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INTENSITY FLUCTUATIONS OF LIGHT PROPAGATED IN SYSTEM OF LARGE SCATTERERS. II. MULTIPLE SCATTERING

A. Yu. Potekhin

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An equation is derived for the propagation of the autocorrelation function of radiant intensity in an optically thick layer of discrete scatterers with a prolate indicatrix. An effective method for its numerical solution is constructed. The autocorrelation function is computed for the light intensity in a medium containing large absorbing and scattering particles. Comparisons are made with known theoretical and experimental results.

An autocorrelation function (ACF) for the intensity fluctuations of a wave field behind a layer of large scatterers was found and its qualitative features were discussed earlier [1]. A single-scattering approximation was used that was valid for $\sigma h << 1$, where σ is the extinction coefficient and h is the thickness of the scattering layer. Here we shall remove this constraint and take into account multiple scattering in such a medium.

Similar problems have been solved [2-5] using an approximation of a delta-correlated random field (DCRF)* by extending it from the theory of light propagation in a turbulent medium to the case of a medium with particles. Svirkunov [2] introduced an equation for the statistical moments of the wave amplitude in a layer of optically soft, statistically independent particles. Conversely, Borovoi [3] examined optically hard particles. The scattered field was divided into shadow-forming and refracted parts and it was shown that first part, which is chiefly responsible for fluctuations, could be described in a direct-path approximation if the scatterers were large ($2\pi R/\lambda >> 1$) and the layer thickness $h << a/\lambda$, where $a = \pi R^2$ is the area of a particle and λ is the wavelength of the light. If the layer is of greater thickness ($h \geq a/\lambda$), small-angle diffraction must be taken into account. This idea was developed further by Barabanenkov and Kalinin [4], who took into account correlations in the locations of the scattering centers. Finally, Apresyan [5] recently examined the most general (and frequently encountered in practice [9]) case of the presence, along with arbitrarily correlated particles, of smooth refractive-index inhomogeneities and derived an equation that covered all of the previous results. It must be noted that numerical solutions have not yet been obtained for the equations [2-5] for the fourth-order coherence function in a medium with particles.

In all of the cited work, light propagation was described by means of a parabolic equation for amplitude. As is known [6], the DCRF method assumes that fluctuations of the dielectric constant are fairly smooth. This condition is satisfied only for optically soft particles [2, 5], and not for optically hard particles [3, 4], which makes the applicability limits uncertain. The first problem to be considered here is that of deriving an equation that would be suitable for optically hard and strongly absorbing particles. We shall employ nonclassical photometry [10], whose advantages in similar problems was discussed earlier [1]. The second and principal problem is that of constructing an effective method for the calculation of intensity fluctuations under conditions of multiple scattering by optically hard or absorbing particles whose dimensions are not negligible. As a practical application, a numerically confirmed explanation is given for the results of the experiments of Trabka and Doerner [11], who detected large deviations of the variance of intensity fluctuations in a layer of "black" particles from those predicted by the direct-path model.

The derivation of an equation for the ACF of radiant intensity in an optically thick layer of discrete scatterers is discussed and the main condition of its applicability is determined quantitatively in Section 1. In Section 2, this equation

*Or a "Markov approximation" [6]. The terminology "DCRF," which was introduced by Kopilevich [7], is clearer, since it cannot be confused with the more-general version of a Markov approximation [8] used to describe light propagation in a two-scale random medium.

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is defined concretely in a small-angle approximation for optically hard particles. In Section 3, it is compared with others [3, 4] and with a model [12] that does not take scattering into account, and the limits of applicability of this model are determined. A method for numerical solution of the obtained equation is described in Section 4. In Section 5, calculation results are presented and discussed and directly compared with experimental results.

1. Transport Equation for Radiant-Intensity Autocorrelation Function. Let us examine the propagation in the positive direction of the z axis of an average radiant intensity $\langle I(\vec{\rho}, z, \vec{s})\rangle$ and its ACF $W(z; \vec{\rho}, \vec{s}; \vec{\rho}', \vec{s}')$, where $\vec{\rho}$ is the "transverse" radius vector of point $\vec{r} = (\vec{\rho}, z)$, $\vec{s} = (\vec{s}_\perp, s_z)$ determines the ray direction, $\vec{s}_\perp$ is its transverse component, and the angle brackets indicate averaging. Just as earlier [1], we shall consider variable z to be a parameter. We shall employ the main idea of Chernov's "local method" [13], in which a single-scattering approximation is applied to an arbitrarily selected thin layer in the medium, which is assumed to be statistically independent from the remaining part of the medium. For this, it is necessary that

$$\sqrt{\sigma l} \ll 1, \tag{1}$$

where l is the effective thickness of the scatterer [1] (which in the case of a large "black" particle is considerably smaller than its geometric thickness, since scattering comes down to "diffraction by the contour"). Condition (1) expresses in quantitative form Apresyan's "Condition 2" [5] and makes it possible to select a scale L such that $l << L << \sigma^{-1} \leq n$. It follows from the inequality $l << L$ that in a layer of thickness L only a small fraction of the scatterers are located close enough to its boundaries to interact with the electromagnetic field beyond its limits. If we ignore these scatterers, we can assume that the radiant intensity $I(z)$ is statistically independent of the scatterers that lie ahead on the luminous-flux path and, in particular, in the layer $(z, z + L)$. The inequality $L << \sigma^{-1}$ means that a single-scattering approximation can be used in this layer. Consequently, $\langle I(z + L)\rangle$ and $W(z + L)$ can be related to $\langle I(z)\rangle$ and $W(z)$ using formulas obtained earlier [1]. It follows from the inequality $\sigma L << 1$ that in the layer in question the contribution of scattering to the total radiation flux is small, and we can use differential forms of the relationships between $\langle I(z + L)\rangle$ and $W(z + L)$ and $\langle I(z)\rangle$ and $W(z)$ (see Appendix).

As a result, equations are obtained that generalize [2-5] in two aspects. Firstly, $I(z)$ can be understood as the generalized [10] or the classical photometric intensity. In the latter case, expressing phenomenologically the scattering coefficient $\sigma(\vec{\rho}, \vec{s}, \vec{s}')$, we obtain a description of the situation in which the scatterers could be entire clouds [14, 15]. Secondly, we can go beyond the scope of a small-angle approximation if the difference of $s_z = \cos\theta$ from unity in (A.10) is taken into account (for example, expand in powers of $(\sin\theta)^2$ [16]). Here, however, we shall restrict ourselves to solving the equation for the ACF of the generalized radiant intensity in a small-angle approximation.

2. Small-Angle Approximation in Statistically Homogeneous Problem for Large Optically Hard Particles. We shall define concretely the form of the obtained equations in a small-angle approximation ($s_z \approx 1$) for the case — examined in detail earlier [1] — in which scattering is chiefly due to diffraction by the particle contour, where the location of a particle in the layer and its illumination are statistically uniform. With allowance for (A.11), Eq. (A.8) becomes a small-angle radiation-transport equation (RTE):

$$(\partial/\partial z + \sigma) \langle I(z, \vec{s}_\perp)\rangle = \sigma\Lambda \int_{\mathbb{R}^2} X(|\vec{s}_\perp - \vec{s}'_\perp|) \langle I(z, \vec{s}'_\perp)\rangle d^2s'_\perp, \tag{2}$$

where Λ is the scattering albedo and $X(\cdot)$ is the indicatrix. The solution of this equation in quadratures is well-known [17]:

$$\langle I(z, \vec{s}_\perp)\rangle = \int_{\mathbb{R}^2} G(z, |\vec{s}_\perp - \vec{s}'_\perp|) I(0, \vec{s}'_\perp) d^2s'_\perp \equiv \hat{G}\cdot I(0),$$

where

$$G(z, q) = \frac{e^{-\sigma z}}{2\pi} \int_0^\infty p \exp[\sigma\Lambda\tilde{X}(p)z]\, J_0(pq)\, dp. \tag{3}$$

The tilde indicates the Fourier transform of the angle variable:

$$\tilde{f}(\vec{p}) \equiv \int_{\mathbb{R}^2} f(\vec{s}_\perp) \exp(-i\vec{p}\vec{s}_\perp) d^2s_\perp. \tag{4}$$

Let us define Eq. (A.14) concretely. On the basis of (A.5) and (3) we conclude that $\hat{B}$ and $\hat{G}$ are commutative, and $\hat{B}_G = \hat{B}$ follows from (A.15). Converting in (A.14) to variables

$$\vec{\rho} = \vec{\rho}_2 - \vec{\rho}_1, \quad \vec{v} = (\vec{s}_{1\perp} + \vec{s}_{2\perp})/2, \quad \vec{w} = (\vec{s}_{2\perp} - \vec{s}_{1\perp}), \tag{5}$$

with allowance for (A.11) we obtain

$$(\partial/\partial z + \vec{w}\cdot\nabla_\rho) W_0(z, \vec{\rho}, \vec{v}, \vec{w}) = \frac{\sigma}{2} \int_{\mathbb{R}^4} \beta(\vec{\rho}, \vec{v} - \vec{v}', \vec{w} - \vec{w}') \times$$
$$\times \left[W_0(z, \vec{\rho}, \vec{v}', \vec{w}') + 1_W(0, \vec{v}', \vec{w}') \right] d^2v' d^2w'. \tag{6}$$

Equation (6) coincides in form with RTEs that take into account frequency redistribution, except that the analog of frequency $\vec{v}'$ is two-dimensional in our case.

We shall examine the simplest case of circular monodisperse particles of radius R in a layer illuminated by a light beam with angular width γ:

$$I(\vec{\rho}, 0, \vec{s}) = I_0 \cdot \theta(\gamma - s_\perp), \tag{7}$$

where θ is the Heaviside function.

For the average radiant intensity, on the basis of (2) and (3) we have

$$\langle I(z, \vec{s}_\perp)\rangle = I_0 e^{-\sigma z}\left\{\theta(\gamma - s_\perp) + \gamma \int_0^\infty J_1(p\gamma) \times \left(\exp(\sigma\Lambda\tilde{X}(p) - 1\right) J_0(ps_\perp) dp\right\} \equiv \bar{I}_\gamma(z, \vec{s}_\perp), \tag{8}$$

where $J_{0,1}(x)$ are Bessel functions.

The optical signal E was expressed in terms of radiant intensity I earlier [1]. We shall find its ACF. From (A.9), (A.10), and (8) it follows that

$$W_E(\vec{\rho}, \vec{v}, \vec{w}) = C_\Pi^2 R_0^2 \int_{\mathbb{R}^6} \bar{I}_\alpha(h, \vec{s}_{1\perp} - \vec{q}_1) \bar{I}_\alpha(h, \vec{s}_{2\perp} - \vec{q}_2) H(|\vec{\rho} - \vec{\rho}'|/R_0) \times$$
$$\times W_0(h, \vec{\rho}' - f\vec{q}_2 - \vec{q}_1), \quad (\vec{q}_1 + \vec{q}_2)/2, \quad (\vec{q}_2 - \vec{q}_1) d^2\rho' d^2q_1 d^2q_2 \tag{9}$$

(variables (5) are used here; just as earlier [1], α is the aperture angle of the receiver and f is the location of the plane of its best focus).

3. Comparison with Wave Theory and with "Direct-Path" Model. We shall show that Eq. (6) is equivalent to an equation proposed by Borovoi [3]. Applying Fourier transform (4) to both sides of (6) for each of the angle variables, we obtain

$$\left[\frac{\partial}{\partial z} - i\nabla_\rho\nabla_p - \frac{\sigma}{2}\tilde{\beta}(\vec{\rho}, \vec{q}, \vec{p})\right]\tilde{W}_0(z, \vec{\rho}, \vec{q}, \vec{p}) = \frac{\sigma}{2}\tilde{\beta}(\vec{\rho}, \vec{q}, \vec{p})\tilde{I}_W(0, \vec{q}, \vec{p}), \tag{10}$$

where

$$\vec{q} = \vec{p}_1 + \vec{p}_2, \quad \vec{p} = (\vec{p}_2 - \vec{p}_1)/2. \tag{11}$$

Formulas for function $\tilde{\beta}$ were presented earlier [1].

Advantages of Eq. (10) over (6) include the absence of an integral term and the fact that it can be solved independently for each $\vec{q}$, if $\vec{q}$ is considered a parameter. Its main shortcomings include the presence of second derivatives and the need for a four-dimensional Fourier transform to convert to radiant intensities.

From (A.9), (A.10), (3), and (11),

$$\tilde{W} = \exp\left[2\sigma z - \sigma\Lambda z(\tilde{X}(p_1) + \tilde{X}(p_2))\right]\tilde{W}_0,$$

$$\tilde{I}_W = \exp\left[\sigma\Lambda z(\tilde{X}(p_1) + \tilde{X}(p_2))\right]\tilde{I}(0, p_1)\tilde{I}(0, p_2).$$

We introduce the function $\Gamma_4(\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4) = \tilde{W} + I_W$, where

$$\{\vec{y}_1, \vec{y}_2, \vec{y}_3, \vec{y}_4\} \equiv \left\{-\frac{\vec{\rho}}{2R} \pm \frac{\vec{q} + 2\vec{p}}{4kR}, \frac{\vec{\rho}}{2R} \pm \frac{\vec{q} - 2\vec{p}}{4kR}\right\}.$$

Considering that $\vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \vec{y}_4 = 0$, we have

$$\left[\frac{\partial}{\partial z} + \frac{i}{2kR^2}\left(\nabla^2_{y1} - \nabla^2_{y2} - \nabla^2_{y3} + \nabla^2_{y4}\right) + 2\sigma\right]\Gamma_4 = \sigma\tilde{\beta}\Gamma_4. \tag{12}$$

Equation (12) coincides with the equation for the fourth statistical moment of the wave field obtained by Borovoi [3]. They coincide because the small-angle approximation that we used to move from (A.15) to (6), just as the parabolic equation employed by Borovoi [3], is equivalent to a Fresnel approximation in the description of wave propagation in free space. Our derivation method has some advantages, however. Firstly, we did not employ the concept of the delta-correlated random field of the dielectric constant, which Borovoi [3] extended to the case of optically hard particles from the theory of light fluctuations in a continuous turbulent medium [6]. Accordingly, the constraints on the magnitude and "abruptness" of the dielectric-constant fluctuations that are characteristic of this theory lose significance. Secondly, more-general equations are derived (see Appendix), which, in particular, enable us to go beyond the limits of a small-angle approximation.

Equations (6) and (10) permit passage to the limit of a model of the fluctuations of light transmitted by a layer of particles, which is known in photography as the "random-disk model" [12] and in atmospheric optics corresponds to the "direct-path approximation" [3]. This model does not allow for deformation of the scattered wave in its propagation inside the layer, which corresponds to the solution of Eq. (10) without the term $i\nabla_\rho\nabla_p W_0$:

$$\vec{W}_0(h, \vec{\rho}, \vec{p}, \vec{q}) = I_0^2\left(\exp(n\alpha\tilde{\beta}(\vec{\rho}, \vec{p}, \vec{q})) - 1\right), \tag{13}$$

where $n = n_V h$ is the number of scatterers per unit of layer surface. Equality (13) corresponds to the formula for the ACF derived by Benton [12] and is widely used in atmospheric optics [18]. The method of its derivation allows us to determine the limits of applicability of this approximation.

In fact, the discarded term $i\nabla_\rho\nabla_p = W_0$ is equal in order of magnitude to $W_0/\rho_0 p_0$, where ρ_0 and p_0 are the characteristic scales of variation of W_0 with respect to variables $\vec{\rho}$ and $\vec{p}$. Considering this term as a small correction and integrating (10), we see that the relative error is on the order of $h/\rho_0 p_0$. It follows from (7) that the characteristic scale of $\tilde{I}_W$ variation is equal to γ^{-1}, and it is apparent from the earlier formulas [1] that the scales of $\tilde{\beta}$ variation with respect ρ and p are equal to R and kR, respectively. Thus, $p_0 \sim \min(\gamma^{-1}, kR)$ and $\rho_0 \sim R$, and for the validity of approximate solution (13) we obtain the condition

$$h \ll \min(R/\gamma, kR^2).$$

If (6) is considered an RTE, it is easy to understand the physical meaning of this condition. It consists of the requirement that the layer thickness h be small enough to make the displacement of the direct ($\sim\gamma h$) and scattered ($\sim h/kR$) rays negligible.

4. Method for Numerical Calculation of Intensity ACF. Now we shall solve Eq. (10) beyond the framework of the direct-path approximation. A similar equation was solved earlier as applied to problems of light propagation in a turbulent atmosphere.

The first numerical solutions for the fourth-order coherence function were obtained [19] by means of a net method, which, however, was not economical in the sense of Samarskii [20], since the explicit difference scheme used [19] was not unconditionally stable. A stable inexplicit scheme has been used [21-23], but only for a two-dimensional scattering medium. Direct numerical modeling of realizations of the dielectric-constant field and phase advances has been employed [24-26]. Using a Fourier transform, Elepov and Mikhailov [27] reduced a DCRF equation to a transport equation similar to (6), which was solved by statistical modeling. But in the case of scattering by particles, the integral kernel β in (6) has an oscillatory nature, but, as is known [28], in the presence of bipolar contributions to the solution, this method loses effectiveness, unlike a net method.

We shall construct a net method for solution of Eq. (10), in which for simplicity we shall assume $q = 0$, which corresponds to the case in which we are interested only in the correlation function of the radiant intensity $W(\vec{\rho}, \vec{v}, \vec{w})$, which is integrated within infinite limits with respect to $\vec{v}$. This is entirely sufficient for calculation of the ACF of the signal W_E is the aperture angle of the receiver α is much wider than the brightness body in the medium: in (9), $\bar{I}_\alpha$ varies slowly in comparison with W_0, and $\bar{I}_\alpha$ can be removed from under the integral sign. The receiver in this case detects the total intensity. We denote the relative ACF of intensity as

$$W_I(\rho) = W_E(\vec{\rho}, 0, 0)/\langle E\rangle^2 = \tilde{W}_0(h, \vec{\rho}, 0, 0)/I_0^2.$$

As in the case of the Schrödinger equation examined by Samarskii [20], it can be shown that an explicit difference scheme for Eq. (10) is unstable. An inexplicit scheme for a similar equation has been used [21-23] to examine a two-dimensional model of a turbulent medium and develop an efficient algorithm for solution of the corresponding system of difference equations. With transition to a three-dimensional medium, the dimensionality of the space in which the operator $\nabla_\rho \nabla_p$ acts increases from 2 to 4, and the algorithm of Liu et al. [22] loses strength. For this reason, we turned to a sum-approximation method [20]. It requires that the multidimensional differential operator in the equation to be solved be represented as a sum of one-dimensional operators, for which we move to new variables:

$$\vec{\rho}' = \vec{\rho}/R + \vec{p}/kR, \qquad \vec{\rho}'' = \vec{\rho}/R - \vec{p};$$

$$\rho' \equiv |\vec{\rho}'|, \qquad \rho'' \equiv |\vec{\rho}''|, \qquad \varphi = \arccos\left[((\rho/R)^2 - (p/kR)^2)/(\rho'\rho'')\right].$$

Then (10) can be written as follows:

$$\hat{D}w = b(w + \Phi), \qquad w|_{\tau=0} = 0, \tag{14}$$

where

$$\hat{D} \equiv \frac{\partial}{\partial\tau} - ig(\nabla^2_{\rho'} - \nabla^2_{\rho''}) = \frac{\partial}{\partial\tau} - ig\left[\frac{1}{\rho'}\frac{\partial}{\partial\rho'}\left(\rho'\frac{\partial}{\partial\rho'}\right) - \frac{1}{\rho''}\frac{\partial}{\partial\rho''}\left(\rho''\frac{\partial}{\partial\rho''}\right) + \left(\frac{1}{\rho'^2} - \frac{1}{\rho''^2}\right)\frac{\partial^2}{\partial\varphi^2}\right], \tag{15}$$

$\tau = \sigma z$ is the optical thickness, $g = 1/(\sigma k R^2)$ is a parameter that characterizes the diffraction spreading of the light beam between successive acts of scattering,

$$b(\rho', \rho'', \varphi) \equiv \tilde{\beta}(R(\vec{\rho} + \vec{\rho}')/2, \ 0, \ kR(\vec{\rho}' - \rho'')/2) \tag{16}$$

(note that explicit expressions for β were presented earlier [1]), and

$$w(\rho', \rho'', \varphi) = \tilde{W}_0(\tau/\sigma, \ R(\vec{\rho}' + \vec{\rho}'')/2, \ 0, \ kR(\vec{\rho}' - \vec{\rho}'')/2),$$

$$\Phi(\rho', \rho'', \varphi) = \tilde{I}'_w(0, \ 0, \ kR(\vec{\rho}' - \vec{\rho}'')/2) = \left[4\pi\gamma J_1(kR\gamma|\vec{\rho}' - \vec{\rho}''|/2)/(k|\vec{\rho}' - \vec{\rho}''|)\right]^2 \tag{17}$$

The operator $\hat{D}$ in (15) is represented as a sum of one-dimensional differential operators, which permits the use of a decomposition method [20]. Without providing a detailed description of the locally one-dimensional difference scheme, which was done earlier [29], we shall point out its main steps. A net is introduced that is uniform with respect to each of the variables; the second-order differential operators in (15) are replaced in a standard manner by three-point difference operators, after which a "layer-by-layer" (with respect to τ) solution is carried out. In each layer, instead of a multidimensional difference equation, a chain of one-dimensional equations with difference operators that approximate the one-dimensional components of operator $\hat{D}$ is solved. An inexplicit scheme is constructed for each equation of this chain and is solved by a drive-through method [20].

The stability of such an algorithm has been proved, the degree of approximation has been determined, and the problem of the boundary conditions for w for large ρ' and ρ'' has been considered earlier [29]. Below we shall examine the results of numerical calculation and their physical consequences.

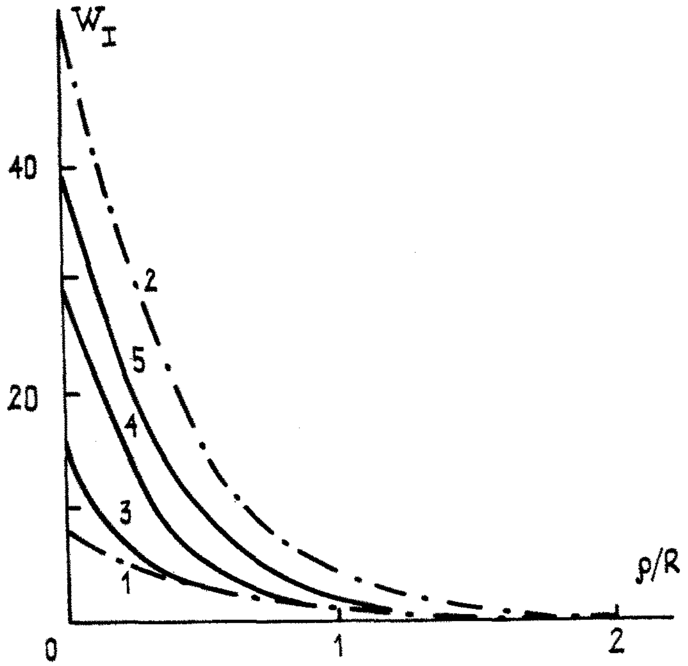

Fig. 1

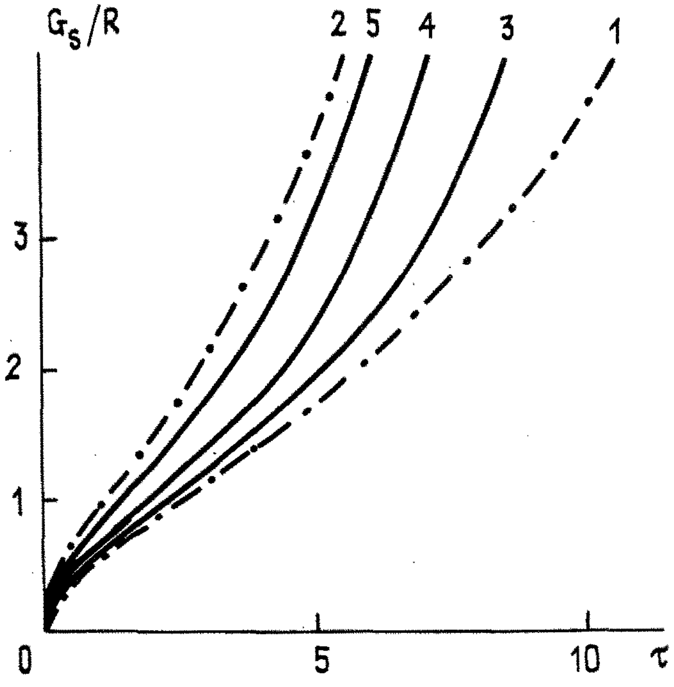

Fig. 2. The curves correspond to those in Fig. 1.

5. Dependence of Optical Fluctuations on Parameters of Scattering Layer. The procedure described in Section 4 was used to calculate the ACF of light intensity $W_I(\rho)$ in an optically thick layer with circular absorbing particles. The calculation accuracy was controlled by variation of the external boundary conditions and the mesh size of the grid. Below are the intensity ACF for layer optical thickness $\tau = 4$ (the results were similar for other thicknesses) and the dependence on τ of Selvin's parameter [11, 12], which determines the standard deviation of the optical density σ_D for reading from a large area A:

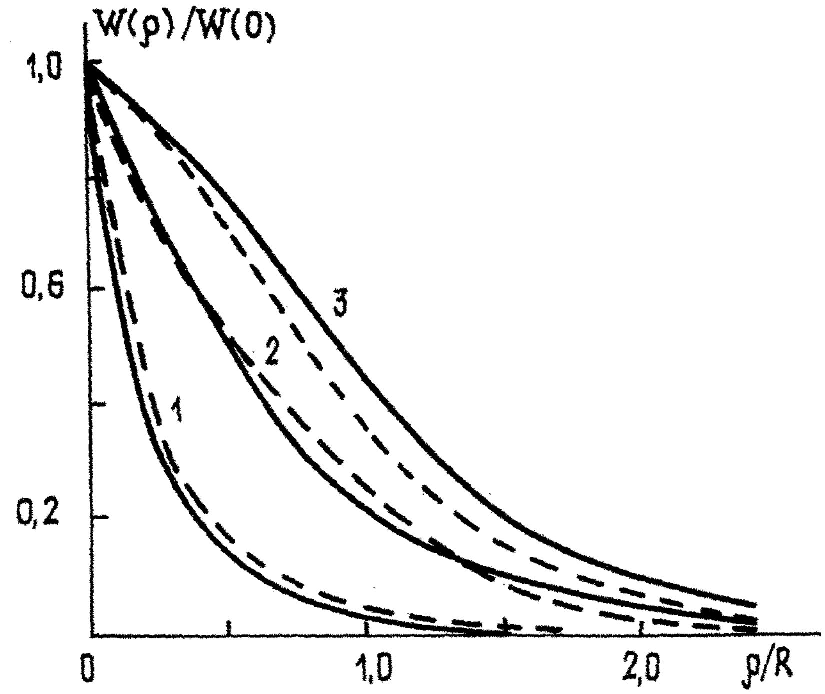

Fig. 3. Solid curves) $g = 0.5$; dashed curves) $g = 0.2$; 1) $\gamma_* = 0$; 2) $\gamma_* = 0.2$; 3) $\gamma_* = 1$.

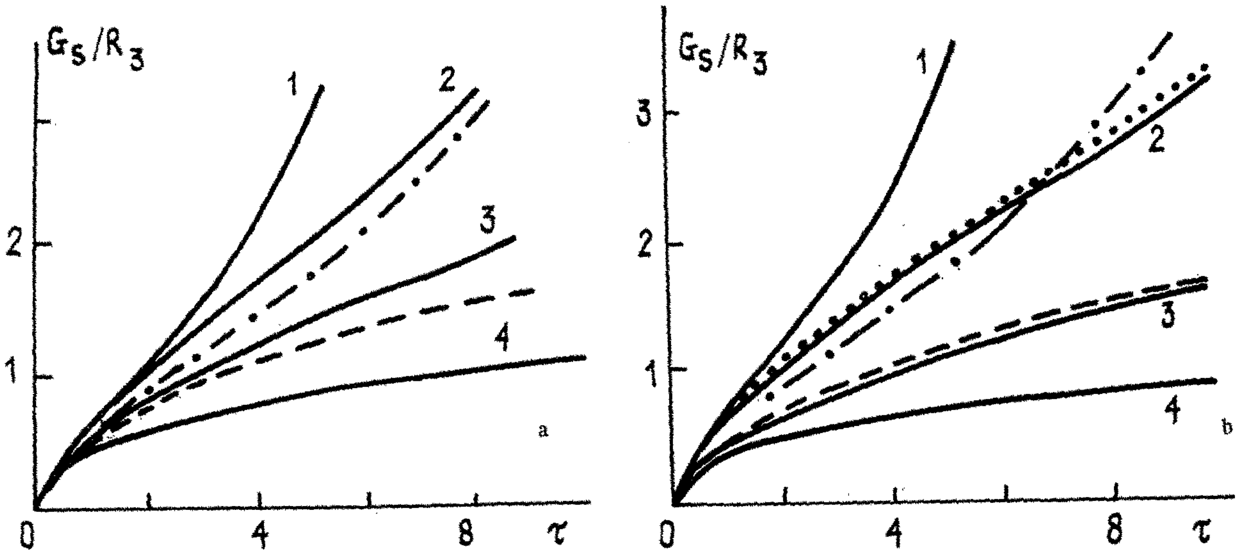

Fig. 4. Dot—dash curves) "disk model" [12]; dashed curves) "disk model" without "overlap effects" [12]; points) approximation of experimental data [11]; solid curves) calculation results: a) $g = 0.5$; b) $g = 1$; 1) $\gamma_* = 0$; 2) $\gamma_* = 0.5$; 3) $\gamma_* = 1$; 4) $\gamma_* = 2$.

$$G_S = \sigma_D\sqrt{A} = \lg e\left[2\pi \int_0^\infty \rho W_1(\rho)\,d\rho\right]^{1/2}. \tag{18}$$

The first series of calculations corresponds to collimated illumination: $\gamma << \gamma_1 \equiv \lambda/2R$. In Fig. 1 is the intensity ACF referred to the square of the average intensity for various values of the light-scattering parameter g. The dot—dash curves correspond to a direct-path model [3] that ignores scattering. With an increase in parameter g, an increase in the central maximum of the ACF is observed, which is explained by the appearance of a speckle structure with a long free path $l_p \equiv 1/\sigma$; conversely, this structure is "blurred" when collisions are frequent (i.e., with short l_p). Figure 2 shows

Selvin's parameter as a function of optical thickness for various g values; for comparison, the dot—dash curves represent corresponding "disk models" [12] with areas of πR^2 (lower curve) and $2\pi R^2$ (upper curve). A doubled "disk" area corresponds to a doubling of the energy from the incident beam with allowance for diffraction (along with absorption). The actual curve of $G_S(\tau)$ is close to the lower and upper curves, respectively, for small and large g values, which is obviously due to scattering: for small g, during the free-path time, most of the scattered photons do not deviate appreciably from a direct path.

The second series of calculations was performed for illumination that was not strictly collimated: $\gamma \geq \gamma_1$. The normalized intensity ACF is shown in Fig. 3 for various values of parameters g and $\gamma_* \equiv \gamma/\gamma_1$. It can be seen how the central peak of the ACF, which corresponds to a speckle structure, is suppressed and the correlation radius is simultaneously increased with an increase in γ_*.

In Fig. 4 is a graph of parameter (18) as a function of optical thickness for various values of g and γ_*. The dot—dash curves correspond to the model without scattering [12]. It has been found [11] that the experimental dependence $G_S(\tau)$ is best approximated by the model without scattering, in which the absorbing disks have an absorption coefficient of 0.41 and dimensions that exceed the true particle dimensions by a factor of 2 or 3. The corresponding dependence, which is represented by points in Fig. 4, practically coincides with one of our calculated curves, from which it is apparent that the deviations from the model [12] are explained by multiple small-angle diffraction by the particles. This conclusion was first advanced elsewhere [30]. It follows that from an examination that the values of the fitting parameters found by Trabka and Doerner [11] are not universal but are functions of the experiment conditions, since the graph of $G_S(\tau)$ are affected by the aperture angle of the illumination and parameters g and γ_1, which characterize the scattering.

Conclusions. Under fairly general assumptions about the effect of scatterers on the radiation, an operator equation is obtained for the propagation of the autocorrelation function (ACF) of radiant intensity. The concrete definition of this equation as applied to optically hard or absorbing particles using a small-angle approximation results in an equation that was introduced earlier [2, 5] for optically soft particles and extended [3, 4] to the case of optically hard particles by a model approach. A procedure for calculation of the radiant-intensity ACF was developed and implemented that can be effective for both disperse and turbulent scattering media.

The calculation results provide information on the autocorrelation functions of optical signals recorded behind a layer of chaotically situated large absorbing particles, which is necessary for interpretation of the results of optical measurements in stochastic media with discrete scatters and for prediction of the noise properties of such optical media.

APPENDIX

Derivation of Equations for Autocorrelation Function of Generalized Radiant Intensity in Disperse Medium

Let, as earlier [1], in the absence of scatterers

$$I(z + \Delta z) = \hat{T}(z, \Delta z) I(z), \qquad I(z) \equiv I(\vec{\rho}, z, \vec{s}), \tag{A.1}$$

and $\hat{\sigma}_n$ is the operator for scattering and absorption by the n-th particle [1], which is an integral operator with respect to the angle variables, with kernel $\sigma_n(\vec{\rho} - \vec{\rho}_n, \vec{s}, \vec{s}\,')$. Then from the single-scattering formula

$$I(h) = \hat{T}(0, h) I(0) + \sum_n \hat{T}(z_n, h - z_n)\hat{\sigma}_n \hat{T}(0, z_n) I(0)$$

it is easy to formulas for the average radiant intensity

$$\langle I(h)\rangle = \left[\hat{T}(0, h) + \int_0^h \hat{T}(z, h - z)\hat{S}\hat{T}(0, z)dz\right]\cdot\langle I(0)\rangle, \tag{A.2}$$

where $\hat{S}$ is an integral operator for $\vec{s}$ with kernel

$$S(\vec{s}, \vec{s}\,') = \int n_v \langle \sigma_n(\vec{\rho}, \vec{s}, \vec{s}\,')\rangle d^2\vec{\rho}. \tag{A.3}$$

Similarly, for the radiant-intensity ACF

$$W(z;\ \vec{\rho}, \vec{s};\ \vec{\rho}', \vec{s}') \equiv \langle I(\vec{\rho}, z, \vec{s}) I(\vec{\rho}', z, \vec{s}')\rangle - I_W(z;\ \vec{\rho}, \vec{s};\ \vec{\rho}', \vec{s}'),$$

where

$$I_W(z;\ \vec{\rho}, \vec{s};\ \vec{\rho}', \vec{s}') \equiv \langle I(\vec{\rho}, z, \vec{s})\rangle\langle I(\vec{\rho}', z, \vec{s}')\rangle,$$

we find [1]

$$\begin{aligned} W(h) = &\Big\{\hat{T}(0, h) \otimes \hat{T}(0, h) + \hat{T}(0, h) \otimes \int_0^h \hat{T}(z, h-z)\cdot\hat{S}\cdot\hat{T}(0, h)dz + \\ &+ \Big[\int_0^h \hat{T}(z, h-z)\cdot\hat{S}\cdot\hat{T}(0, z)dz\Big] \otimes \hat{T}(0, h)\Big\}\cdot W(0) + \\ &+ \Big[\int_0^h \Big(\hat{T}(z, h-z) \otimes \hat{T}(z, h-z)\Big)\cdot\hat{B}\cdot(\hat{T}(0, z) \otimes \hat{T}(0, z))dz\Big]\cdot(W(0) + I(0)), \end{aligned} \tag{A.4}$$

where $\otimes$ represents an operator direct product, W and I_W are considered functions of the pair of points $(\vec{\rho}, \vec{s}; \vec{\rho}', \vec{s}')$, which are functions of the parameter z, and the operator of scattered-radiation covariance $\hat{B}$ is an integral operator with respect to the angle variables $(\vec{s}, \vec{s}')$ with kernel

$$\hat{B}(\vec{\rho}, \vec{\rho}', \vec{s} \leftarrow \vec{s}'', \vec{s}' \leftarrow \vec{s}''') = \int n_v \langle\sigma_n(\vec{\rho} + \vec{\rho}'', \vec{s}, \vec{s}'')\sigma_n(\vec{\rho}' + \vec{\rho}'', \vec{s}', \vec{s}''')\rangle d^2\rho''. \tag{A.5}$$

The conditions under which (A.2) and (A.4) were derived were explained in greater detail earlier [1].

It follows from the inequality $\sigma L << 1$ that the integral terms in these equations are small. We shall linearize them with respect to the parameter σL:

$$\langle I(z + L)\rangle - \langle I(z)\rangle = (\hat{T}(z, L) - 1 + L\hat{S})\ \langle I(z)\rangle; \tag{A.6}$$

$$\begin{aligned} W(z + L) - W(z) = &(\hat{T}(z, L) \otimes \hat{T}(z, L) - 1)W(z) + \\ &+ L[\hat{T}(z, L) \otimes \hat{T}(z, L)]\cdot[(\hat{S}_1 + \hat{S}_2)W(z) + \hat{B}(W(z) + I_W(z))], \end{aligned} \tag{A.7}$$

where $\hat{S}_1 = \hat{S} \otimes 1$ and $\hat{S}_2 = 1 \otimes \hat{S}$.

It is obvious that $\hat{T}(z, 0) = 1$. We shall assume that operator $\hat{T}(z, \Delta z)$ is differentiable with respect to parameter Δz and that the layer thickness L is small enough that $\hat{T}(z, L)$ has little effect on functions $\langle I\rangle$ and W:

$$\|(\hat{T}(z, L) - 1)\langle I\rangle\| \ll \|\langle I\rangle\|, \qquad \|(\hat{T}_{1,2}(z, L) - 1)W\| \ll \|W\|.$$

We move to differential forms of Eqs. (A.6) and (A.7):

$$d\langle I(z)\rangle/dz = (\hat{t}(z) + \hat{S})\langle I(z)\rangle; \tag{A.8}$$

$$dW(z)/dz = (\hat{t}_1 + \hat{t}_2 + \hat{S}_1 + \hat{S}_2)W(z) + \hat{B}(W(z) + I_W(z)), \tag{A.9}$$

where $\hat{t}_1 = \hat{t}(z) \otimes 1$, $\hat{t}_2 = 1 \otimes \hat{t}(z)$, and

$$\hat{t}(z) = \frac{d\hat{T}(z, \varepsilon)}{d\varepsilon}\Big|_{\varepsilon \to +0}. \tag{A.10}$$

If the scatters are located in a homogeneous nonabsorbing medium. then, according to (A.1) and (A.10),

$$\hat{t} = -\frac{\vec{s}_\perp}{s_z}\nabla_\rho. \tag{A.11}$$

It follows from (A.3) that operator $\hat{S}$ describes both the scattering and absorption of light by discrete inhomogeneities; therefore, (A.8), with allowance for (A.11), becomes the classical transport equation for the average radiant intensity [17]. Note that here it is derived without use of the concept of an elementary volume (such a result has been obtained for optically soft particles [5]).

Let us consider the case of uniform illumination and a statistically homogeneous scattering layer. Then $\hat{t}\langle I\rangle = 0$, and

$$<I(\vec{\rho}, z, \vec{s})> = \int_{\Omega} G(z; \vec{s}, \vec{s}\,')<I(\vec{\rho}\,', 0, \vec{s}\,')>d\Omega_{s'} \equiv \hat{G}(z)<I(0)>, \tag{A.12}$$

where G is the surface Green's function, which satisfies the transport equation

$$\frac{dG(z)}{dz} = \hat{S}G(z)$$

with the singular boundary condition

$$G(0; \vec{s}, \vec{s}\,') = \sigma_{\Omega}(\vec{s}, \vec{s}\,').$$

Representing function W as

$$W(z) = (\hat{G}(z)\otimes\hat{G}(z))W_0(z), \tag{A.13}$$

from Eqs. (A.8), (A.9), and (A.12) we obtain

$$\frac{dW_0(z)}{dz} = (\hat{t}_1 + \hat{t}_2 + \hat{B}_G)W_0(z) + \hat{B}_G I_W(0), \tag{A.14}$$

where

$$\hat{B}_G \equiv (\hat{G}(z)\otimes\hat{G}(z))^{-1}\cdot\hat{B}\cdot(\hat{G}(z)\otimes\hat{G}(z)). \tag{A.15}$$

If function G is known or is easily computed, Eq. (A.14) can be more useful than (A.9).

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