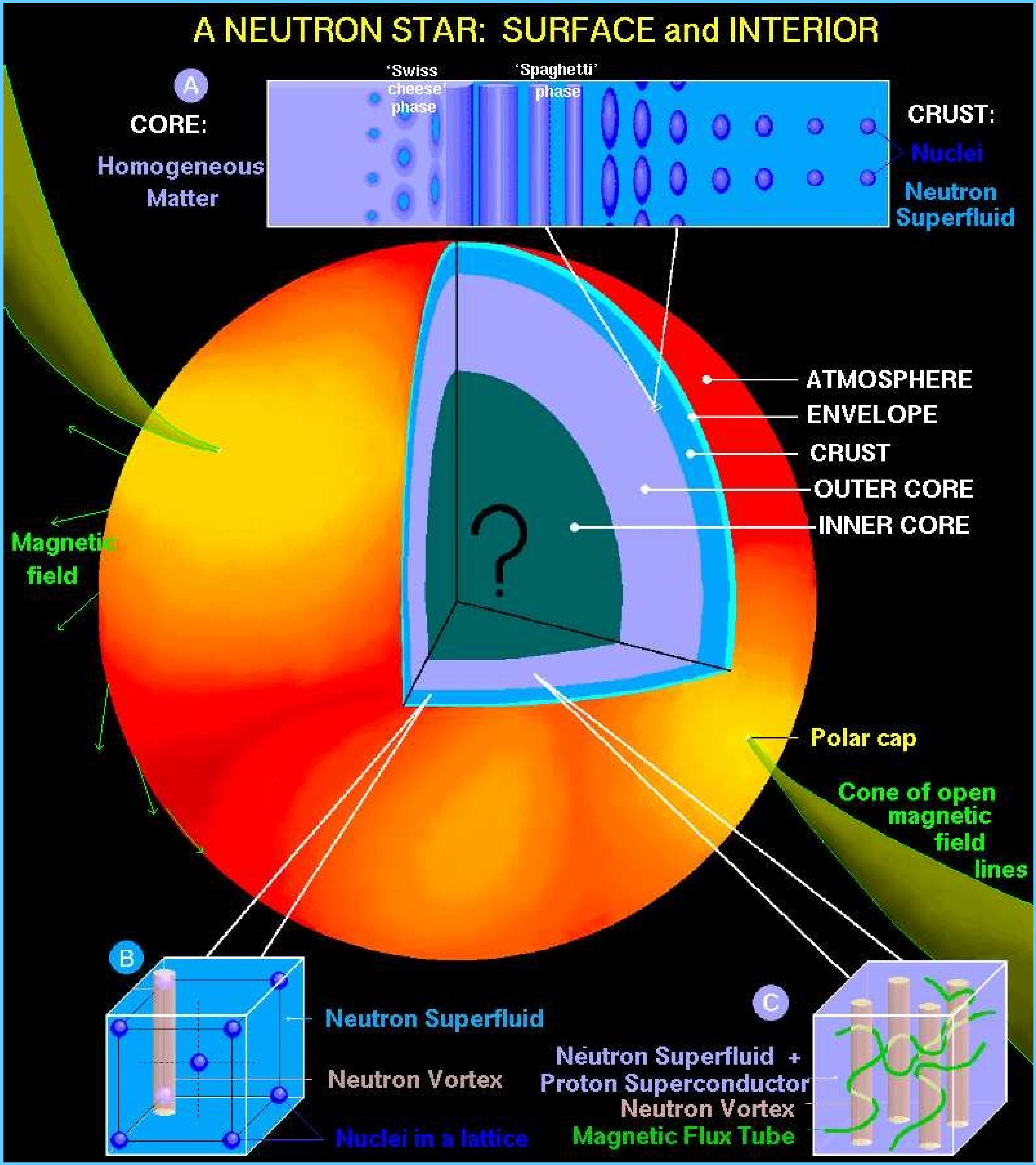
Neutron matter Equation of State at very low density

StPetersburg june 2008

A section (schematic)

of a neutron star



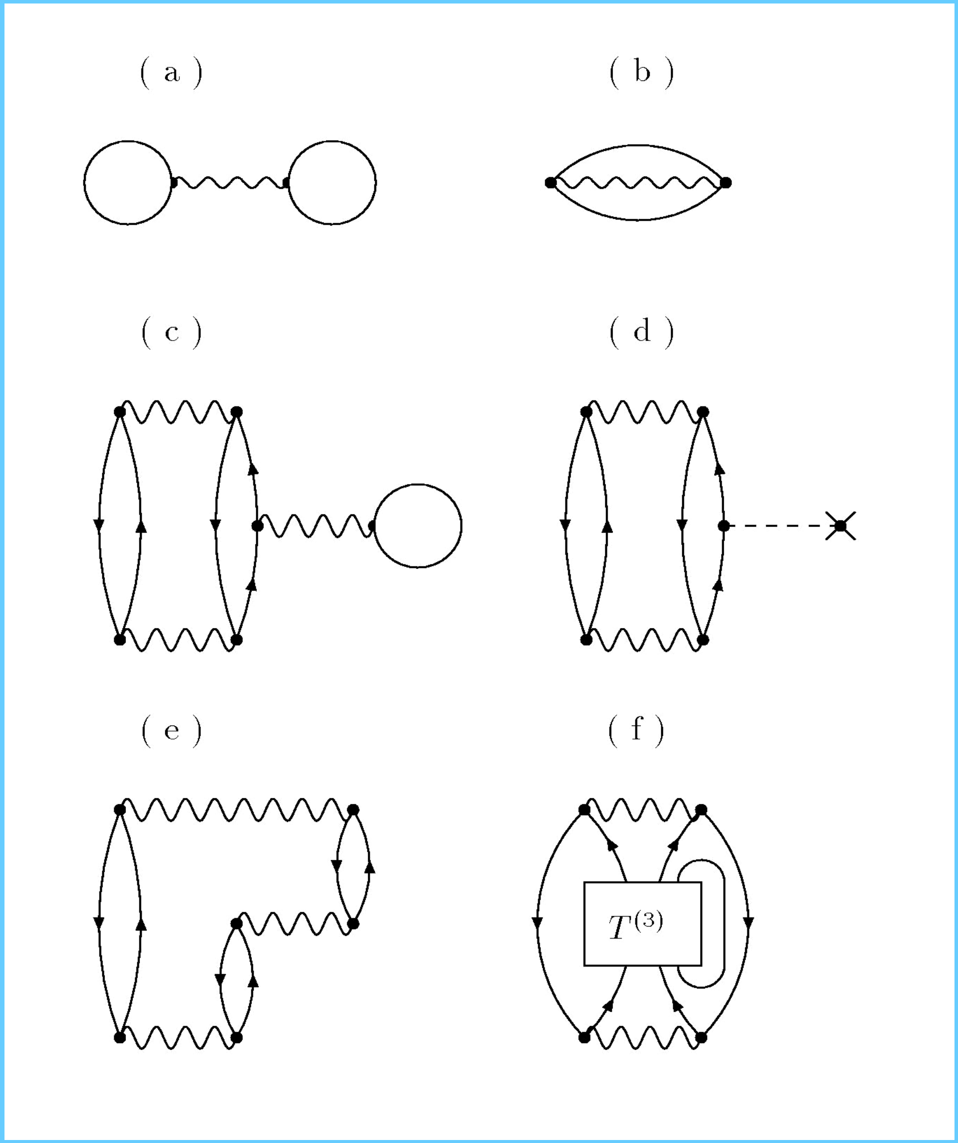
Neutron matter studies

Motivations

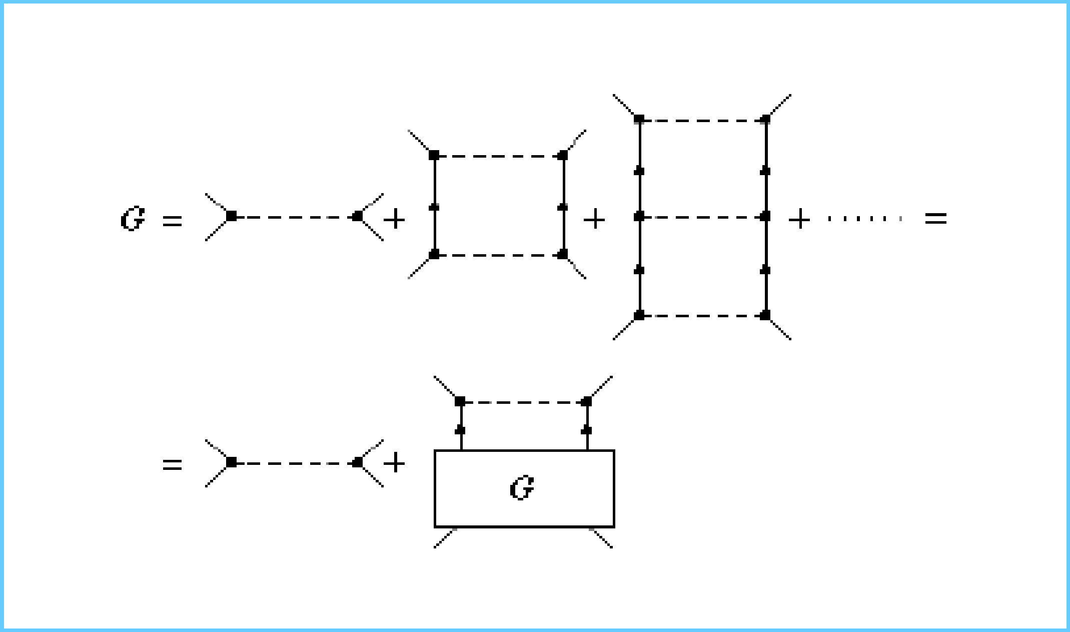
1. Low density neutron matter in the drip region

3. "Close" to the unitary limit (-a → ∞)

3. Set the uncertainity in the many-body treatment

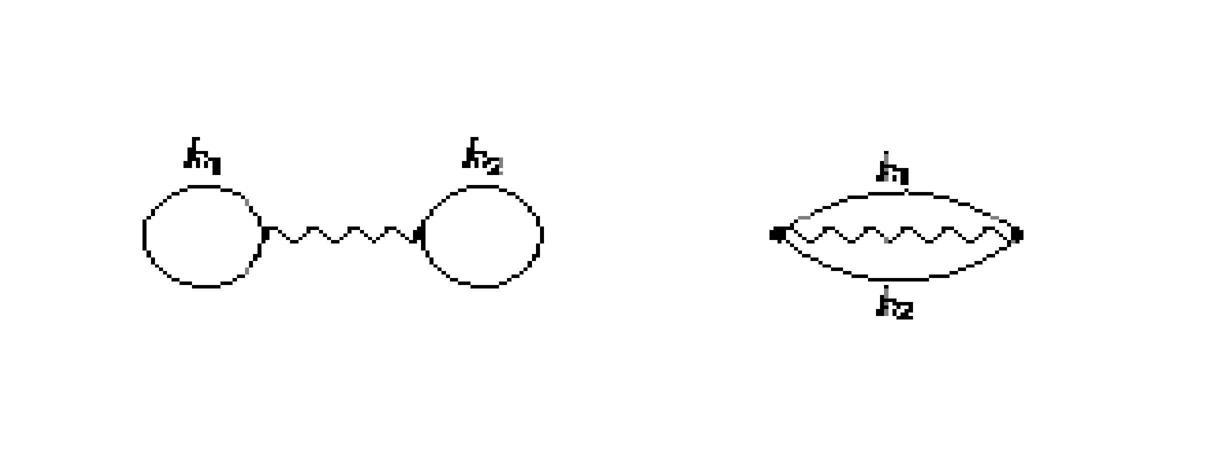


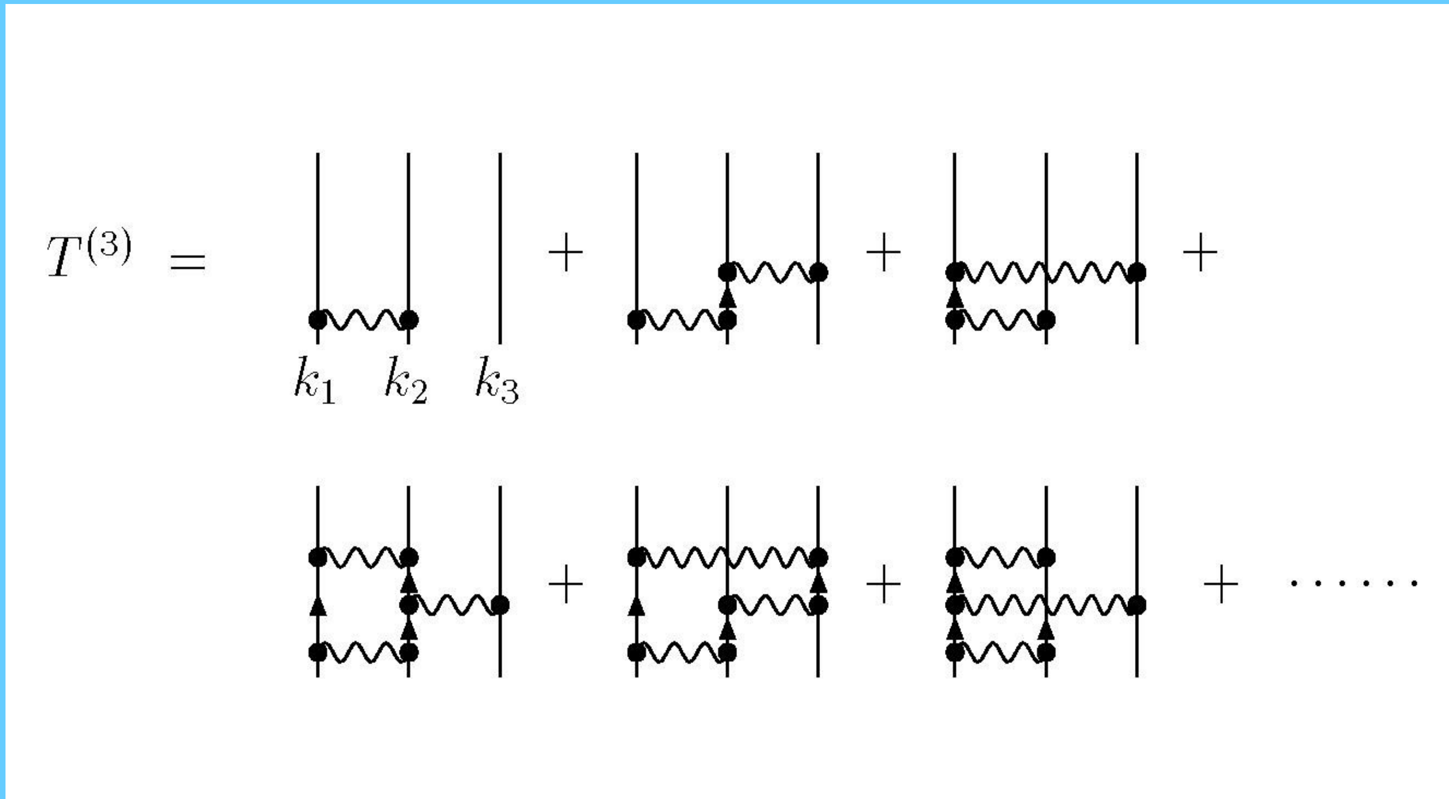
Two and three hole-line diagrams in terms
of the Brueckner G-matrixs



Ladder diagrams for the scattering G-matrix

$$G = V + V \frac{Q}{e} G$$





The ladder series for the three-particle scattering matrix

$$T_3 = G + GX\frac{Q_3}{e}T_3$$

$$E_{3h} =$$

$$\frac{1}{2}\sum_{k_1k_2k_3}\sum_{[k'k'']} < k_1k_2 | G | k_1'k_2' >_A$$

$$\frac{1}{e} < k_1'k_2'k_3 | XT_3X | k_1''k_2''k_3 > \frac{1}{e'}$$

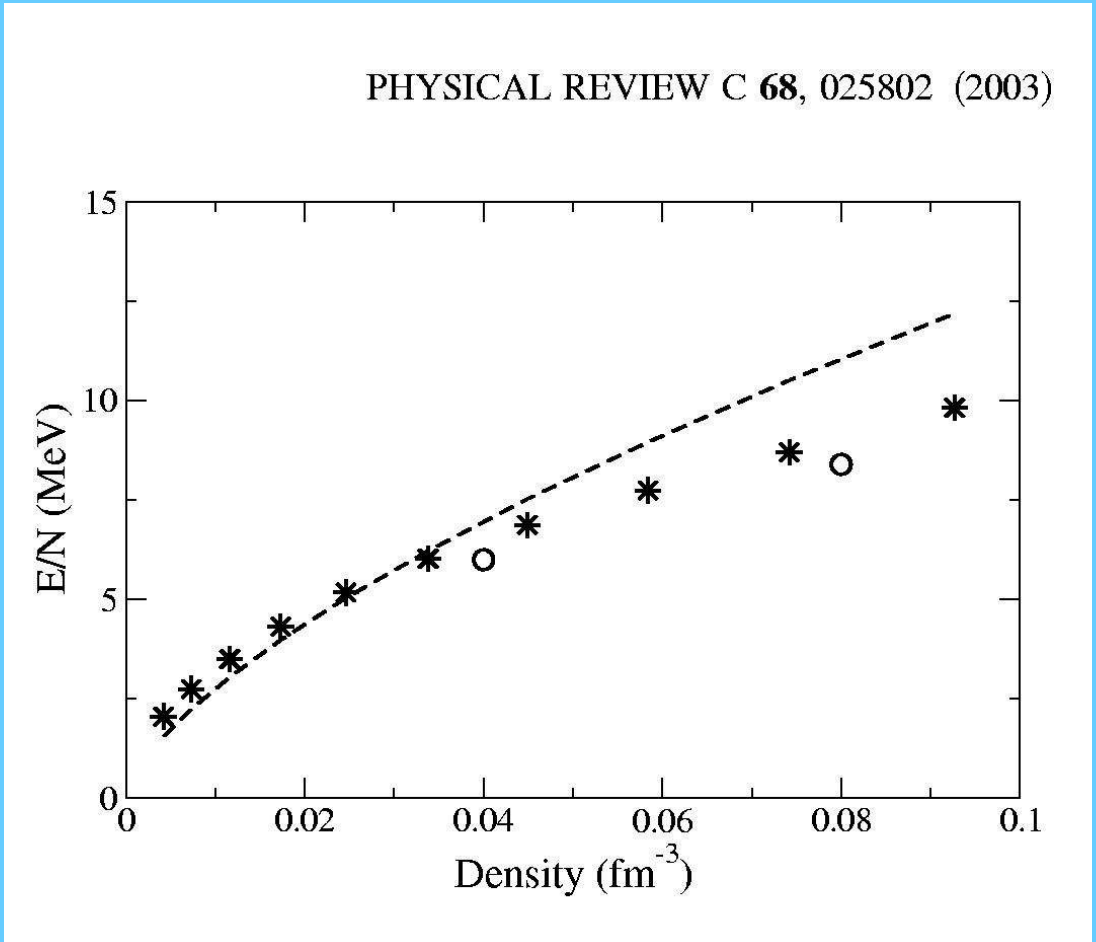
$$< k_{1''}k_2'' | G | k_1k_2 >_A$$

$$k_1, k_2, k_3 \leq k_F$$

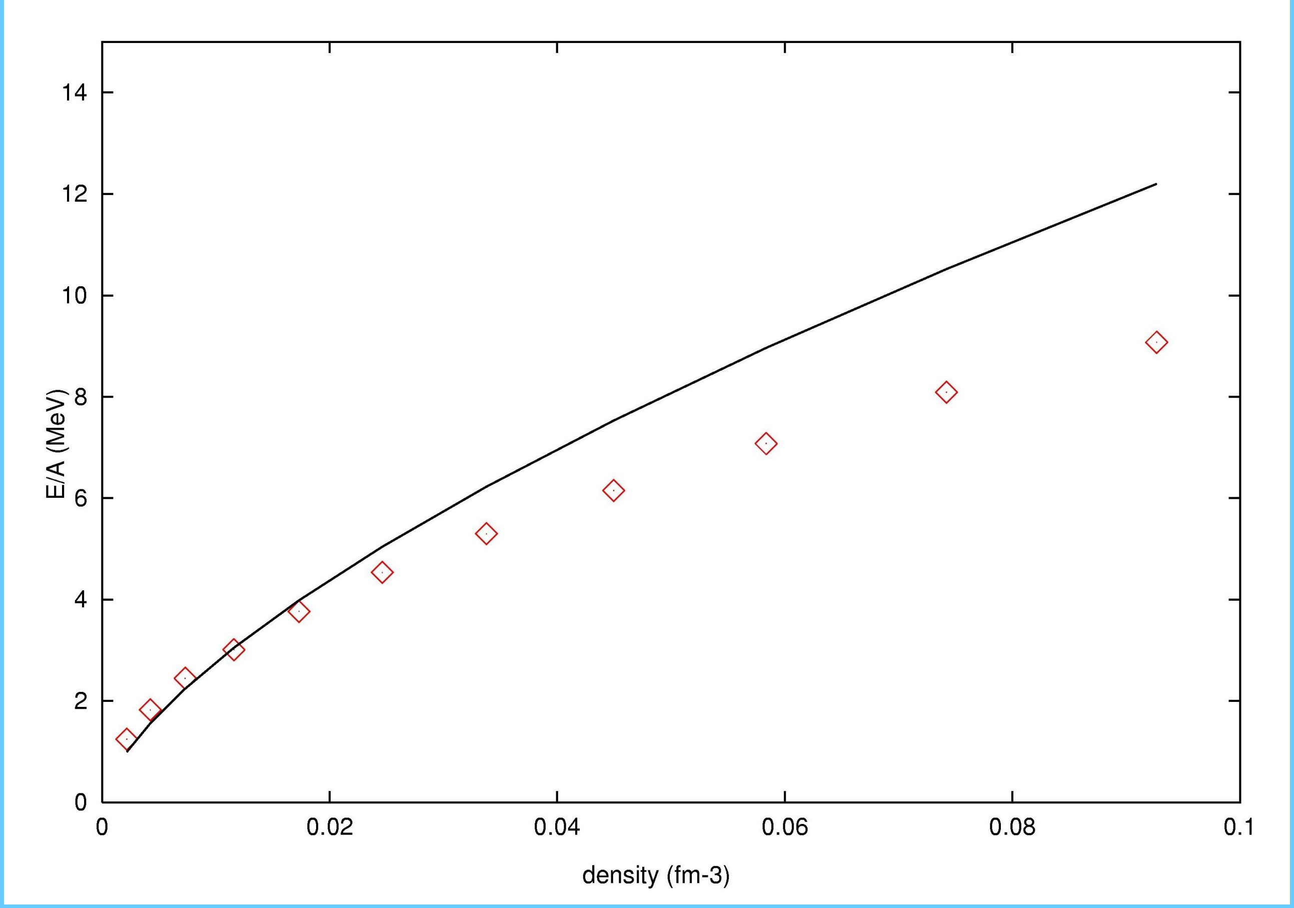
$$k_1', k_2', k_1'', k_2'' \geq k_F$$

Three hole-line contribution

“Low” density



Puzzling “quadratic” behaviour of interaction energy
Dotted line : ½ E(free gas)
Stars : Friedman & Pandharipande, Nucl. Phys. A361,501(1981)
Urbana potential



Similar behaviour in BBG calculations (v18 potential)

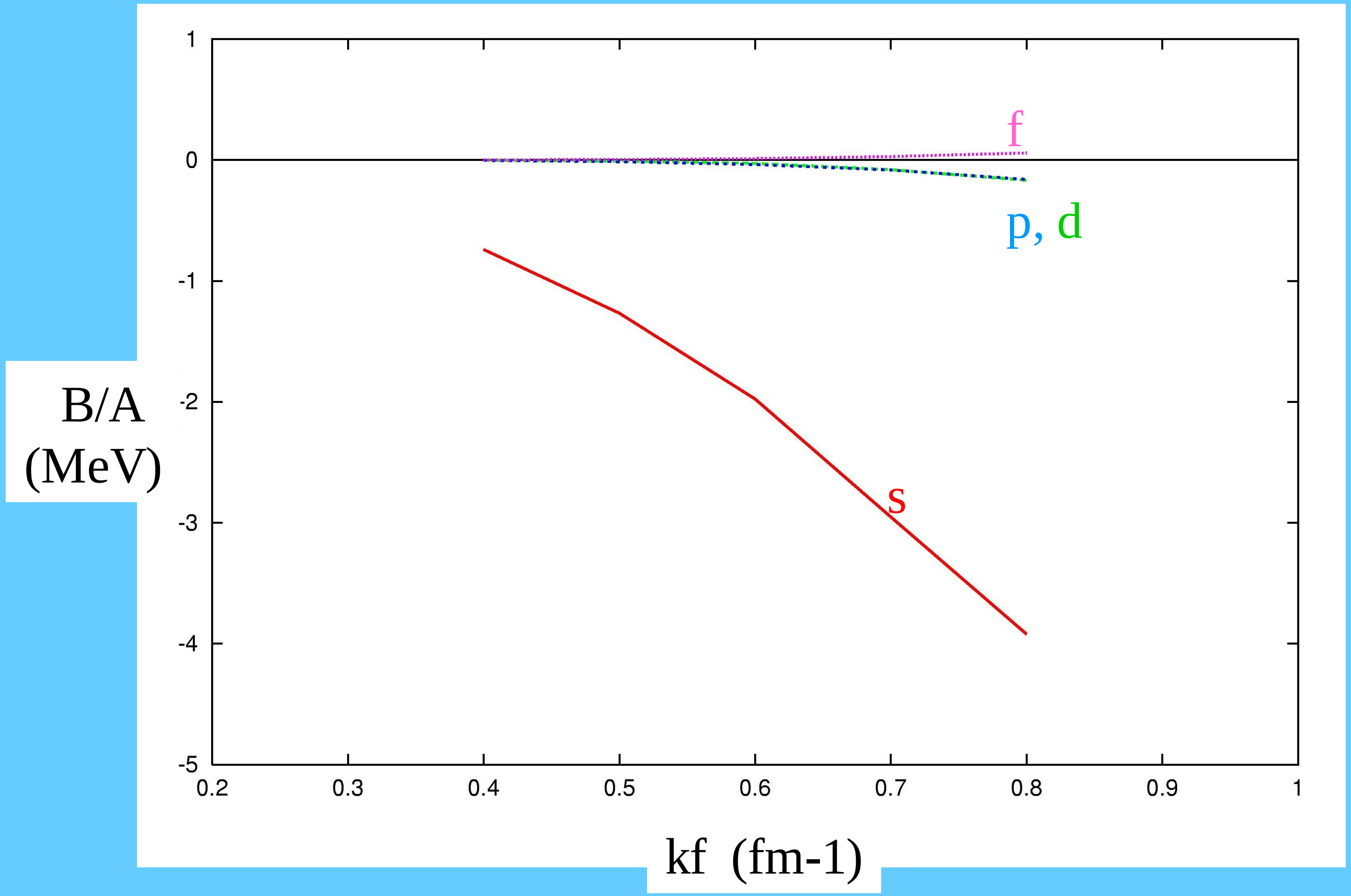
These results are suggestive of the “unitary limit” behaviour of neutron matter :

$$-a \gg d \gg r$$

where a is the scattering length, r is the effective range and d is the average distance between particles. For $a \to -\infty$ the only scale is given by $d \approx 1/k_F$ and the corresponding energy scale can be only the kinetic energy

$$E = \xi E_F$$

with ξ a density independent factor



"Low" density region

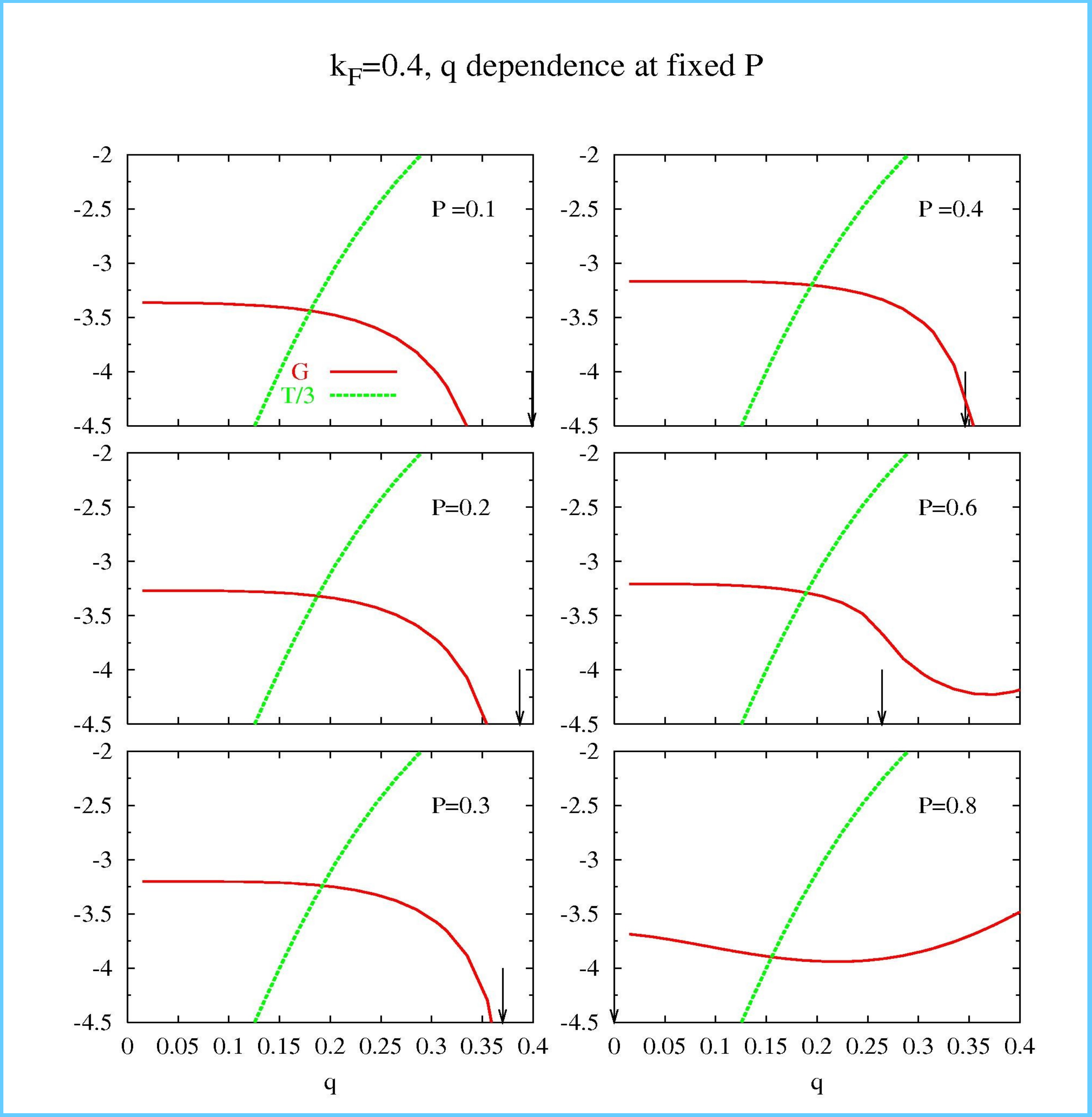
1. The s-wave dominates
2. The thre hole-lines are small (< 0.2 MeV)
3. Three-body forces are negligible (< 0.01 MeV)
4. Effect of self-consistent U is small (see later)

M.B. & C. Maieron

Three hole-line contribution

k_F	0.4	0.5	0.6	0.7	0.8	(fm-1)
D_3	0.023	0.091	0.107	0.153	0.148	(MeV)
B	-0.630	-0.416	-0.526	-0.611	-0.592	
BU	0.485	0.389	0.515	0.648	0.651	
R	0.156	0.122	0.123	0.121	0.095	
H	0.012	-0.004	-0.005	-0.005	-0.006	

M.B. & C. Maieron, PRC 77, 015801 (2008)



M.B. & C. Maieron, PRC 77, 015801 (2008)

Using effective theory

Pethick and Schwenk PRL **95** 160401 (2005)

(Pauli operator only)

Introduce a local and energy dependent interaction

$$\mathbf{V(E) = \delta(r - r').g^2/(\Delta - E)}$$

together with a cutoff $\mathbf{\Lambda}$ and adjust $\mathbf{g}$ and $\mathbf{\Delta}$ to reproduce the scattering length $\mathbf{a}$ and effective range $\mathbf{r_e}$ to lowest order in $\mathbf{\Lambda}$. Taking only cutoff independent terms in the ladder sum one gets

$$\mathbf{T_{med} = 4\pi/D}$$

$$\mathbf{D = \frac{1}{a} - \frac{1}{2} r_e k^2 - \frac{k_f + 0.5P}{\pi} + \frac{k}{\pi}\log(\frac{k_F + 0.5P + k}{k_F + 0.5P - k}) + \frac{k^2 + 0.25P^2 - k_F^2}{\pi P}\log(\frac{(k_F + 0.5P)^2 - k^2}{(k_F - 0.5P)^2 - k^2})}$$

The "in medium" scattering length and effective range turn out to be

.

$$\mathbf{a' = \frac{a}{1 - 2\,a\,k_F/\pi} \qquad r'_e = r_e - \frac{4}{\pi\,k_F}}$$

For $\mathbf{a\,k_F >> 1}$, $\mathbf{a' \sim -\frac{1}{k_F}}$. However this is valid only if the term in the effective range $\mathbf{r_e}$ is small, and this implies

$$\mathbf{-\frac{1}{a} << k_F << \frac{2}{r_e}}$$

For the **n-n** system $\mathbf{1/a \sim -0.2}$ and $\mathbf{\frac{2}{r_e} \sim 0.7}$, and the conditions cannot be satisfied.

A simple exercise in nuclear matter
Calculate the neutron matter EOS at low density

Take a separable representation for the 1S0 channel

$$V(k,k') = \lambda \varphi(k) \varphi(k') \quad \text{with e.g.} \quad \varphi(k) = [k^2 + \beta^2]^{-1}$$

for which the free scattering matrix reads

$$T(k,k') = \lambda \varphi(k) \varphi(k') / [1 - \lambda < \varphi | G_0(E) | \varphi >]$$

where $G_0(E)$ is the free two-body Green's function. Then fix the parameters λ, β in order to reproduce the scattering length and effective range for this channel (low energy data)

The in-medium G-matrix reads

$$G(k,k') = \lambda \varphi(k) \varphi(k') / [1 - \lambda < \varphi | Q\, G_0(E) | \varphi >]$$

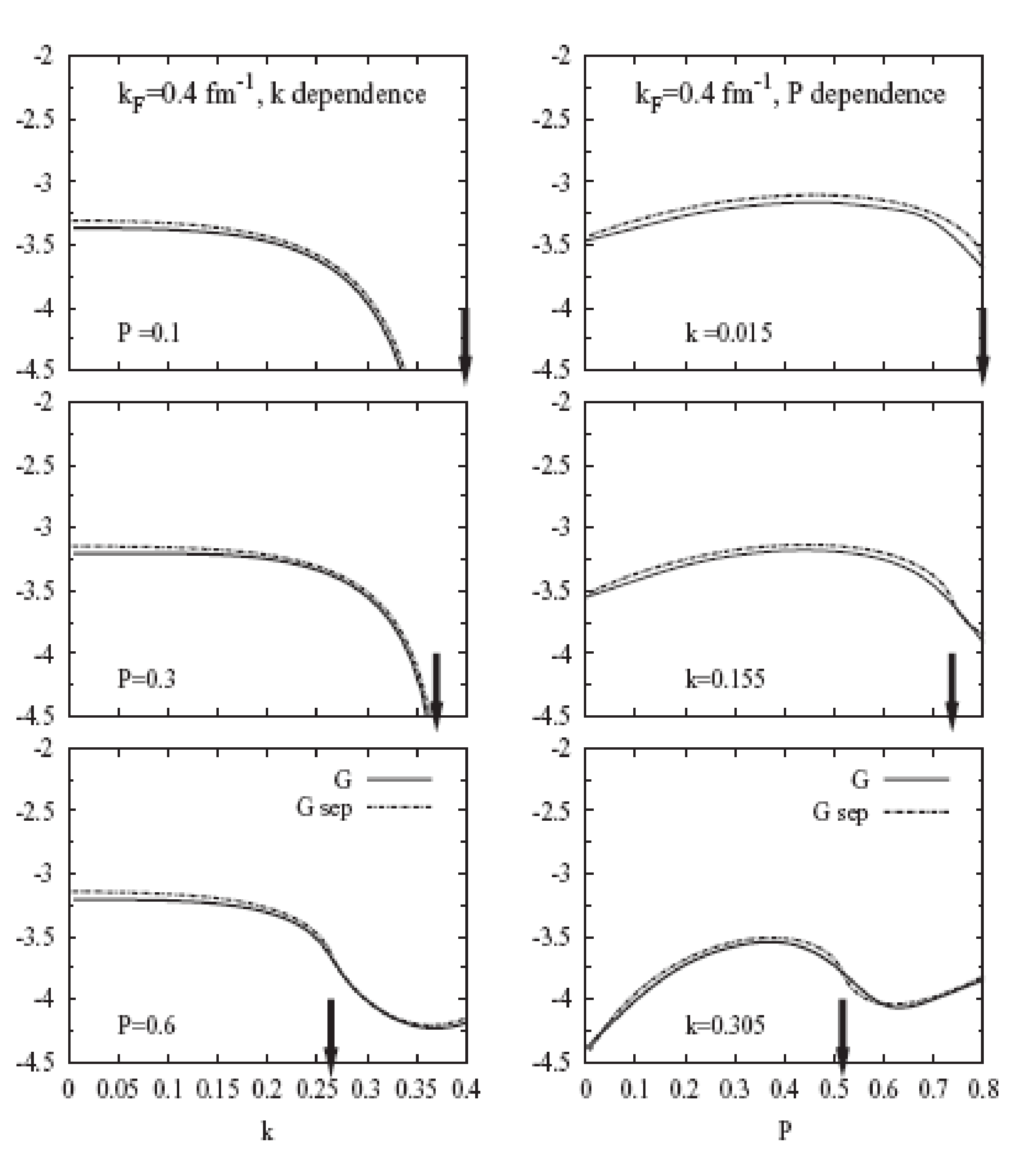
where Q is the Pauli operator. Compare G-matrix and T-matrix. Everything is analytical. The neutron matter energy can be calculated by simple integration.

Explicit expression of the separable G-matrix

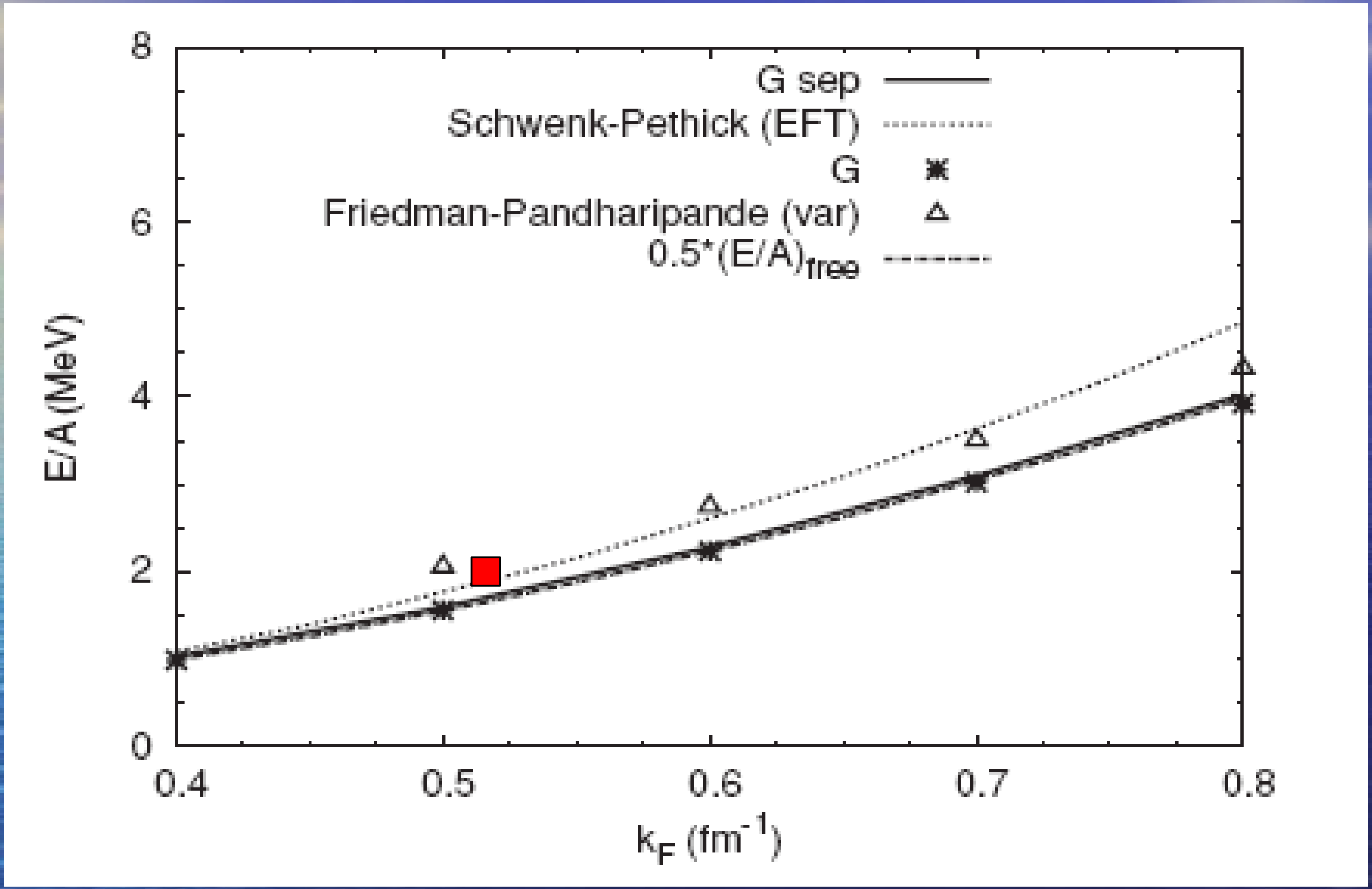
$$(k|G(P,k_F)|k) = 1/\Big[\big(1/a - \tfrac{1}{2}r_0k^2 + \tfrac{1}{2}k^4/(b^3\beta)\big) + A(k,P,k_F)\Big].$$

$$A = -\frac{1}{\pi b}(b^2 - k^2)\arctan\left(\frac{k_F + P/2}{b}\right) + \frac{1}{\pi}k\log\left(\frac{k + k_F + P/2}{-k + k_F + P/2}\right) + \frac{1}{\pi P}(k_F^2 - P^2/4 - k^2) \times \log\left|\frac{(k_F + P/2)^2 + b^2}{k_F^2 - P^2/4 + b^2}\cdot\frac{k_F^2 - P^2/4 - k^2}{(k_F + P/2)^2 - k^2}\right|,$$

$$a = \frac{1}{b}\left(\frac{2\beta}{1+\beta}\right); \quad r_0 = \frac{1}{b}\left(\frac{\beta - 2}{\beta}\right).$$

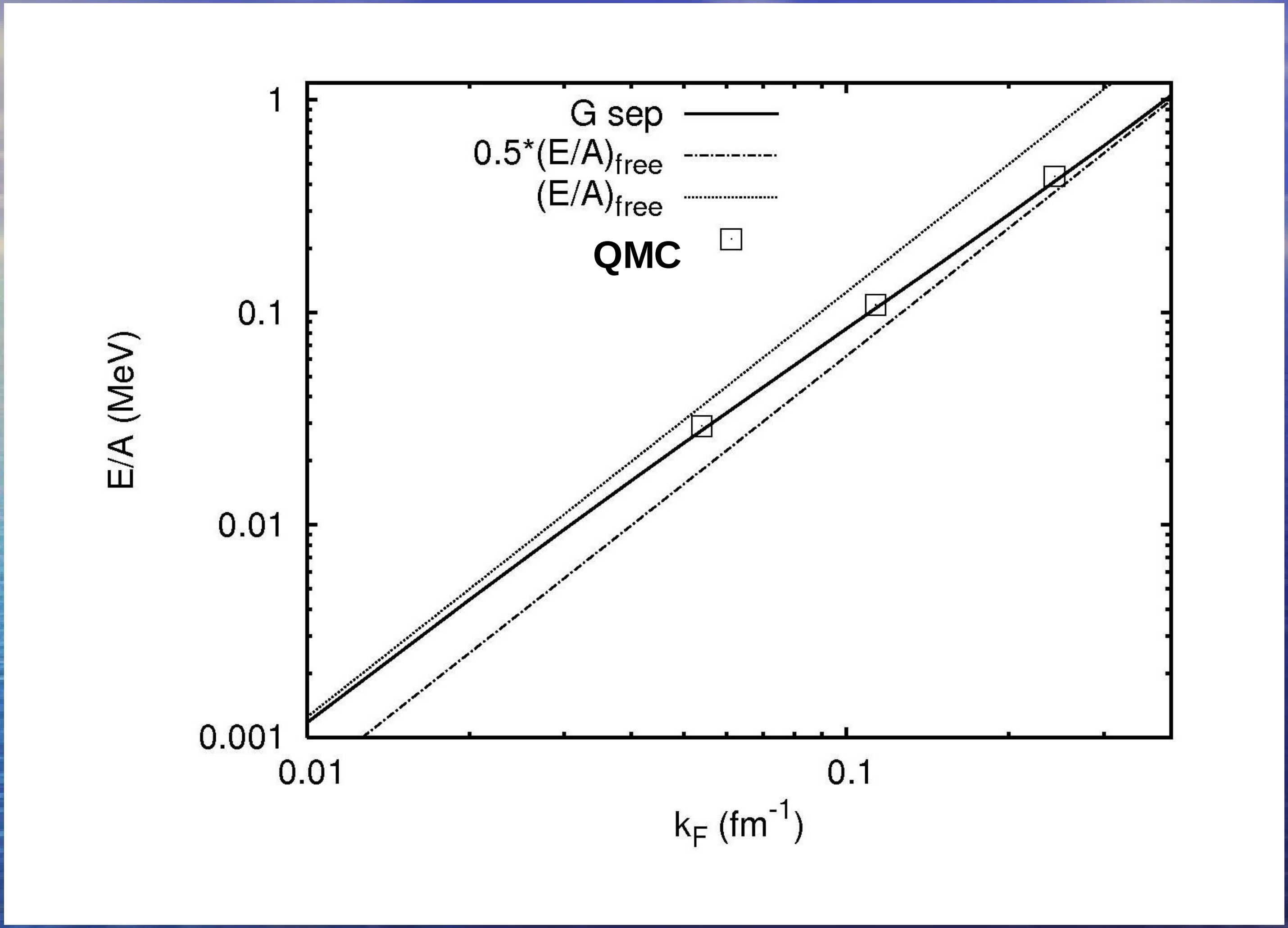


M.B. & C. Maieron, PRC 77, 015801 (2008)



M.B. & C. Maieron, PRC 77, 015801 (2008)

■ A. Gezerlis and J. Carlson, Pnys. Rev. C 77,032801 (2008)
Quantum Monte Carlo calculation



M.B. & C. Maieron, PRC 77, 015801 (2008)

Conclusions for the "very low" density region

1. Only s-wave matters, but the "unitary limit" is actually never reached. Despite that the energy is ½ the kinetic energy in a wide range of density (for unitary 0.4-0.42 from QMC).

5. The dominant correlation comes from the Pauli operator

7. Both three hole-line and single particle potential effects are small and essentially negligible

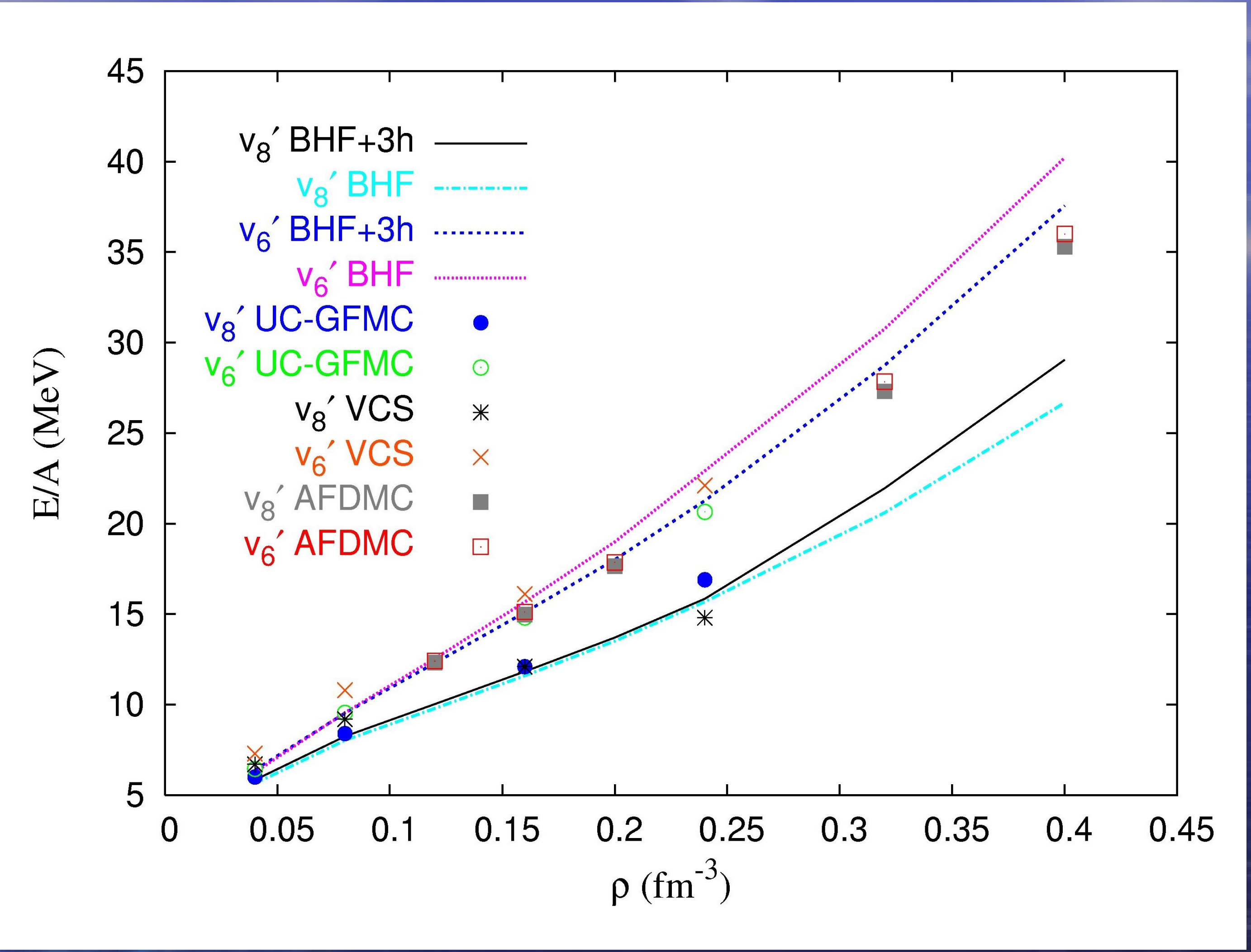
10. Three-body forces negligible

12. The rank-1 potential is extremely accurate : scattering length and effective range determine completely the G-matrix.

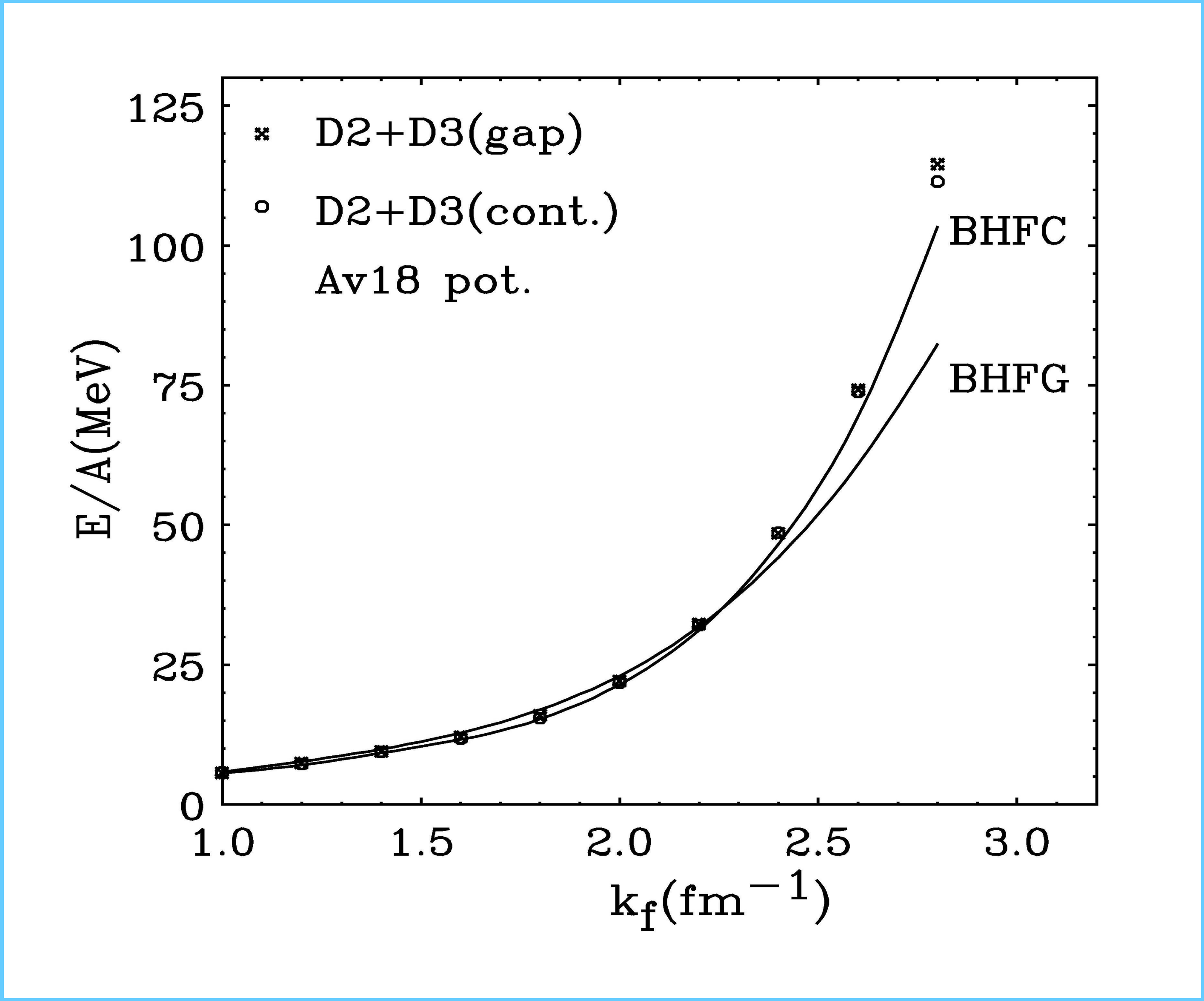
15. Variational calculations are slightly above BBG. Good agreement with QMC.

In this density range one can get the "exact" neutron matter EOS

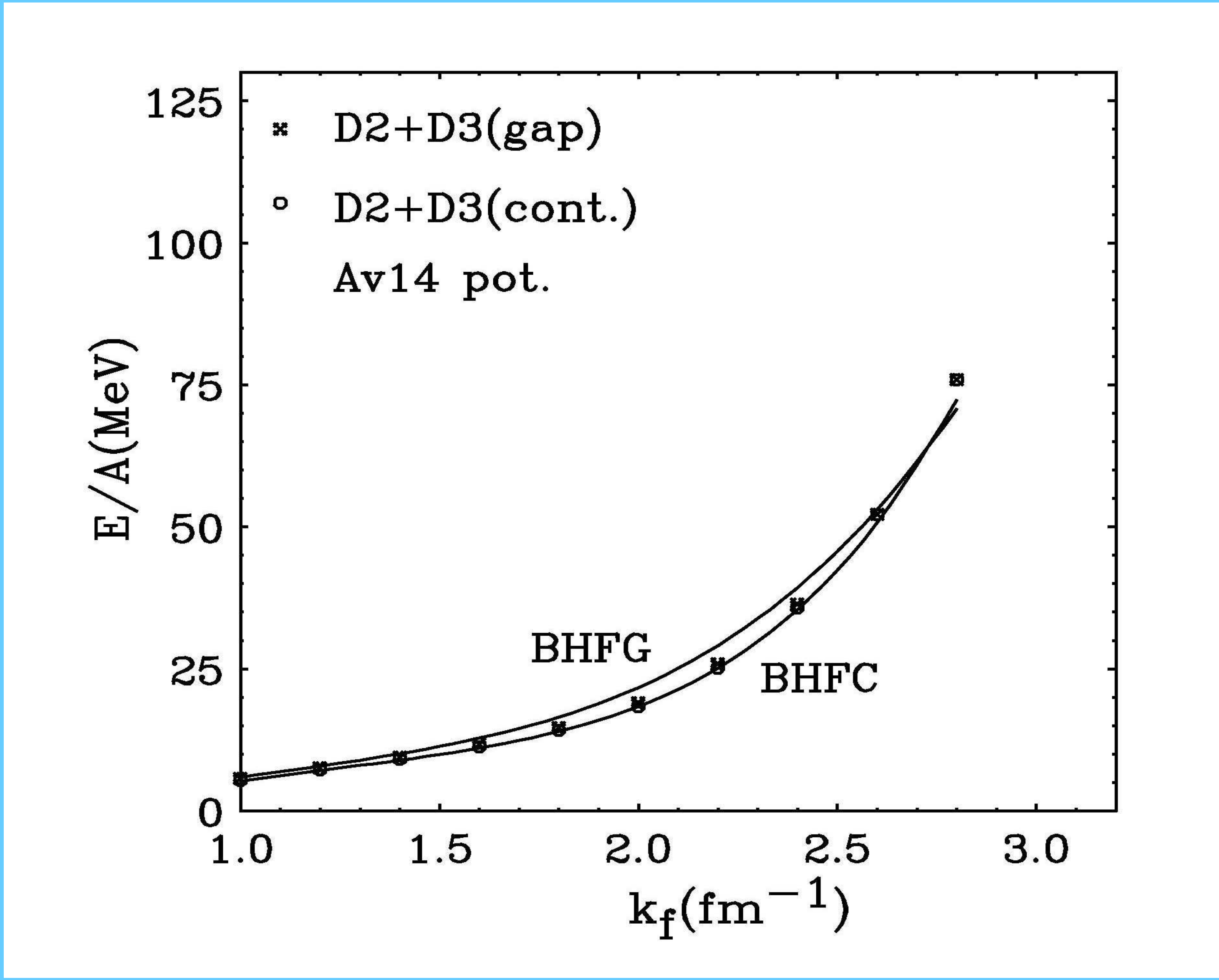
Confronting with "exact" GFMC for v6 and v8



Variational and GMFC : Carlson et al. Phys. Rev. C68, 025802(2003)
BBG : M.B. and C. Maieron, Phys. Rev. C69,014301(2004)



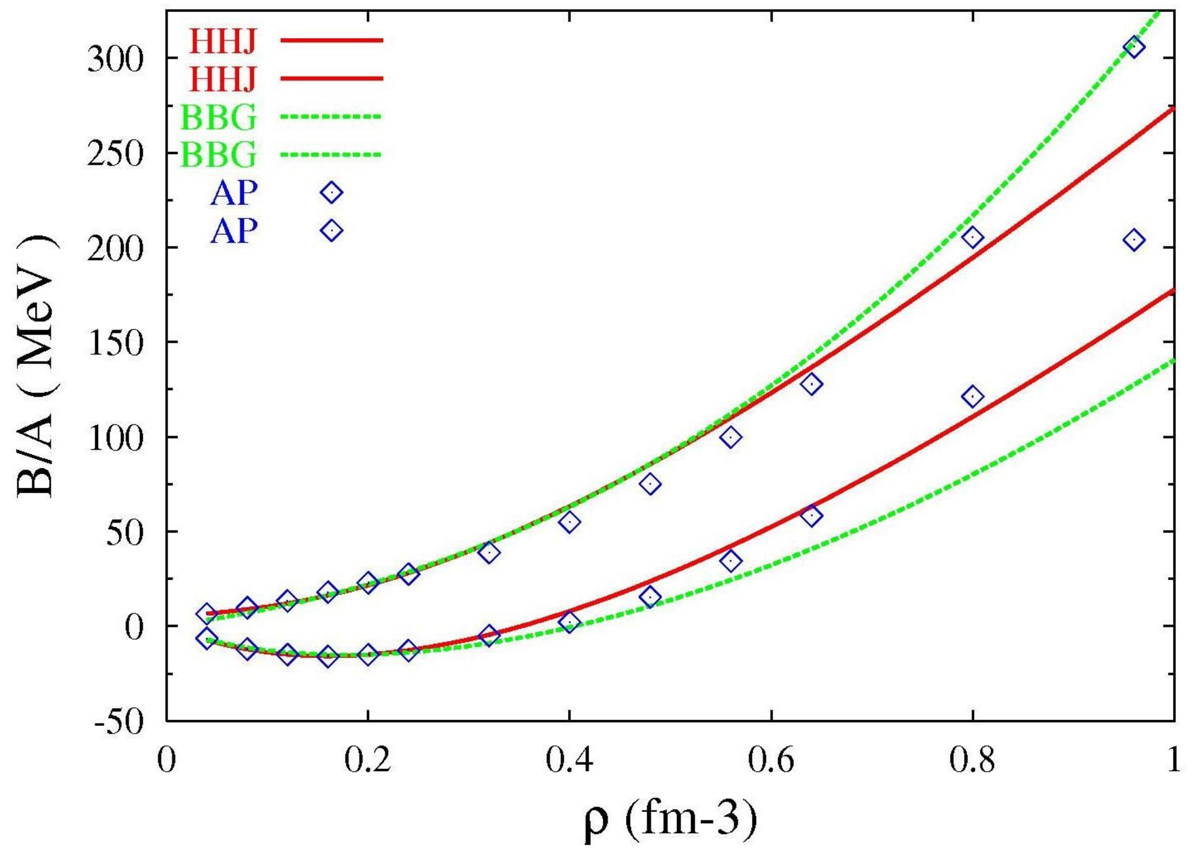
Neutron matter



Neutron matter

The baryonic Equations of State

V18 NN potential + TBF



HHJ : Astrophys. J. 525, L45 (1999)
BBG : PRC 69 , 018801 (2004)
AP : PRC 58, 1804 (1998)