S)$

$\uparrow$

$Slope = \tan\theta$

step3: analyze the force-balance at point 2

$\rightarrow V_{out,m}(S)$

step4: connecting these two quantities to get the rate~

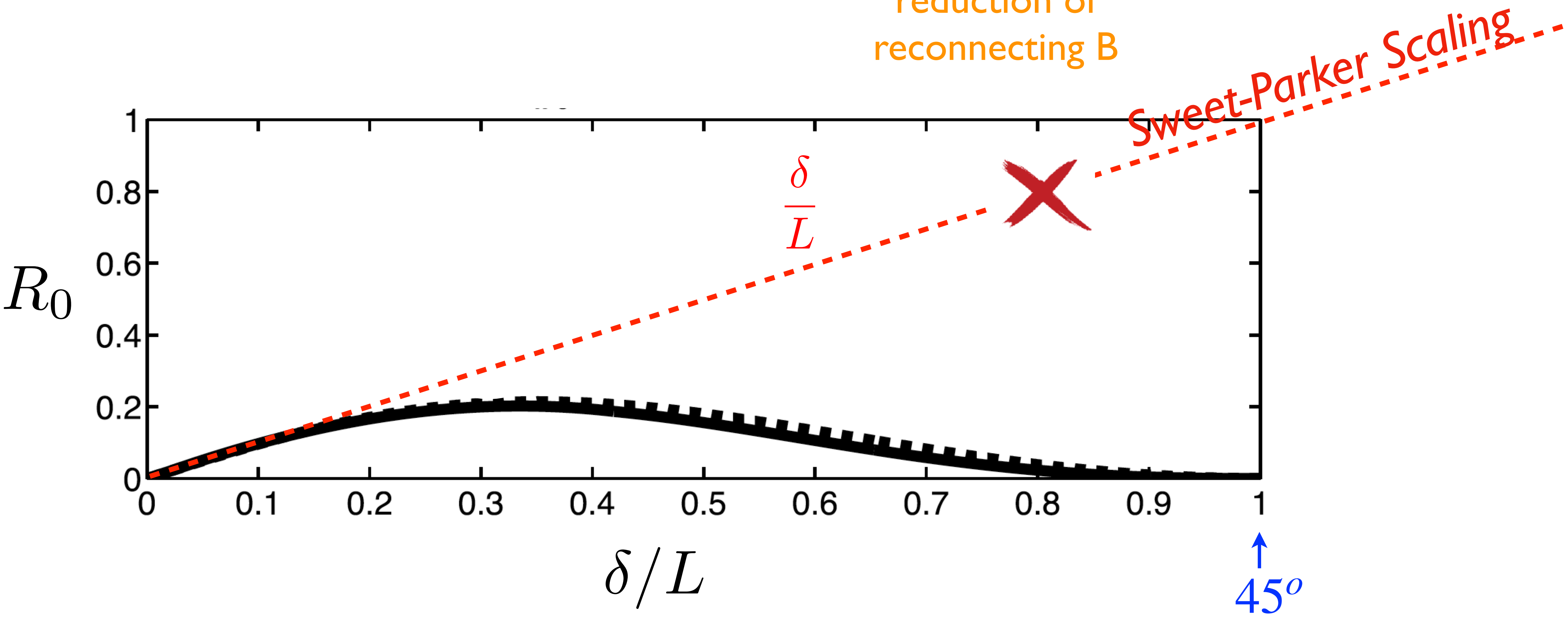
$$\rightarrow E_y(S) = B_{zm}V_{out,m}/c$$

'm' indicates quantities at the microscopic scale

Explanation of the fast rate ~ 0.1

-- Geometrical consideration!

$$R_0 \equiv \frac{cE_y}{B_{x0}V_{A0}} = \left(\frac{B_{zm}}{B_{x0}}\right)\left(\frac{V_{out,m}}{V_{A0}}\right) \simeq \frac{\delta}{L}\underbrace{\left[\frac{1-(\delta/L)^2}{1+(\delta/L)^2}\right]^2}_{\text{reduction of reconnecting B}}\overbrace{\sqrt{1-\left(\frac{\delta}{L}\right)^2}}^{\text{reduction of } V_{out}}$$



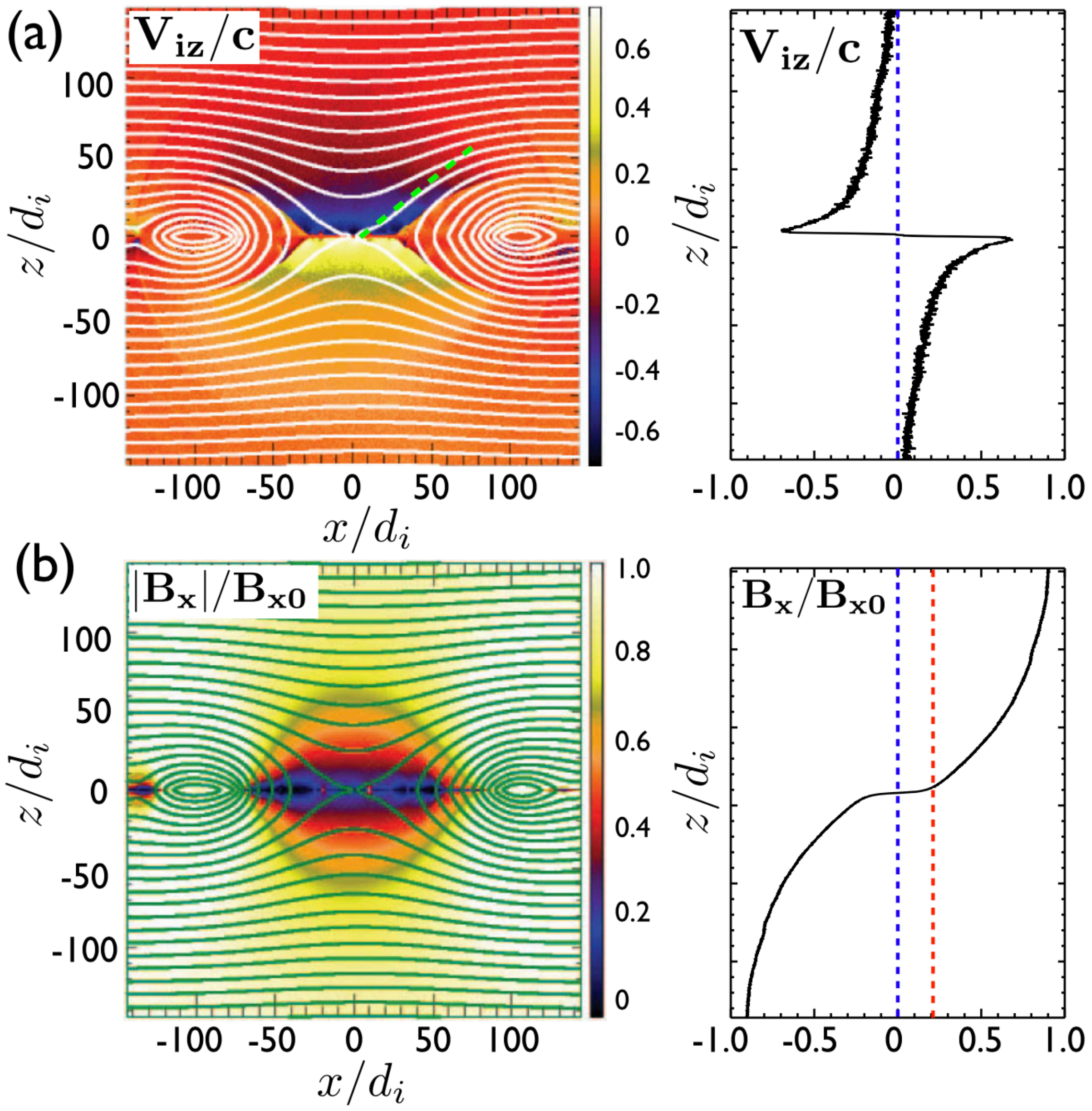
(Liu+ PRL 2017)

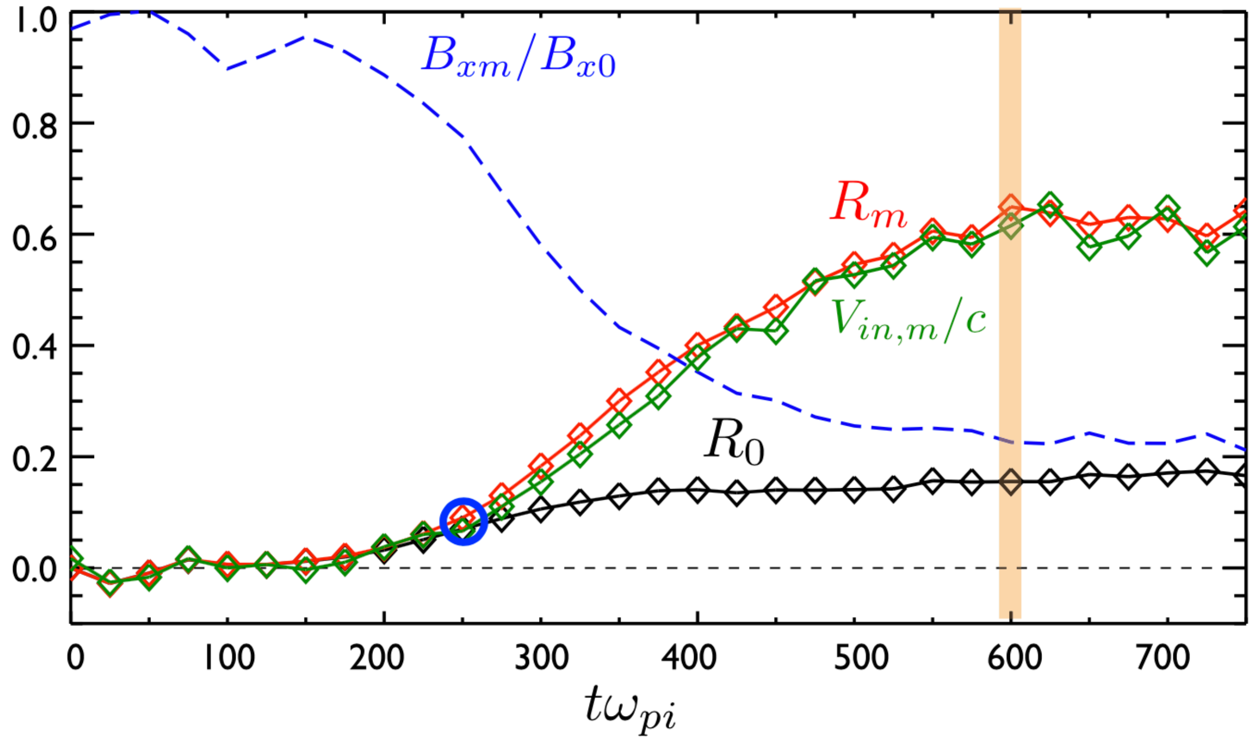
- Fast rate R~ O(0.1) is an upper bound value.
- Reconnection tends to proceed near the most efficient state, which has R ~ O(0.1).
- Nicely, rate is insensitive to δ/L near this state.

Relativistic Reconnection

(e.g., Crab Nebula's superflares)

2D PIC simulation $$\sigma = \frac{B_x^2}{4\pi nmc^2} \simeq 100$$





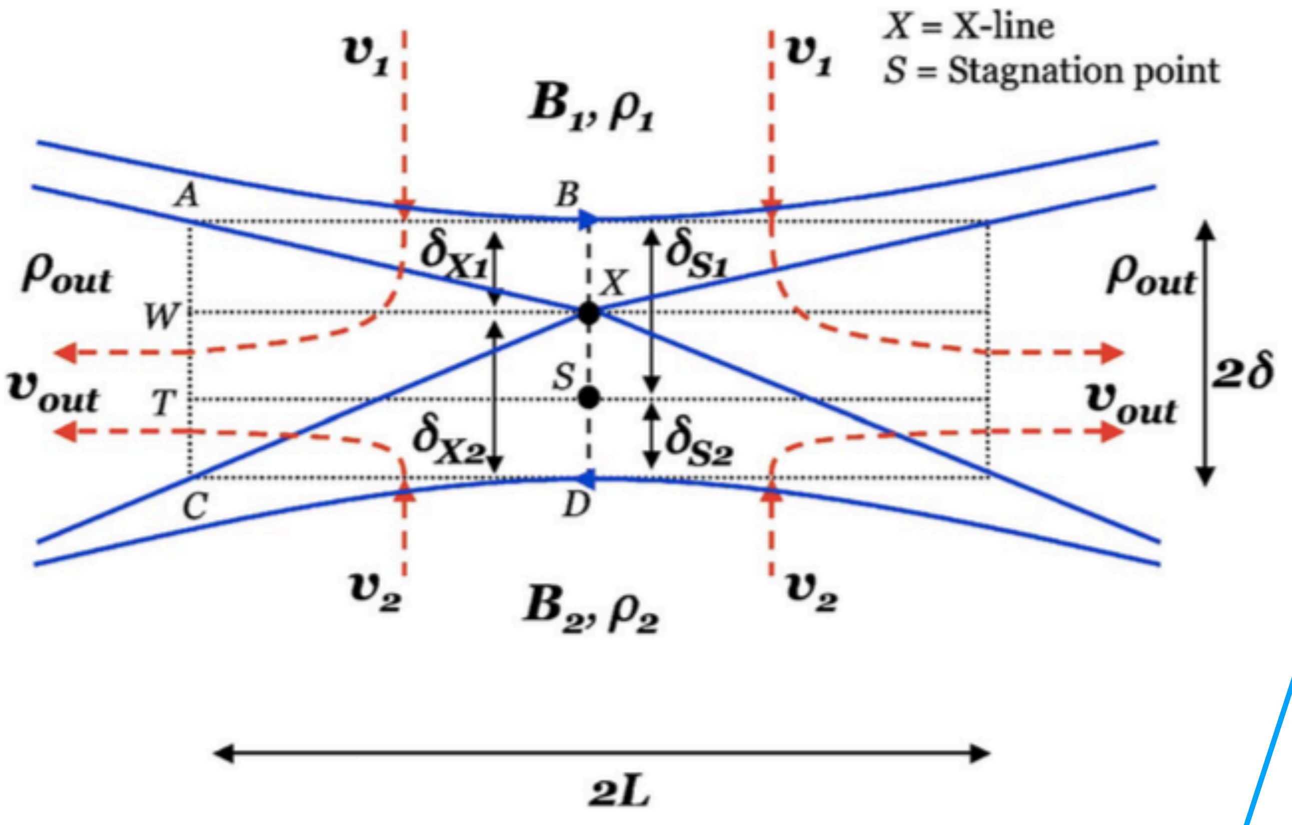
(Liu+ PRL 2017,
Guo, Liu+ PoP 2020)

- Outflow speed approaches the speed of light!
- Although the rate normalized to microscopic quantities can go up to ~ O(1) (i.e., inflow speed can also approaches the speed of light), the rate normalized to the asymptotic quantities is still bounded by O(0.1).
- Clear "scale-separation" is seen! (i.e., B_{xm} vs. B_{x0})

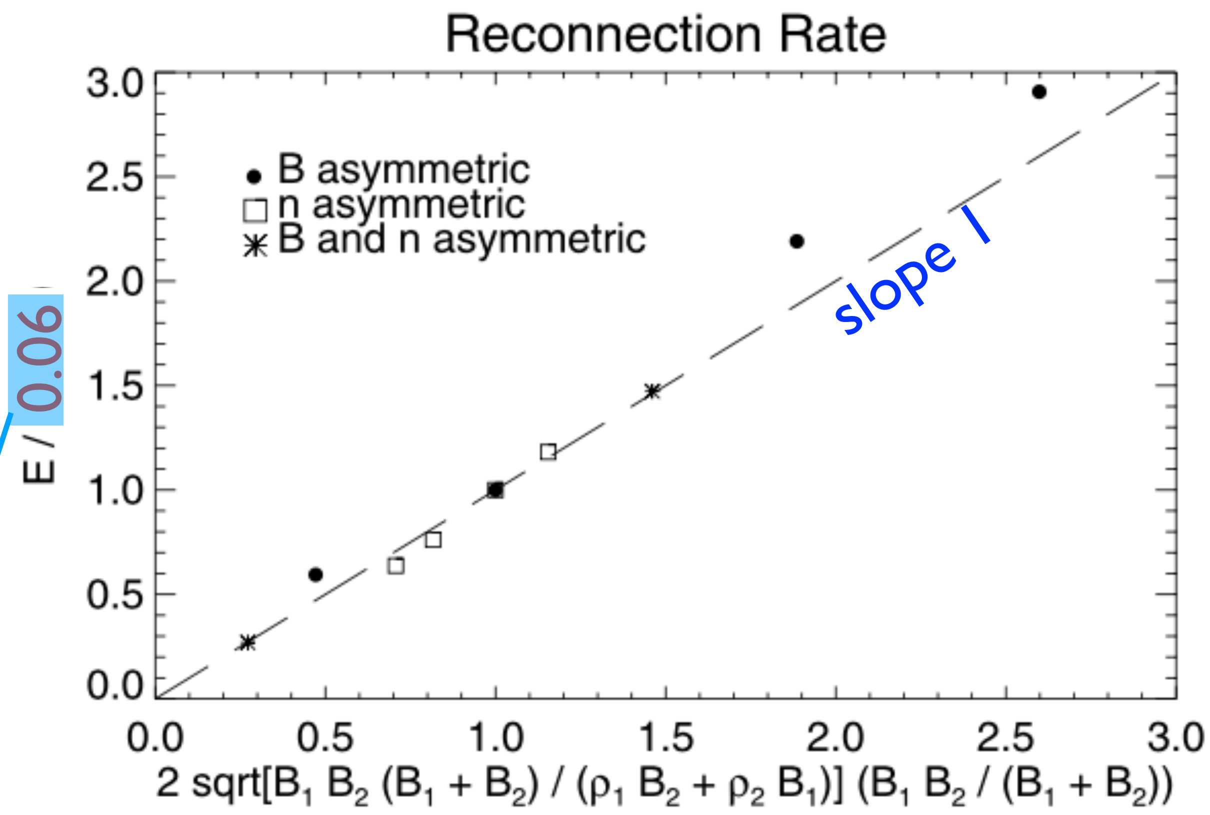
Okay~ how about asymmetric reconnection?

Asymmetric Reconnection

(e.g., Earth's magnetopause)



Two-fluid simulations



Cassak & Shay GRL (2008)

Cassak-Shay formula

$$E_{CS} = 2\left(\frac{B_1 B_2}{B_1 + B_2}\right)\left(\frac{V_{out}}{c}\right)\left(\frac{\delta}{L}\right)_{eff}$$

Cassak & Shay, PoP (2007)

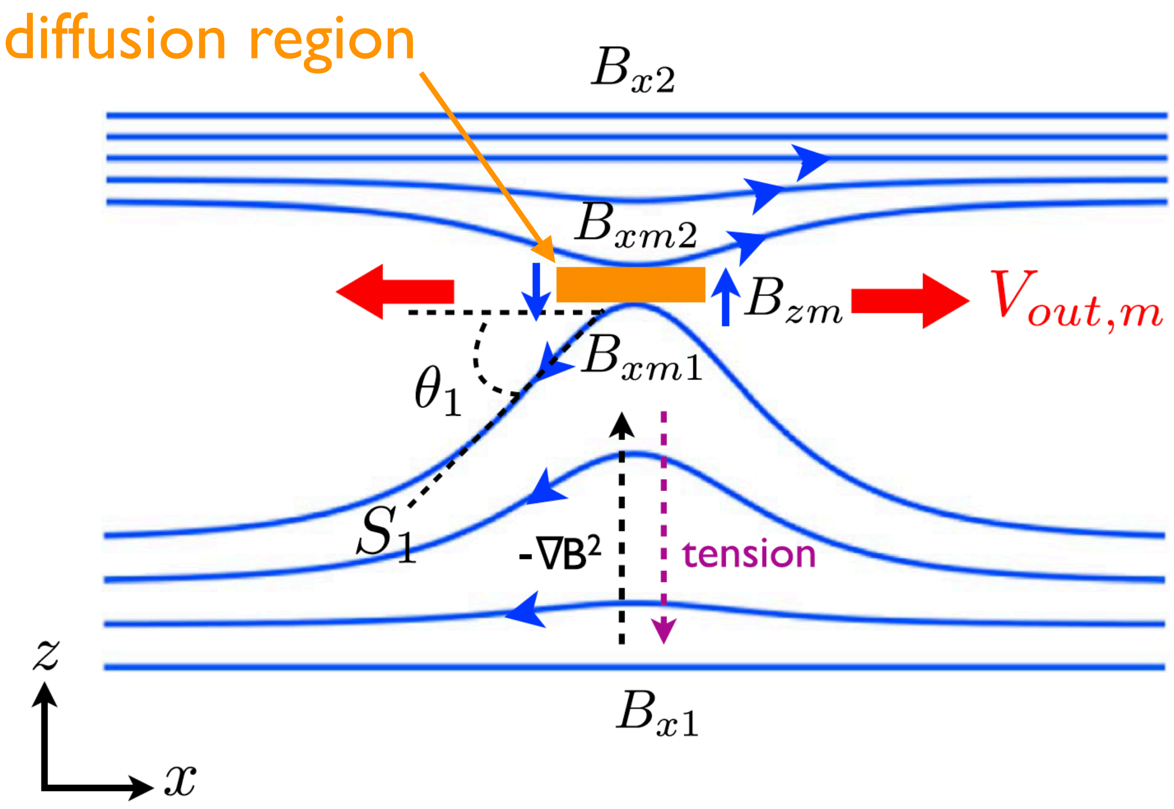
where

$$\frac{V_{out}}{c} = \sqrt{\frac{B_1 B_2}{4\pi}\left(\frac{B_1 + B_2}{B_1\rho_2 + B_2\rho_1}\right)}$$

Swisdak & Drake, GRL (2007)

→ $(\delta/L)_{eff} \sim \mathcal{O}(0.1)$! **but why?**

Constraint at the inflow region (step1&2)



'*m*' indicates quantities at the microscopic scale

From the force-balance along the inflow,

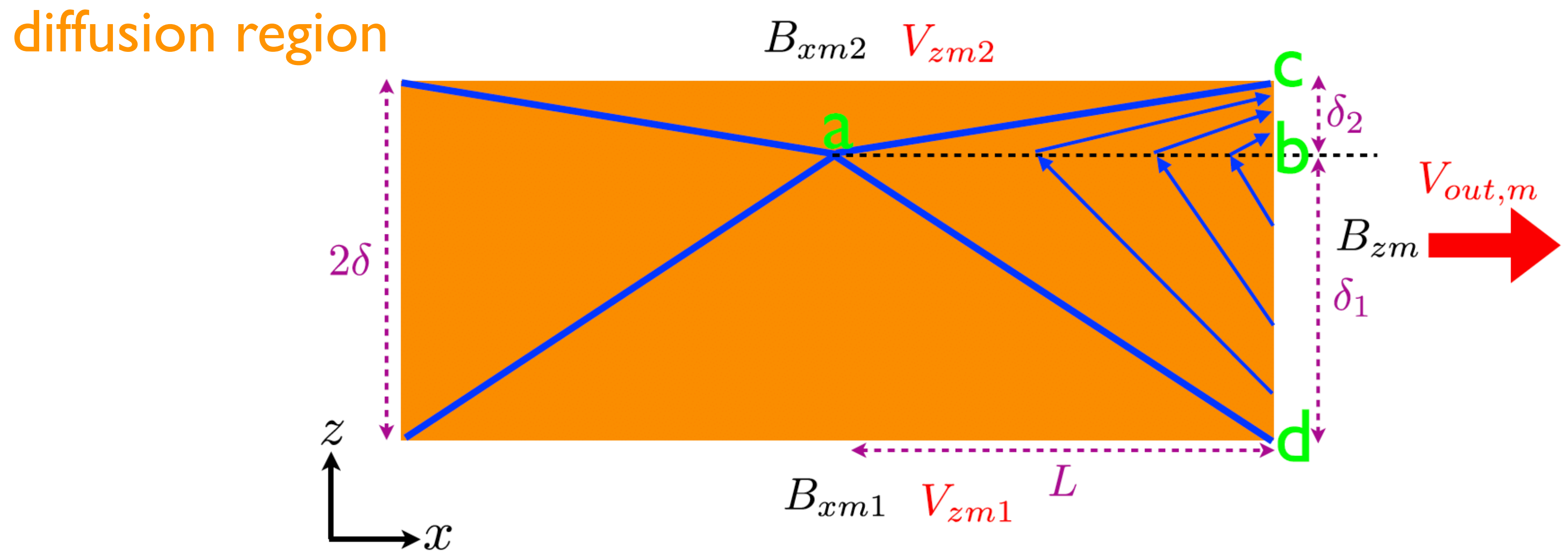
$$B_{xmi} \simeq B_{xi} \frac{1 - S_i^2}{1 + S_i^2}$$

slope of the upstream field lines

$$i = 1{,}2$$

- With a strong B asymmetry, the reduction of the reconnecting field primarily comes from the weak field side (sheath side).

Constraint at the outflow region (step 3)



From the momentum equation along the outflow,

$$V_{out,m} \simeq \sqrt{\frac{B_{xm1}B_{xm2}}{4\pi\bar{\rho}} - \frac{1}{\pi\bar{\rho}}\left(\frac{B_{xm1}B_{xm2}}{B_{xm1}+B_{xm2}}\right)^2\left(\frac{\delta}{L}\right)^2}$$

tension

magnetic pressure

Swisdak & Drake, GRL (2007)
Cassak & Shay, PoP (2007)

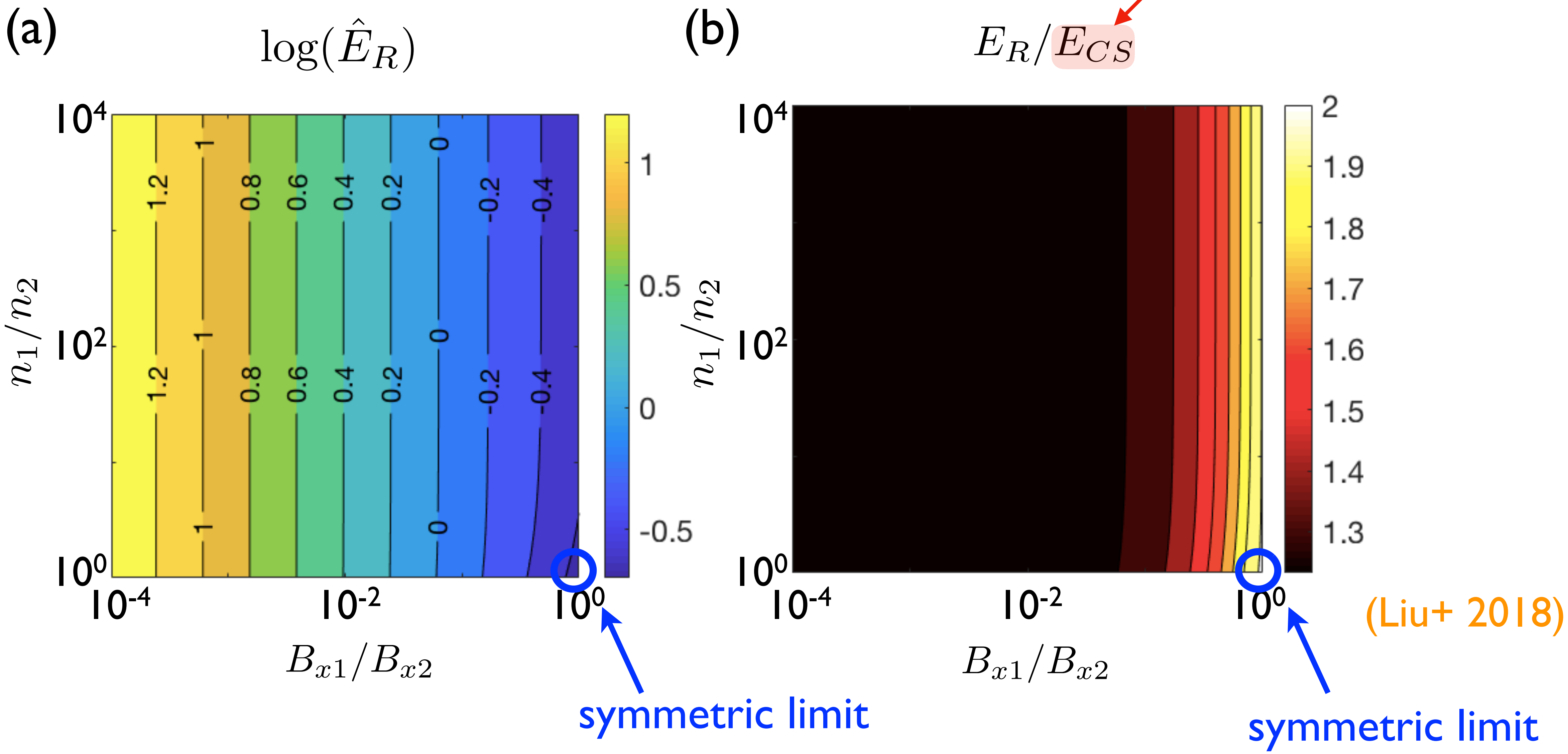
- Outflow can be reduced in the large δ/L limit
  - -- but its effect is (B_{xm1}/B_{xm2}) times smaller with a strong B asymmetry.
  - -- thus the reduction will be negligible in this limit.

Predicted asymmetric reconnection rate (step 4)

$$\hat{E}_R \equiv \frac{cE_R}{V_{Ax1}B_{x1}}$$

$$E_y(S_1) \simeq B_{zm}(S_1)V_{out,m}(S_1)/c$$

$(\delta/L)_{eff}$ ~0.1 is assumed.



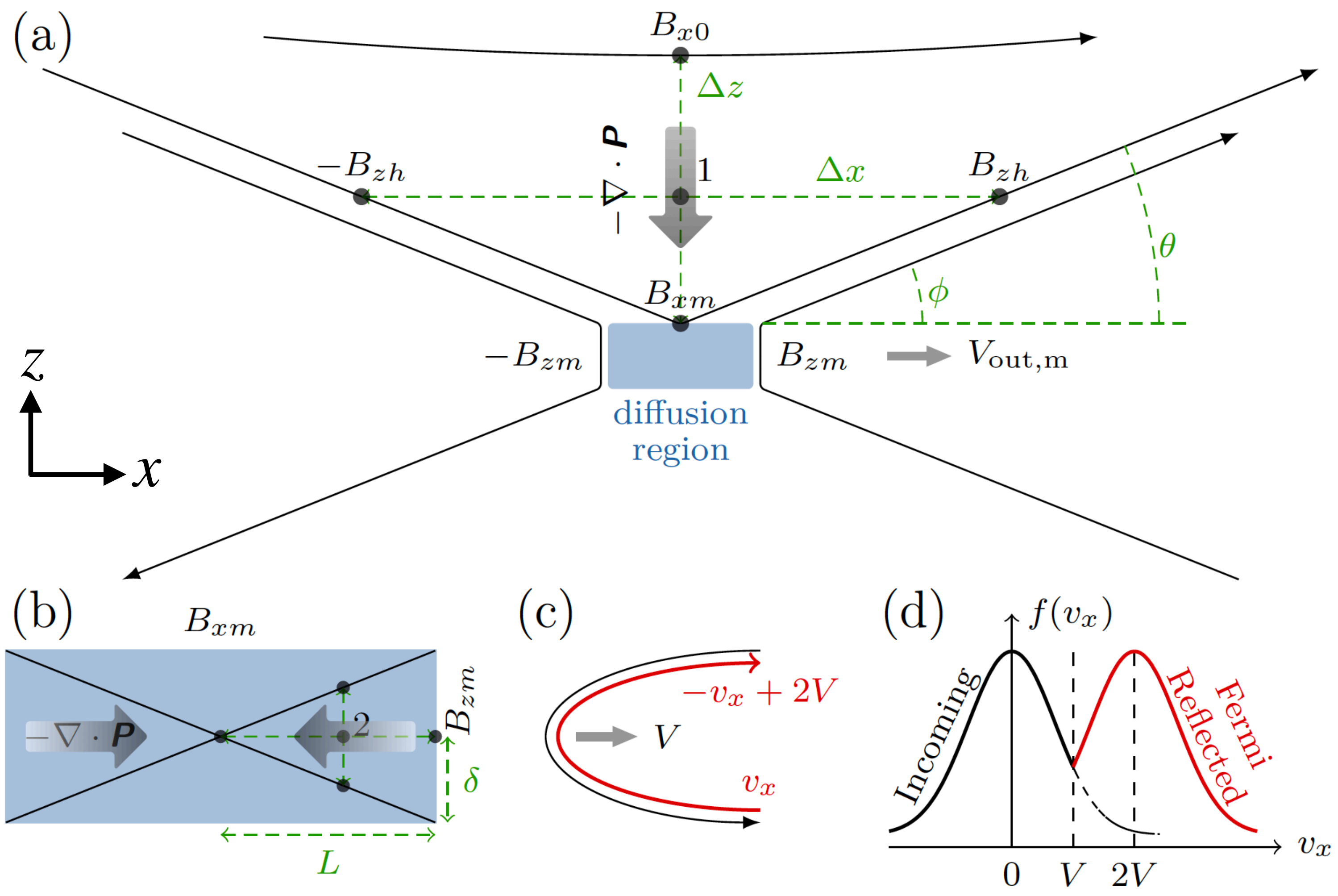
symmetric limit

symmetric limit

(Liu+ 2018)

- The prediction is obtained by maximizing the reconnection rate as a function of the opening angle.
- This prediction agrees well with the Cassak-Shay scaling!
  -- this further suggests that $(\delta/L)_{eff}$ ~0.1 arises from constraints at the inflow & outflow.

Including thermal pressure effects…



$$nm_i\frac{\partial \boldsymbol{V}}{\partial t} + nm_i\boldsymbol{V}\cdot\nabla\boldsymbol{V} + \nabla\frac{B^2}{8\pi} + \nabla P_\perp \simeq \nabla\cdot\left(\frac{\varepsilon}{4\pi}\boldsymbol{B}\boldsymbol{B}\right)$$

$$\varepsilon = 1 - 4\pi(P_\parallel - P_\perp)/B^2$$

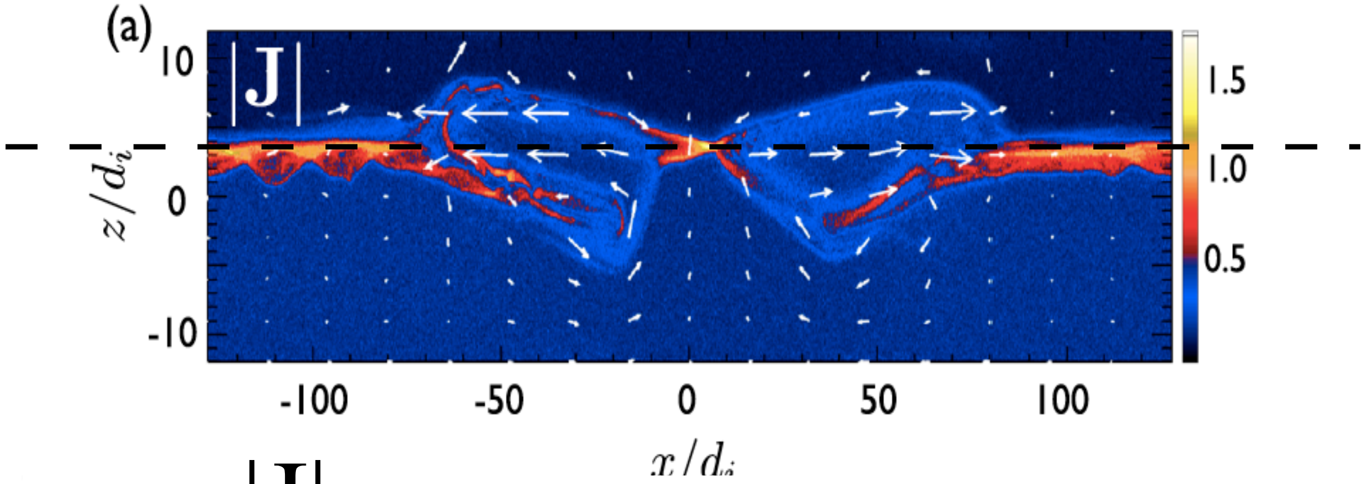
(Xiaocan Li +, *in preparation*)

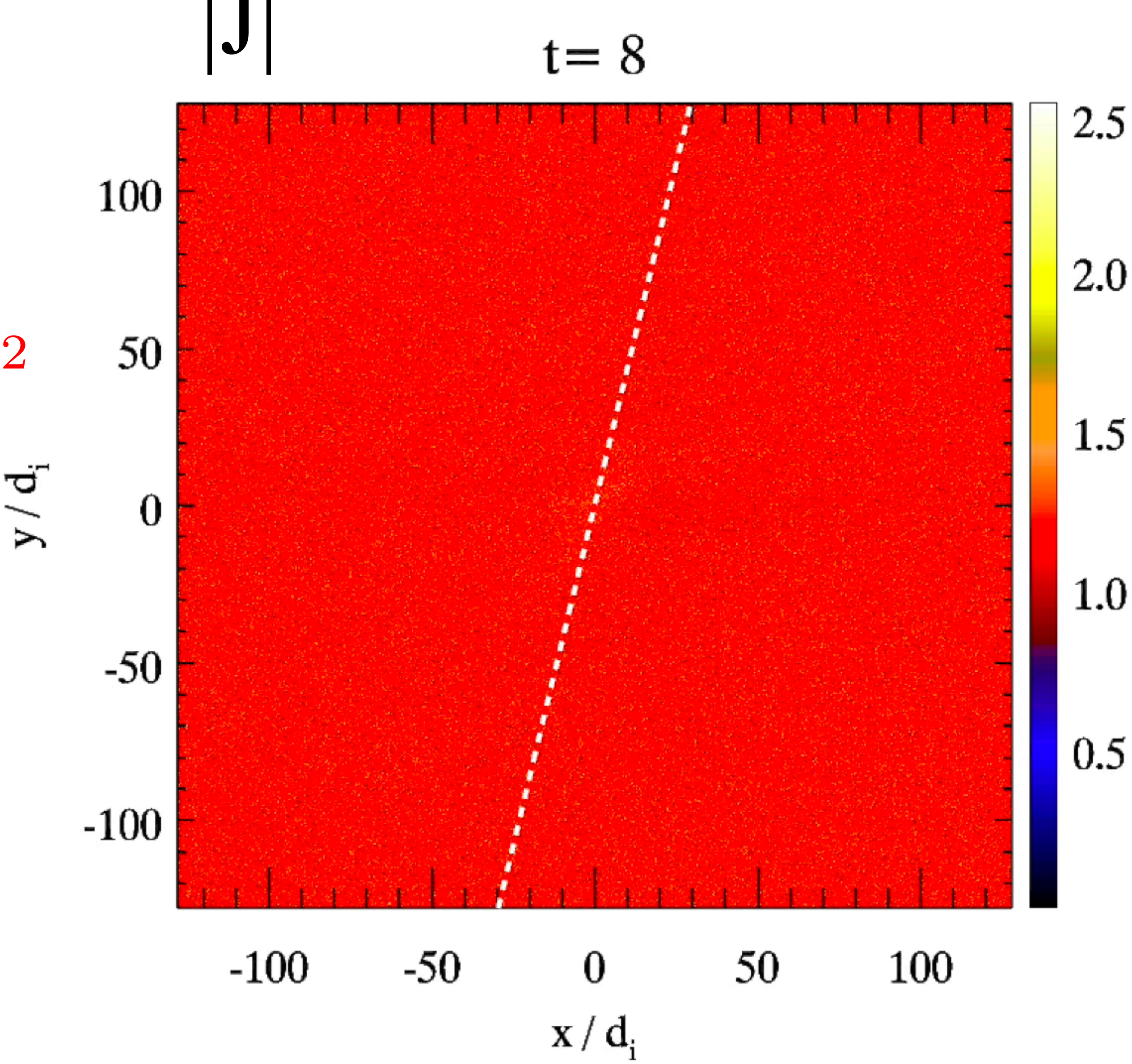
Orientation & Spread of the Reconnection X-line

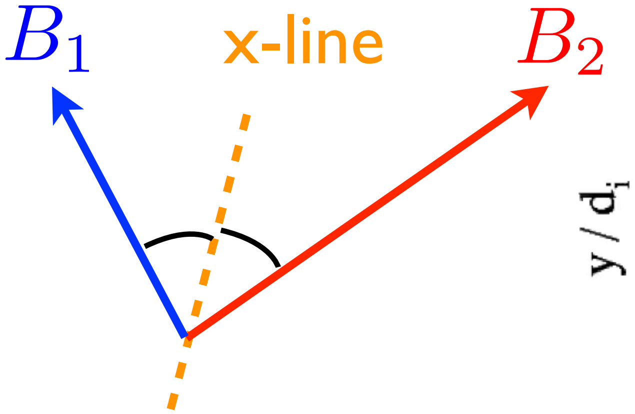
Overview of 3D (less-turbulent) reconnection in PIC

mi/me=25
256 x 256 x 24 di

b_g=1, asymmetric





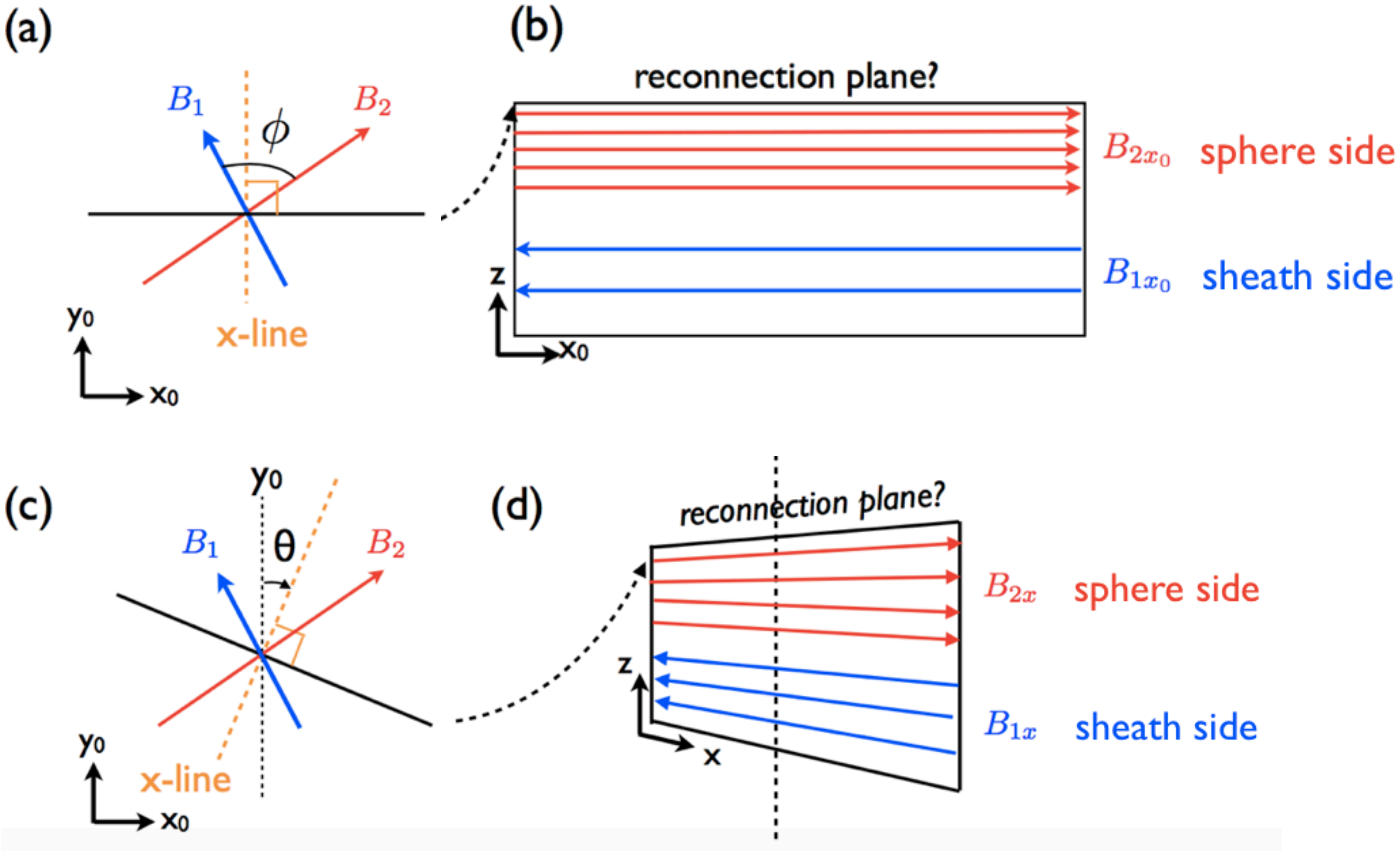


(T. Li + 2020, Liu + 2018)

- Check the orientation & spread of the reconnection x-line!

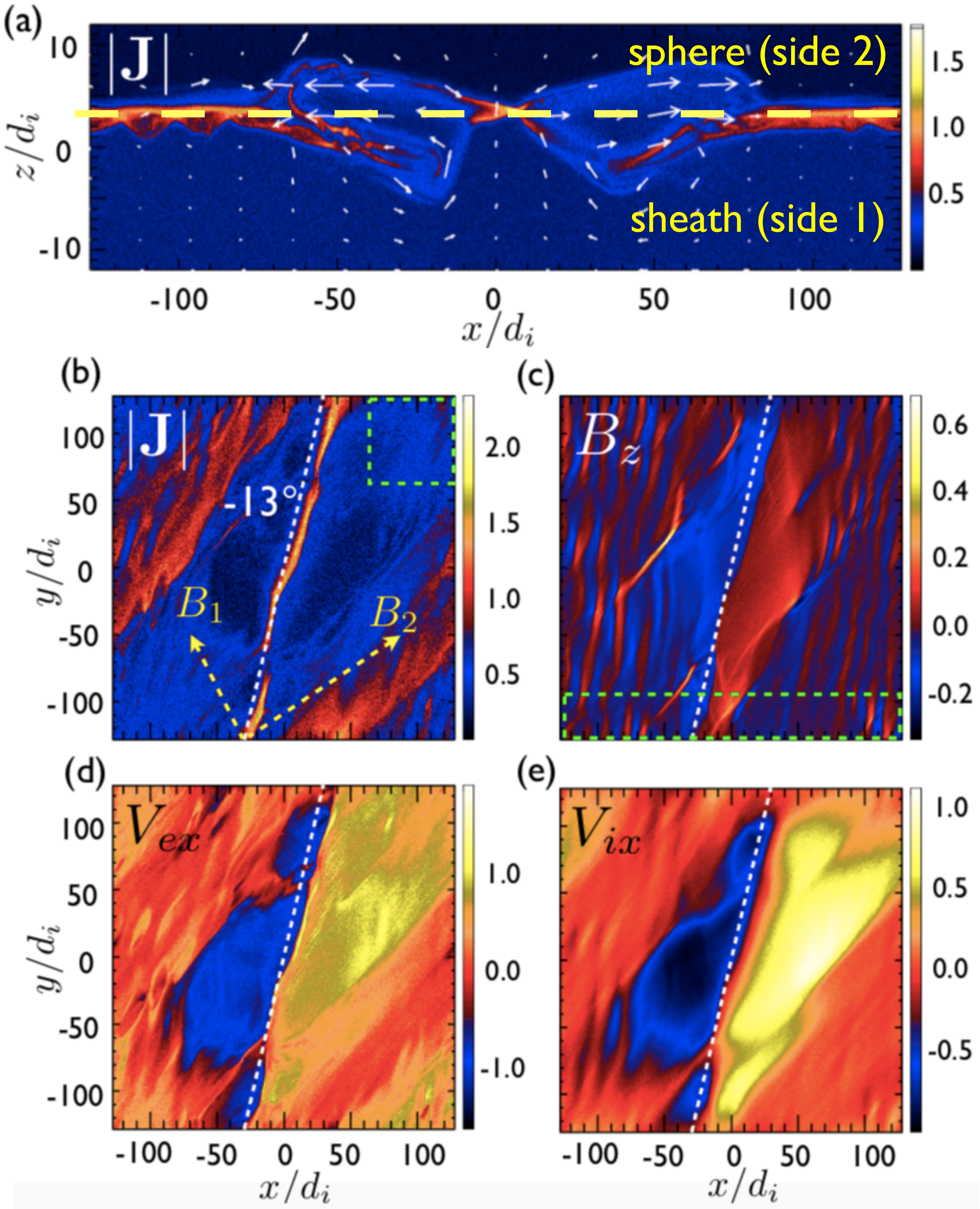
On the X-line Orientation…..

Q: Which plane does reconnection "prefer"??



- Once preferred reconnection plane is determined, the x-line orientation is determined~

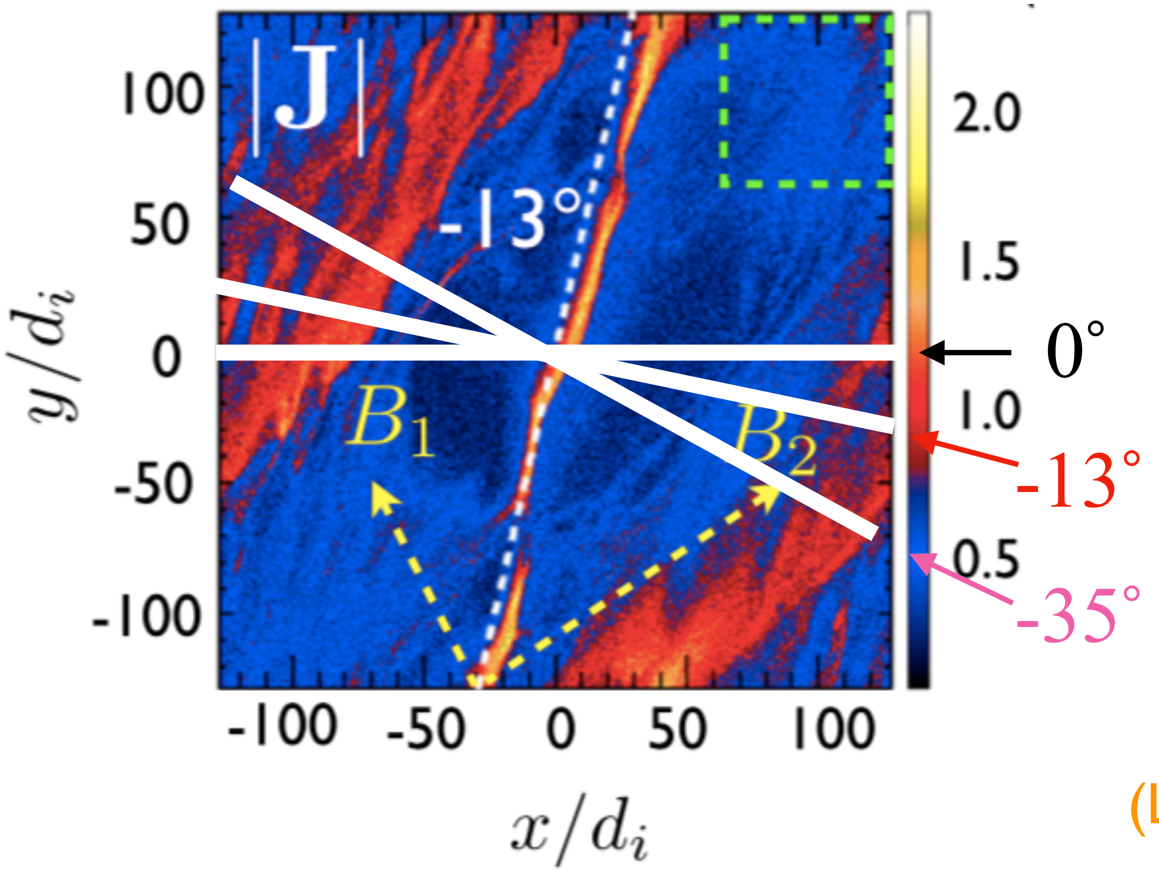
Measurement of the X-line orientation



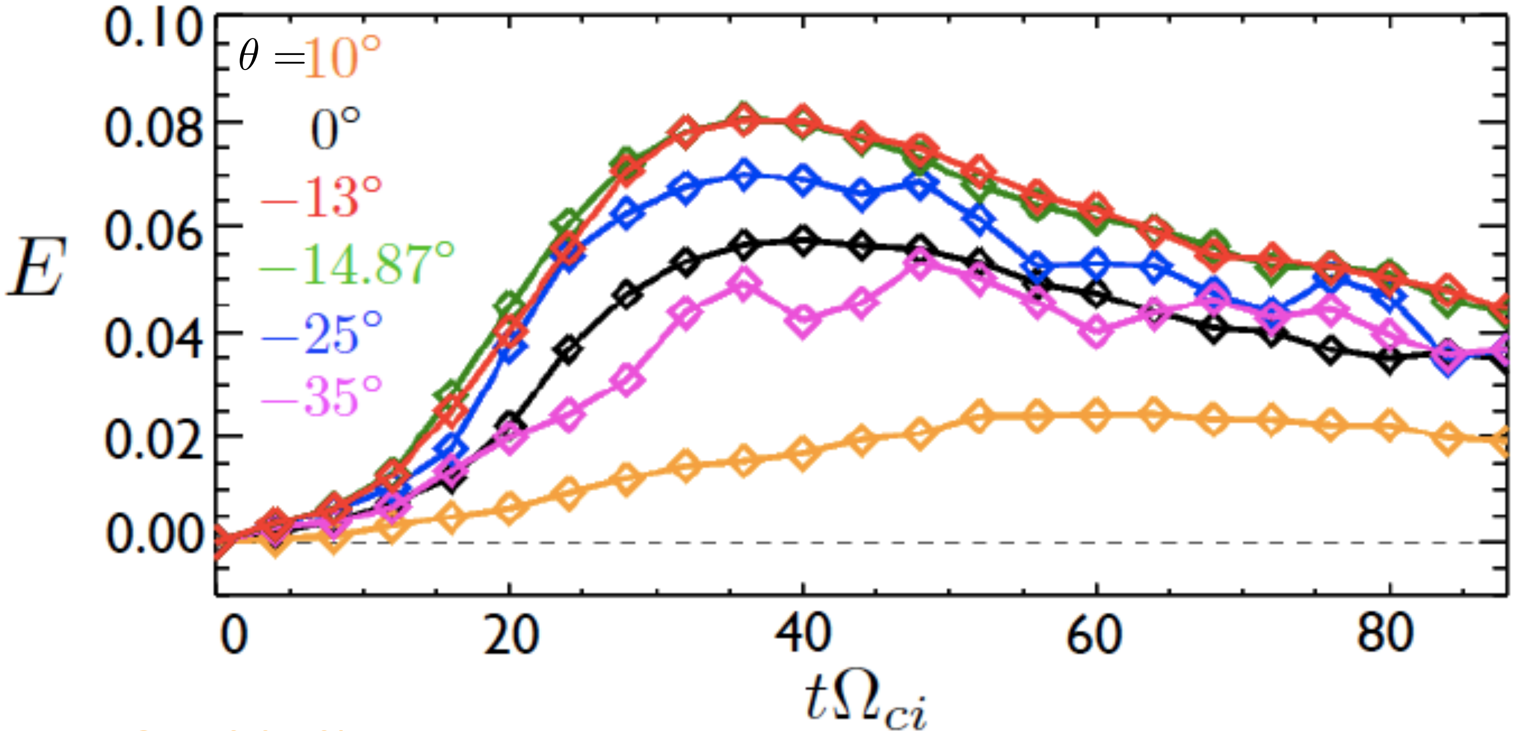
- X-line develops with a well-defined orientation ~ -13°.

What can companion 2D simulations tell us?

3D



2D oblique planes



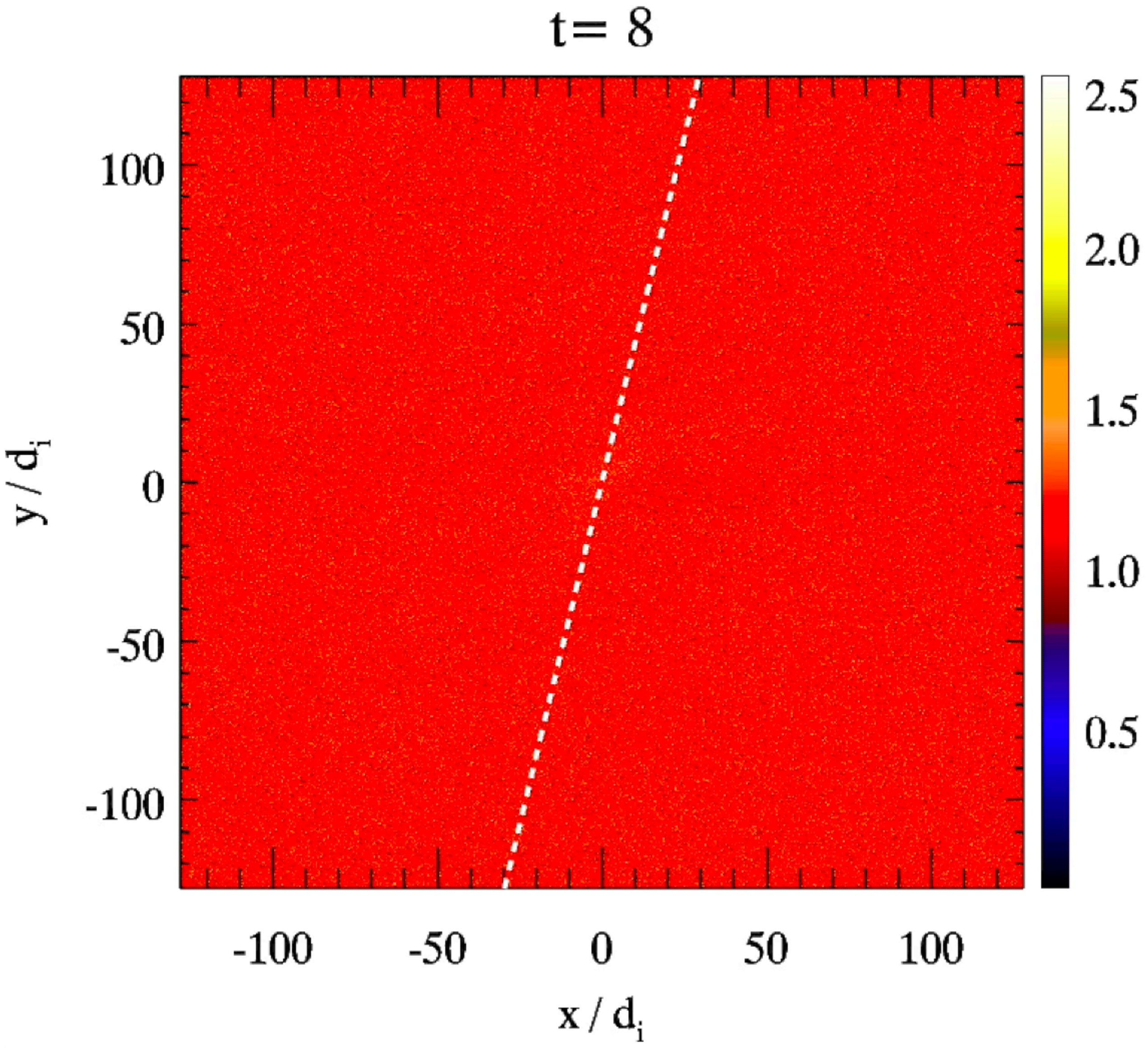
(Liu+ JGR, 2018)

- This 2D vs 3D comparison suggests that the system tends to maximize the reconnection rate!

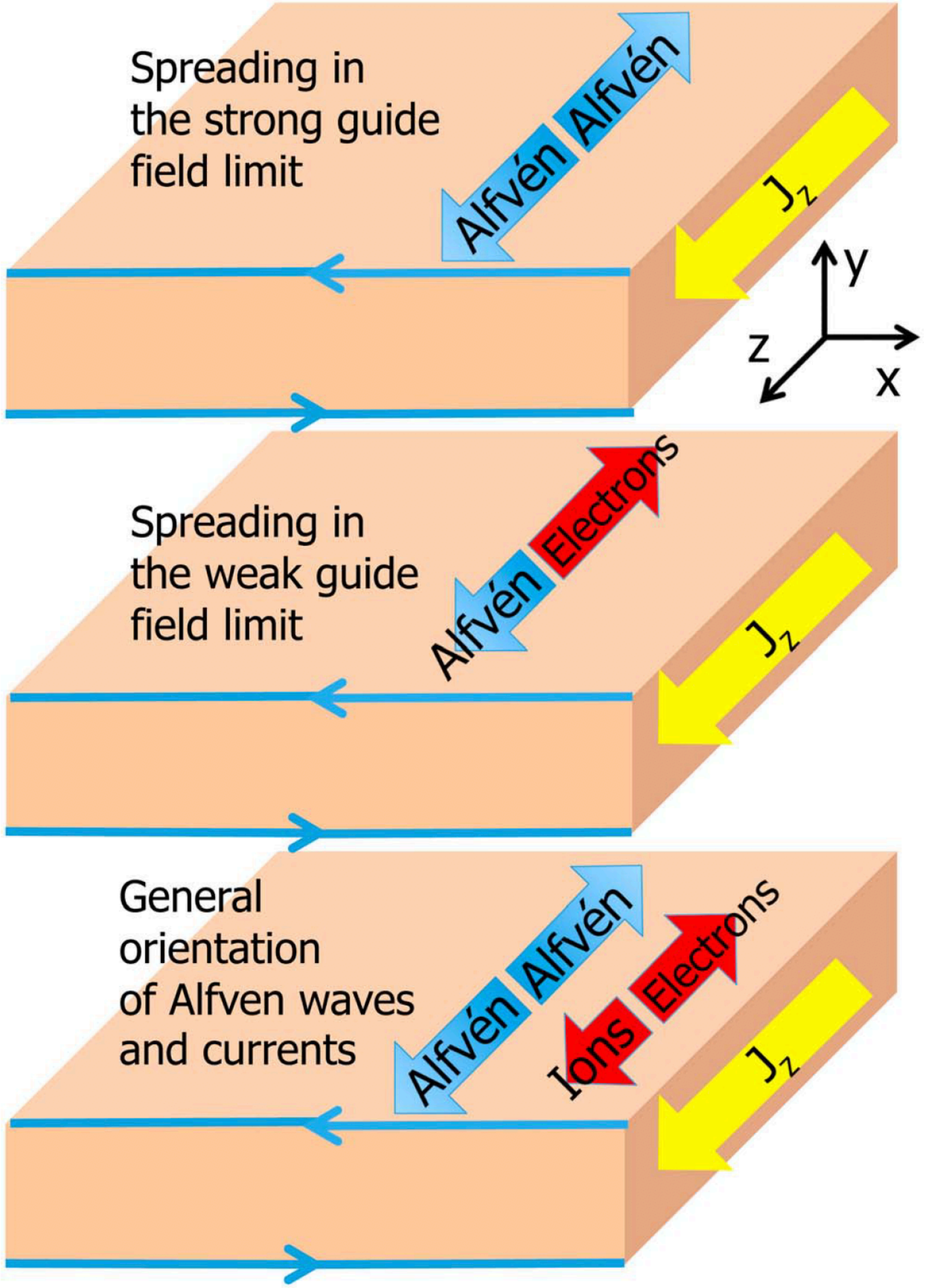
— other possibilities include maximizing outflow speed, energy density, tearing growth rate…..etc

(Swisdak & Drake 2007; Schreier+2010; Hesse+ 2013; Liu+JGR 2015,2018)

On the X-line Spread…



Plausible spread speeds



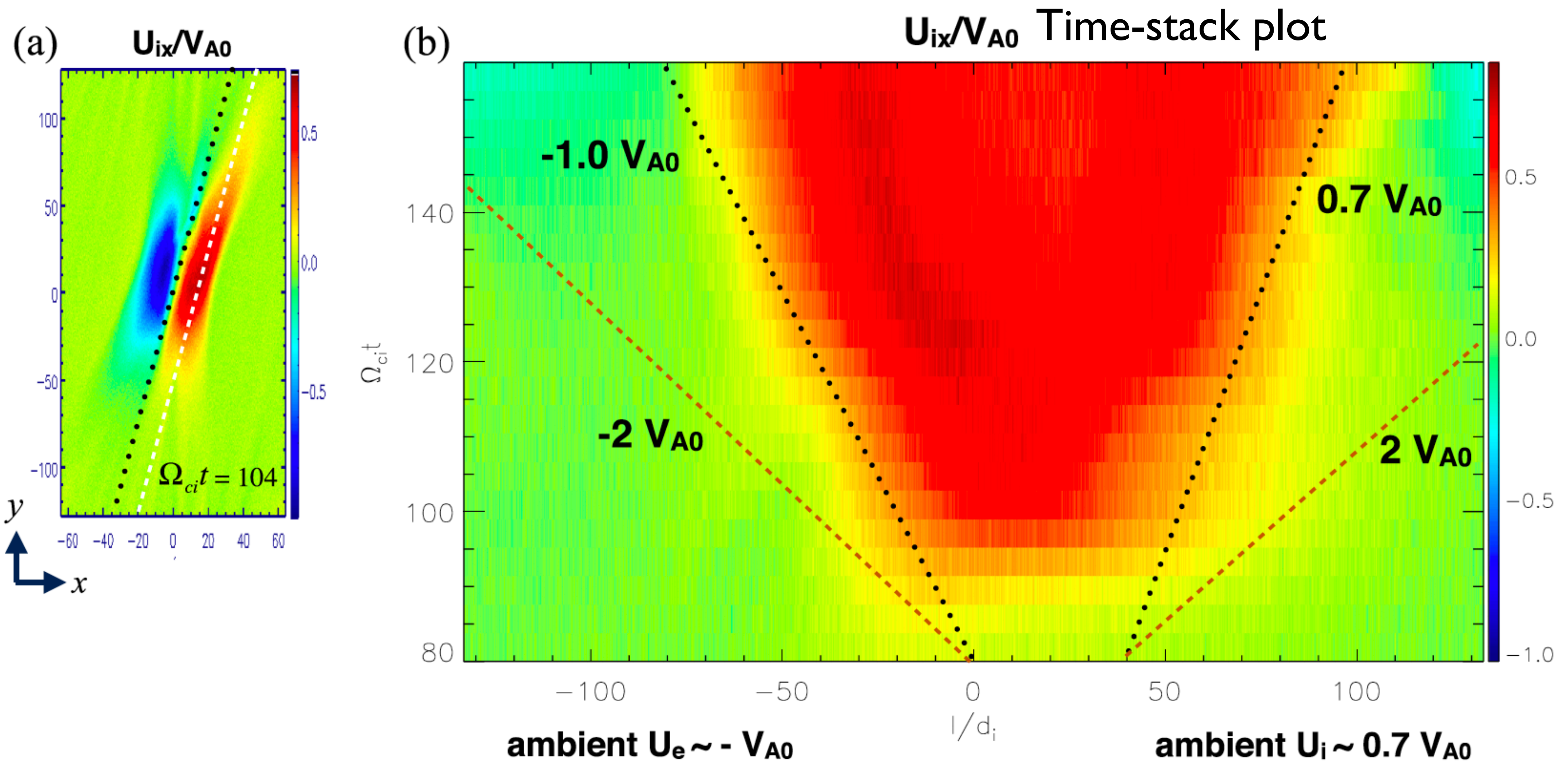
$$v_{Xe} = \max\left\{v_{eg}, c_{Ag}\right\},$$
$$v_{Xi} = \max\left\{v_{ig}, c_{Ag}\right\},$$

(Sherpherd & Cassak 2012)

- X-line spread takes the maximum of "guide-field Alfvén" speed and "current carrier" speed.

Measuring the spread speed

$$b_g = 2, \quad \delta_0 = 0.8 d_i$$



ambient $U_e \sim -V_{A0}$
$V_{Ag} = \pm 2 V_{A0}$

ambient $U_i \sim 0.7 V_{A0}$

(T. Li+ 2020)

- The spread speed is lower than the guide-field Alfvén speed (=2V_{A0} in this case)!

The current sheet thickness δ_0 matters…

The tearing growth rate

$$\gamma_{max}/\Omega_{ci} \propto \frac{1}{(\delta_0/d_i)^3 b_g}$$

strongly depends on the thickness….

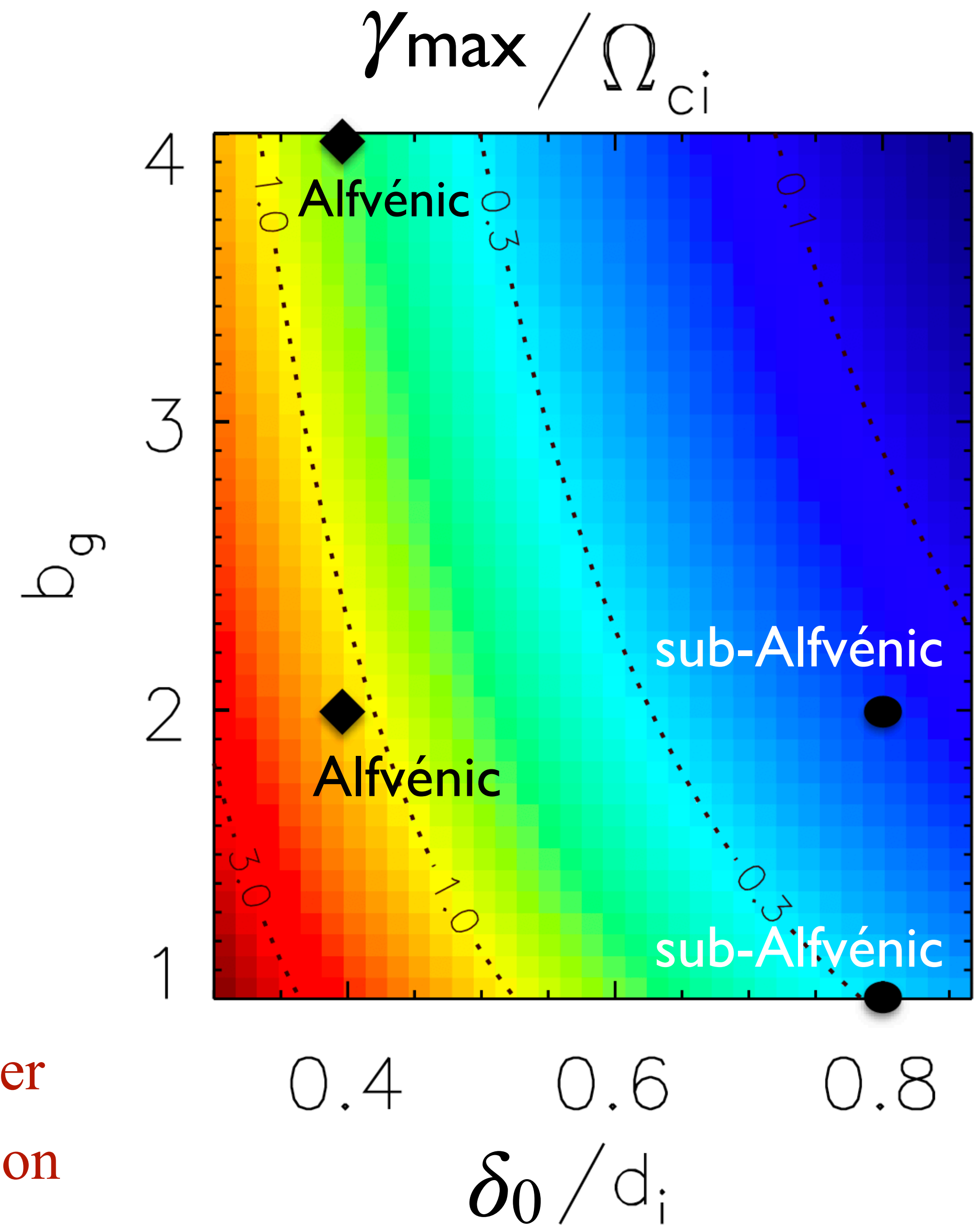
We found… Alfvénic spread occurs when

$$\boxed{\gamma_{max}/\Omega_{ci} \gtrsim \mathcal{O}(1)}$$

Hypotheses

Reconn. is easy to onset if tearing time-scale ($1/\gamma_{\rm max}$) is shorter than the plasma convection time-scale ($\tau_{\rm A}$) within the diffusion region…. (e.g. Pucci & Velli 2014)

$$\gamma_{max}\tau_A \simeq \gamma_{max}d_i/V_A \simeq \gamma_{max}/\Omega_{ci} \gtrsim \mathcal{O}(1)$$

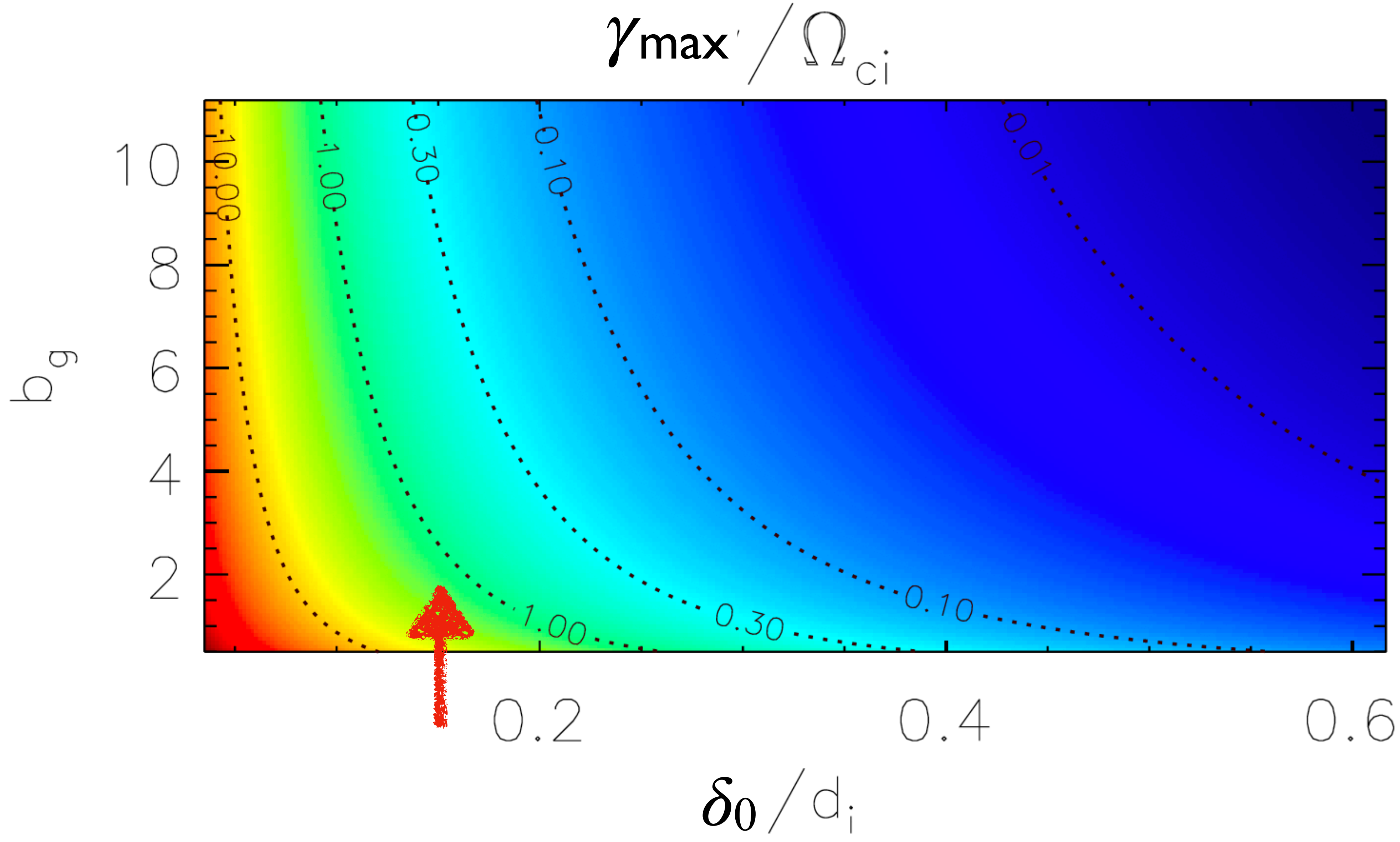


(T. Li+ 2020)

- The efficiency of continuous-reconnection-onset (at two ends of the x-line) determines whether the x-line spreads in Alfvénic speed, or sub-Alfvénic speed.

Alfvénic Spread can be difficult…

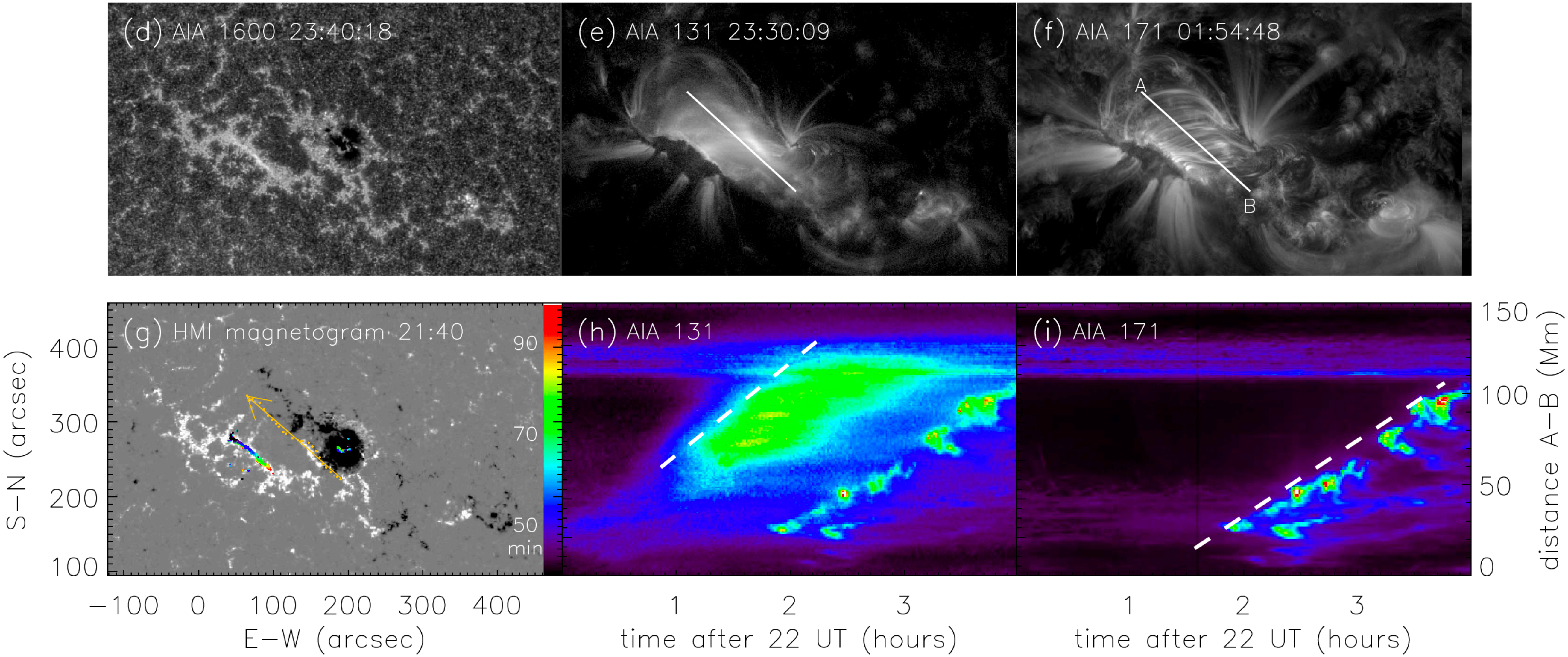
With real mass ratio $m_i/m_e = 1836$



- Because it requires a very thin current sheet $\delta_0 \lesssim 0.2d_i$ to begin with ~~

Solar Observations

2011 Sep. 13

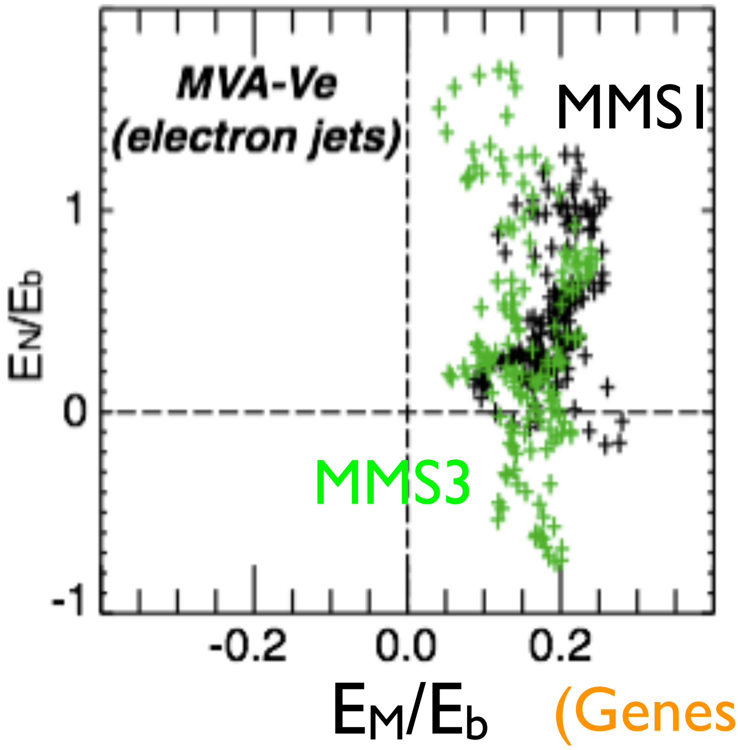


(Jiong Qiu+ APJ 2017)

- Bi-directional spread, Parallel (w.r.t. the current) spread, Anti-parallel spread are all possible in flare loop observation.
- Will be exciting to learn more details!

Magnetospheric Multiscale Mission (MMS) & Solar Observations of Reconnection Rates

MMS Observations
7/11 event

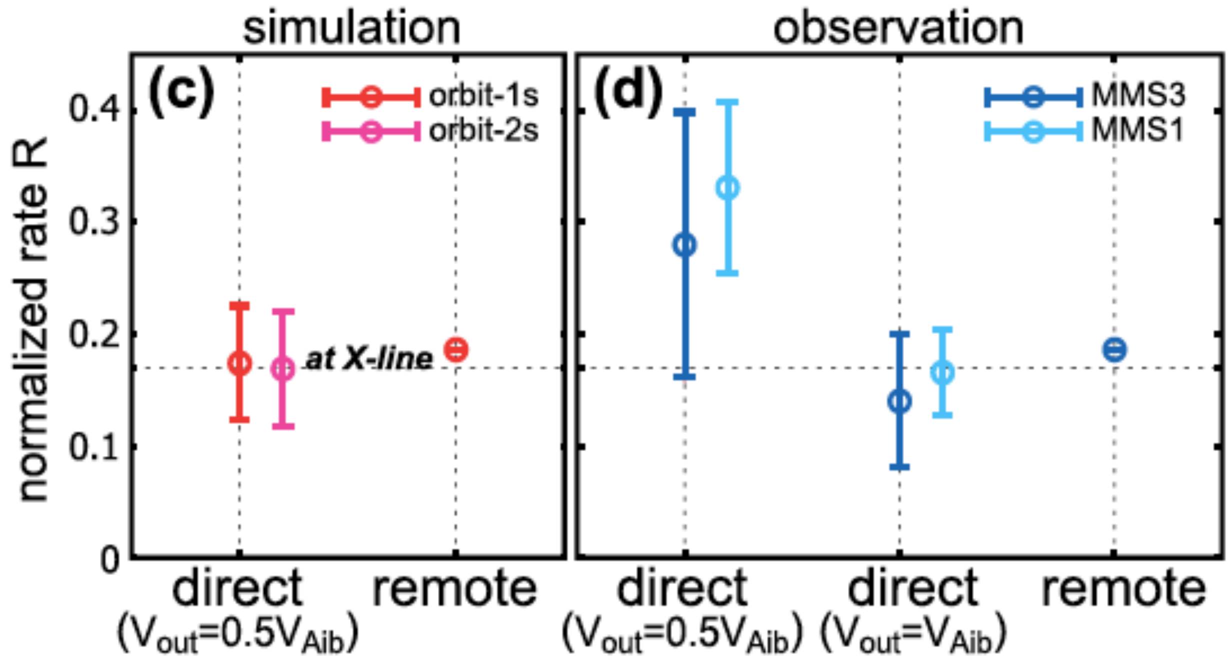


(Genestreti+ JGR 2018)

Measuring E_M

—Tried 14 different LMN coordinate systems

R~0.18 ± 0.035



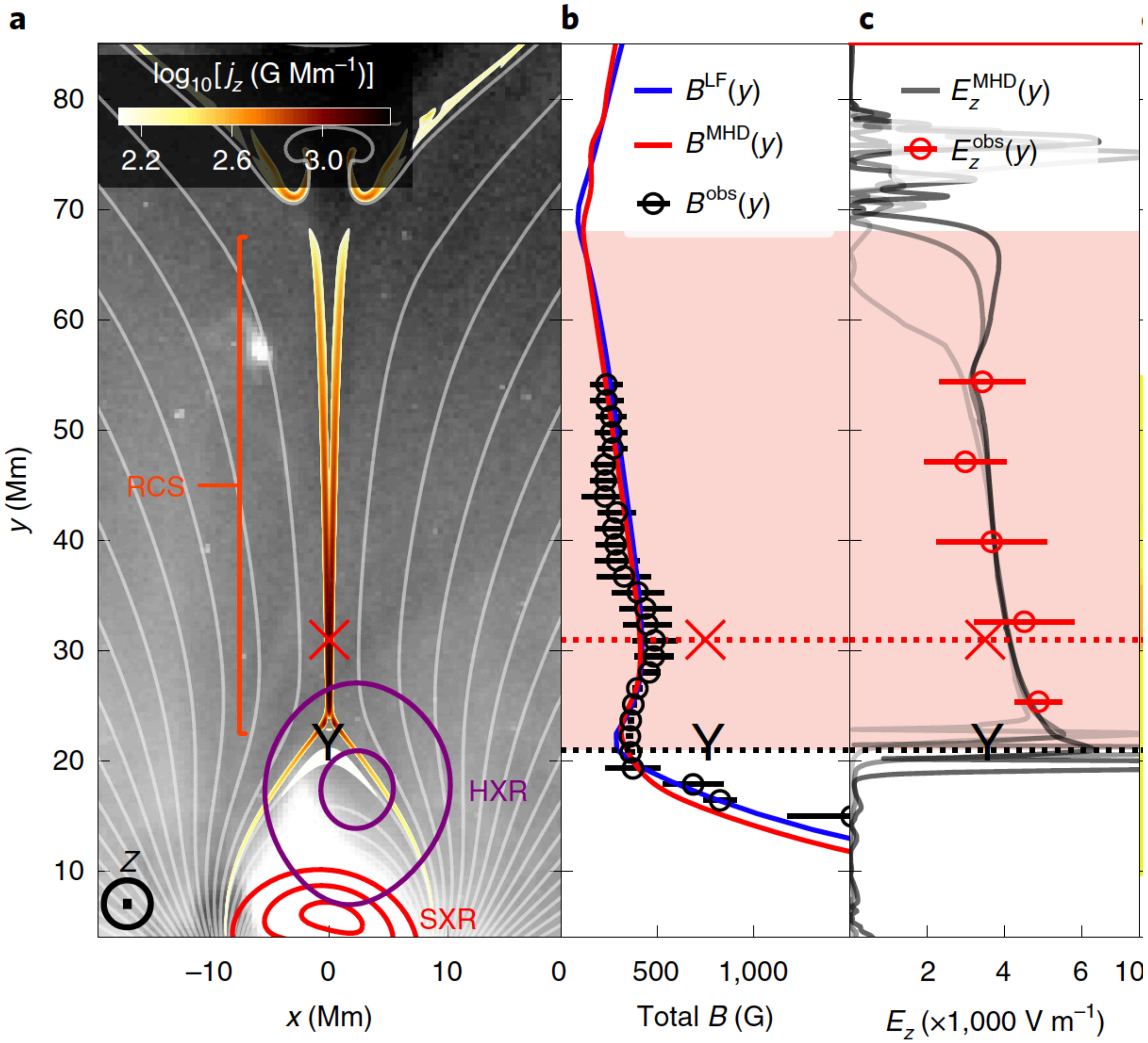
(Nakamura+ JGR 2018)

Measuring E_M

—Took advantage of the close comparison with 2D PIC simulations~

Solar Observations

2017 Sep. 10



(Bin Chen + Nature 2020)

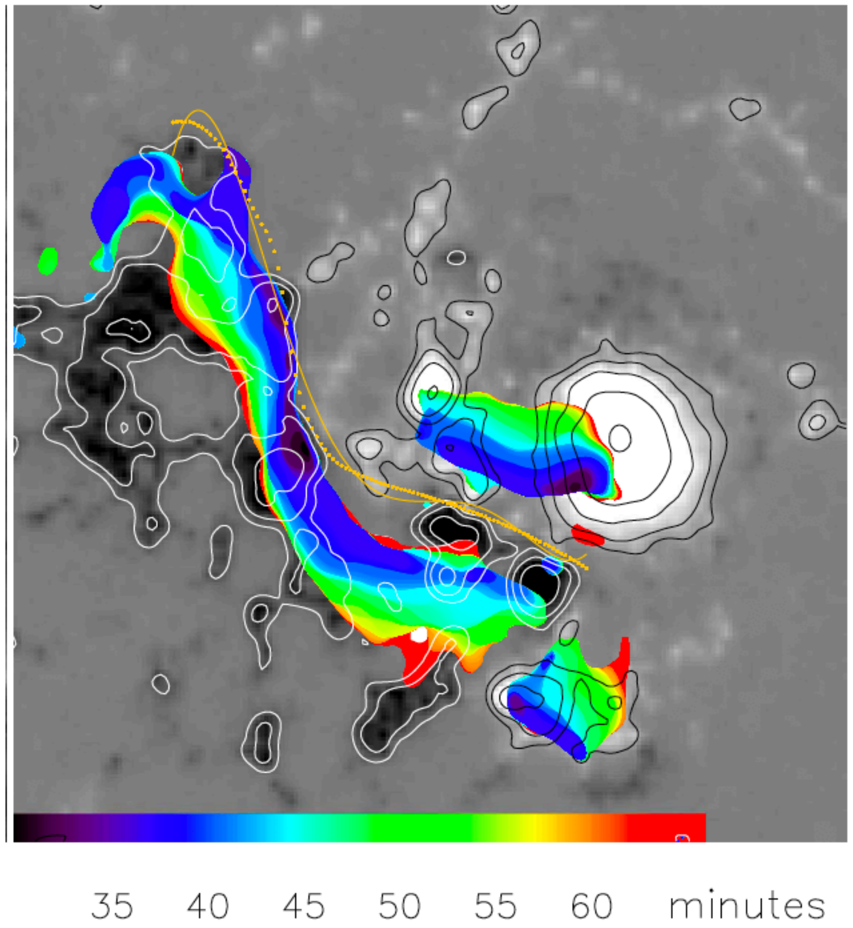
- Microwave imaging from *Expanded Owens Valley Solar Array* (*EOVSA*)
- Measuring the magnetic field strength & inflow speed

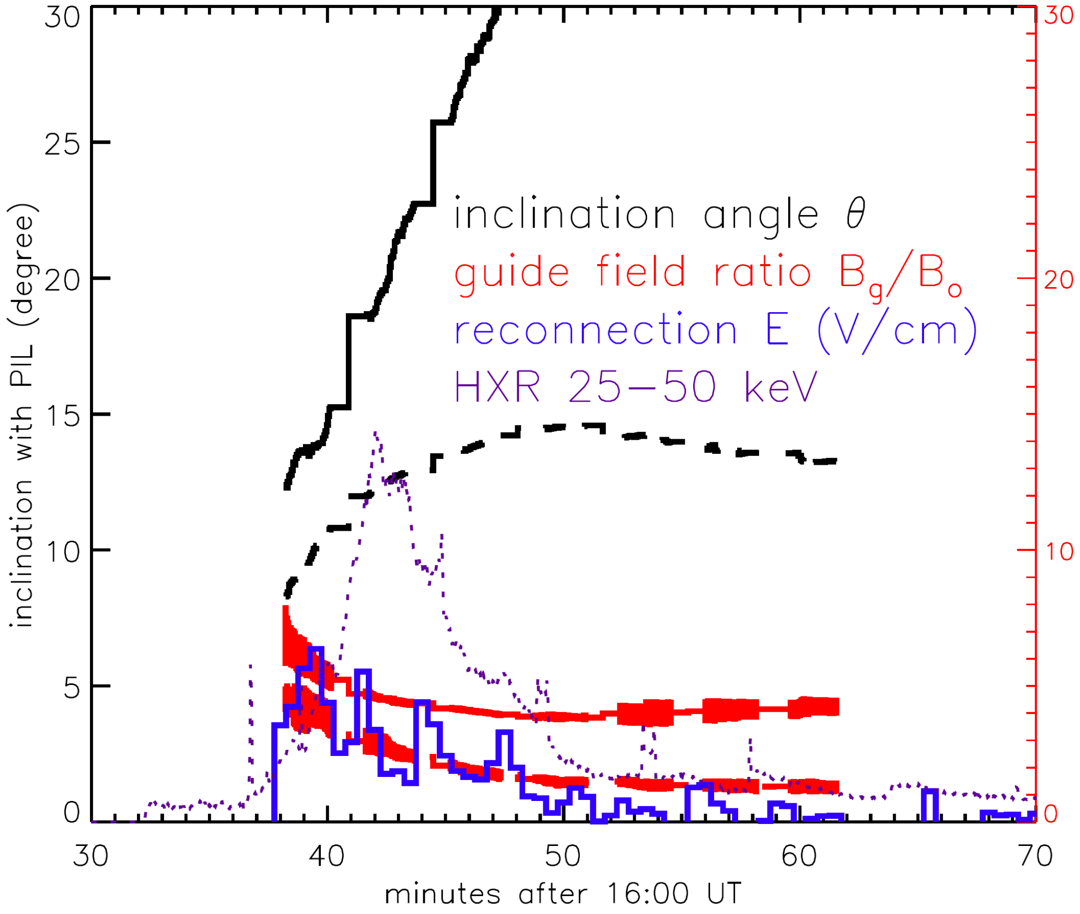
— E=VxB, super-Dreiser regime!

— the normalized rate R~ 0.01

Solar Observations

2005 May 13





(Jiong Qiu+ APJ 2017)

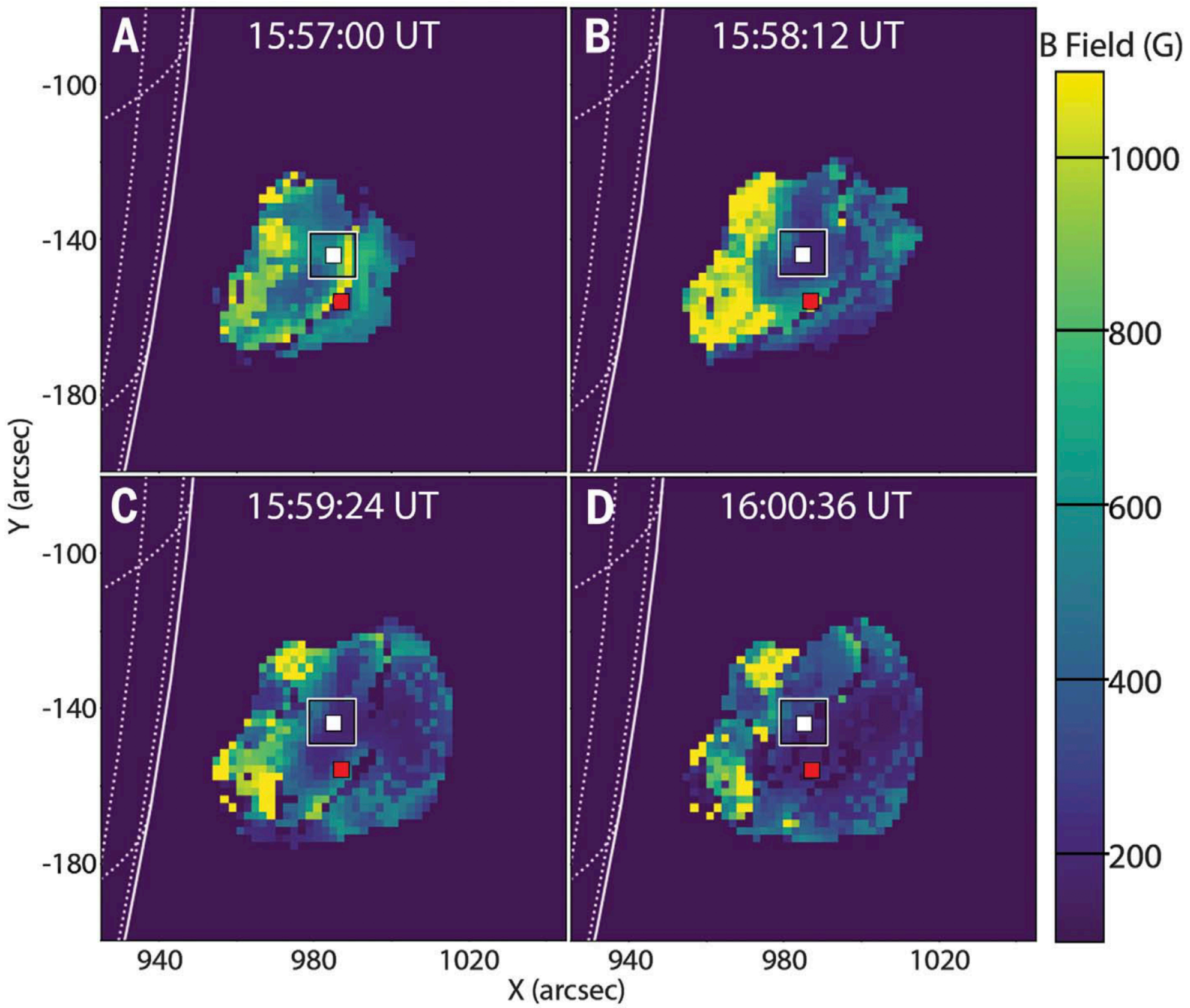
- UV-1600 Å imaging from *TRACE*.
- Measuring the perpendicular expansion of ribbon

— how fast reconnected flux accumulates~

— the normalized rate R~ 0.01-0.1??

Solar Observations

2017 Sep. 10



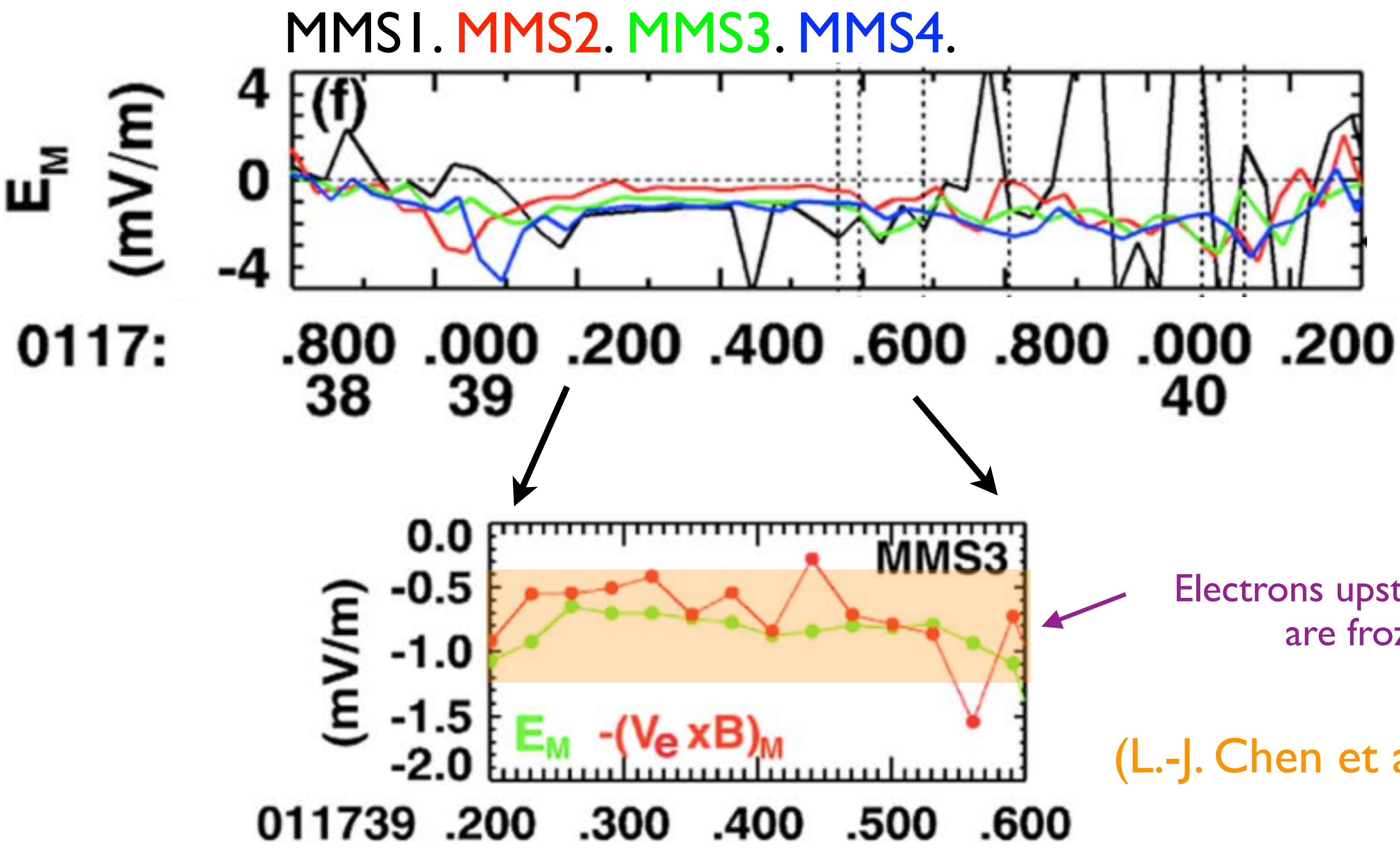
(G. Fleishman+ Science 2020)

- Microwave imaging from *Expanded Owens Valley Solar Array* (*EOVSA*)
- May be possible to infer the reconnection rate from magnetic energy decaying rate.?

— the normalized rate R~ ?

MMS Observations

December 14, 2015 event: B_g~ 0.2, B_{L2}/B_{L1}~1.3, n_2/n_1~6.8



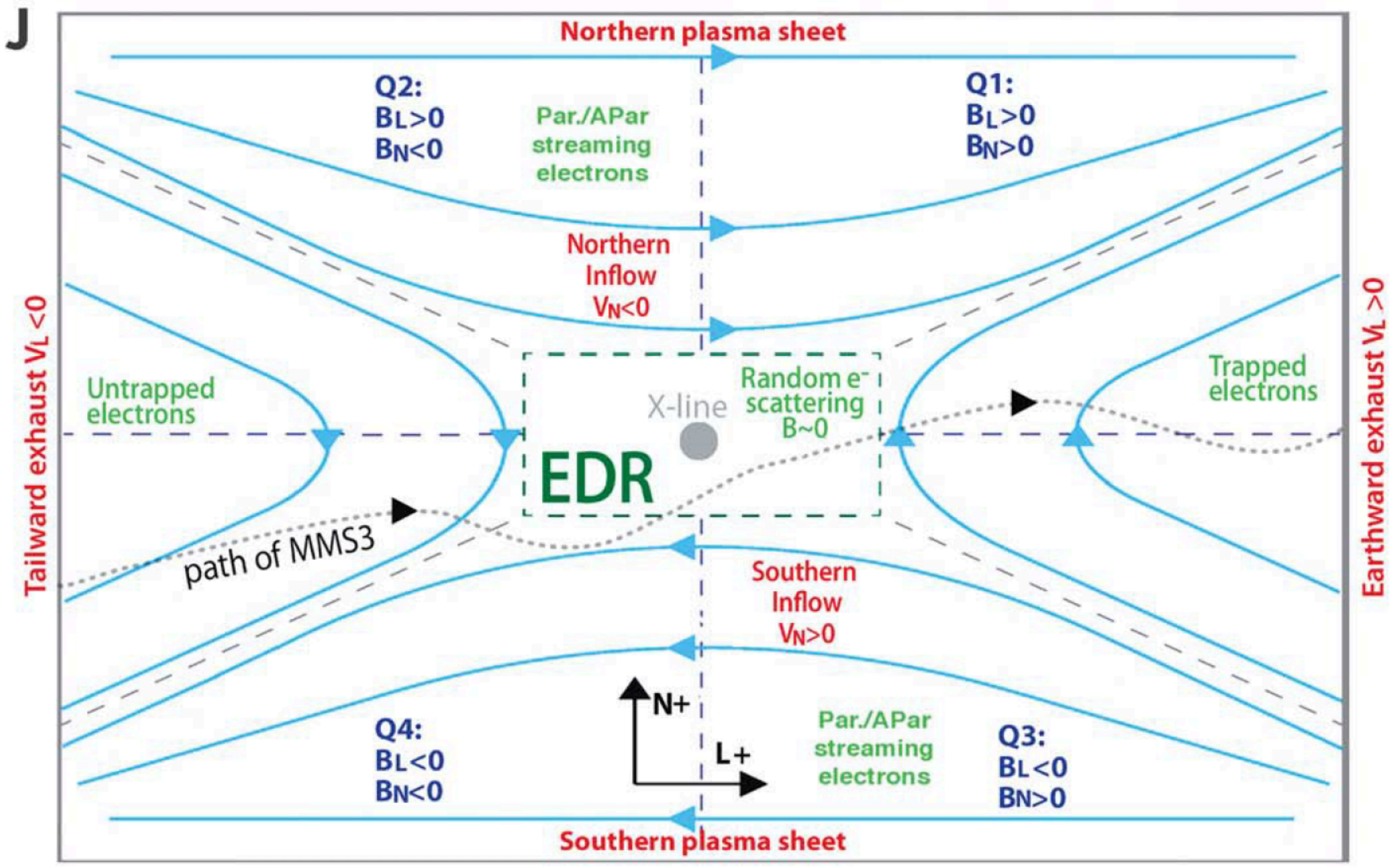
Electrons upstream of EDR
are frozen-in!

(L.-J. Chen et al., JGR 2017)

- An uniform electric field over at least 8 electron skin depths corresponds to a normalized rate ~ 0.1.
- The rate of the October 16, 2015 event was estimated to be ~ 0.3. (Burch+ Science 2016)

MMS Observations

7/11 event



[LMN] ~ [XYZ] GSM

(Torbert+ Science 2018)

- Measuring the aspect ratio of EDR~ 0.1–0.2

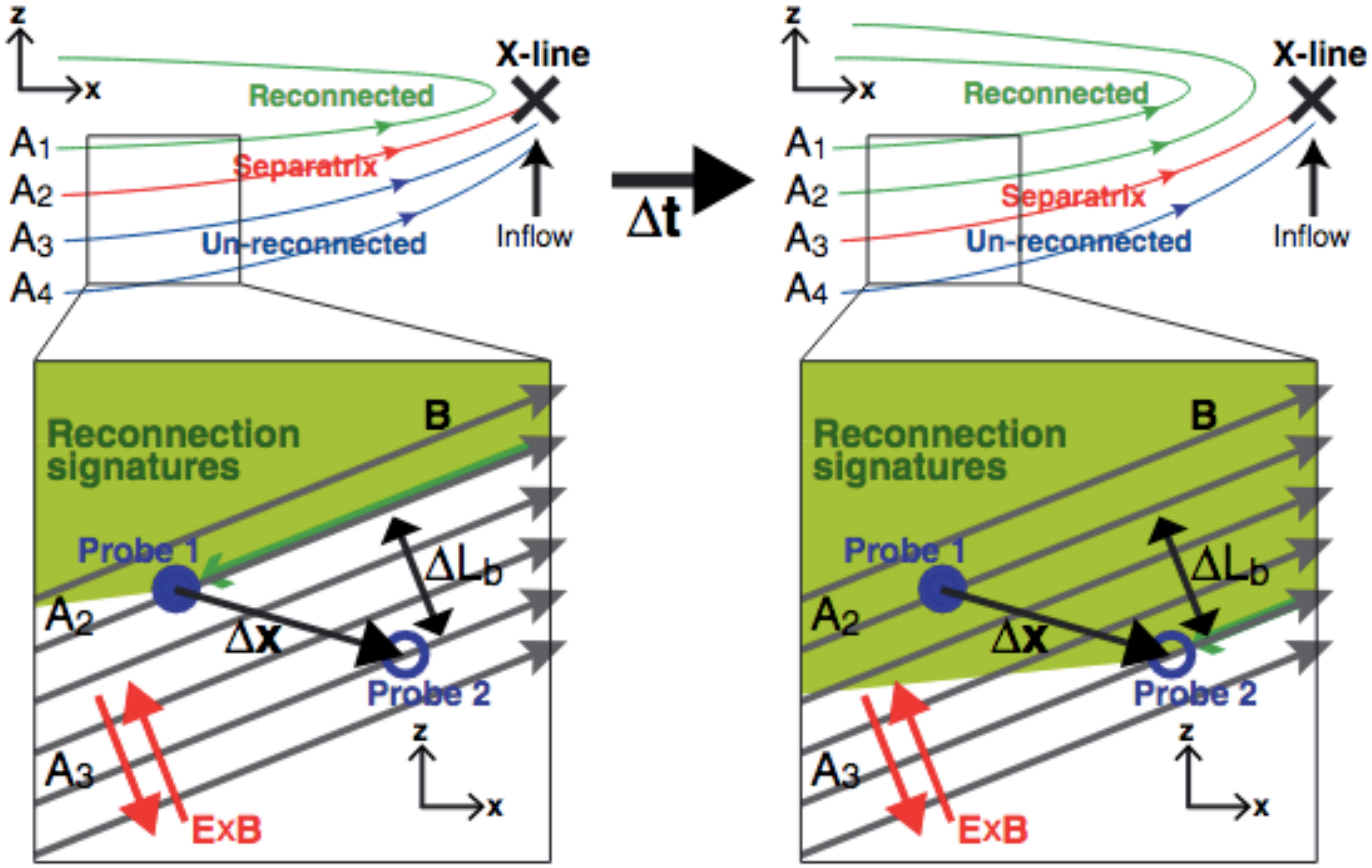
— Using timing analysis to get L.

— Using current density to get δ.

MMS Observations

— new technique in measuring the rate

t_0 $t_0 + \Delta t$



(Nakamura+ JGR 2018)

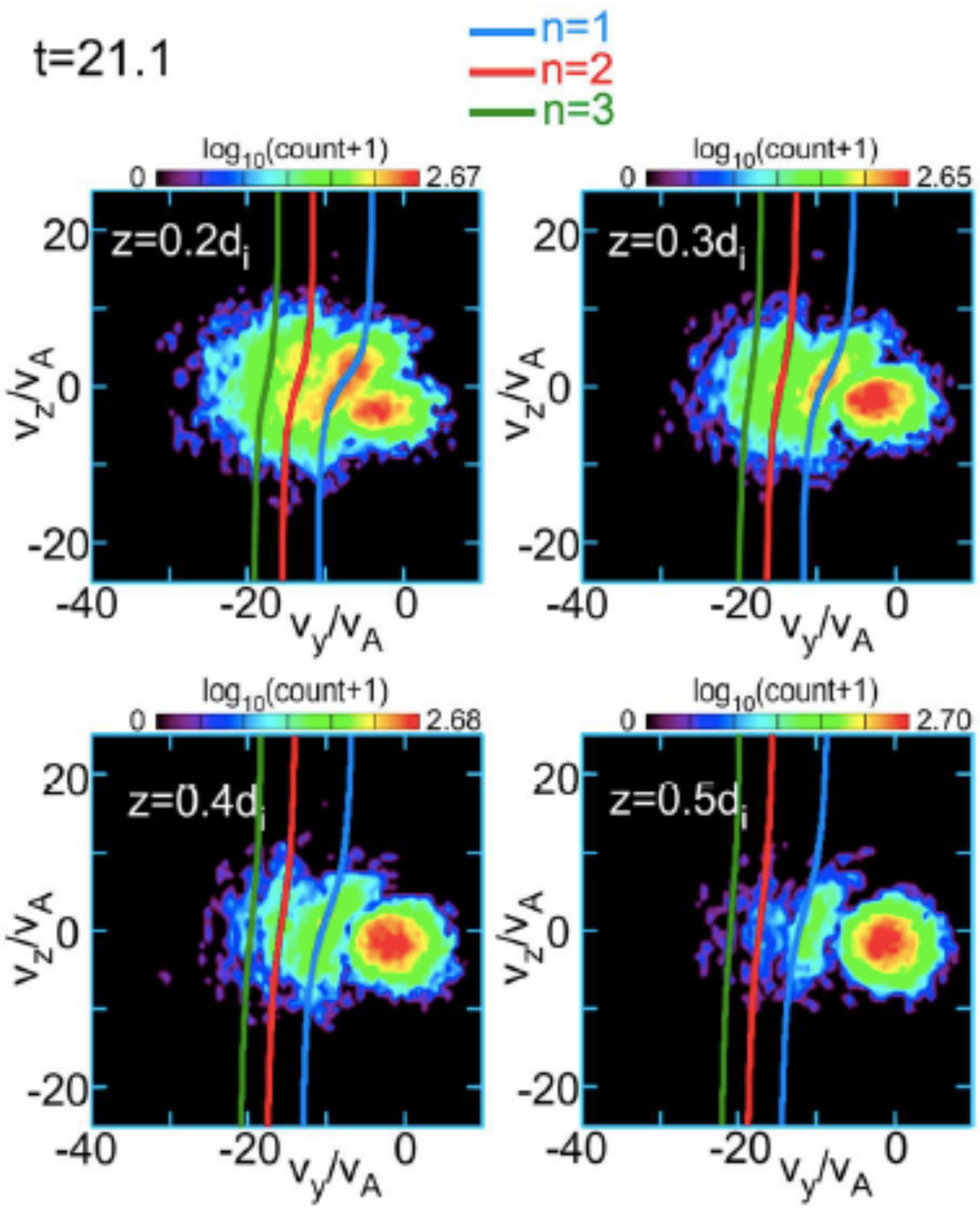
$$E_R \simeq -\frac{\Delta L_b B}{\Delta t} = -\frac{(\Delta \mathbf{X} \times \mathbf{B})_y}{\Delta t} \simeq -[(\mathbf{V}_{timing} - \mathbf{V}_{convection}) \times \mathbf{B}]_y$$

convection of the magnetic flux
respect to the probe

- Measuring the flux difference at separatrix to infer the reconnection rate remotely!

MMS Observations

— new technique in measuring the rate



(Bessho+ GRL 2018)

$b = dB_x/dz$

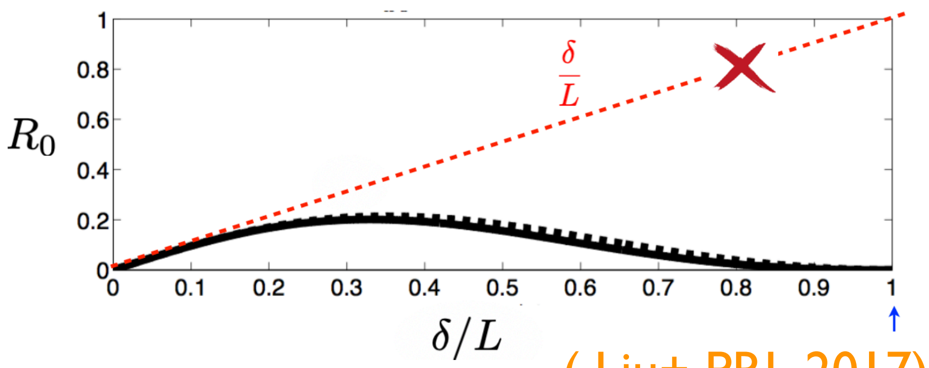
$k = dE_z/dz$

$$\Delta v_y = \left[\left(-v_{y0} - \frac{ck}{b}\right)^{3/2} + \frac{3\pi}{4}(2n+1)\left(\frac{mc}{eb}\right)^{1/2}\frac{eE_r}{m}\right]^{2/3} - \left[\left(-v_{y0} - \frac{ck}{b}\right)^{3/2} + \frac{3\pi}{4}(2n-1)\left(\frac{mc}{eb}\right)^{1/2}\frac{eE_r}{m}\right]^{2/3}.$$

- Inferring reconnection rate from particle distributions at the diffusion region.
  - — E_R accelerates electrons in the out-of-plane (-y) direction.
  - — R~ 0.22-0.28 for the 7/11 event.

Summary & future (unsolved questions)

★ 0.1 is an upper bound value.



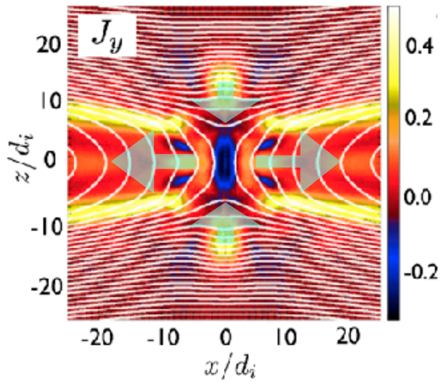
(Liu+ PRL 2017)

★ What is the primary localization mechanism?

— Why is the geometry in uniform-resistivity MHD so different?

(the only exception.?)

— While a localization mechanism is needed for fast reconnection, different systems may have different localization mechanisms.



(Liu+, PoP, 2018)

★ Turbulence!? if yes, how does it affect reconnection rate?

— 3D rate in turbulent-reconnection appears to be similar to the 2D rate.

(Daughton+, PoP 2014, Le+ PoP, 2018)

★ X-line orientation can be determined by maximizing the rate.

(Liu+ JGR 2018)

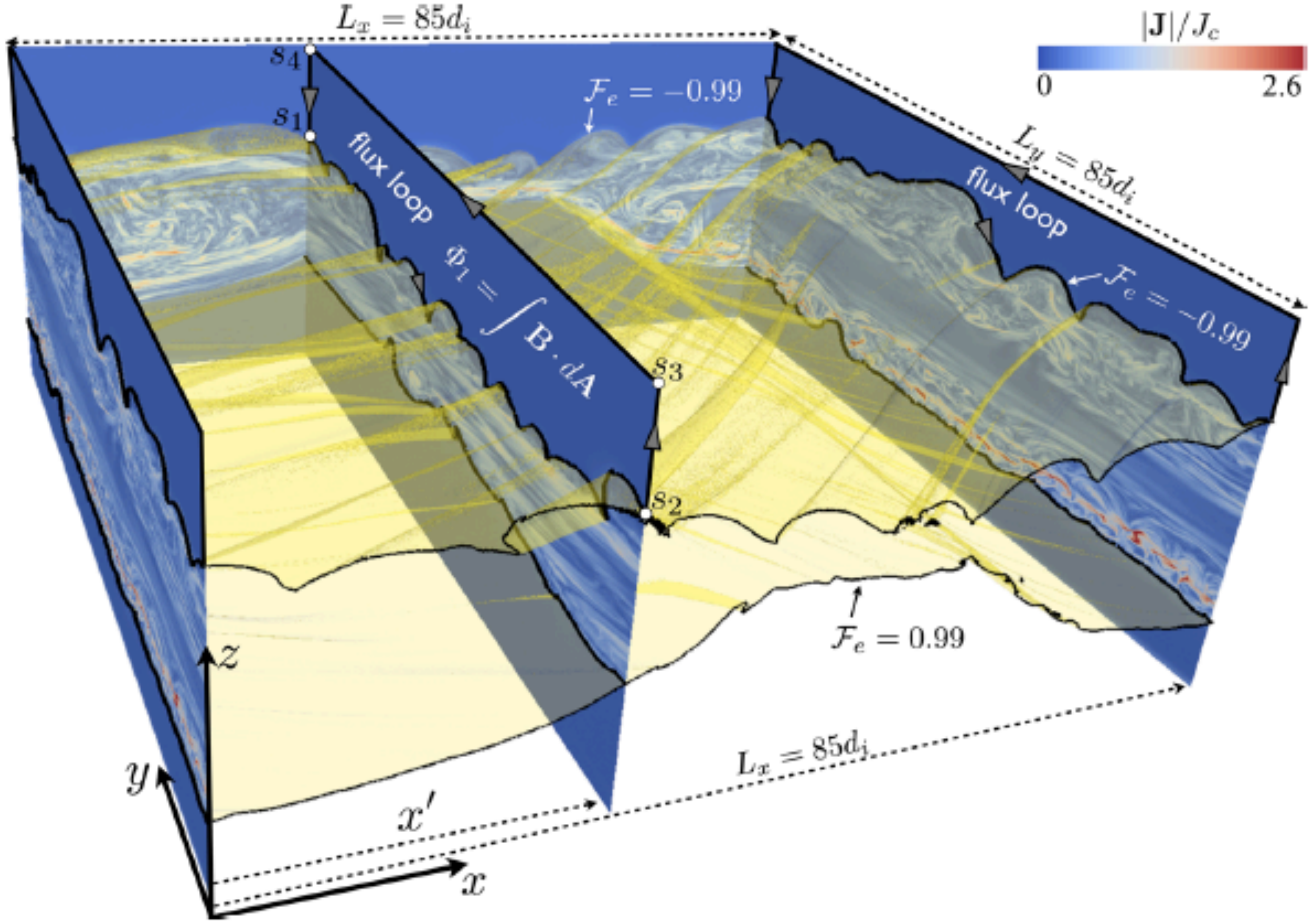
★ X-line spread can be dictated by the “Onset physics”

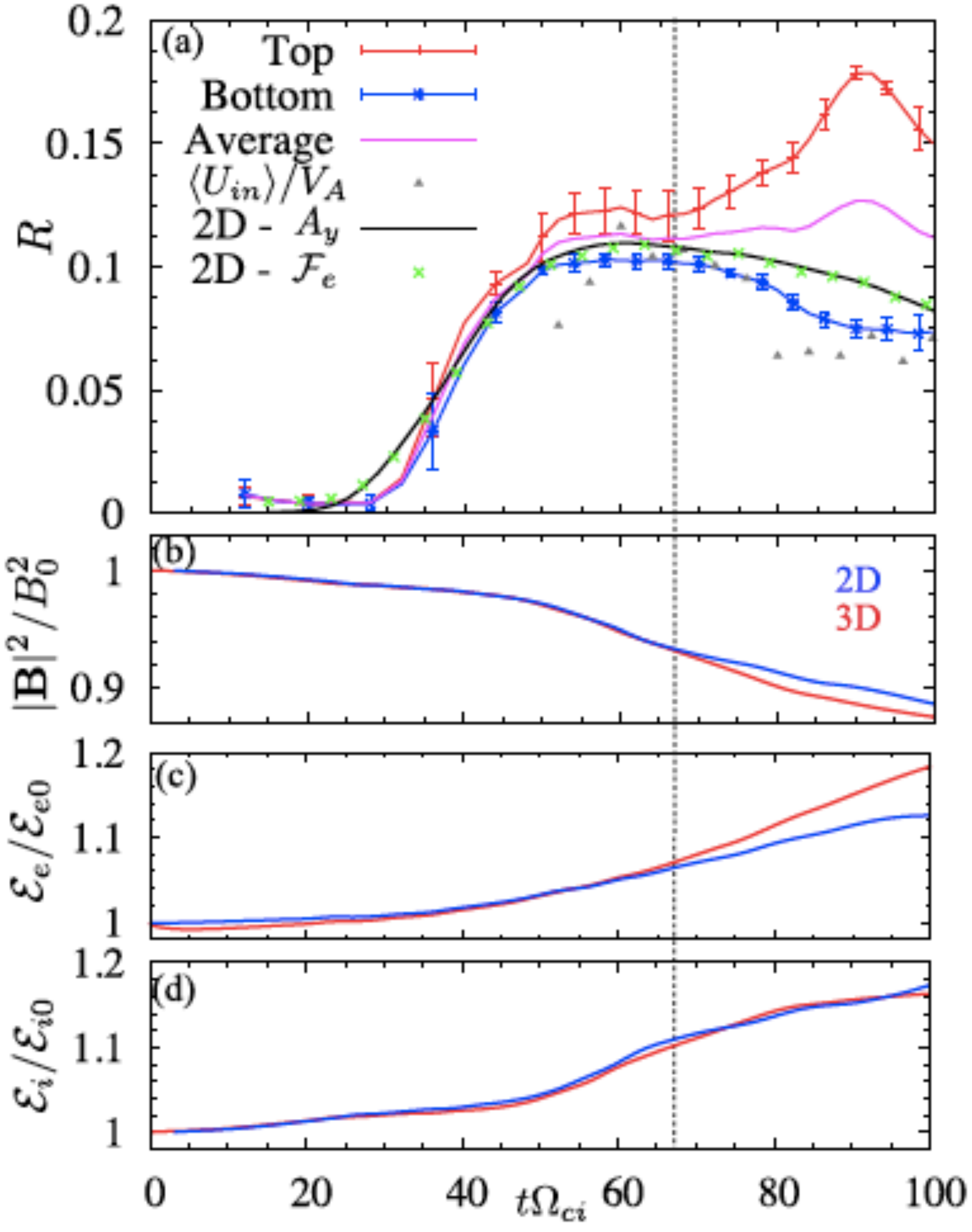
— difficult to spread in Alfvénic speed if sheet thickness $\gtrsim d_i$

(Li+ JGR 2020)

Backup slides

★ Turbulence!? if yes, how does it affect reconnection rate?





$$\mathcal{F}_e = \frac{n_e^{bot} - n_e^{top}}{n_e^{bot} + n_e^{top}}$$

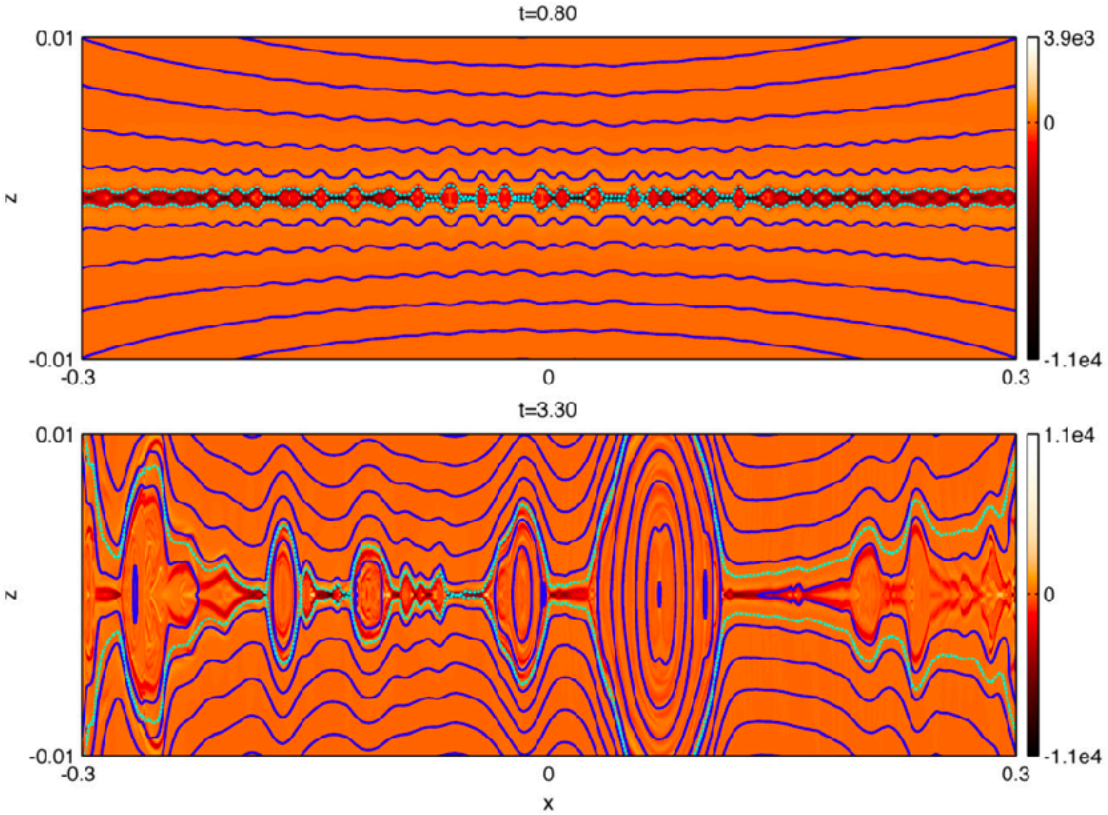
mi/me=100
85 x 85 x 35 di

(Daughton et al. 2014)

- 3D & 2D rates are similar~~~

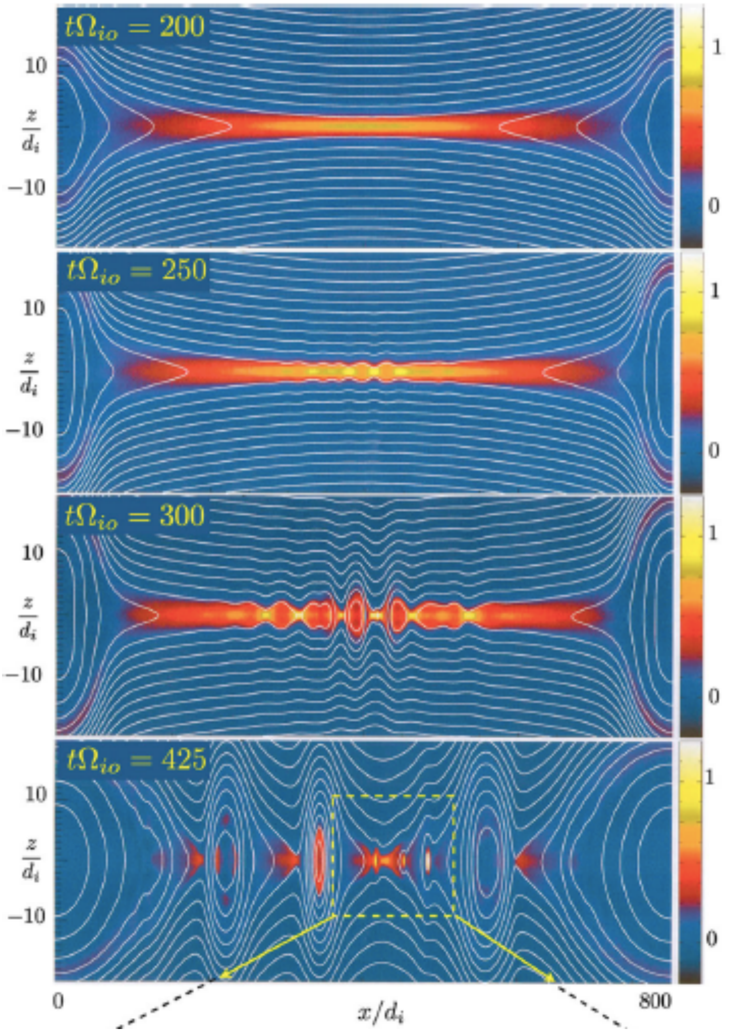
★ Plasmoids (i.e., secondary tearing modes)

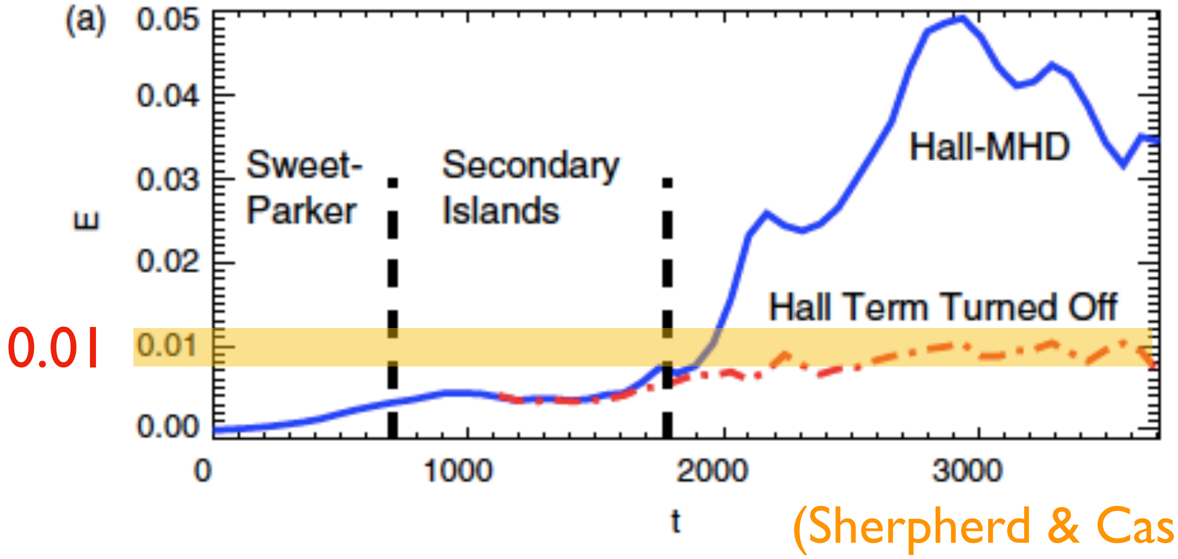
resistive-MHD when η is very small



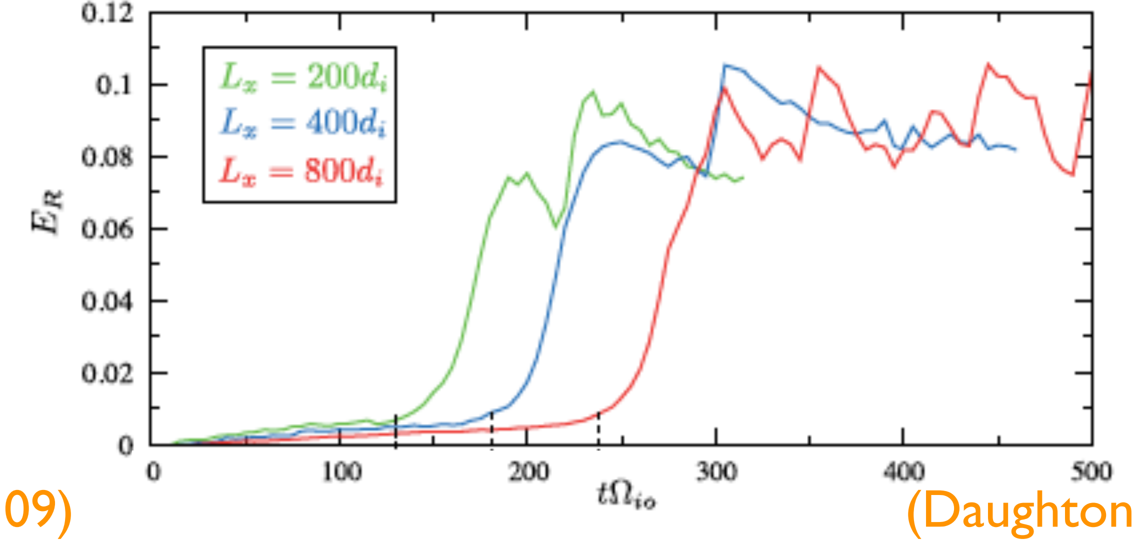
(Y. -M. Huang+ 2010, Loureiro+ 2007…)

PIC with collisions





(Sherpherd & Cassak+ 09)

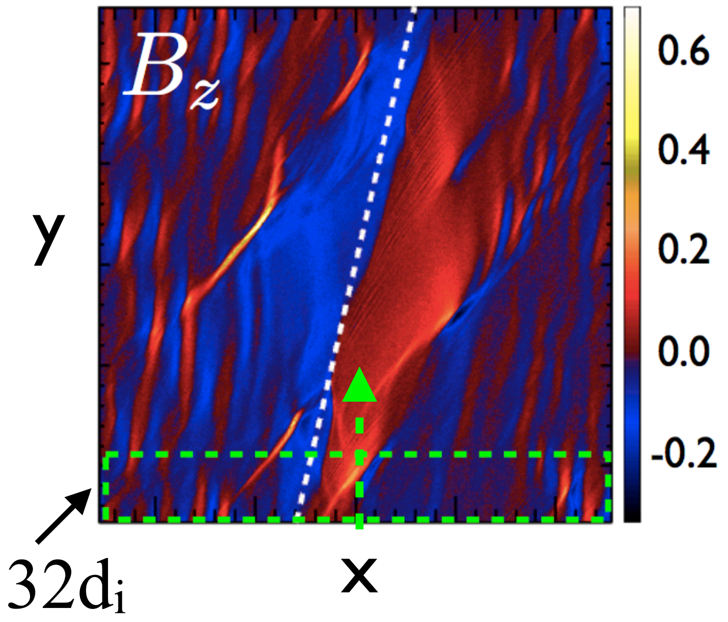


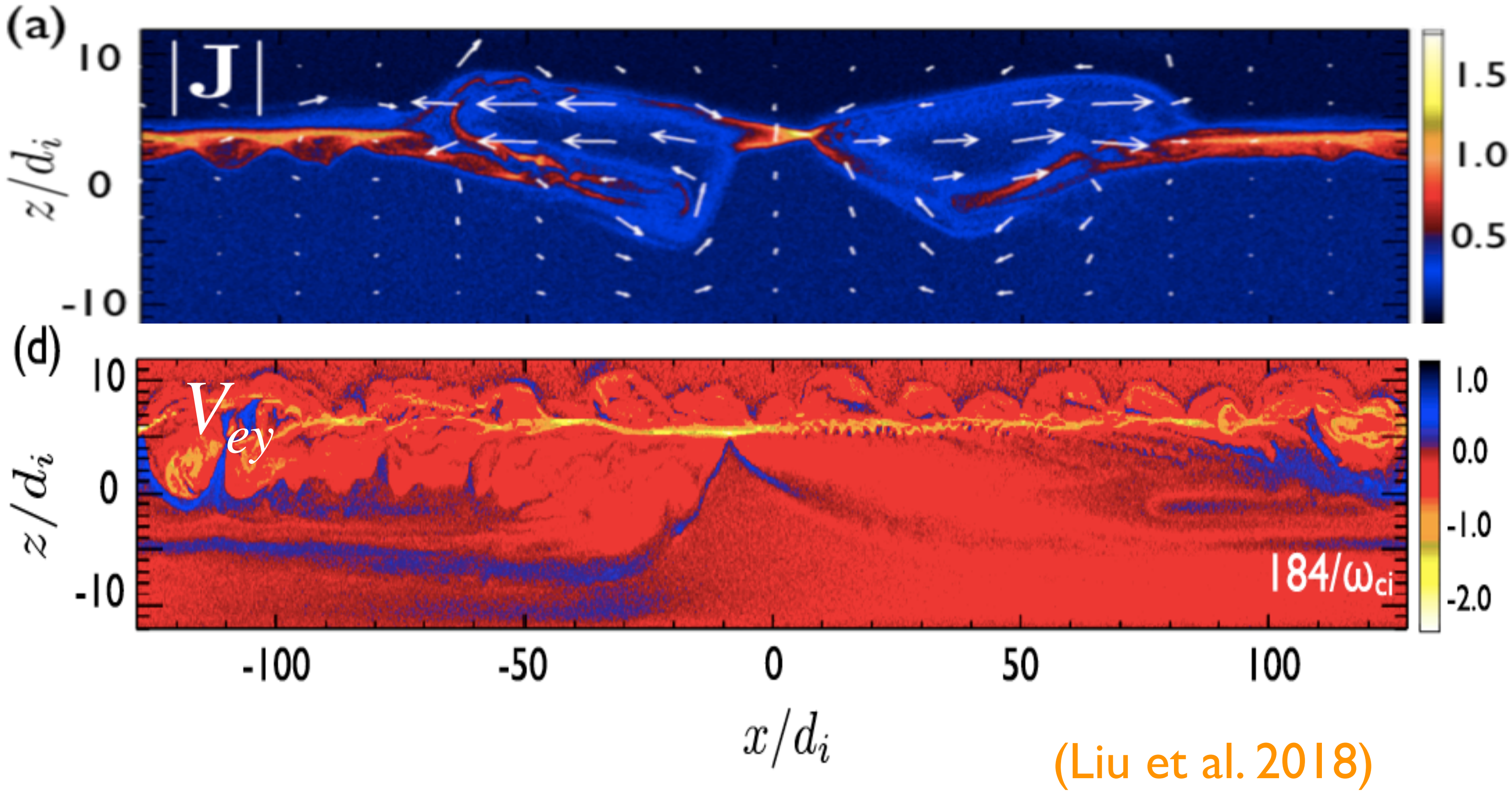
(Daughton+ 09)

- (my opinion) Tearing may provides the localization, enhancing the rate, but cannot explain the fast rate value ~ O(0.1).

After all, be careful about the periodic boundary ~~~

mi/me=25
256 x 256 x 24 di





(Liu et al. 2018)

- Be careful, because the periodic boundary inside a small box can make everything turbulent quickly…
- Reconnection could be more laminar than we previously thought…?