Cosmological Distributions and Evolution of Gamma-ray Bursts and their relations to Star Formation Rate and Gravitational Waves

Vahe' Petrosian
Stanford University
with
Maria Dainotti
and Ellie Kitanidis, Brad Efron
OUTLINE

I. General Remarks
   Correlations and standard candles

II. The Luminosity Function and its evolution

III. Procedures:
   Forward Fitting vs Non-parametric methods

IV. Results of Applications
   A. Long GRBs
   B. Short AGNs
I. General Remarks

Cosmology with discrete sources
Cosmology with Standard “Candles?”

Method For Measuring Cosmological Distance

\[ d_m(z) = \frac{(c/H_0)}{\int_0^z \frac{dz'}{\sqrt{\Omega(z')}}} \]

1. Standard Candle: **Constant Luminosity**
   \[ d_m(z)(1+z) = \left[ \frac{L}{(4\pi f)} \right]^{1/2} \]

2. Standard Yardstick: **Constant Diameter**
   \[ d_m(z)/(1+z) = D/\theta \]

3. OR: Find a **tight** relation between a **distance dependent** and a **distance independent** parameter

Well known examples:

A. Cepheids: **Luminosity-Period relation**
B. Type Ia Supernovae: **Peak luminosity-Light profile width**
GRB Correlations as SC?

Examples of Correlations After Few Redshifts

1. Variability-Luminosity \textbf{(Reichart et al. 2001)}
2. Lag-Luminosity \textbf{(Norris, Maeani & Bonnell 2000)}
3. $E_{\text{peak}} - \varepsilon_{\text{iso}}$ or $E_{\text{peak}} - \varepsilon_{\gamma} \textbf{(Amati; Ghirlanda et al.)}$
4. And Several Variations on These \textbf{(see Schaeffer et al.)}
SOME RELEVANT EQUATIONS

1. “Luminosity Function” and Correlation
\[ \psi(\mathcal{E}_{\text{iso}}, E_p) = \phi(\mathcal{E}_{\text{iso}}[E_p]) \xi(E_p) \]
\[ \phi(\mathcal{E}_{\text{iso}}) \propto \delta(\mathcal{E}_{\text{iso}} - \mathcal{E}_0 f(E_p/E_0)), \quad \text{e.g., } f(x) = x^n \]

2. COSMOLOGY
\[ \mathcal{E}_{\text{iso}} = 4\pi d_m^2(1 + z)F_{\text{tot}}, \quad \text{Define } F_0 = 4\pi(c/H_0)^2F_{\text{tot}} \]
\[ d_m = (c/H_0) \int_0^z dz/\sqrt{\Omega(z)}, \quad \text{with } \Omega = \rho/\rho_0 \]
\[ \int_0^z dz'/\sqrt{\Omega(z')} = \left( \frac{f[E_P^{\text{obs}}(1 + z)/E_0]}{(1 + z)F_{\text{tot}}/F_0} \right)^{1/2} \]

POSSIBLE EVOLUTIONS

\[ \mathcal{E}_{\text{iso}} = \mathcal{E}_0 \times g(z)f \left( z, \frac{E_P}{E_0 \times h(z)} \right) \]

SHAPE

SPECTRAL AND ENERGY

Figure 1: Schematic shape (left), spectral (right, red) and energy (right, black) Evolutions.

\[ \left( \int_0^z \frac{dz'}{\sqrt{\Omega(z')}} \right)^2 = f \left( \frac{E_P^{\text{obs}}(1 + z)}{E_0 h(z)} \right) \frac{F_0 g(z)}{(1 + z)F_{\text{tot}}} \]
Problems With These Correlations

in particular with \( E_{\text{peak}} - \varepsilon_{\text{iso}} \) or \( E_{\text{peak}} - \varepsilon_{\gamma} \)

Band and Preece

Nakar and Piran

Pseudo-Redshifts (Ghirlanda et al)

Li et al.

Konus-WIND at 25
Cosmology with Discrete Sources

Determination of Global Cosmological Parameters

1. Type Ia Supernovae: *Standard Candle and well understood*
   
   *BUT Low z*

2. Galaxies and Quasars (AGNs): *High z but broad distributions*

   *Galaxies least understood astrophysical sources*

3. Gamma-Ray Bursts: *High z and not well understood*

   Question: *SN-like or Galaxy-like?*
Cosmology with Discrete Sources

First Step

_Determination of the Luminosity Function_

Without loss of generality we can write

$$\Psi(L, z) = \rho(z)\psi\left(\frac{L}{g(z)}\right)/g(z)$$

- $\rho(z)$ is the (co-moving) Density Evolution
- $g(z)$ is the Luminosity Evolution
II. The Luminosity Function and its Evolution
The required data: Bivariate $L-z$ distribution
Bi-variate Luminosity-redshift Distribution

Redshift vs. Luminosity for 200 Swift Bursts

Missing sources: Malmquist or Eddington Bias

Konus-WIND at 25
III. Procedures

Forward Fitting vs Non-parametric EPL Methods

Lynden-Bell 1973

Konus-WIND at 25
The common practice is to assume forms for the GRB “Luminosity” Function

\[ \Psi(L, z) = \rho(z) \psi\left(\frac{L}{g(z)}\right) / g(z) \]

Luminosity Evolution

\[ L(z) = L_0 g(z); \quad g(z) = (1 + z)^k \]

Density Evolution

\[ \rho(z) \]

Energy Spectrum  *Power-law, Broken Power-law, etc*

Difficulty: *Involves many functions each with several parameters*

Uniqueness??
Procedures: 2. Non-parametric

Some past non-parametric methods

Schmidt (1968) $V/V_{\text{max}}$ or Lynden-Bell (1973) $C$- methods

These however assume that Luminosity and Redshift are

*Uncorrelated or are Independent variables*
Procedures: 2. Non-parametric

More recently (Efron and VP, 1992, 1999) have developed a method that first determines the $L-z$ correlation; i.e.

$$g(z)$$

Then remove this correlation by defining

$$L_0 = \frac{L}{g(z)}$$

Which is now independent of redshift and allows

*Determination of all distributions non-parametrically and directly from the data with very few assumptions or prescribed functional forms*
1. Test of Independence

Spearman Rank Order Test: Distribution of Ranks

Kendall's tau Statistic

\[ \tau = \frac{\Sigma_j (R_j - E_j)}{\sqrt{\Sigma_j V_j}} \]

associated sets of \( L_i, z_i \) with \( N_i \) and \( M_i \) sources in the sets
Test of Independence

Remove the correlation by a variable transformation e.g.

\[ L'_i = \frac{L_i}{g_i(z)} \]

\[ g(z) = (1 + z)^k \]
Test of Independence

Remove the correlation by a variable transformation e.g.

\[ L'_i = \frac{L_i}{g_i(z)} \]

\[ g(z) = (1 + z)^k \]

\[ \Psi(L', z) = \psi(L') \rho(z) \]
Given uncorrelated or independent variables
Can account for truncation

\[ X = N_3 \left( \frac{N_2}{N_1} \right) \]

Petrosian, 1993

Konus-WIND at 25
2. The Bivariate Distributions

Based on the associated sets

associated sets of $L_i, z_i$

With

$N_i$ and $M_i$

Sources in the sets
The single variable distributions

The method gives the cumulative $L$ and $z$ distributions

Non-parametrically and with no binning

$$\Phi(L_i) = \int_{L_i}^{\infty} \Psi(L) dL = \Pi_1^i (1 + 1/N_j)$$

$$\sigma(z_i) = \int_0^{z_i} \rho(z) (dV/dz) dz = \Pi_1^i (1 + 1/M_j)$$

From these we get the sought differential distributions

$$\Psi(L) \text{ and } \rho(z)$$
IV. Application to Swift Long Gamma-ray Bursts

Density (rate) Evolution vs Star Formation Rate
Caveats: Selection Effects and Truncations

1. Gamma-ray trigger
   Peak count or flux threshold

2. Localization
   X-ray flux threshold

3. Optical follow-up and Redshift
   Optical Magnitude etc
2. Bias Due to X-ray Observations

Strong *Correlation* between Gamma and X-rays
Same for GRBs with or without redshift.

_Thus, Small bias if any_

*(data from Nysewander et al. 2009)*
3. Optical and Redshift Bias

There is no good criteria for redshift bias.

The *optical flux* can be used as indicator but there is no well defined limit.

Opt.-X-ray fluxes *correlated*

So *use X-ray threshold* to correct for this bias

*(data from Nysewander et al. 2009)*
1. Test of independence

*Luminosity evolution*
Cumulative Distributions

Luminosity Function

Rate Density Evolution

\[ \phi(L) = \phi_0 \left( \frac{L}{L_0} \right)^{-\alpha} e^{-L/L_0} \]

\( L_0 = 10^{51.16} \) erg/s, \( \alpha = 0.48 \), \( \phi_0 = 10^{1.86} \)

\[ \phi(L) = \phi_0 \left( \frac{L}{L_0} \right)^{-\delta} \left[ 1 + \left( \frac{L}{L_0} \right)^{\delta - 4} \right] \]

\( L_0 = 10^{51.0} \) erg/s, \( \delta_1 = 0.5 \), \( \delta_2 = 2.2 \), \( \phi_0 = 10^{1.89} \)

Konus-WIND at 25
GRB and Star Formation Rates

Density Rate Evolutions $\dot{\rho}(z)$

- No Luminosity Evolution
- $\gamma$ and $\gamma$-ray Thresholds
- $\gamma$-ray Threshold Only
- Raw Observed
- SFR Madau and Dickinson, 2014

Redshift $Z = 1 + z$
GRB and Star Formation Rates

Figure 8. Comparison between GRB formation rate $\rho(z)$ (blue) and the observed SFR. The SFR data are taken from Hopkins & Beacom (2006), which are shown as red dots. The SFR data from Bouwens et al. (2011) (stars) and Wang (2013) (open circles) are also used. All error bars are 1σ errors.
GRB and Star Formation Rates

Density Rate Evolutions $\dot{\rho}(z)$

Redshift $Z = 1 + z$
SUMMARY on Long GRBs

1. In order to use GRBs as Cosmological Tools we need a better understanding of the *distribution and evolution* of their characteristics.

2. We have emphasized the advantages of *non-parametric approach* and demonstrated how to determine luminosity and rate density evolutions.

   **GRB Formation Rate very different than the Star Formation Rate.**

3. Further studies can improve our understanding of the phenomenon which will help in using them as tools to explore

   *The high redshift universe.*

4. In the long run, GRBs may prove to be useful for

   *GLOBAL cosmological studies.*
1. Sample selection

Short GRBs and Gravitational Waves

Preliminary results from a Swift sample of SGRB

More uncertain because fewer SGRBs
Short GRBs and Gravitational Waves

Preliminary results from a Swift sample of SGRB

More uncertain because fewer SGRBs

1. Sample selection

Konus-WIND at 25
Short GRBs and Gravitational Waves

Preliminary results from a Swift sample of SGRB

2. Results:
   a. Luminosity Evolution
      \[ L(z) \propto Z^k/(1 + Z/Z_c)^k; \quad k = 3.6 \]

   b. Cumulative Luminosity Function \( \Phi(L) \)

   c. Density Rate Evolution \( \dot{\rho}(z) \)

Konus-WIND at 25
Short GRBs and Gravitational Waves

Preliminary results from a Swift sample of SGRB

2. Results:
   a. Luminosity Evolution
   \[ L(z) \propto Z^k/(1 + Z/Z_c)^k; \quad k = 3.6 \]
   b. Cumulative Luminosity Function \( \Phi(L) \)
   c. Density Rate Evolution \( \dot{\rho}(z) \)
SGRB and Star Formation Rates

Paul atXiv:1710.05620

Konus-WIND at 25
Comparison: Short and Long GRBs

Redshift distribution

Konus-WIND at 25
SUMMARY: Short GRBs

1. Small samples that can be considered “complete”

2. Preliminary results show
   a. *Similar luminosity evolution as Long GRBs*
   b. *Luminosity function steeper at low luminosities*
   c. *Rate evolution similar to the low redshift part of the LGRBs:*
      
      Perhaps delayed SFR

3. The high rates of both at low redshift will have important consequence for gravitational wave rate.
Backups
Further Testing of the Results

Assume GRB rate = SFR

\[ \frac{d\sigma}{dz} = SFR \times \frac{(dV/dz)}{Z} \]

\[ SFR \propto Z^{2.7}/\left[1+(Z/2.9)^{5.6}\right] \]

\[ \frac{dN}{dz} = \frac{d\sigma}{dz} \times \Phi(l_{min}(z)) \]
Further Testing of the Results

Log $N$-Log $S$ Test
GRBs: As Cosmological Probes

GRBs Can be useful probes for study of the early universe such as Reionization, Star Formation Rate, Metallicity Evolution

However

For this we need to determine the evolution of their characteristics (e.g. Formation Rate, Luminosity, .....)

This requires a large sample with redshifts and well defined observational selection criteria and data truncation

Konus-WIND at 25