Studying the internal structure and dynamics of the Sun

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HELIOSEISMOLOGY

*Helios*: Classical Greek for the Sun

*Seismos*: Classical Greek for tremors

*Logos*: Classical Greek for reasoning or discourse
The Sun and the stars oscillates in normal modes.

Normal modes in an oscillating system are special solutions where all the parts of the system are oscillating with the same frequency (called "normal frequencies" or "allowed frequencies").

Stellar oscillations are standing waves.

A standing wave, also known as a stationary wave, is a wave that remains in a constant position.

This phenomenon can occur because the medium is moving in the opposite direction to the wave, or it can arise in a stationary medium as a result of interference between two waves travelling in opposite directions.
In the Sun, sound waves are generated at the top of the Convection Zone...
A Crash Course in Helioseismology

- The Sun oscillates in millions of different modes.
- The oscillations are linear and adiabatic.
- All observed modes are acoustic i.e., \( p \)-modes or surface modes i.e. \( f \)-modes.
- Each mode is characterized by three numbers:

  1. \( n \): the radial order, the number of nodes in the radial direction
  2. \( l \): the degree, the number of nodal circles on the circumference
  3. \( m \): the azimuthal order. \( m \) goes from \(+l\) to \(-l\)

\(|m|\) = no. of node circles crossing a latitude 
\(l-|m|\) = no. of node circles crossing a longitude.

\( l=2, m=1 \) \( l=3, m=0 \) \( l=3, m=-3 \)
• If the Sun were spherically symmetric and did not rotate, all modes with the same \( l \) and \( n \) but different \( m \) would have the same frequency.

• Rotation lifts this degeneracy, giving rise to “rotational splittings” of the modes:

\[
D_{nlm} = \frac{\nu_{nlm} - \nu_{nl-m}}{2m} = \int_0^1 \int_0^1 dr \, d \cos \theta \, K_{nlm}(r, \theta) \Omega(r, \theta)
\]

\[
\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{\text{max}}} a_j(n, l) P_j^{(l)}(m).
\]

• The central frequency is used to determine the spherically symmetric structure of the Sun.

• Odd order splitting coefficients (corresponding to the symmetric part of the splitting coefficients) are used to determine the rotation rate inside the Sun.

• Even-order splitting coefficients (corresponding to the antisymmetric part of the splitting coefficients) are used to determine asphericity.
Different modes penetrate to different depths
Describing the modes

Model of present Sun

\[
\frac{d^2 \xi_r}{dr^2} \approx -\frac{\omega^2}{c^2} \left( \frac{S_l^2}{\omega^2 - 1} \right) \left( \frac{N^2}{\omega^2 - 1} \right) \xi_r
\]

Eigenfunction oscillates as function of \( r \) when

\[\omega^2 > S_l^2, N^2\]  \hspace{1cm} \text{p modes}

\[\omega^2 < S_l^2, N^2\]  \hspace{1cm} \text{g modes}

\[S_l^2 = \frac{l(l + 1)c^2}{r^2}\]

\[N^2 = g \left( \frac{1}{\Gamma_1} \frac{d \ln p}{dr} - \frac{d \ln \rho}{dr} \right) \approx \frac{g^2 \rho}{p} (\nabla_{ad} - \nabla + \nabla_{\mu})\]
What do solar observations look like?
A Solar Power Spectrum

MDI Medium-l Power Spectrum

frequency, mHz

angular degree, ℓ
A sample of helioseismic data

MDI data, 360-day time series, 1000 σ errorbars
What have we learned?

- Solar structure
- Solar dynamics
  - Physics!

and

- Solar-cycle related changes
Comparing frequencies does not work
Changes in surface physics changes frequencies

Graph (a) shows the relationship between $Q_m^\delta \nu$ (in $\mu$Hz) and $\nu$ (in $\mu$Hz), decreasing as $\nu$ increases from 2000 to 4000 $\mu$Hz. Graph (b) plots $\delta c / c$ against $r / R_\odot$, where $\delta c / c$ remains close to 0 until $r / R_\odot$ reaches 0.8, after which it quickly decreases to 0.
Complication: the Surface Term
We therefore resort to inversions
The inverse problem for structure

Relationship between structure and frequencies:

\[-\omega^2 \rho \ddot{\xi} = \nabla \left( c^2 \rho \nabla \cdot \dot{\xi} + \nabla p \cdot \dot{\xi} \right) - g \nabla \cdot (\rho \dot{\xi}) - G \rho \nabla \left( \int_{\nu} \frac{\nabla \cdot (\rho \dot{\xi}) d^3 \vec{r}}{|\vec{r} - \vec{\nu}|} \right)\]

A Hermitian Eigenvalue problem, therefore use the variational principle:

\[\frac{\delta \omega_i}{\omega_i} = \int K_{c^2, \rho}(r) \frac{\delta c^2}{c^2} dr + \int K_{\rho, c^2}(r) \frac{\delta \rho}{\rho} dr + \frac{F_{\text{surf}}(\omega_i)}{I_i}\]
We can actually invert for structure differences
Inversion results showing differences between models and the Sun

Models from Bahcall, Basu & Serenelli (2005)
Inversions from Basu et al. (2009)
We can determine which physical processes are important

Models from Basu, Pinsonneault & Bahcall 2000
We can determine the helium abundance of the solar convection zone

\[ W(r) = \frac{r^2}{Gm} \frac{dc^2}{dr} \]

\[ c^2 = \Gamma_1 \frac{p}{\rho} \]
More physics: The equation of state

From Elliott & Kosovichev 1998
Can we eliminate the composition term?

\[ \frac{\delta \Gamma_1}{\Gamma_1} = \left( \frac{\partial \ln \Gamma_1}{\partial Y} \right)_{P,\rho} \delta Y + \left( \frac{\partial \ln \Gamma_1}{\partial \ln P} \right)_{\rho,Y} \frac{\delta P}{P} \]

\[ + \left( \frac{\partial \ln \Gamma_1}{\partial \ln \rho} \right)_{P,Y} \frac{\delta \rho}{\rho} \left( \frac{\delta \Gamma_1}{\Gamma_1} \right)_{\text{int}} \]

Results from Basu & Christensen-Dalsgaard 1997
The current headache: What is the metallicity of the Sun?

<table>
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<tr>
<th>Element</th>
<th>GS98</th>
<th>AGS05</th>
<th>Caffau et al. (2010)</th>
<th>Asplund et al. (2009)</th>
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The sound speed and density profiles differ.
What about Solar dynamics?
The surprise of rotation

Differential Rotation

Glatzmaier 1985

Antia & Basu 2018

26 days
29 days
33 days
38 days
A bit more detail

Tachocline

Near-surface shear layer

\[ \frac{\Omega}{2\pi} \text{ (nHz)} \]

\[ \frac{r}{R} \]

Howe 2015
The tachocline is extremely thin
Meridional flows
Where do the meridional flows return?

Jackiewicz et al. 2015

Chen & Zhao 2017
Solar activity related changes
Solar oscillation frequencies change with solar cycle
Solar sound-speed changes as a function of latitude

Baldner & Basu 2008
There are cycle-to-cycle differences

Howe et al. 2017
Changes in dynamics are much more interesting!
Changes in zonal flows
Antia & Basu 2008
Meridional flows change too

Basu & Antia
2010
And the swirls near the poles

From Bogart, Baldner & Basu 2015, 2018
What next?

There are a number of unanswered questions when it comes to helioseismic analysis of the Sun:

– Where do the meridional flows close?
– What happens at the poles
– What came first, the zonal flows of the sunspot butterfly diagram?