INTENSITY FLUCTUATIONS OF LIGHT PROPAGATING IN SYSTEM OF LARGE SCATTERERS. I. SINGLE SCATTERING

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An autocorrelation function for wave-field fluctuations detected by a square-law receiver behind a layer of large scatterers is found by means of nonclassical photometry in a single-scattering approximation.

In the solution of a number of applied problems, it is necessary to take into account the fluctuations of light passing through a medium containing random discrete scatterers whose dimensions cannot be ignored. Such scatters are aerosol and precipitation particles in optical communications and location [1, 2], entire clouds in meteorology [3, 4], and emulsion grains in photography [5]. The existing methods for calculation of optical fluctuations in media with discrete scatterers are mainly based on either determination of the light field in the far zone, where the scatterers show up as point centers [3, 6-9], or a "straight-path approximation" [1], in which deformation of the scattered wave by its propagation is ignored [4, 10-12]. An equation that makes it possible to go beyond these approximations has been proposed [1] and extended [13], but corresponding quantitative results are practically absent.

The problem is made geometrically clear and them simplified with the aid of nonclassical photometry [14], in terms of which single scattering of light by large scatterers with an extended indicatrix will be examined here. In Sec. 1, general formulas will be written for the mean value and autocorrelation function (ACF) of brightness. In Sec. 2, these formulas will be made specifically applicable to large optically rigid particles. The ACF of a optical signal will be examined in Sec. 3 in various limiting cases and an agreement between newly obtained results and previously known data will be established. Numerical results will be presented in Sec. 4. And the formulas required to determine the correlation coefficients for the brightness of light diffracted by circular particles will be derived in the Appendix.

1. Single-Scattering Approximation. We shall examine light propagation through a layer $0 < z < h$ containing random scatterers each of which is characterized by the location of its conditional "center" and a set of other parameters. Let the scattering take place at small angles to the z axis, so that backscattering can be ignored. The mathematical expectation and ACF of the brightness are specified at the illuminated boundary $z = 0$; the problem consists of finding them at the boundary $z = h$. With satisfaction of the conditions of quasi-homogeneity [14], the formulas derived below will be equally valid for the conventional (photometric) and generalized brightness.

Let $\mathbf{s}_\perp$ be the projection of the vector $\mathbf{s} = (\mathbf{s}_\perp, s_z)$, which determines the beam direction, onto the plane $z = \text{const}$, let $I(\mathbf{r}, \mathbf{s}) = I(\boldsymbol{\rho}, z, \mathbf{s})$ be the brightness in that plane ($\boldsymbol{\rho} \in \mathbf{R}^2$), and let in the absence of scatterers the brightness propagation in the medium be described by a linear operator $\hat{T}$:

$$I(z + \Delta z) = \hat{T}(z, \Delta z) I(z).$$

Here, the brightness $I(z)$ is considered a function of $\boldsymbol{\rho}$ and $\mathbf{s}$, which is a function of the parameter z, and $\hat{T}(z, \Delta z)$ is an operator that is a function of the parameters z and Δz. For empty space,

$$\hat{T}(z, \Delta z) f(\vec{\rho}, \vec{s}) = f(\vec{\rho} - \Delta z \cdot \vec{s}_\perp / s_z, \vec{s})$$

and, in the more general case, $\hat{T}$ can take into account, for example, the effect of absorption or turbulence.

Let the radiation interact with the n-th scatterer inside the layer $(z_n - l/2, z_n + l/2)$ in accordance with a law that is invariant with respect to displacements of the center $(\boldsymbol{\rho}_n, z_n)$. We shall also assume that the indicated layer is thin [16], i.e., $\hat{T}(z, l) I(z) \approx I(z)$, and that the effect of the scatterer is local with respect to $\boldsymbol{\rho}$:

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$$I(\vec{\rho}, z_n+1/2, \vec{s}) - I(\vec{\rho}, z_n-1/2, \vec{s}) = \hat{\sigma}_n I(z_n - 1/2) \equiv$$

$$\equiv \int \sigma_n(\vec{\rho} - \vec{\rho}_n, \vec{s}, \vec{s}')I(\vec{\rho}, z_n-1/2, \vec{s}')d\Omega_{s'}.$$

The kernel σ_n of the scattering operator $\hat{\sigma}_n$ can be generalized.

The condition of applicability of the single-scattering approximation, as is known [17], can be written as

$$\sigma h = n_v a_e h \ll 1, \tag{1}$$

where σ is the extinction factor, n_v is the numerical volume concentration of the scatterers, and a_e is their mean effective cross-section: $a_e = Q_{att}a$, where a is the geometrical cross-section and Q_{att} is the attenuation-efficiency factor [18].

Condition (1) makes it possible to examine scatterers that do not irradiate one another. In this case, the generalized brightness of the scattered radiation is additive and thus

$$I(h) = \hat{T}(0, h)I(0) + \sum_n \hat{T}(z_n, h-z_n)\hat{\sigma}_n\hat{T}(0, z_n)I(0) + \Delta, \tag{2}$$

where Δ is a correction, which is ignored in the single-scattering approximation. Note that Δ can be large in individual random realizations, but it makes a negligible contribution with conversion to mean values.

Let the scatterers be arranged independently. Then, according to Isimaru [17], with averaging the sum in (2) becomes an integral with respect to the coordinates of the centers of the scatterers with a weight equal to the density of their arrangement n_v, and for the mean brightness we obtain

$$<I(h)> = \left[\hat{T}(0, h) + \int_0^h \hat{T}(z, h-z)\hat{S}\hat{T}(0, z)dz\right]<I(0)>, \tag{3}$$

where $\hat{S}$ is an integral operator with respect to s with the kernel

$$S(\vec{s}, \vec{s}') = \int_{R^2} n_v<\sigma_n(\vec{\rho}, \vec{s}, \vec{s}')>d^2\vec{\rho}, \tag{4}$$

and the angle brackets indicate averaging.

To find the ACF

$$W(z; \vec{\rho}, \vec{s}; \vec{\rho}', \vec{s}') \equiv <I(\vec{\rho}, z, \vec{s})I(\vec{\rho}', z, \vec{s}')> - I_w(z; \vec{\rho}, \vec{s}; \vec{\rho}', \vec{s}'),$$

where

$$I_w(z; \vec{\rho}, \vec{s}; \vec{\rho}', \vec{s}') \equiv <I(\vec{\rho}, z, \vec{s})><I(\vec{\rho}', z, \vec{s}')>,$$

we multiply the corresponding parts of (2), which has been written for points (ρ, s) and (ρ', s') and perform the averaging. The product of the sums on the right sides is represented as

$$\sum_{n,m} A_n(\vec{\rho}, \vec{s})A_m(\vec{\rho}', \vec{s}') = \sum_n A_n(\vec{\rho}, \vec{s})A_n(\vec{\rho}', \vec{s}') + \sum_n \sum_{m\neq n} A_n(\vec{\rho}, \vec{s})A_m(\vec{\rho}', \vec{s}'), \tag{5}$$

where $A_k \equiv \hat{T}(z_k, h - z_k)\hat{\sigma}_k\hat{T}(0, z_k)I(0)$. With averaging, the first sum on the right side of (5) corresponds to single and the second to double integration with weight n_v with respect to the scatterer coordinates. Using condition (1), we ignore the second sum in comparison with the first. Taking (3) into account, we have

$$W(h) = \Bigg\{\hat{T}(0, h) \bullet \hat{T}(0, h) + \hat{T}(0, h) \bullet \int_0^h \hat{T}(z, h-z)\circ\hat{S}\circ\hat{T}(0, z)dz +$$

$$+ \left[\int_0^h \hat{T}(z, h-z)\circ\hat{S}\circ\hat{T}(0, z)dz\right] \bullet \hat{T}(0, h)\Bigg\}\cdot W(0) + \tag{6}$$

$$+\left[\int_0^h \left(\hat{T}(z,h-z)\otimes\hat{T}(z,h-z)\right)\circ\hat{B}\circ\left(\hat{T}(0,z)\otimes\hat{T}(0,z)\right)dz\right]\cdot(W(0)+I(0)),$$

where $\otimes$ indicates the direct product of the operators, W and I_v are considered functions of the pair of points (ρ, s; ρ', s') of the space $\mathbf{R}^2\times\Omega$, which are functions of the parameter z, and $\hat{B}$ is an integral operator with respect to the angle variables (s, s') with the integral kernel

$$B(\vec{\rho},\vec{\rho}',\vec{s}\leftarrow\vec{s}'',\vec{s}'\leftarrow\vec{s}''')=\int_{\mathbf{R}^2} n_v\langle\sigma_n(\vec{\rho}+\vec{\rho}'',\vec{s},\vec{s}'')\sigma_n(\vec{\rho}'+\vec{\rho}'',\vec{s}',\vec{s}''')\rangle d^2\rho''.$$

If $\hat{S}$ is the scattering operator, $\hat{B}$ is the covariance operator of the scattered radiation.

2. Small-Angle Scattering by Optically Rigid Particles. Limiting ourselves to a small-angle approximation, we make the obtained relations apply specifically to "optically rigid" particles that are large in comparison with wavelength λ, the scattering by which was examined earlier by Borovoi [1]. In that study, the wave field scattered by a particle was divided into two components – shadow-forming and refracted – and it was argued that the first component was chiefly responsible for the optical fluctuations detected after the layer, which is a result of small-angle diffraction by the particle contour. Considering this, we shall limit the examination to the shadow-forming component. Regardless of these arguments, such an examination is clearly valid for "black" particles [18], for which a refracted field is practically absent. Such particles include sooty aerosols [2] and the grains of chemically developed photographic emulsions [5].

In examining the propagation of radiation through a randomly inhomogeneous medium, we must distinguish the statistical ensemble of microscopic sources that determines the coherence properties of the incident radiation and the ensemble of realizations of the medium. An optical instrument can only record values that have been averaged over the ensemble of microscopic sources [15] (such averaging will be indicated by angle brackets with the subscript "s"), but it is fully capable of responding to perturbations produced in the transition from one realization of a random medium to another (averaging over such a "macroscopic" ensemble was indicated above by angle brackets without subscripts).

According to Apresyan and Kravtsov [14], when the quasi-homogeneity conditions are satisfied, the propagation and small-angle scattering of a wave field with a complex amplitude $u(\mathbf{r})$ can be described in simple language with the aid of the concept of generalized brightness, which is the space–angle power spectrum:

$$I(\vec{r},\vec{s}):=\frac{s_z}{\lambda^2}\int_{\mathbf{R}^2}\langle u^*(\vec{\rho}-\vec{\rho}'/2,z)u(\vec{\rho}+\vec{\rho}'/2,z)\rangle_S e^{ik\vec{s}_\perp\vec{\rho}'}d^2\rho'. \tag{7}$$

Let a nontransparent screen of size $R \gg \lambda/2\pi$ be located in the plane $z = 0$, and let a quasi-homogeneous wave with brightness $I_0(\mathbf{r}, \mathbf{s}_\perp)$ be incident from the direction of negative z. Let $\chi_R(\rho)$ be a function that is equal to unity on the screen and zero outside of it. We shall find the brightness for $z > 0$ in the phase-space region in which the quasihomogeneity conditions are satisfied (using the Babinet principle [15], we can show that the conditions hold [14], p. 107). In a Kirchhoff approximation [15], we can write $u(\rho, +0) = u(\rho, -0)\chi_R(\rho)$. We find the correlation function of the field $u(\rho, -0)$ by transforming (7), which is written for I_0. Using (7) again, in a small-angle approximation ($s_x \approx 1$) we find

$$I(\vec{\rho},+0,\vec{s}_\perp)-I_0(\vec{\rho},\vec{s}_\perp)=\int_{\mathbf{R}^2}\sigma_n(\vec{\rho},\vec{s}_\perp-\vec{q}')\bar{I}_0(\vec{\rho},\vec{q}')d^2\vec{q}', \tag{8}$$

where

$$\sigma_n(\vec{\rho},\vec{q}')=2\left\{\Lambda\sigma_0(\vec{\rho},\vec{q})-\frac{1}{\lambda^2}\,\mathrm{Re}\left[e^{-2ik\vec{\rho}\cdot\vec{q}}\int_{\mathbf{R}^2}\chi_R(\vec{\rho}'/2)\,e^{ik\vec{\rho}'\vec{q}}d^2\rho'\right]\right\},$$

$$\sigma_0(\vec{\rho},\vec{s}_\perp)=\frac{1}{\lambda^2}\int_{\mathbf{R}^2}\chi_R(\vec{\rho}-\vec{\rho}'/2)\chi_R(\vec{\rho}+\vec{\rho}'/2)\exp(ik\vec{\rho}'\vec{s}_\perp)d^2\rho', \tag{9}$$

and the quantity $\Lambda = 1/2$, as we shall see, has the meaning of the scattering albedo. Expression (8) is identical in form to the conventional photometric formula for small-angle scattering by an extended object. This makes it possible to examine from unique points of view problems of classical transport theory and diffraction problems. The role of wave effects is clearly distinguished, facilitating interpretation. Another advantage of the photometric description is simplicity of determination of the optical-receiver response, which is characterized in the language of geometrical optics. For example, let the brightness distribution on the plane $z = h$ be known, and the receiver be located in empty space $z > h$ and focused on the plane $z = f + h$ and have on that plane a field of vision of radius R_0 with center ρ_0 and aperture angle α, where the optical axis passes through ρ_0 in the direction s_0. Ignoring diffraction by the elements of the optical system, which is permissible for $\alpha >> \lambda/R_0$, we find the instrument response in a paraxial approximation:

$$E(\vec{\rho}_0, \vec{s}_0) = C_i \int\limits_{\rho<R_0,\ |(\vec{s}-\vec{s}_0)_\perp|<\alpha} d^2\rho d\Omega_s\ I(\vec{\rho}_0 + \vec{\rho} - f\vec{s}/s_z, h, \vec{s}), \tag{10}$$

where C_i is the instrument constant.

Let us consider the simplest case of circular monodisperse particles situated statistically uniformly in a layer that is evenly illuminated by a light beam with angular width γ:

$$I(\vec{\rho}, 0, \vec{s}) = I_0\theta(\gamma - s_\perp), \tag{11}$$

where θ is the Heaviside function. For particles of radius R, formula (4) becomes

$$S(\vec{s}, \vec{s}') = \sigma\cdot\left[\Lambda X(|\vec{s}_\perp - \vec{s}'_\perp|) - \delta^2(\vec{s}_\perp - \vec{s}'_\perp)\right],$$

where $\sigma = 2n_v a$, $a = \pi R^2$, the albedo $\Lambda = 1/2$, and X is the Fraunhofer diffraction indicatrix [18]:

$$X(|\vec{q}|) = \frac{1}{a}\int\limits_{R^2} \sigma_0(\vec{\rho}, \vec{q})d^2\rho = \frac{a}{\lambda^2} D^2(q), \qquad D(q) = \frac{2J_1(kRq)}{kRq},$$

and J_n is an n-th-order Bessel function. Similarly, for the kernel of operator B in formula (6) we have:

$$B(\vec{\rho}_1, \vec{\rho}_2, \vec{s}_1 \leftarrow \vec{s}'_1, \vec{s}_2 \leftarrow \vec{s}'_2) =$$

$$= \frac{1}{2}\sigma\beta(\vec{\rho}_2 - \vec{\rho}_1, (\vec{s}_{1\perp} + \vec{s}_{2\perp} - \vec{s}'_{1\perp} - \vec{s}'_{2\perp})/2, \vec{s}_{1\perp} - \vec{s}'_{1\perp} - \vec{s}_{2\perp} + \vec{s}'_{2\perp}); \tag{12}$$

$$\beta(\vec{\rho}, \vec{v}, \vec{w}) = \Lambda^2\beta_0(\vec{\rho}, \vec{v}, \vec{w}) - 2\Lambda\beta_-(\vec{\rho}, \vec{v}, \vec{w}) +$$
$$+ \beta_+(\vec{\rho}, \vec{v})\delta(\vec{w}) + \frac{1}{4}\beta_+(\vec{\rho}, \vec{w}/2)\delta(\vec{v}). \tag{13}$$

Formulas for β_0, β_-, and β_+ are provided in the Appendix. The function β_0 results from the product of the first term on the right side of (9) taken at the point $(\rho_1, s_{1\perp})$ multiplied by the same term at $(\rho_2, s_{2\perp})$ and describes the incoherent autocorrelation of the brightness scattered by a particle, and the function β_+, which are produced by multiplication of the second terms, and β_-, which correspond to the cross products, are required to allow for absorption and interference.

Introducing the more convenient variables $\boldsymbol{\rho} = \rho_2 - \rho_1$, $\mathbf{v} = (s_{1\perp} + s_{2\perp})/2$, and $\mathbf{w} = s_{2\perp} - s_{1\perp}$, with the aid of formulas (6), (11), and (12) we obtain

$$W(h, \vec{\rho}, \vec{v}, \vec{w}) = I_0^2\frac{\sigma}{2}\int\limits_0^h dz \int\limits_{|\vec{v}'\pm\vec{w}'/2|<\gamma} d^2v' d^2w' \beta(\vec{\rho} - \vec{w}z, \vec{v} - \vec{v}', \vec{w} - \vec{w}'). \tag{14}$$

The ACF of a signal detected by the receiver examined above is, according to (10) and (14),

$$W_E(\vec{\rho},\vec{v},\vec{w}) = C_i^2 R_0^2 I_0^2 \frac{\sigma}{2} \int_0^h dz \int_{\mathbb{R}^2} d^2\rho' H(|\vec{\rho}-\vec{\rho}'|/R_0) \times$$
$$\times \int_{\{|\vec{v}-\vec{v}'|\pm(\vec{w}-\vec{w}')/2|<\alpha,\ |\vec{v}''\pm\vec{w}''/2|<\gamma\}} d^2v'\, d^2w'\, d^2v''\, d^2w''\, \beta(\vec{\rho}'+(z-f)\vec{w}',\ \vec{v}'-\vec{v}'',\ \vec{w}'-\vec{w}''), \tag{15}$$

where $H(y)$ is the intersection area of two unit circles whose centers are separated by distance y (see Appendix).

3. Recorded Optical Noise in Limiting Cases. We shall assume that the receiver does not resolve individual particles: $R_0 >> R$. Then, the slow variation of H as compared with β makes it possible to remove H from under the integral with respect to ρ' in (15) and then integrate with respect to ρ' using the formulas provided in the Appendix. If it is also assumed that all particles lie within the focusing depth ($\max(f, h-f) << \max(kR^2, R_0/\alpha)$), we can let $(z-f) \approx 0$ in (15), and, as a result

$$W_E(\vec{\rho},\vec{v},\vec{w}) = E_0^2 \sigma h (R/R_0)^2 \frac{1}{\pi} H(\vec{\rho}/R_0) \times g(\vec{v}-\vec{w}/2, \alpha) g(\vec{v}+\vec{w}/2, \alpha), \tag{16}$$

where $E_0 = (\pi\gamma R_0)^2 I_0$ is the luminous flux that passes through the reading area in the absence of scatterers, and

$$g(q, \gamma) \equiv \theta(\gamma - q) - \Lambda \int_0^{2\pi} d\varphi \int_0^{\gamma} q'\, dq'\, X(\sqrt{q^2+q'^2+2qq'\cos\varphi}).$$

Formula (16) is obtained more simply in an approximation of Fraunhofer diffraction. Correlation in it is entirely dependent on coverage of the apertures in reading. This fact is a consequence of the condition $R_0 >> R$. Let us examine the space ACF of the signal $W_E(|\boldsymbol{\rho}|) \equiv W_E(\rho, 0, 0)$ in another limiting case: $R >> \lambda/\alpha$. In this case, all of the scattered light is incident on the receiver, and α can approach infinity in (15). In this case,

$$W_E(\rho) = \frac{\sigma C_i^2}{2\pi^4} E_0^2 \int_0^h dz \int_{\mathbb{R}^4} d^2y\, d^2y' H(y) H(y') \times$$
$$\times \bar{\beta}(|\vec{\rho} + R_0\vec{y} + (z-f)\gamma\vec{y}'|, z-f), \tag{17}$$

where

$$\bar{\beta}(|\vec{\rho}|, z) \equiv \int_{\mathbb{R}^4} \beta(\vec{\rho} + \vec{w}z, \vec{v}, \vec{w})\, d^2v\, d^2w.$$

Substituting (13) into (17) and integrating term by term, it is easy to express the function $\bar{\beta}$ in terms of single integrals of the product of the Bessel functions and elementary functions. The explicit form of these functions was given elsewhere [19]. For $|z| >> kR^2$, these formulas are simplified:

$$\bar{\beta}(\rho, z) = \frac{1}{2}\left(\frac{kR^2}{\pi z}\right)^2 \int_0^2 xH^2(x) J_0\left(\frac{kRx\rho}{|z|}\right) dx - \left[\frac{kR^2}{|z|} D\left(\frac{\rho}{z}\right)\right]^2 + \frac{kR^2}{4|z|} D\left(\frac{\rho}{2|z|}\right) \cos\frac{k\rho^2}{4z} + \frac{2}{\pi} H(\rho/R). \tag{18}$$

The first term in formula (18) describes the ACF of the intensity of light scattered at angles that differ from one another by $(\rho/|z|)$. This function was obtained in an approximation of Fraunhofer diffraction earlier [8]. The second term, which is omitted here, describes the cross-correlation of the scattered and absorbed radiation. Unlike the first two terms, the last two terms in the Fraunhofer approximation degenerate into a deltoid singularity, which corresponds to autocorrelation of the absorbed radiation, while in our formula (18) they describe the ACF of the specular structure observed behind a coherently illuminated particle layer. Three correlation scales are clearly distinguished in this case: the particle size R in the last term, the radius of the first Fresnel zone $(\lambda|z|)^{1/2}$ in the penultimate term, and the scattering spot $(\lambda|z|/R)$ in the first two terms.

$$W_{rel}(\rho/R) = W_E(\rho)/\langle E\rangle^2,$$

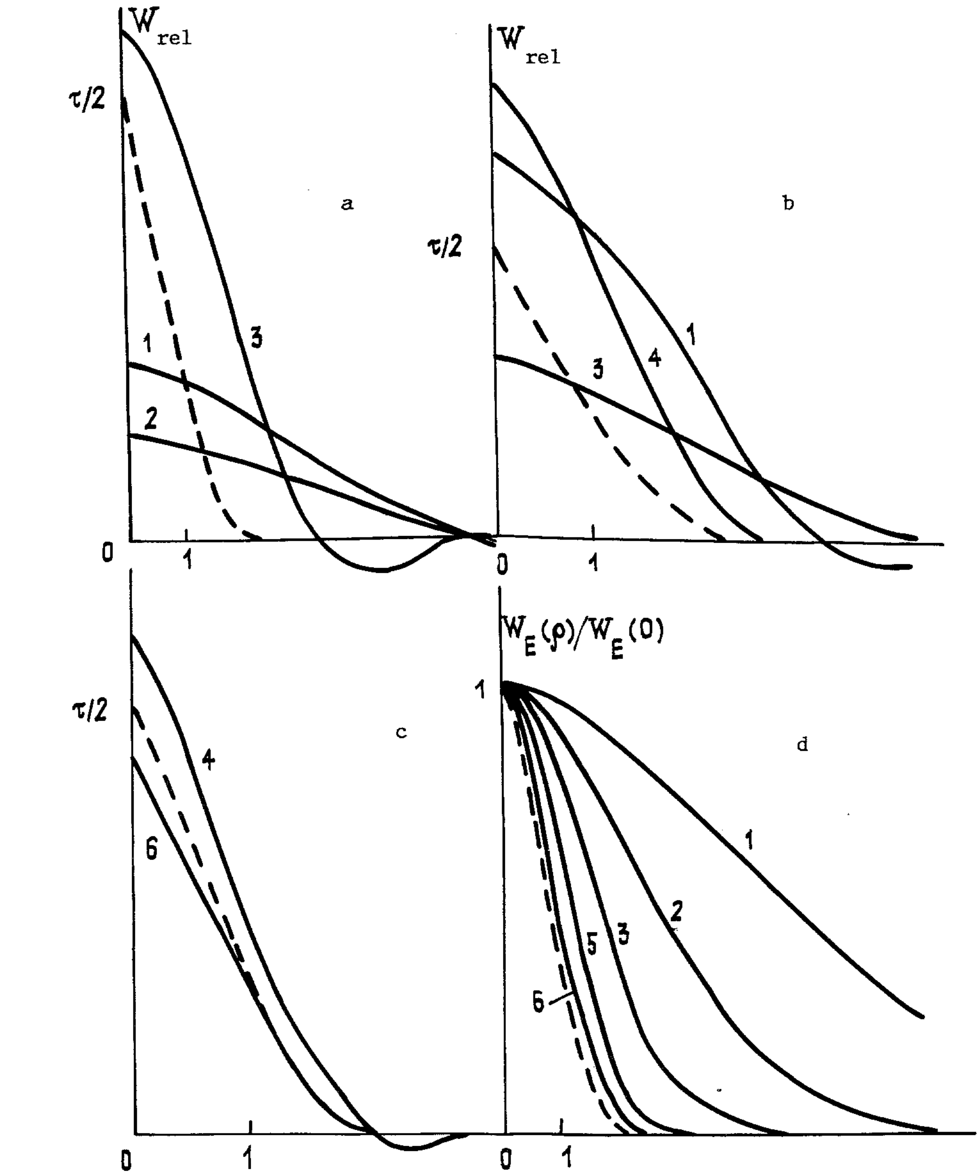

Fig. 1. ACF of optical signal: a) $\alpha_* = 0.125$; b) $\alpha_* = 0.25$; c) $\alpha_* = 1$; d) $\alpha_* = \gamma_*$; 1) $\gamma_* = 0.0625$; 2) $\gamma_* = 0.125$; 3) $\gamma_* = 0.25$; 4) $\gamma_* = 0.5$; 5) $\gamma_* = 1$; 6) $\gamma_* = 4$.

When all of the particles lie within the effective focusing depth ($\max(f, h - f) << \max(kR^2, R_0/\alpha)$), the formulas for the function $\bar{\beta}$ are radically simplified, and we obtain

$$W_E(\rho) = E_0^2 C_1^2 \sigma h \frac{1}{2\pi} H(\rho/R).$$

The latter equality can also be obtained without allowance for scattering by the particles, as distinct from all previous results.

4. Dependence of Optical Noise on the Conditions of Its Recording. Let us consider the case in which both layer boundaries are within the limits of the receiver's focusing depth. In this case, in (15) the integral with respect to z is replaced by the factor h, and the integral with respect to ρ' corresponds to purely spatial averaging of the ACF with weight $H(\cdot)$, which corresponds to overlapping reading areas. Since its smoothing effect on the ACF is obvious, we shall not examine this averaging, formally introducing a "point" reading area: $R_0 \to 0$. Let the receiver be oriented orthogonally to the layer: v = w = 0. Then,

$$W_E(|\vec{\rho}|) = \left(\frac{E_0}{\pi\gamma^2}\right)^2 \frac{\tau}{2} \int\limits_{\substack{|\vec{v}' \pm \vec{w}'/2| < \alpha \\ |\vec{v}'' \pm \vec{w}''/2| < \gamma}} d^2v' d^2w' d^2v'' d^2w'' \beta(\vec{\rho}, \vec{v}' - \vec{v}'', \vec{w}' - \vec{w}''),$$

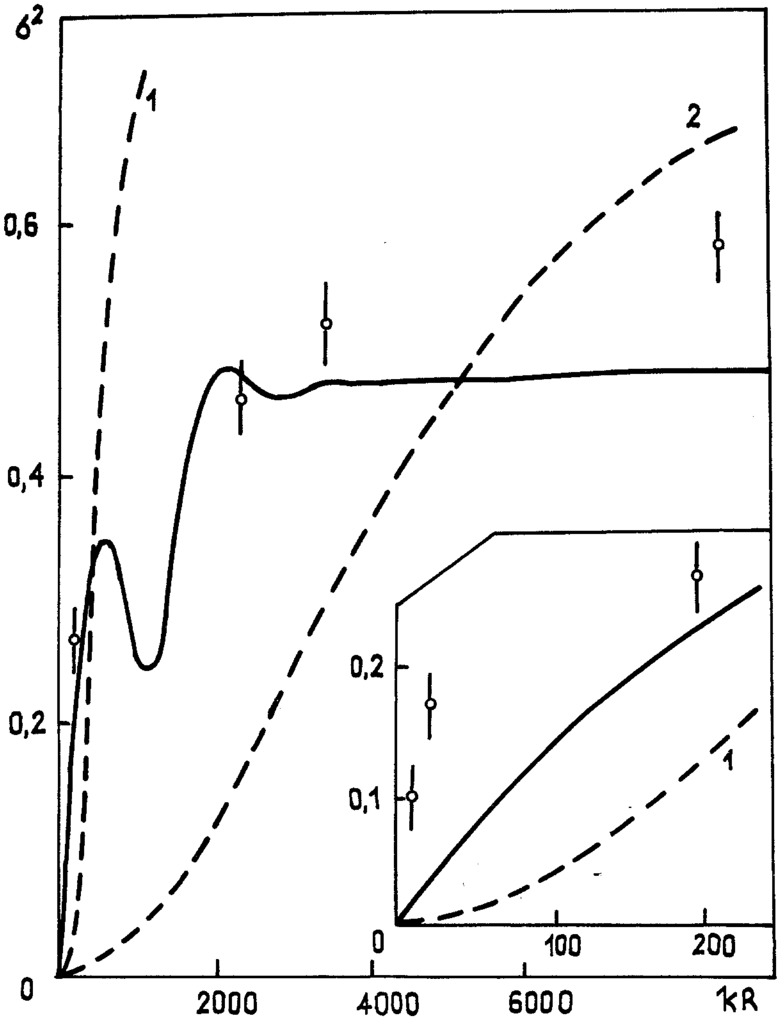

Fig. 2. Relative variance of intensity fluctuations as a function of diffraction parameter of particles according to formula (17) (solid curves) and direct-path approximation (dashed curves). 1) kR_0 = 500; 2) 5000.

where $\tau = \sigma h$ is the optical thickness of the layer. We shall express the function β in terms of its Fourier transform $\tilde{\beta}$. We shall employ the following scaling to eliminate the inexplicit dependence of the integrand on R and λ:

$$\beta_*(\vec{\rho}_*, \vec{q}_*, \vec{p}_*) \equiv \tilde{\beta}(R\vec{\rho}_*, kR\vec{q}_*, kR\vec{p}_*).$$

Substituting the variables accordingly, we obtain

$$W_E(|\vec{\rho}|) = \frac{E_0^2 \tau \alpha_*^2}{2\pi^2 \gamma_*^2} \int\limits_{R^4} d^2x d^2y/(xy)^2 J_1(\pi\alpha_* x) J_1(\pi\alpha_* y) \times J_1(\pi\gamma_* x) J_1(\pi\gamma_* y) \beta_*(\vec{\rho}, (\vec{x}+\vec{y})/2, (\vec{x}-\vec{y})/2),$$

where $\gamma_* \equiv \gamma/\gamma_1$ and $\alpha_* \equiv \alpha/\gamma_1$ are the apertures of the illuminator and receiver referred to the average angle of single diffraction $\gamma_1 \equiv \lambda/2R$, $x \equiv |\mathbf{x}|$, and $y \equiv |\mathbf{y}|$.

The relative ACF is of greatest interest:

$$W_{rel}(\rho/R) = W_E(\rho)/\langle E\rangle^2.$$

where the average signa; $\langle E \rangle$ is expressed with the aid of formulas (3), (10), and (11). With accuracy to terms that are linear with respect to τ, we have

$$W_{\mathrm{rel}}(\rho_*) = \frac{\tau \max(\alpha_*^4, \gamma_*^4)}{2\pi^2\alpha_*^2\gamma_*^2} \int_{\mathbf{R}^4} d^2x d^2y J_1(\pi\alpha_* x) J_1(\pi\alpha_* y) \times \tag{19}$$

$$\times J_1(\pi\gamma_* x) J_1(\pi\gamma_* y) \beta_*(\vec{\rho}_*, (\vec{x} + \vec{y})/2, (\vec{x} - \vec{y})/2)/(xy)^2.$$

We calculated integral (19) by the trapezoid rule, introducing polar coordinates for the vectors **x** and **y** and taking into account the symmetry of the integrand with respect to their transposition.

The variation of W_{rel} as a function of the reading aperture angle α is illustrated in Figs. 1a-c. Since formula (19) is symmetrical with respect to transposition of α and γ, these same results demonstrate the dependence of the ACF on the convergence angle of the illuminating beam γ, which, according to Sec. 1, is related to the degree of coherence of the illumination. The case in which the illumination and reading apertures coincide is shown in Fig. 1d.

Most of the published experimental studies of intensity fluctuations in scattering by particles involve natural media (see bibliographies in [2, 20, 21]). The set of factors acting in such media that are not precisely known (differences in particle shape and size, particle movement, and turbulence of the medium) make direct comparisons of theory with experiment difficult. At the same time, for the specific interpretation of full-scale experiments, it is necessary to know the dependence of each of the factors individually, which demonstrates the usefulness of the study of model situations, some of which are examined here. At the same time, we are unaware of any published results of experiments performed with model media that would permit direct comparison with the results shown in Fig. 1. In Fig. 2, the dependence of the flicker index σ^2 on the diffraction parameter of the particles according to formula (17) is compared with experimental data [22] for optical thickness $\tau = 1$. Formula (17), which corresponds to a broad light beam can be used to describe experiments [22] with a narrow laser beam if the principle of optical reciprocity is employed (then the parameter R_0 has the meaning of the beam radius). The condition $f >> R_0/\gamma$ and the parameter values $kf = kh = 10^6$ and $\gamma = 10^{-2}$ rad, which are consistent with the experiment conditions [22], were used in the calculation. Some models that have been utilized [6-12, 21, 22] do not describe the observed rapid rise in the flicker index for small kR with subsequent much greater smooth saturation near 0.5. For example, the dashed curves in Fig. 2 correspond to the "direct path" approximation for two values of R_0. It is apparent that formula (17) provides a better description of the experimental data. The deviation of the calculated curve for large kR, which is expressed in a rise in the number of points over the theoretical value, is explained by the increasing role of multiple scattering, which is no longer negligible for $\tau = 1$ and large R. These effect will be examined in a subsequent article.

Conclusions. Nonclassical photometry has made it possible to obtain in a single-scattering approximation fairly intuitive formulas for the mean value and autocorrelation function (ACF) of the optical noise in a layer containing discrete scatterers with a strongly extended indicatrix. The case in which scattering was due to small-angle diffraction was examined using these formulas. It was shown, for example, that if the reading area of the optical receiver is much greater than the scatterer dimensions and all scatterers lie within the limits of the focusing depth, the ACF is determined by Fraunhofer diffraction. It was also shown that the ACF of the intensity in the far zone with respect to the particles oscillates and is characterized by three scales (particle size, the radius of the first Fresnel zone, and the scattering-spot size); but in the opposite limiting case, it degenerates to the ACF of overlapping disks. The calculations that were performed demonstrated the dependence of the optical correlation characteristics on the angular width of the illuminating beam, receiver aperture, and particle radius.

APPENDIX

Correlation Coefficients of Small-Angle Diffraction by Black Disks

Let $H_N(\boldsymbol{\rho}_1, \ldots, \rho_N)$ be the intersection area of N circles of unit radius with centers ρ_n:

$$H_N(\vec{\rho}_1, \ldots, \vec{\rho}_N) = \int \chi_1(\vec{\rho} + \vec{\rho}_1) \ldots \chi_1(\vec{\rho} + \vec{\rho}_N) d^2\rho,$$

$$\chi_R(\rho) = \theta(R - |\vec{\rho}|),$$

$$H(2\rho) \equiv H_2(-\vec{\rho}, \vec{\rho}) = 2(\arccos\rho - \rho^2\sqrt{1-\rho^2})\theta(1-\rho).$$

We shall calculate the function in (13)

$$\beta(\vec{\rho}, \vec{v}, \vec{\nu}) = \frac{1}{a} \int_{\mathbf{R}^4} \sigma_n(\vec{\rho}', \vec{v} - \vec{\nu}/2)\sigma_n(\vec{\rho} + \vec{\rho}', \vec{v} + \vec{\nu}/2)d^2\rho,$$

where the scattering index σ_n is determined by formula (9). A Fourier transform with respect to the angle variables gives

$$\tilde{\beta}(\vec{\rho}, \vec{q}, \vec{p}) \equiv \int_{\mathbf{R}^4} \exp[ik(\vec{q}\vec{v} + \vec{p}\vec{\nu})]\beta(\vec{\rho}, \vec{v}, \vec{\nu})d^2v d^2\nu =$$

$$= \Lambda^2\tilde{\beta}_0(\vec{\rho}, \vec{q}, \vec{p}) - 2\Lambda\tilde{\beta}_-(\vec{\rho}, \vec{q}, \vec{p}) + \tilde{\beta}_+(\vec{\rho}, \vec{q},) + \tilde{\beta}_+(\vec{\rho}, 2\vec{p}),$$

where

$$\tilde{\beta}_0(\vec{\rho}, \vec{q}, \vec{p}) = \frac{4}{\pi}H_4(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4),$$

$$\tilde{\beta}_-(\vec{\rho}, \vec{q}, \vec{p}) = [H_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) + H_3(\vec{y}_2, \vec{y}_3, \vec{y}_4) +$$

$$+ H_3(\vec{y}_3, \vec{y}_4, \vec{y}_1) + H_3(\vec{y}_4, \vec{y}_1, \vec{y}_2)]/\pi,$$

$$\tilde{\beta}_+(\vec{\rho}, \vec{p}) = [H(|\vec{\rho} + \vec{p}/2k|/R) + H(|\vec{\rho} - \vec{p}/2k|/R)/\pi,$$

$$\{\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4\} \equiv \left\{-\frac{\vec{\rho}}{2R} \pm \frac{\vec{q}+2\vec{p}}{4kR}, \frac{\vec{\rho}}{2R} \pm \frac{\vec{q}-2\vec{p}}{4kR}\right\}.$$

We renumber the points $\mathbf{y}_i$ such that the broken line $\mathbf{y}_1 \to \mathbf{y}_2 \to \mathbf{y}_3 \to \mathbf{y}_4$ does not intersect itself. For convenience, we continue the numbering of the vertices according to the cycle: $\mathbf{y}_0 = \mathbf{y}_4, \mathbf{y}_1 = \mathbf{y}_5$, etc. Let $\mathbf{y}_{ij} = \mathbf{y}_j - \mathbf{y}_i, y_{ij} = |\mathbf{y}_{ij}|$. Note that $H_4(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4) = 0$ if at least one of $y_{ij} \geq 2$. Let all $y_{ij} < 2$. It is obvious that $H_4(\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3, \mathbf{y}_4) = H_3(\mathbf{y}_{i-1}, \mathbf{y}_{i+1}, \mathbf{y}_{i+2})$ if of the unit circles with centers $\mathbf{y}_1, \dots, \mathbf{y}_4$ one circle with center $\mathbf{y}_i$ covers the intersection of the other three. The condition of such coverage

$$F_c(\vec{y}_{i-1}, \vec{y}_i, \vec{y}_{i+1}) \equiv \vec{y}_{i,i-1}\vec{y}_{i,i+1}(c_{i,i-1} + c_{i,i+1}) + F_3(\vec{y}_{i-1}, \vec{y}_i, \vec{y}_{i+1})(4c_{i,i-1}c_{i,i+1} - 1) \leq 0,$$

where $c_{ij} \equiv \sqrt{y_{ij}^{-2} - 1/4}$, and $F_3(\vec{y}_i, \vec{y}_j, \vec{y}_k) = \frac{1}{2}\sqrt{y_{ji}^2 y_{jk}^2 - (\vec{y}_{ji}\vec{y}_{jk})^2}$ is the area of a triangle with vertices $\mathbf{y}_i, \mathbf{y}_j, \mathbf{y}_k$.

Let $F_c(\mathbf{y}_{i-1}, \mathbf{y}_i, \mathbf{y}_{i+1}) > 0$ for any i. Then, the intersection of the four circles in question is a quadrangle with segments joined to it; the lengths of its sides

$$l_i = \Big[2 - (1/4)(y_{i,i-1}^2 + y_{i,i+1}^2 - y_{i-1,i+1}^2) + 2\vec{y}_{i,i-1}\vec{y}_{i,i+1} \times$$

$$\times c_{i,i-1}c_{i,i+1} - 2F_3(\vec{y}_{i-1}, \vec{y}_i, \vec{y}_{i+1})(c_{i,i-1} + c_{i,i+1})\Big]^{1/2},$$

and it area

$$F_4'(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) = (1/2)(F_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) + F_3(\vec{y}_2, \vec{y}_3, \vec{y}_4)) +$$

$$+ (1/4)\Big[(\vec{y}_{34} - \vec{y}_{12})(c_{12}\vec{y}_{12} - c_{34}\vec{y}_{34}) + (\vec{y}_{23} - \vec{y}_{41})(c_{41}\vec{y}_{41} -$$

$$- c_{23}\vec{y}_{23})\Big] + c_{12}c_{23}F_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) + c_{23}c_{34}F_3(\vec{y}_2, \vec{y}_3, \vec{y}_4) +$$

$$+ c_{34}c_{41}F_3(\vec{y}_3, \vec{y}_4, \vec{y}_1) + c_{41}c_{12}F_3(\vec{y}_4, \vec{y}_1, \vec{y}_2).$$

To obtain H_4, it is necessary to add to F_4' the total area of all segments that are based on chords l_i, $i = 1$-4. Finally, we have:

1) If $y_{ij} \geq 2$ for some i, j, then $H_4(y_1, y_2, y_3, y_4) = 0$.

2) If $y_{ij} < 2$ for all i, j, then two cases are possible:

2a) If $\exists\ i$: $F_c(y_{i-1}, y_i, y_{i+1}) \leq 0$, then

$$H_4(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) = H_3(\vec{y}_{1-1}, \vec{y}_{1+1}, \vec{y}_{1+2}),$$

2b) If $F_c(y_{i-1}, y_i, y_{i+1}) > 0$ for all i, then

$$H_4(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) = F_4'(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) + \sum_{1=1}^{4}\left[\arcsin\frac{l_1}{2} - \frac{l_1}{2}\sqrt{1 - l_1^2/4}\right].$$

A formula for $H_3(y_1, y_2, y_3)$ is thus derived:

1) If $y_{ij} \geq 2$ ($\{i, j\} \subset \{1, 2, 3\}$) for some i, j, then

$$H_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) = 0;$$

2) If $y_{ij} < 2$ for all i, j, then two cases are possible:

2a) If $\exists\ i \in \{1, 2, 3\}$: $F_c(y_{i-1}, y_i, y_{i+1}) \leq 0$, then

$$H_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) = H_2(\vec{y}_{1-1}, \vec{y}_{1+1}) \text{ (here } \vec{y}_0 = \vec{y}_3,\ \vec{y}_4 = \vec{y}_1);$$

2b) If $F_c(y_{i-1}, y_i, y_{i+1}) > 0$ for all i, then

$$H_3(\vec{y}_1, \vec{y}_2, \vec{y}_3) = F_3(\vec{y}_1, \vec{y}_2, \vec{y}_3)\ (\tfrac{1}{4} + c_{12}c_{13} + c_{13}c_{23} + c_{12}c_{13}) -$$

$$- \tfrac{1}{4}(y_{12}^2c_{12} + y_{13}^2c_{13} + y_{23}^2c_{23}) + \sum_{1=1}^{4}(\arcsin\frac{l_1}{2} - \frac{l_1}{2}\sqrt{1 - l_1^2/4}).$$

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