Phase Transitions in Neutron Stars. Are they non-congruent?

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The base

Non-Congruent Phase Transition in Uranium Dioxide

Hypothetical severe accident at fast breeder nuclear reactor

Expected temperature of nuclear fuel


Cooperation: MIPT - IHED RAS - IPCP RAS - OSEU(Ukraine) - MPEI - ITEP – VNIIEF (Sarov) ⇔ ITU (JRC, Karlsruhe) ⇔ GSI (Darmstadt)

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**Non-congruent phase transition in uranium dioxide**


# Sketch of theoretical approach

## Quasi-chemical representation for liquid & gaseous phases

### Ionic model

*(Liquid)*

\[ U^{6+} + U^{5+} + U^{4+} + U^{3+} + O^{2-} + O^- \]

### Multi-molecular model

*(Liquid & Gas)*

\[ U + O + O_2 + UO + UO_2 + UO_3 \]

\[ U^* + UO^+ + UO_2^+ + O^- + UO_3^- + e^- \]

## Interactions: *(Pseudopotential components)*

- Intensive short-range repulsion
- Coulomb interaction between charged particles
- Short-range effective attraction between all particles

## Interaction corrections: *(Modified for mixtures)*

- Hard-sphere mixture with varying diameters
- Modified Mean Spherical Approximation *(MSAE+DHSE)*
- Modified Thermodynamic Perturbation Theory \{TPT- \( \sigma(T); \varepsilon(T) \)\}

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Forced-congruent evaporation in U-O system

- Stoichiometry of coexisting phases are equal: $x' = x''$
- Van der Waals loops (at $T < T_c$) corrected via the “double tangent construction”
- Standard phase equilibrium conditions:
  $$P' = P'' \quad / \quad T' = T'' \quad / \quad G'(P,T,x) = G''(P,T,x)$$
- Standard critical point:
  $$(\partial P/\partial V)_T = 0 \quad / \quad (\partial^2 P/\partial V^2)_T = 0 \quad / \quad (\partial^3 P/\partial V^3)_T < 0$$
Phase equilibrium conditions in reacting Coulomb system

**Phase - I**

\[ \varphi' \]
\[ n_i' + n_k' + \ldots + n_e' \]

**Heat exchange**

\[ T' = T'' \]

**Impulse exchange**

\[ P' = P'' \]

**Neutral species**

\[ \mu_i'(P,T,x') = \mu_i''(P,T,x'') \]
\[ \mu_z'(P,T,x') = \mu_z''(P,T,x'') \]
\[ \ldots \ldots \ldots \ldots \]
\[ \mu_k'(P,T,x') = \mu_k''(P,T,x'') \]

**Equilibrium reactions**

(reduced number of basic units)

\[ \mu_a'(P,T,x') = \mu_a''(P,T,x'') \]
\[ \mu_b'(P,T,x') = \mu_b''(P,T,x'') \]

**Charged species**

**NB!** - Chemical potentials of charged species are not equal

**Electro-chemical potentials are equal**

\[ \mu_i' + Z_i e \varphi' = \mu_i'' + Z_i e \varphi'' \]

**Potential drop at any mean-phase interface in equilibrium Coulomb system**

\[ \mu_1'(P,T,x') = \mu_1''(P,T,x'') + \Delta \varphi Z_1 e \]
\[ \mu_2'(P,T,x') = \mu_2''(P,T,x'') + \Delta \varphi Z_2 e \]
\[ \ldots \ldots \ldots \ldots \]
\[ \mu_e'(P,T,x') = \mu_e''(P,T,x'') - \Delta \varphi e \]

**Uranium – Oxygen system**

\[ \mu_U'(P,T,x') = \mu_U''(P,T,x'') \]
\[ \mu_O'(P,T,x') = \mu_O''(P,T,x'') \]

**NB!** - Basic units in NS – baryons and electrons!
Electrostatics of Phase Boundaries in Coulomb Systems

**Terrestrial applications**

*Electrostatic (Galvani) potential*

![Graph showing electrostatic potential difference across various temperature points.](image)


**Quark-hadron phase transition in NS**

![Phase diagram showing the quark-hadron phase transition in neutron stars.](image)


\[ e\Delta \phi_{HQ} = (\mu_e)_{\text{Hardron phase}} - (\mu_e)_{\text{Quark phase}} \]

\[ e\Delta \phi_{HQ} \approx 200 \text{ MeV} ! \]

Non-congruent evaporation in U-O system

Pressure - Temperature Diagram

Non-congruent phase boundaries

Double-tangent construction (standard procedure)

Pressure - Density Diagram

Non-congruent phase boundaries

1 – Non-congruent (total) equilibrium
2 – Forced-congruent (partial) equilibrium

NB! 2-dimensional two-phase region instead of standard P-T saturation curve

NB! High pressure level of non-congruent phase decomposition

NB! Critical point should be of non-standard type: \((\partial P/\partial V)_T \neq 0\), \((\partial^2 P/\partial V^2)_T \neq 0\)

It should be instead: \((O/U)_{\text{liquid}} = (O/U)_{\text{vapor}}\) and \(\{||\partial \mu_i/\partial n_k||_T\}_{CP} = 0\)
Non-congruent phase transformation in two-phase region

Phase Diagram $P-T$ of Non-congruent Evaporation

- First liquid droplets in saturated vapor
- Small vapor bubbles in boiling liquid

- Oxygen depleted liquid! Different stoichiometry!
- Oxygen enriched vapor! Different stoichiometry!
Non-congruent evaporation in U – O system

 Isotherms in two-phase region

**Standard** pressure-density diagram

**Non-congruent** pressure-density diagram

- **CP** - Critical point
- **BC** - Boiling curve
- **SC** - Saturation curve
- **T**\(_{\text{max}}\) - Crycondentherm

**Isothermal** phase transition starts and finishes at **different pressures**

**Isobaric** phase transition starts and finishes at **different temperatures**

Chemical composition of coexisting phases

Boiling Conditions

Solid

Liquid (boiling conditions)

Sublimated vapour

Vapor (boiling conditions)

Saturation Conditions

Liquid (saturation conditions)

Vapor (saturation conditions)

First vapor bubbles in boiling UO$_2$ (oxygen enriched)

Last liquid drops in vapor UO$_2$ (oxygen depleted)


Liquid (O/U = 2.0) ↔ Vapor (O/U > 2.0)

Vapor (O/U = 2.0) ↔ Liquid (O/U < 2.0)

$P = \text{const}$
Basic conclusion

- Any phase transition in a system of **two** or **more chemical elements** must be **non-congruent**
- **Congruent** phase transition is **exception**

**Hypothetical example of non-congruent phase transition**

- **Plasma Phase Transition** in $\text{H}_2/\text{He}$ mixture in Jupiter, Saturn, Brown Dwarfs and Extra-Solar Planets . . .
- **Coulomb crystallization** in $\text{C}^{6+}/\text{O}^{8+}/\text{He}^{4+}$ mixture in White Dwarfs and in multi-nuclear envelope of Neutron Stars
- **Quark-hadron** phase transition in interiors of Strange Stars
Neptune and “Hot-water” extrasolar planet GJ436b

Water (phase diagram)

GJ436b
Star: - Gliese 436 (RD)
\[ M \sim 22M_\text{O} \]
\[ R \sim 4R_\text{O} \]
\[ \Delta T \sim 2.6 \text{ days (!)} \]
\[ T_{\text{Surf.}} \sim 500 \text{ K} \]
Main Comp. – H\(_2\)O

(*) Cavazzoni et al., 1999

Any phase transition in a system of two or more chemical elements must be non-congruent

T. Mattsson & M. Desjarlais (Sandia Lab.): High energy-density water: DFT – simulations
PNP-12, 2006, Darmstadt, Germany // WEHS-workshop 2007, Bad Honnef, Germany,
Hypothetical phase transitions in interior of neutron stars: are they **CONGRUENT** or **NON-CONGRUENT**?

First quark droplets in hadron matter

Last hadron bubbles in quark matter
Non-congruence in exotic situations (mesoscopic scenario)

**Structured Mixed Phase ⇔ ‘Pasta’ plasma**

‘Pasta’ plasma – hadron-quark phase transition in interior of neutron stars
(‘Mixed phase’ of Glendenning et al.)
- Charged quark droplets (rods, slabs) in equilibrium hadron matter
- Charged hadron droplets (rods, slabs) in equilibrium quark matter

Fig. 1. Nuclear and quark matter structures in a ~ 1.4M☉ neutron star. Typical sizes of structures are ~ 10⁻¹⁴m but have been scaled up to be seen.

Heiselberg and Hjorth-Jensen
*Phase Transitions in Neutron Stars*

T.Maruyama, T.Tatsumi, T.Endo, S.Chiba
*Pasta structures in compact stars*

Structured Mixed Phase Transitions – are they congruent or not?
Что пользы нам в неконгруэнтности кроме замены терминологии?

What’s the use of talking about non-congruence besides terminology changing?
Hypothetical phase transitions in interior of neutron stars: are they **CONGRUENT** or **NON-CONGRUENT**?

**Melting point**

**Non-congruent phase boundaries**

**Double-tangent construction** (standard procedure)

**UO₂**

**Critical Point**

After Fridolin Weber, WEHS Seminar, Bad Honnef, 2006
After David Blaschke, WEHS Seminar, Bad Honnef, 2007

- Forced-congruent phase transition
- Non-congruent phase transition
What’s a matter of high-temperature phase transformations in NS?
Evaporation of strange matter in the early Universe

Alcock C., Farhi E. (PRD, 1985)
Alcock C., Olinto A. (PRD, 1989)

Strange matter, a stable form of quark matter containing a large fraction of strange quarks, may have been copiously produced when the Universe had a temperature of \( \sim 100 \text{ MeV} \). We study the evaporation of lumps of strange matter as the Universe cooled to 1 MeV. Only lumps with baryon number larger than \( \sim 10^{-6} \) could survive. This places a severe restriction on scenarios for strange-

Strange matter is a form of quark matter that has been conjectured to be stable at zero temperature. If heated to a temperature \( T \geq 2 \text{ MeV} \), a strange-matter lump evaporates nucleons from its surface. We show that at higher temperatures \( (T \geq 20 \text{ MeV}) \), strange matter **boils**, with bubbles of hadronic gas forming and growing throughout the interior. Strange matter, or any other phase which resembles strange matter, could not have survived this process in the early Universe.
Hypothetical phase transitions in interior of neutron stars: are they **CONGRUENT** or **NON-CONGRUENT**?

Main point:
– Phase transformations in NS at high temperature?
Basic feature of non-congruent phase transition dynamics

Parameters of non-congruent phase transformation strongly depend on rapidity of the transition.

Main point – competition between thermal conductivity and diffusion

• There are two limiting regimes of transformation:
  - **Slow**, totally equilibrium transformation ("Global" equilibrium mode)
  - **Fast**, partially equilibrium transformation without change of stoichiometry (Forced-congruent mode - FCM)

• Terrestrial applications:
  - **Evaporation** under the intensive *surface heating*  
    *(laser, electron or ion beam etc.)*

What’s scenario of non-congruent phase transitions in NS and WD

- “Slow” crystallization in WD looks like Global Equilibrium Mode.
- “Fast” crystallization in NS looks like Forced-congruent Mode.
Evaporation of strange matter in the early Universe

Alcock C., Farhi E. (PRD, 1985)
Alcock C., Olinto A. (PRD, 1989)

What’s scenario of hypothetical non-congruent boiling up (foam) of strange matter lumps in early Universe

We show that at higher temperatures (\( T > 8 \text{ MeV} \)), strange matter boils, with bubbles of hadronic gas forming and growing throughout the interior. Strange matter, or any other matter which resembles strange matter, could not have survived this process in the early Universe.
Crystallization on C/O mixture in White Dwarf

Fig. 1.—Phase diagrams for a C/O mixture as computed by Ichimaru et al. (1988, dashed line) and Segretain & Chabrier (1993, solid line), where


“Slow” crystallization in WD looks like Global Equilibrium Mode
Crystallization on C/O mixture in White Dwarf

Oxygen profile in WD

Phase diagram in C/O mixture

Fig. 1. Phase diagram of the carbon-oxygen mixture at constant electronic pressure. $T^* = 1/T$ is the reduced temperature.

J. Barrat, J. P. Hansen, R. Mochkovitch (1988)

“Fast” crystallization in NS looks like **Forced-Congruent Mode**

a) – initial
b) – final (Ichimaru)
b) – final (Segretain & Chabrier)
Conclusion

If one takes into account hypothetical **non-congruence** of phase transitions in neutron stars, he should **revise** totally the **scenario** of all the phase transformations in NS.

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