The polariton physics in strongly-coupled semiconductor microcavities leads to interesting non-linear optical properties: when resonantly pumped with an off-axis laser, signal and idler beams are observed, with one, the signal, always emitted close to the surface normal direction. This behaviour can be interpreted in terms of an optical parametric oscillator (OPO), albeit an unusual one, as the non-linearity is $\chi^3$, rather than the normal $\chi^2$. An immediate consequence of this is that there is a self-interaction (‘blue-shift’) term, $|\phi|^2\phi$, which is not a part of the normal OPO physics.

In this paper I present an analytic treatment of the role of the self-interaction, taking into account all the contributions from the pump, signal and idler fields. This model helps to explain a number of puzzling experimental results: It provides a ‘phase diagram’ of the polariton system, showing the pump angles and energies at which OPO behaviour is expected, and where there is a simple bistability. There is a minimum angle of $\sim 10^\circ$, but at higher angles the OPO threshold simply increases gradually, with no particular significance to the ‘magic-angle’, $\sim 16^\circ$, where the pump, signal and idler are all resonant with the bare dispersion. When the OPO switches on, the self-interaction pulls the signal towards the normal direction, so at higher signal powers the emission is always close to $0^\circ$.

I also consider the case when the fields are not simple plane waves, so that the polaritons can have spatial structure. In this case, the self-interaction term leads to a generalised Gross-Pitaevskii (G-P) equation, for the coupled pump, signal and idler fields. This suggests the possibility of vortices with phase singularities; for example, even with a plane-wave pump, using a singular optical beam as a probe, it may be feasible to create a vortex anti-vortex pair in the signal and idler. I present the results of a two dimensional numerical study of the polariton G-P equations, to investigate the structure and stability of such vortex states.