Neutrino scattering in the theory of neutrino heat conduction

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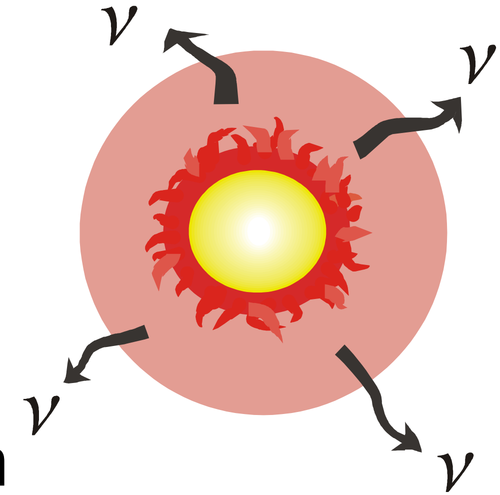
Neutrino transport in collapsing stellar core:

- Determines dynamic and thermodynamic properties of implosion such as density, temperature and lepton number at the moment of bounce;
- Of crucial importance for possible conversion of stalled shock into an outgoing blast wave.
- The major part of neutrino flux is radiated in the regime of neutrino opacity – from neutrinosphere. The flux is controlled by neutrino diffusion in the neutrino-opaque core and can be well described by Neutrino Heat Conduction (NHC) theory.



Equations of neutrino hydrodynamics

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{Gm}{r^2} - \frac{1}{\rho}\mathcal{M}_\nu,$$

$$\frac{dE}{dt} + P\frac{d}{dt}\left(\frac{1}{\rho}\right) = \frac{1}{\rho}\left(u\mathcal{M}_\nu - \mathcal{E}_\nu\right),$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho u\right) = 0,$$

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho,$$

Fixed (laboratory) frame, terms $(u/c)^2$ and $(u_{\text{sound}}/c)^2$ being neglected.

$$\mathcal{M}_\nu = \frac{1}{c^2}\frac{\partial S_\nu}{\partial t} + \frac{\partial K_\nu}{\partial r} + \frac{1}{r}\left(3K_\nu - U_\nu\right),$$

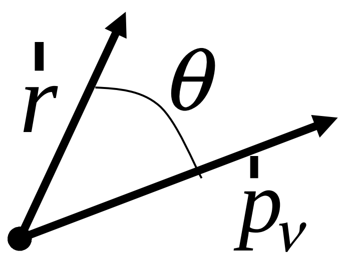
$$\mathcal{E}_\nu = \frac{\partial U_\nu}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 S_\nu\right),$$

$$K_\nu = K_\nu(r,t) = \frac{2\pi}{c}\int_{-1}^{1}\mu^2 \mathbf{I}_\nu d\mu,$$

$$S_\nu = S_\nu(r,t) = 2\pi\int_{-1}^{1}\mu \mathbf{I}_\nu d\mu,$$

$$U_\nu = U_\nu(r,t) = \frac{2\pi}{c}\int_{-1}^{1}\mathbf{I}_\nu d\mu,$$

$$\mu = \cos\theta$$



$$\mathbf{I}_\nu = \int_0^\infty I_\nu \, d\omega_\nu$$

ω_ν – neutrino energy

$$\omega_\nu = c\left|\vec{p}_\nu\right|$$

Transport equation

$$\widehat{\mathrm{D}} I_\nu = -\frac{I_\nu - I_{\nu \mathrm{e}}}{\widetilde{l}_\nu} + \widehat{\mathrm{B}}(I_\nu, I'_\nu),$$

$$\widehat{\mathrm{D}} \equiv \frac{1}{c}\frac{\partial}{\partial t} + \mu \frac{\partial}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial}{\partial \mu}$$

$\tilde{l}_\nu$ – neutrino absorption mean free path corrected for stimulated absorption; neutrino scattering mean free path is in scattering operator $\hat{\mathrm{B}}$

$$\mathcal{E}_\nu = 2\pi \int_0^\infty \int_{-1}^{1} \left[-\frac{I_\nu - I_{\nu\mathrm{e}}}{\widetilde{l}_\nu} + \widehat{\mathrm{B}}(I_\nu, I'_\nu) \right] d\mu d\omega_\nu ,$$

$$\widehat{\mathrm{D}} I_1 = \widehat{\mathrm{D}} I_\mathrm{e} + \widehat{\mathrm{D}} \delta I_1$$

$$\mathcal{M}_\nu = \frac{2\pi}{c} \int_0^\infty \int_{-1}^{1} \mu \left[-\frac{I_\nu - I_{\nu\mathrm{e}}}{\widetilde{l}_\nu} + \widehat{\mathrm{B}}(I_\nu, I'_\nu) \right] d\mu d\omega_\nu$$

successive approximations: $I_1, I_2, \ldots$ (we omit index ν hereafter)

$$\widehat{\mathrm{D}} I_\mathrm{e} = -\frac{I_1 - I_\mathrm{e}}{\widetilde{l}} + \widehat{\mathrm{B}}(I_1, I'_1)$$ integral equation for I_1

$$\widehat{\mathrm{D}} I_1 = -\frac{I_2 - I_\mathrm{e}}{\widetilde{l}} + \widehat{\mathrm{B}}(I_2, I'_2)$$ integral equation for I_2

One has to use I_2 in […] above!

Comoving frame

$$t = t_0, \quad r = r_0,$$

$$\omega = \omega_0 / L, \quad \mu = \mu_0 + (1 - \mu_0^2) u / c,$$

Lorentz factor $L = 1 - \mu u / c = 1 - \mu_0 u / c$

$$I = I_0 / L^3, \quad l = l_0 / L, \quad \widehat{\mathrm{B}} = \widehat{\mathrm{B}}_0 / L^2.$$

$$I_{\mathrm{e}0} = \frac{\omega_0^3}{c^2 h^3} \frac{1}{1 + \exp\left(\frac{\omega_0}{kT} - \psi\right)}$$

$$f_{\mathrm{e}} = \frac{c^2 h^3}{\omega_0^3} I_0 = \frac{1}{1 + \exp\left(\frac{\omega_0}{kT} - \psi\right)}$$

Scattering operator in comoving frame

$$\widehat{\mathrm{B}}(I,I') = \frac{\omega^3}{c^2h^3}\iint\left[e^{\frac{\omega'-\omega}{kT}}f'(1-f) - f(1-f')\right]$$
$$\times R(\omega,\omega',\eta)\, d\Omega' d\omega'.$$

$$d\Omega' = \sin\theta' d\theta' d\phi' = d\mu' d\phi'$$

$$l_{\mathrm{S}}^{-1} = \iint R(\omega,\omega',\eta)\, d\Omega' d\omega'$$

filling factor: $f = \dfrac{c^2h^3}{\omega^3} I$

$$\eta = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\phi-\phi')$$

$$R(\omega',\omega,\eta) = \left(\frac{\omega}{\omega'}\right)^2 e^{\frac{\omega'-\omega}{kT}} R(\omega,\omega',\eta)$$

$$\widehat{\mathrm{D}} I_{\mathrm{e}} = -\frac{I_1 - I_{\mathrm{e}}}{\widetilde{l}} + \widehat{\mathrm{B}}(I_1, I_1')$$

⇧

$$I_1 = I_{\mathrm{e}} + \delta I_1$$

neglecting terms of the order of $(\delta I_1)^2$ in $\widehat{\mathrm{B}}$ one can reduce the integrals for $\mathcal{E}_\nu$ and $\mathcal{M}_\nu$ to the form that results in generalizing of the neutrino heat conduction for the case of neutrino scattering being of importance.

Equations of neutrino heat conduction (NHC)

$$\frac{dr}{dt} = u, \qquad \frac{1}{\rho} = \frac{4\pi}{3}\frac{\partial r^3}{\partial m},$$

$$\frac{du}{dt} = -4\pi r^2 \frac{\partial}{\partial m}\left(P + P_\nu\right) - \frac{Gm}{r^2},$$

$$\frac{d}{dt}\left(E + \frac{U_\nu}{\rho}\right) + \left(P + P_\nu\right)\frac{d}{dt}\left(\frac{1}{\rho}\right) = -4\pi\frac{\partial}{\partial m}\left(r^2 H_\nu\right),$$

$$\frac{d\Lambda_\nu}{dt} + 4\pi m_u \frac{\partial}{\partial m}\left(r^2 F_\nu\right) = 0,$$

$$\Lambda_\nu = \frac{m_u}{\rho}\left(n_- - n_+ + n_\nu - n_{\overline{\nu}}\right)$$

$$U_\nu = 3P_\nu = \frac{15aT^4}{2\pi^4}\left[F_3(\psi) + F_3(-\psi)\right] = \frac{\psi^4}{4} + \frac{\pi^2}{2}\psi^2 + \frac{7\pi^4}{60}$$

H_ν – neutrino energy flux

F_ν – lepton charge flux

Λ_ν – lepton charge

$$H_\nu = -\frac{4\pi}{3h^3c^2}\left[(A_\nu + A_{\overline{\nu}})\frac{1}{kT^2}\frac{\partial T}{\partial r} + (B_\nu - B_{\overline{\nu}})\frac{\partial \psi_\nu}{\partial r}\right]$$

$$F_\nu = -\frac{4\pi}{3h^3c^2}\left[(B_\nu - B_{\overline{\nu}})\frac{1}{kT^2}\frac{\partial T}{\partial r} + (D_\nu + D_{\overline{\nu}})\frac{\partial \psi_\nu}{\partial r}\right]$$

Onsager symmetry priciple

$$A = \int_0^\infty \frac{f_e}{\lambda_m} g_T \omega^3 d\omega, \quad B = \int_0^\infty \frac{f_e}{\lambda_m} g_\psi \omega^3 d\omega, \quad D = \int_0^\infty \frac{f_e}{\lambda_m} g_\psi \omega^2 d\omega$$

$$g_T(\omega) = \omega(1 - f_e) + \int_0^\infty \Phi_1(\omega, \omega') g_T(\omega') \frac{d\omega'}{\lambda_m(\omega')},$$

$$g_\psi(\omega) = (1 - f_e) + \int_0^\infty \Phi_1(\omega, \omega') g_\psi(\omega') \frac{d\omega'}{\lambda_m(\omega')}.$$

integral equations for $g_T(\omega)$, $g_\psi(\omega)$

Coherent and incoherent scattering:

$$R(\omega,\omega',\eta) = R_{\mathrm{nc}}(\omega,\omega',\eta) + R_{\mathrm{cs}}(\omega,\eta)\delta(\omega-\omega')$$

Dirac δ-function

$$\lambda_{\mathrm{m}}(\omega) \equiv \widetilde{l}^{-1} + l_{\mathrm{cs}}^{-1}(1-\langle\eta\rangle) + \frac{1}{1-f_{\mathrm{e}}}\int\limits_0^\infty (1-f'_{\mathrm{e}})\,\Phi_0(\omega,\omega')d\omega'$$

$$\Phi_0(\omega,\omega') = 2\pi \int\limits_{-1}^{1} R_{\mathrm{nc}}(\omega,\omega',\eta)d\eta,$$

$$\Phi_1(\omega,\omega') = 2\pi \int\limits_{-1}^{1} \eta R_{\mathrm{nc}}(\omega,\omega',\eta)d\eta.$$

The incoherent part of the scattering kernel $R_{\mathrm{nc}}(\omega,\omega',\eta)$ enters the NHC equations only through zeroth and first moments of its expansion in terms of Legendre polynomials Φ_0 and Φ_1, respectively.

For pure coherent scattering: $R_{\mathrm{nc}}(\omega, \omega', \eta) = 0$

$$\Phi_0(\omega, \omega') = 0\,, \;\; \Phi_1(\omega, \omega') = 0$$

$$g_T(\omega) = \omega(1 - f_{\mathrm{e}})$$

$$g_\psi(\omega) = (1 - f_{\mathrm{e}})$$

no integral equations!

$$\lambda_{\mathrm{m}}(\omega) = \widetilde{l}^{-1} + l_{\mathrm{cs}}^{-1}(1 - \langle\eta\rangle)$$

$$\langle\eta\rangle = 2\pi l_{\mathrm{cs}} \int_{-1}^{1} \eta R_{\mathrm{cs}}(\omega, \eta) d\eta,$$

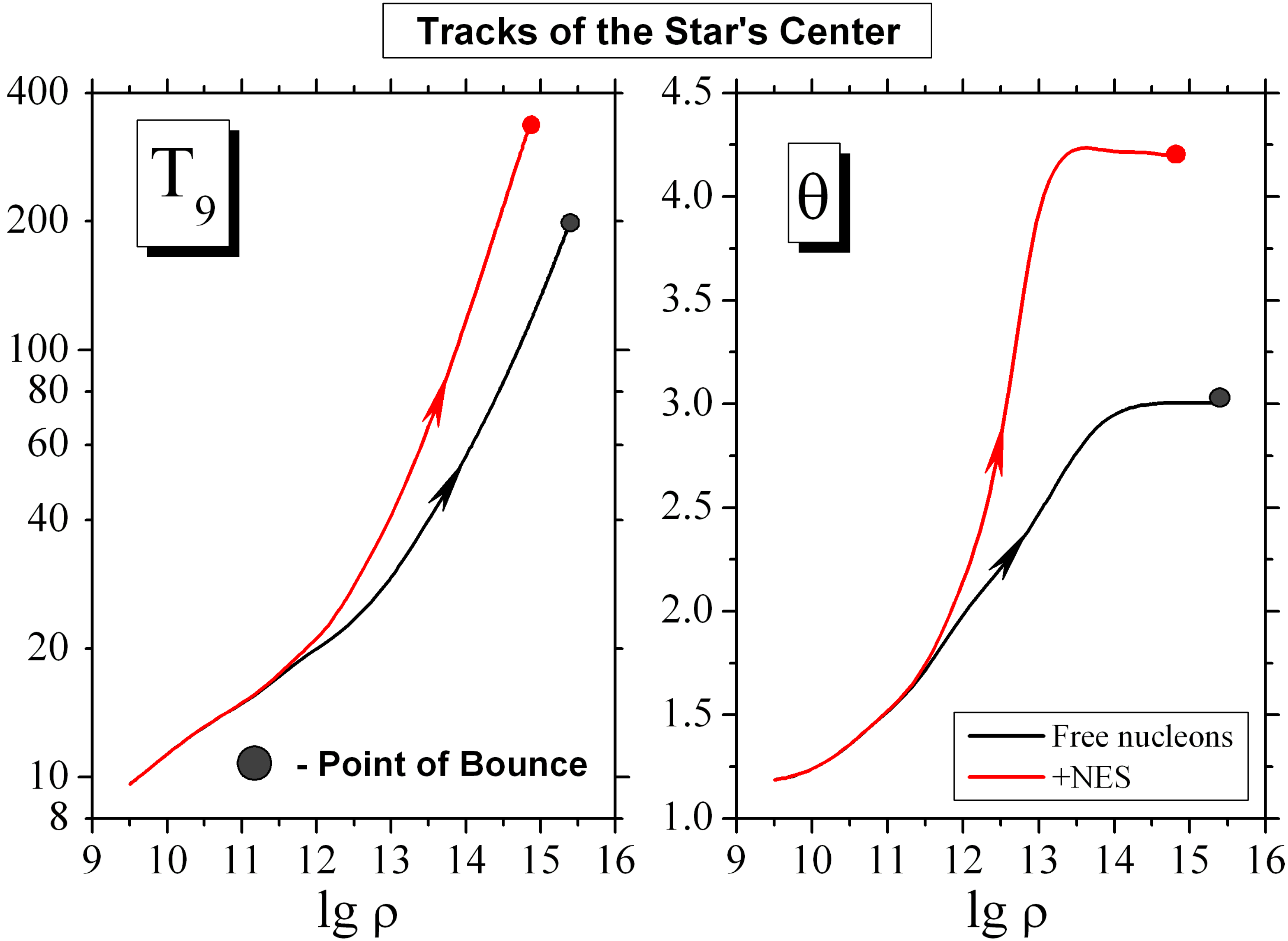
$$l_{\mathrm{cs}}^{-1} = 2\pi \int_{-1}^{1} R_{\mathrm{cs}}(\omega, \eta) d\eta,$$

Transport cross-section:

$$l_{\mathrm{cs}}^{-1}(1 - \langle\eta\rangle) = \sigma_{\mathrm{cs}}^{\mathrm{t}} n_{\mathrm{cs}}$$

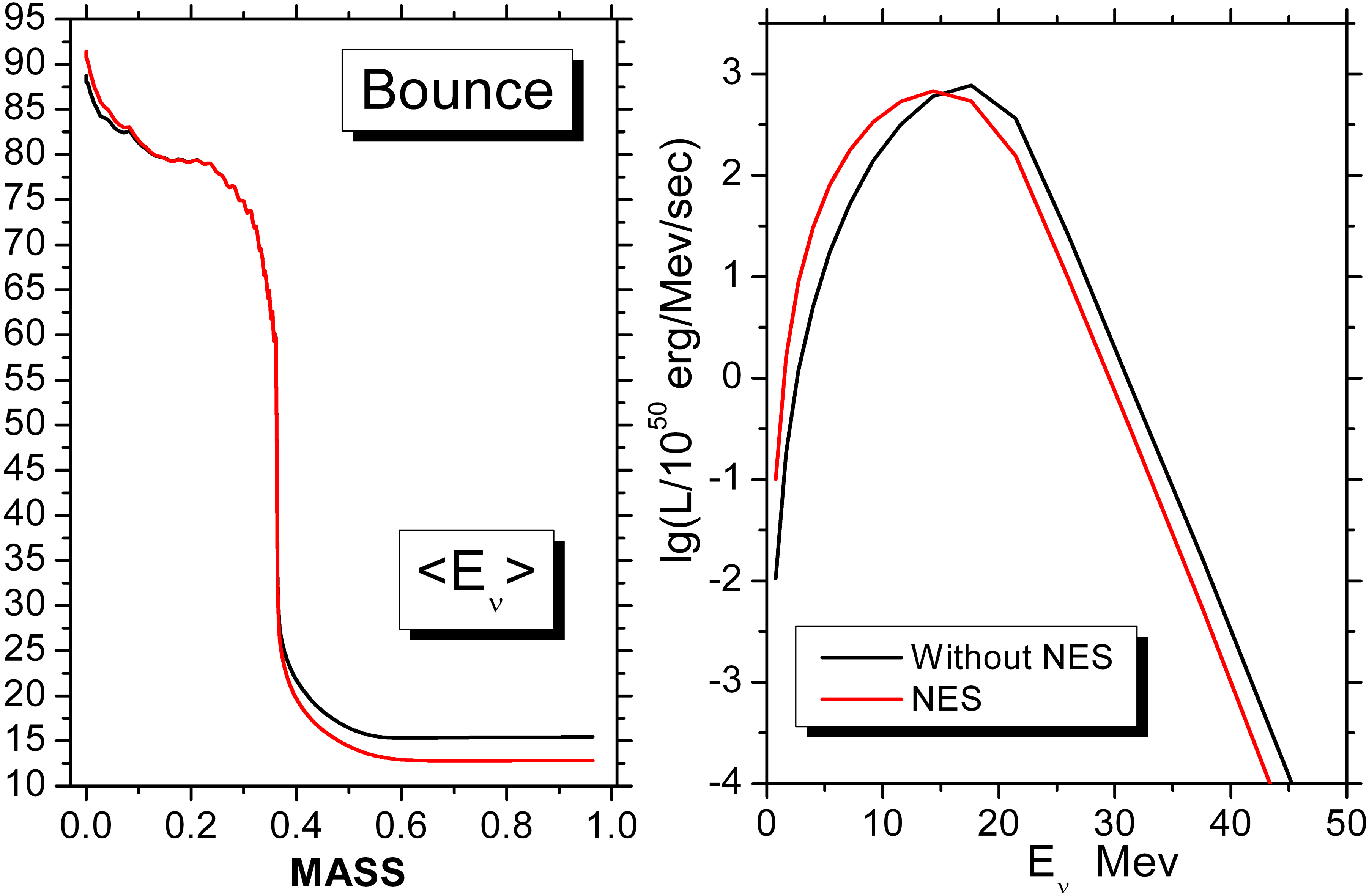
$$\sigma_{\mathrm{cs}}^{\mathrm{t}} = \sigma_{\mathrm{cs}}(1 - \langle\eta\rangle)$$

n_{cs} is the number of coherently scattering particles per unit volume



Tracks of the Star's Center
T9
θ
- Point of Bounce
Free nucleons
+NES
lg ρ
lg ρ
400
200
100
80
60
40
20
10
8
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
9 10 11 12 13 14 15 16
9 10 11 12 13 14 15 16

NES Influence



Влияние NES на коэффициенты нейтринной теплопроводности

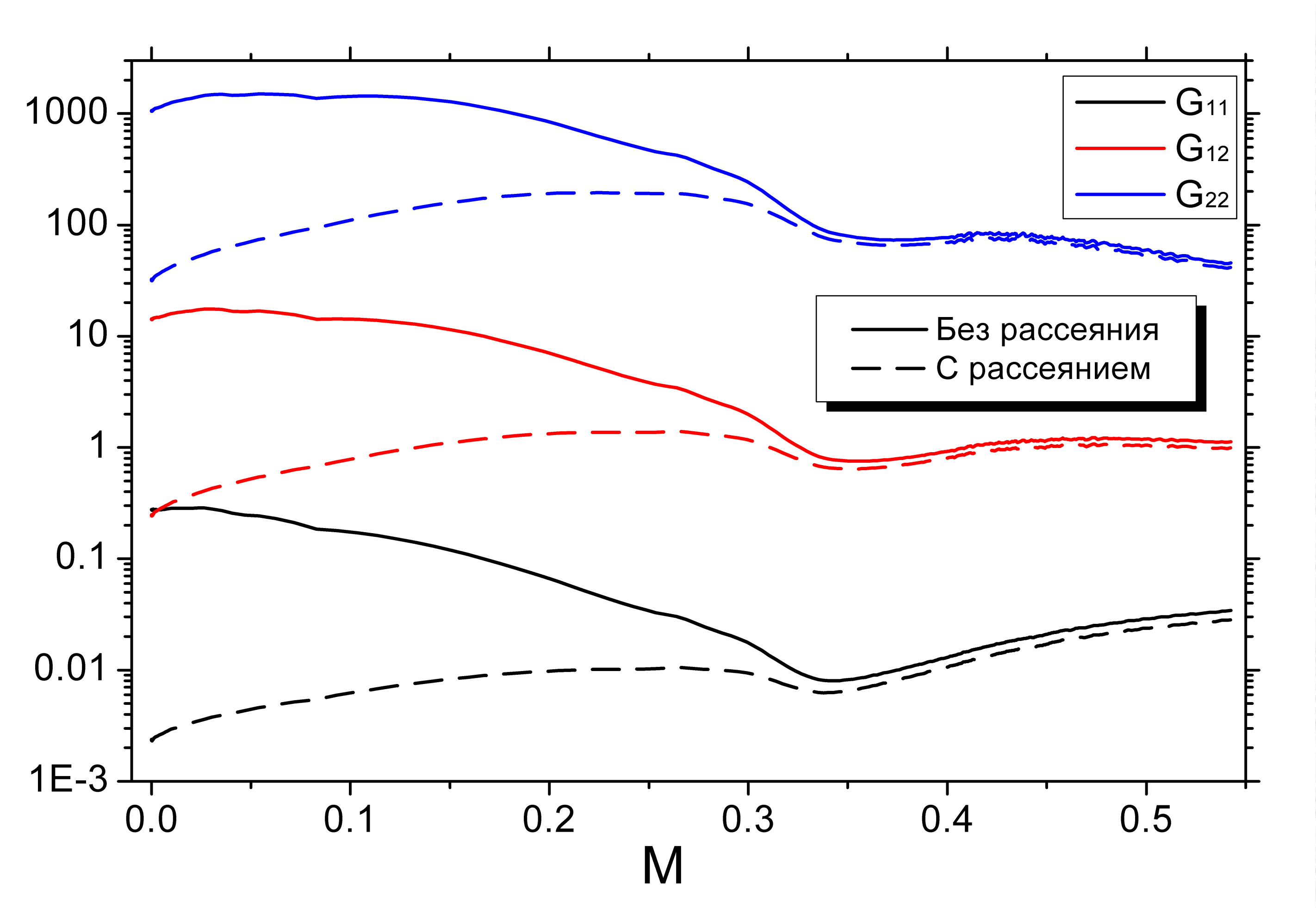
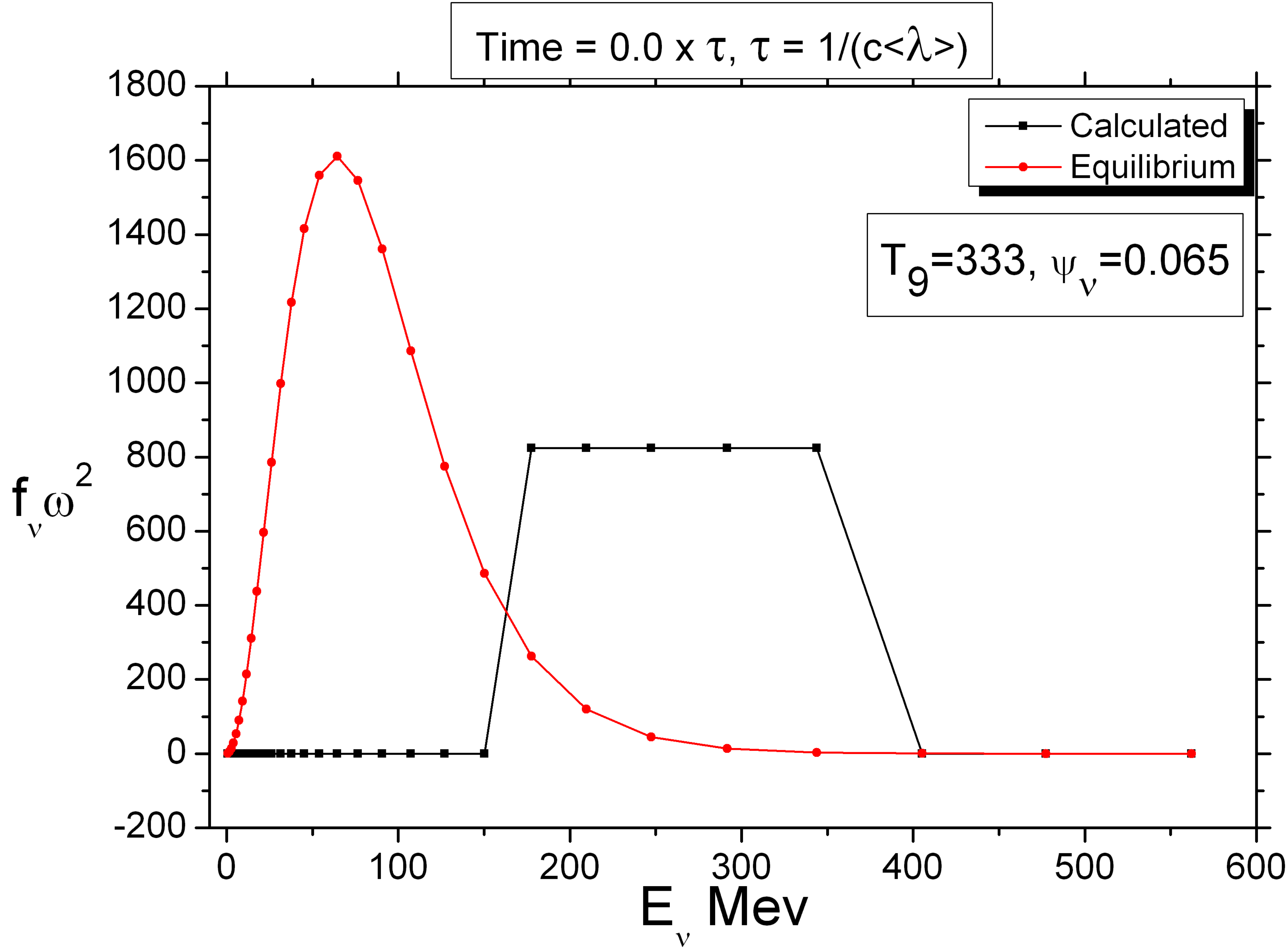


Рисунок соответствует звезде после отскока, более подробно описанной на следующем слайде. Показана только область совместной применимости нейтринной теплопроводности без/с рассеянием.



Time = 0.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T_9=333, ψ_ν=0.065
$f_\nu \omega^2$
E_ν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
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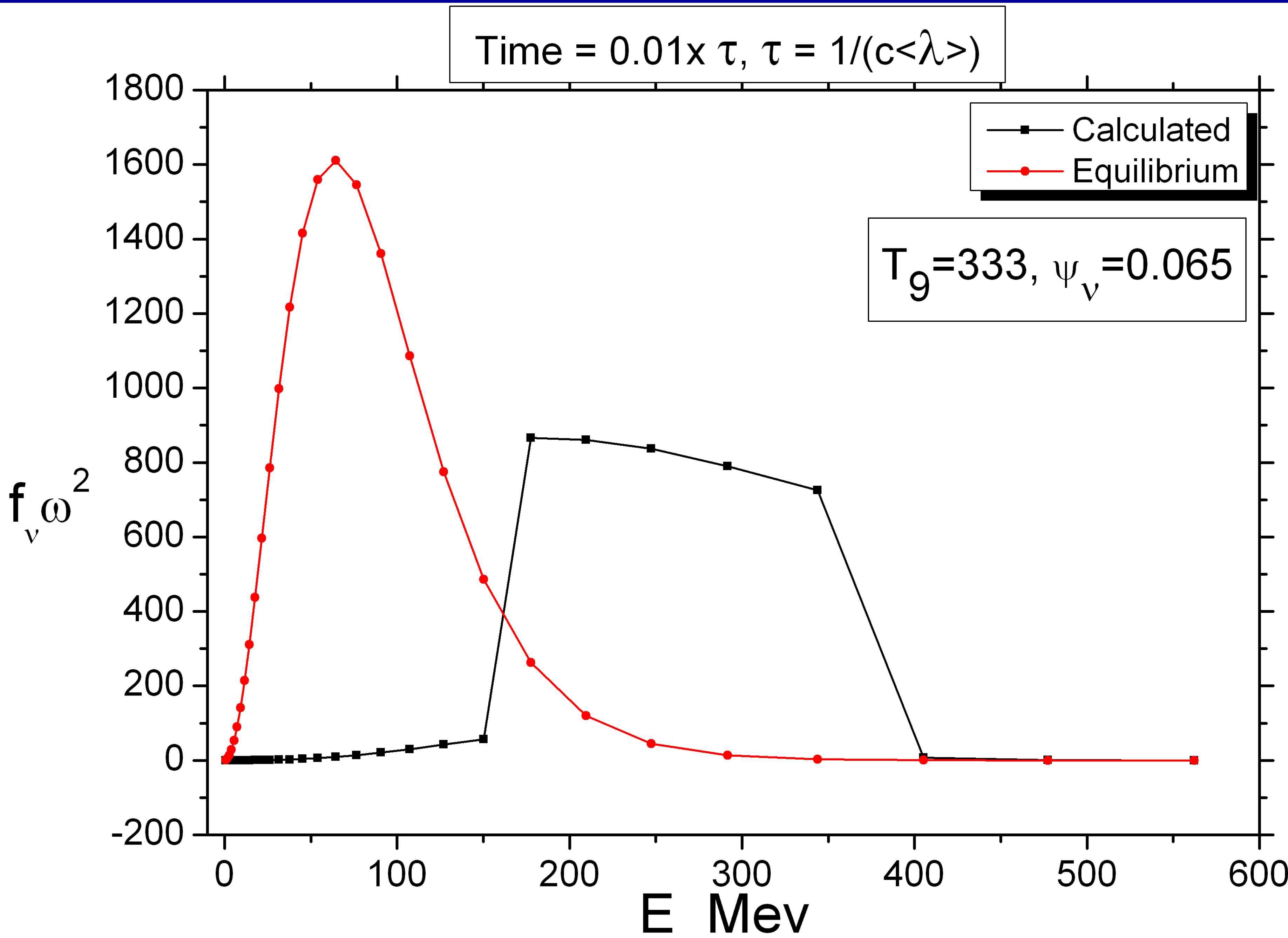
500
600



Time = 0.01x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
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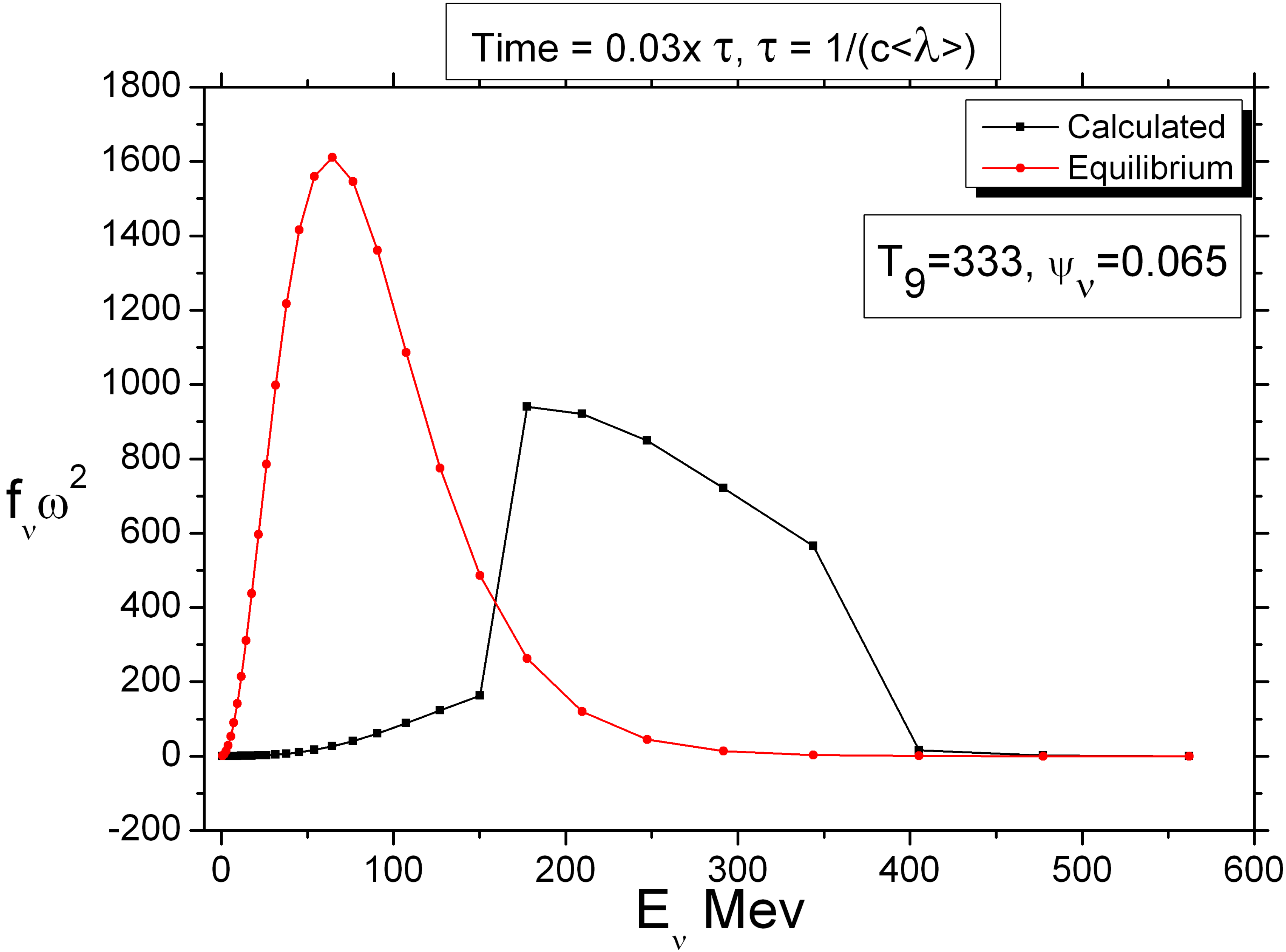
600



Time = 0.03x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
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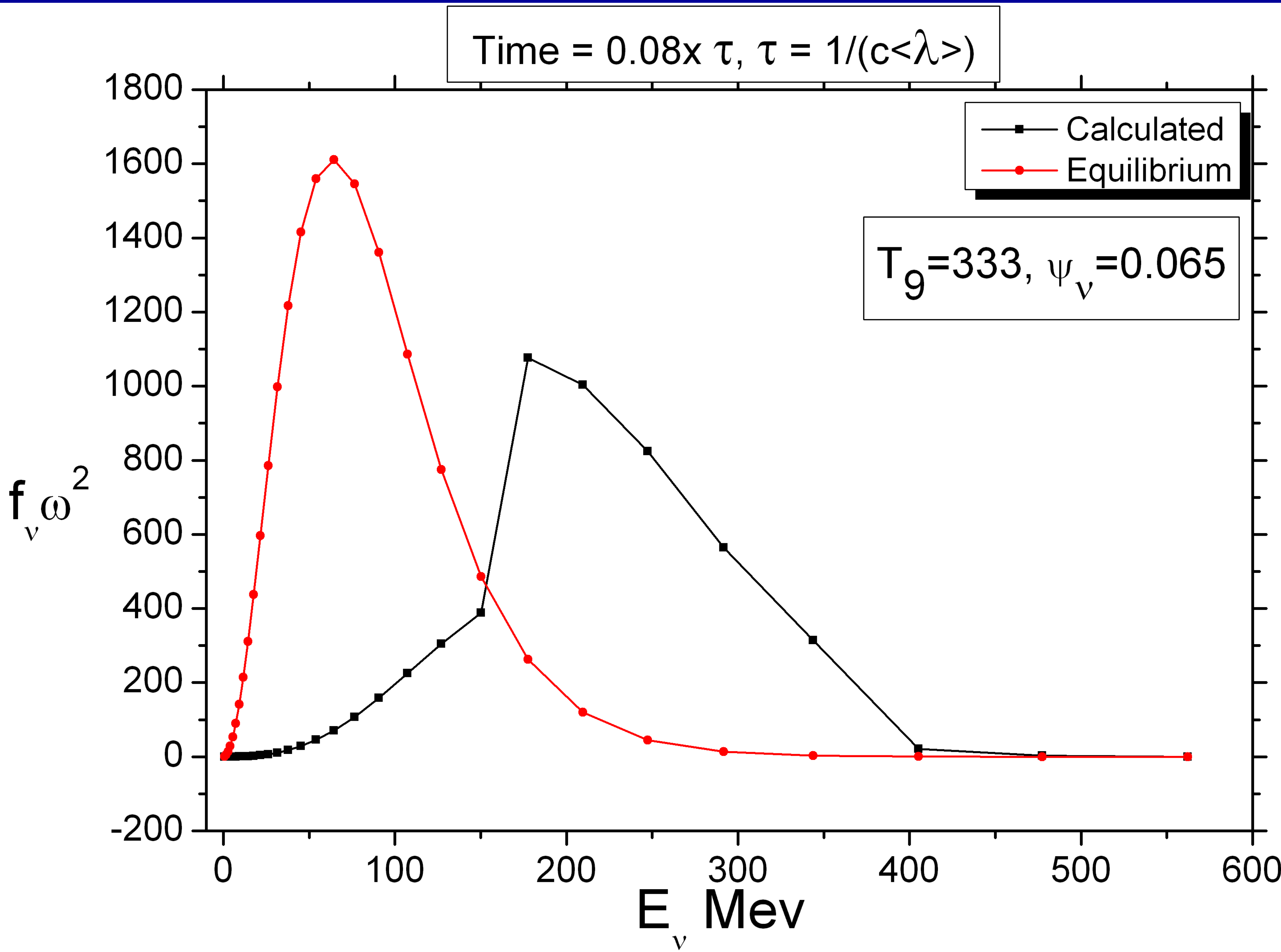
Time = 0.08x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
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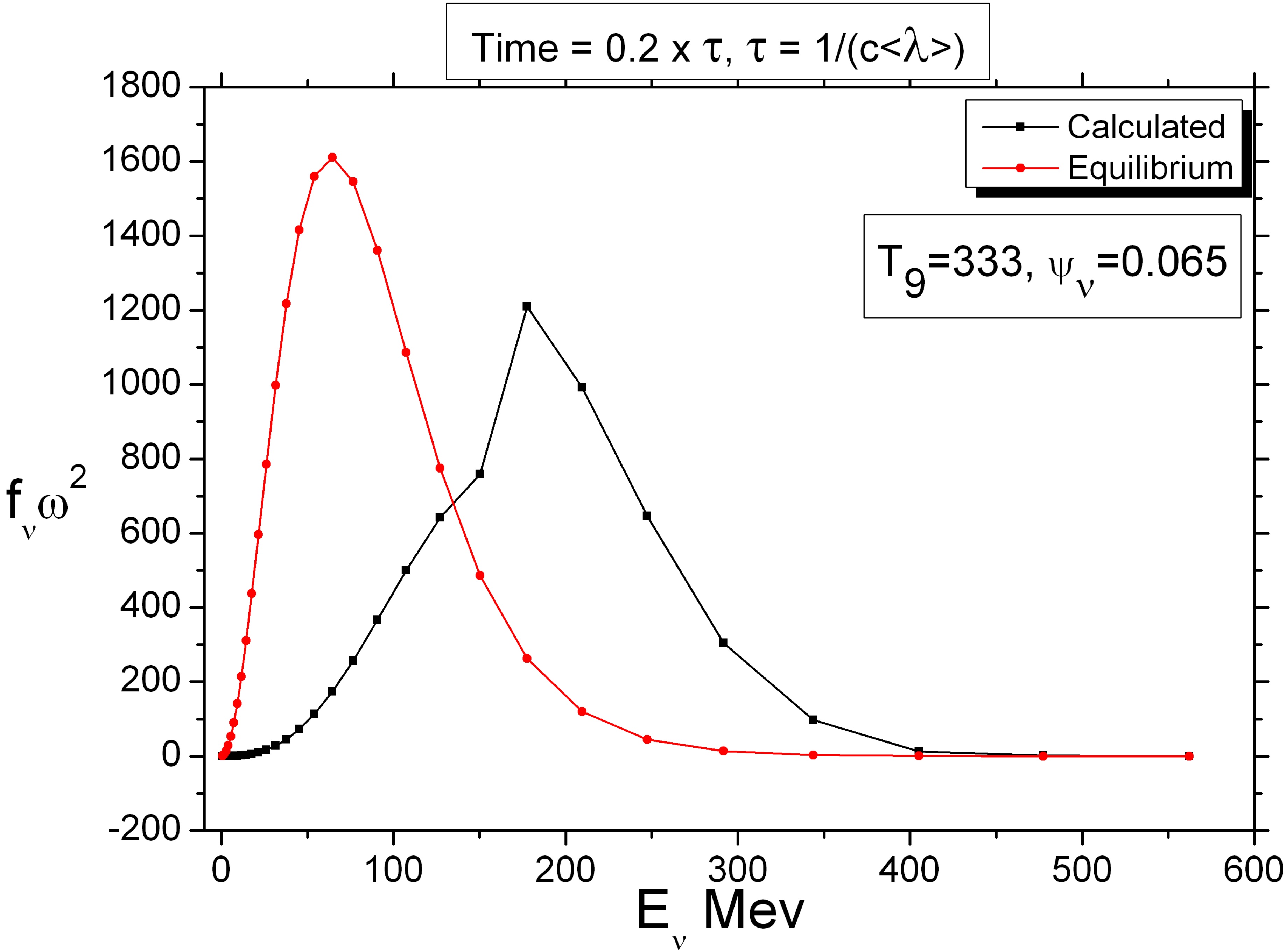

Time = 0.2 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
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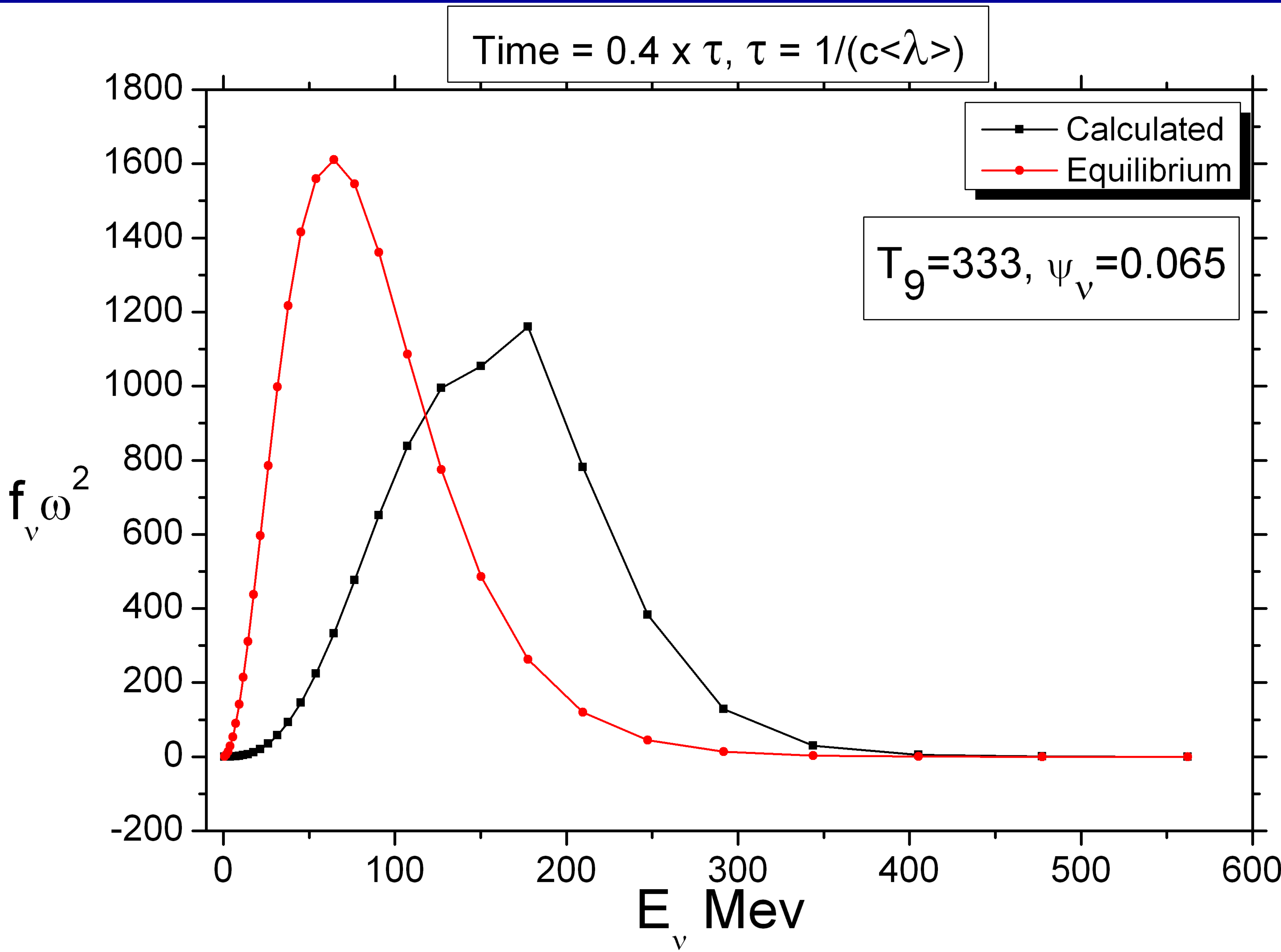
600



Time = 0.4 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
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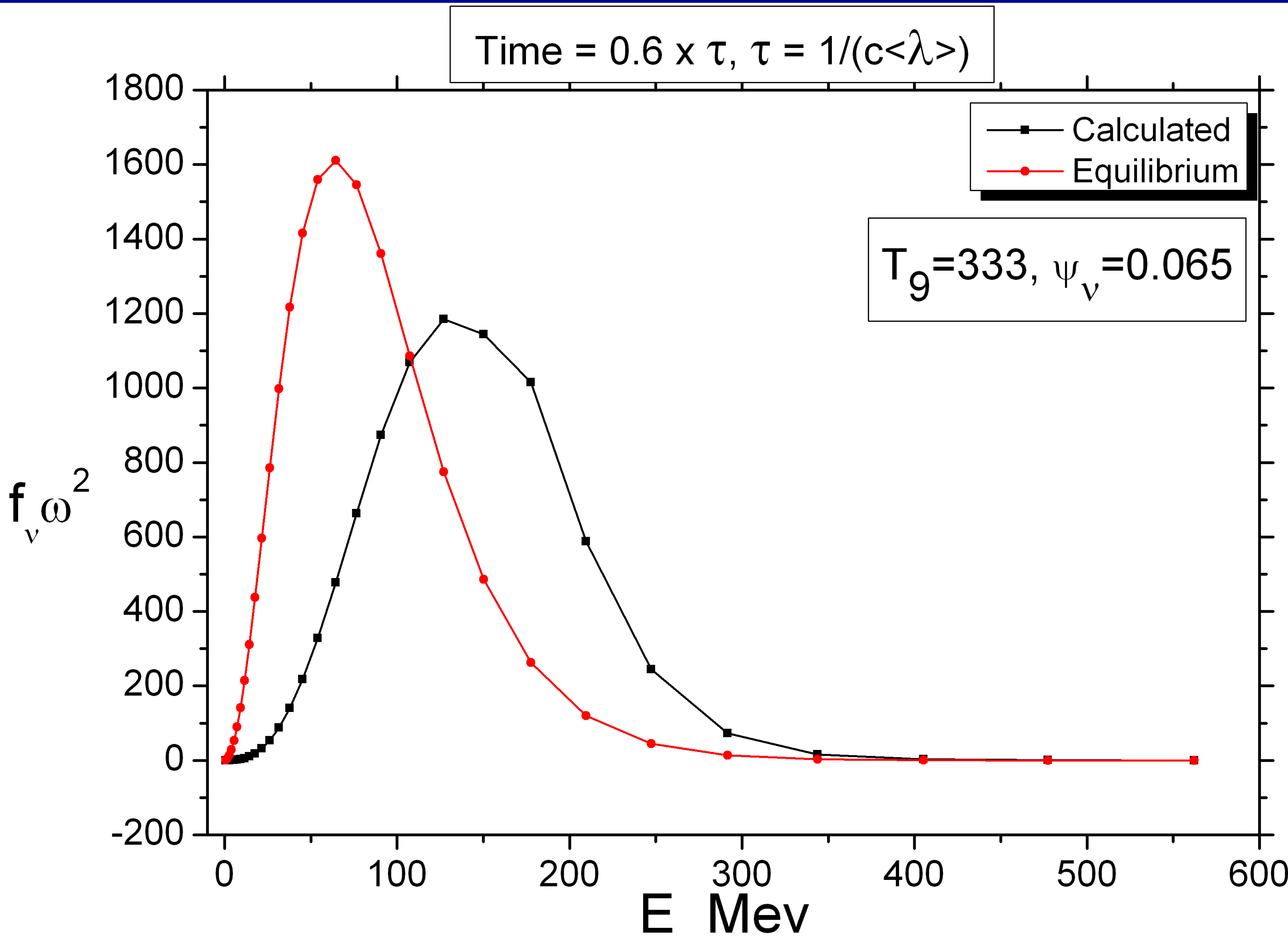
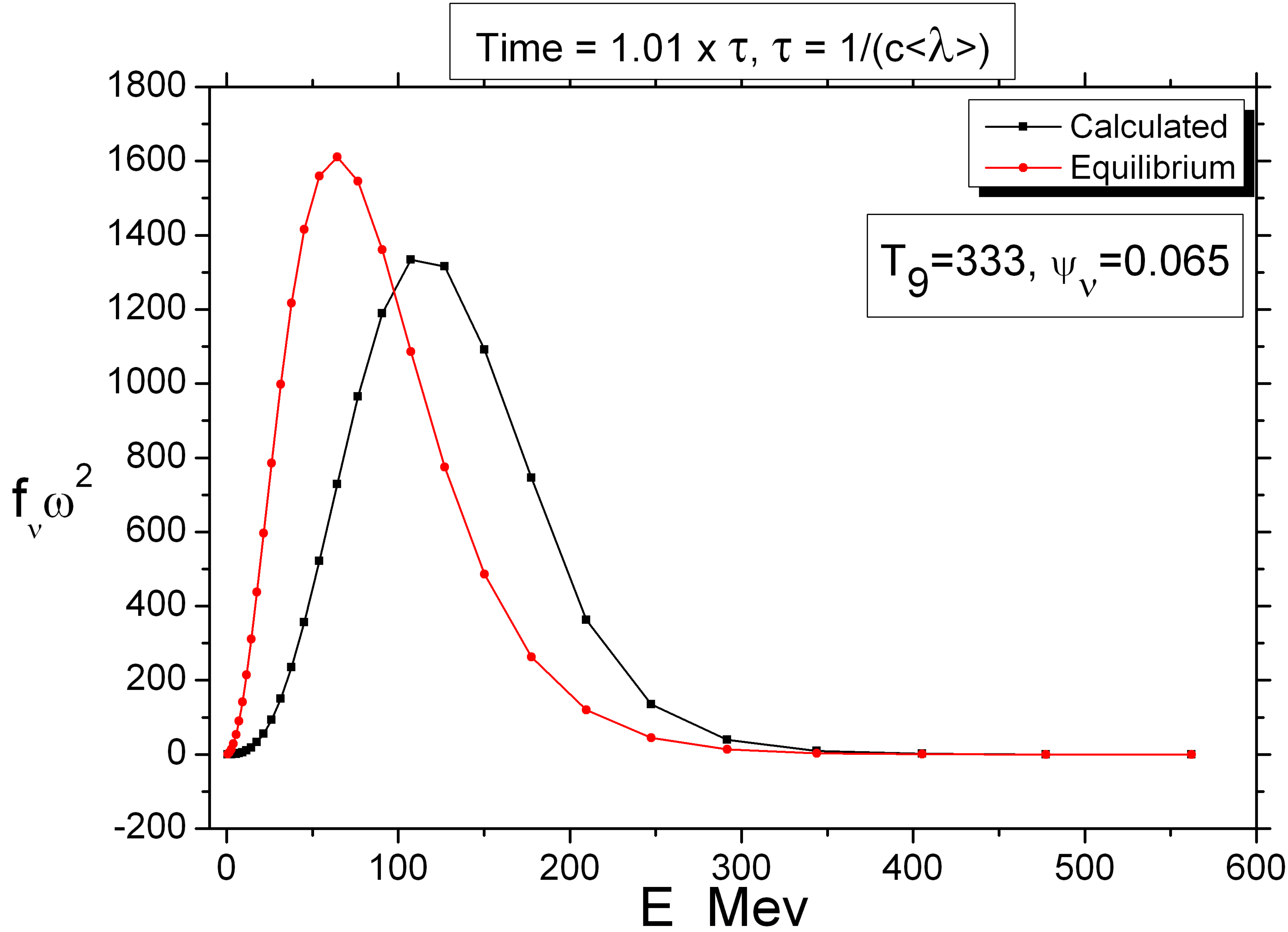
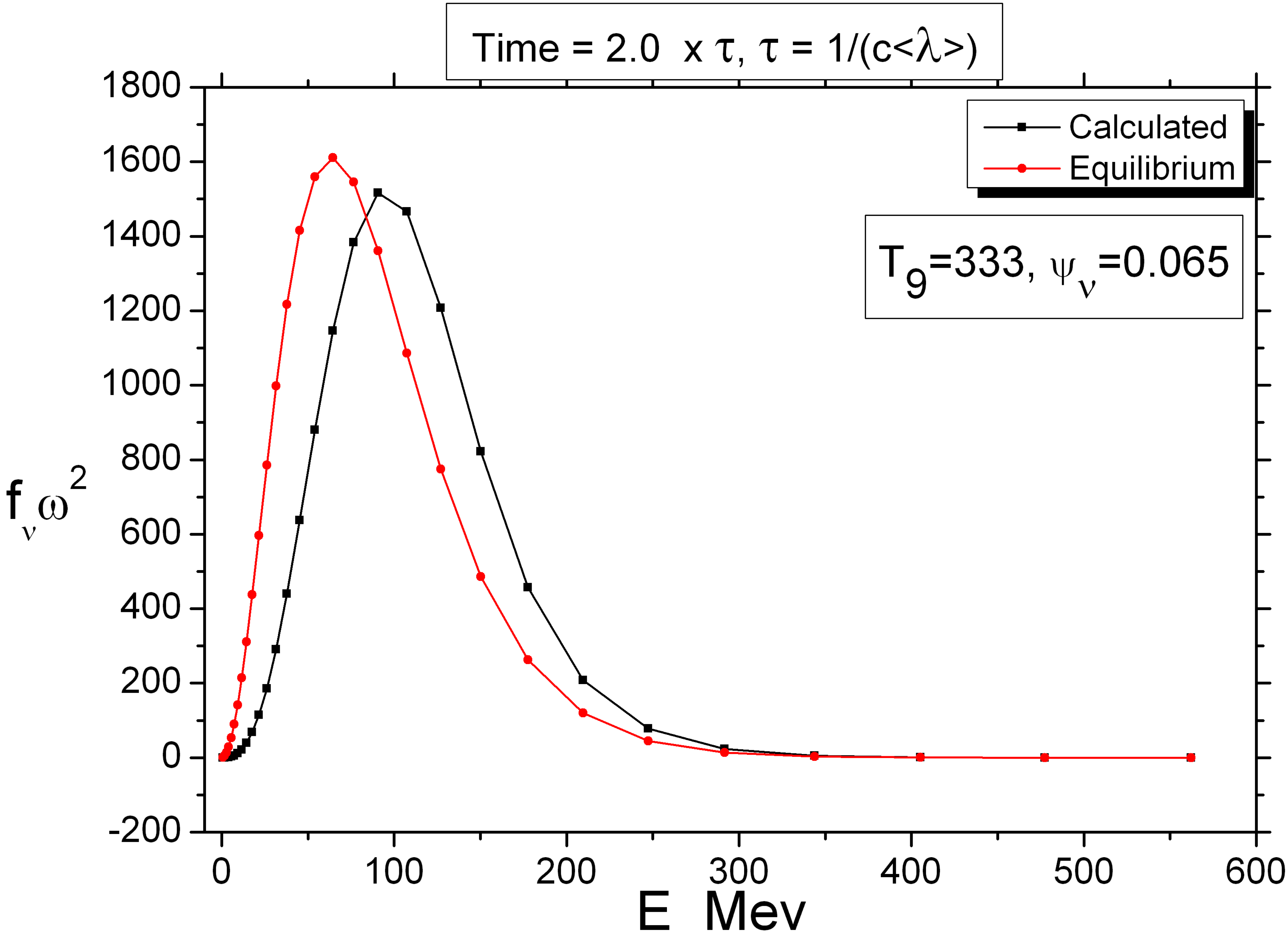
600



Time = 0.6 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
1800
1600
1400
1200
1000
800
600
400
200
0
-200
fνω2
0
100
200
300
400
500
600
Eν Mev



Time = 1.01 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T_9=333, ψ_ν=0.065
$f_\nu \omega^2$
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600
E_ν Mev



Time = 2.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
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400
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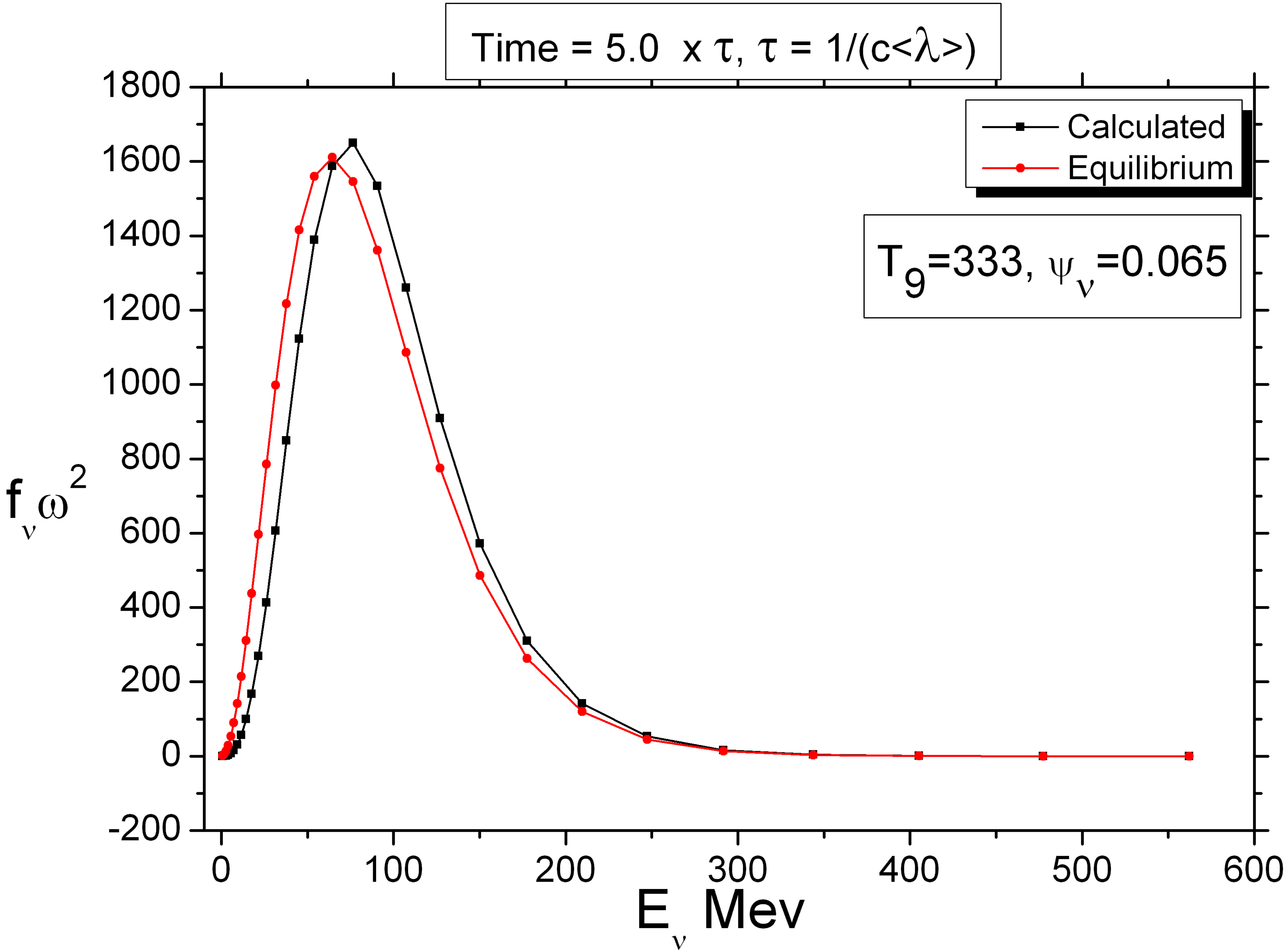
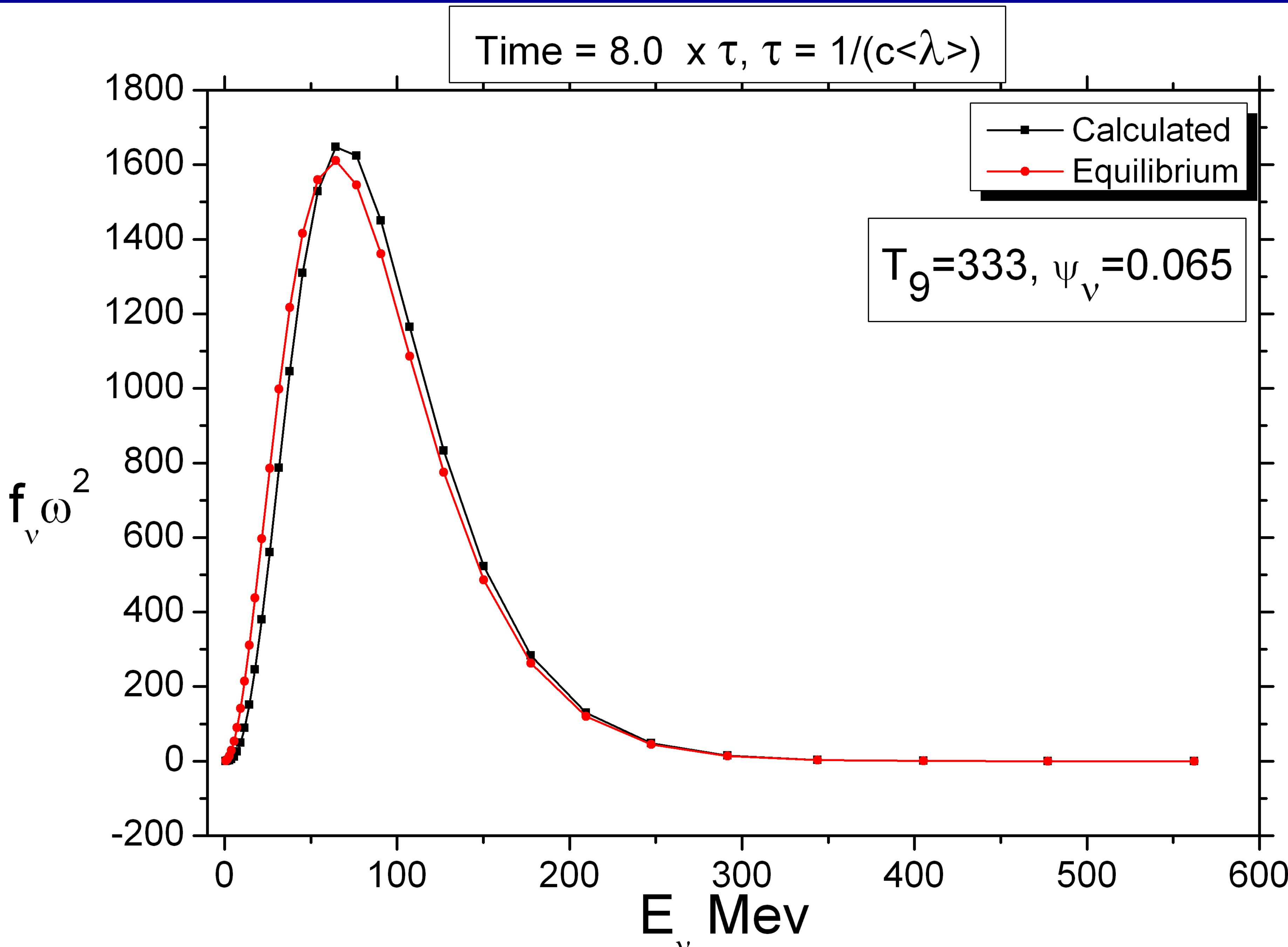


Time = 5.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
1800
1600
1400
1200
1000
800
600
400
200
0
-200
fνω2
0
100
200
300
400
500
600
Eν Mev



Time = 8.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T_9=333, ψ_ν=0.065
$f_\nu \omega^2$
E_ν Mev
1800
1600
1400
1200
1000
800
600
400
200
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-200
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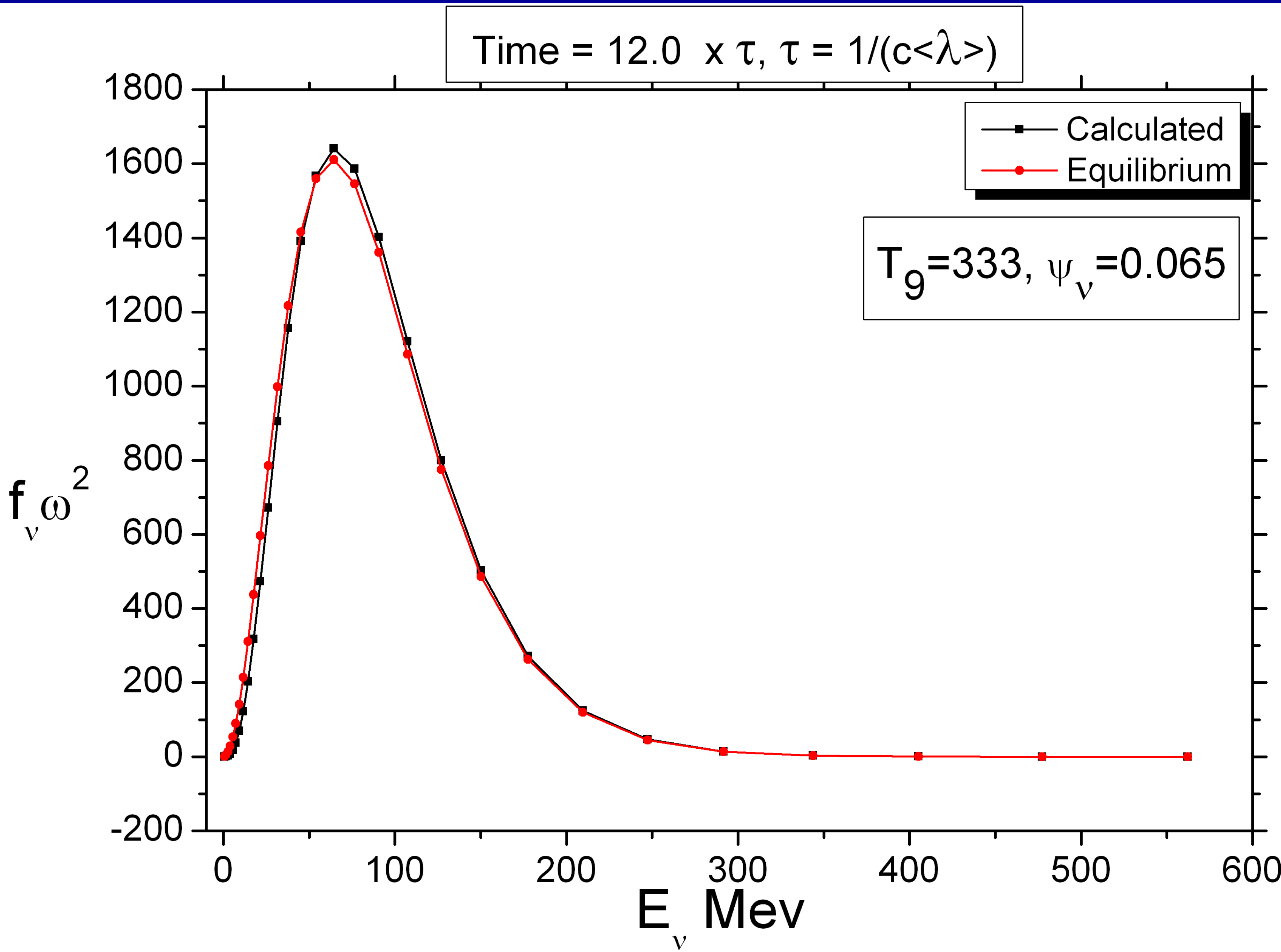
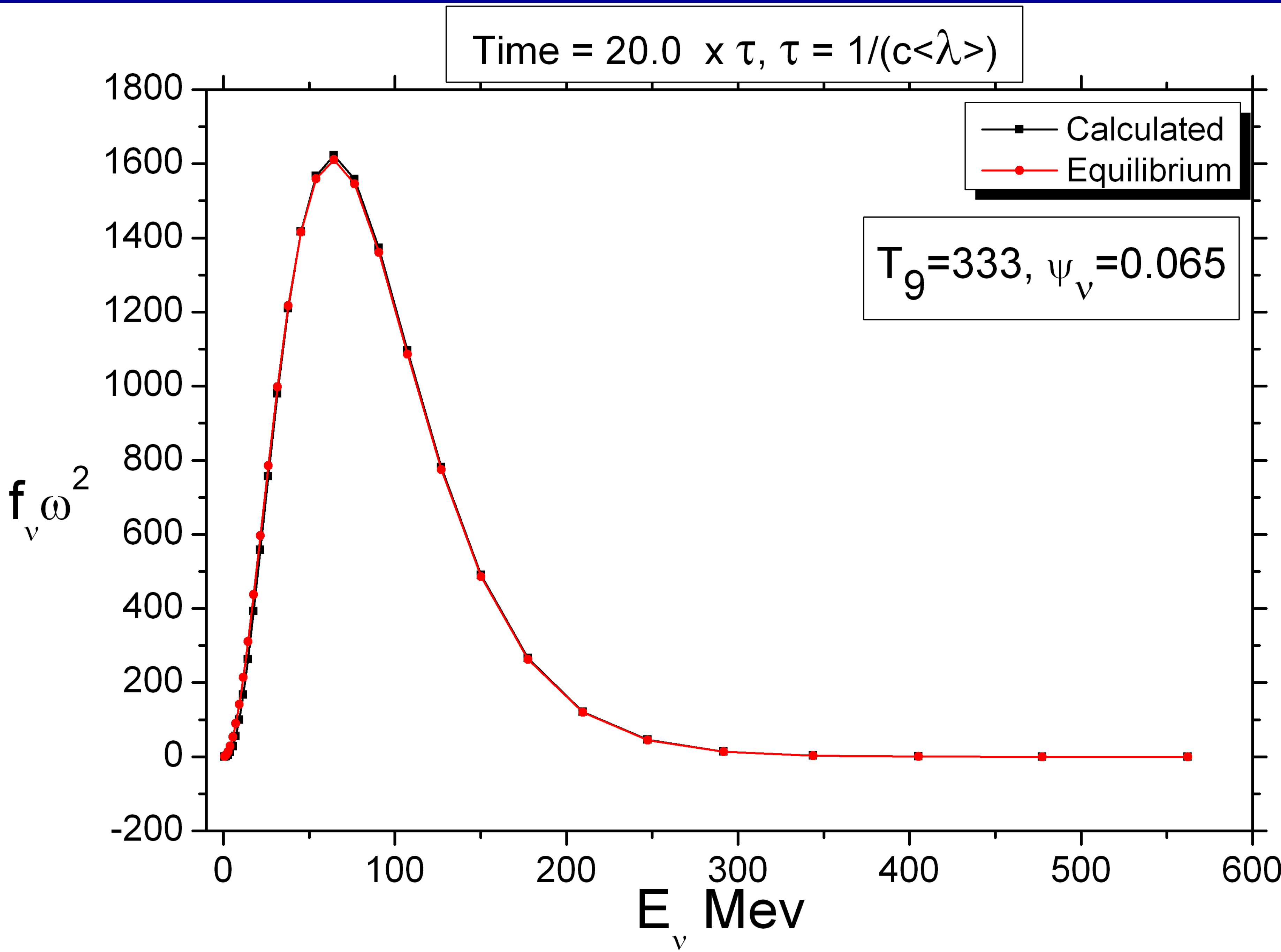


Time = 12.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
1800
1600
1400
1200
1000
800
600
400
200
0
-200
fνω2
0
100
200
300
400
500
600
Eν Mev



Time = 20.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
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-200
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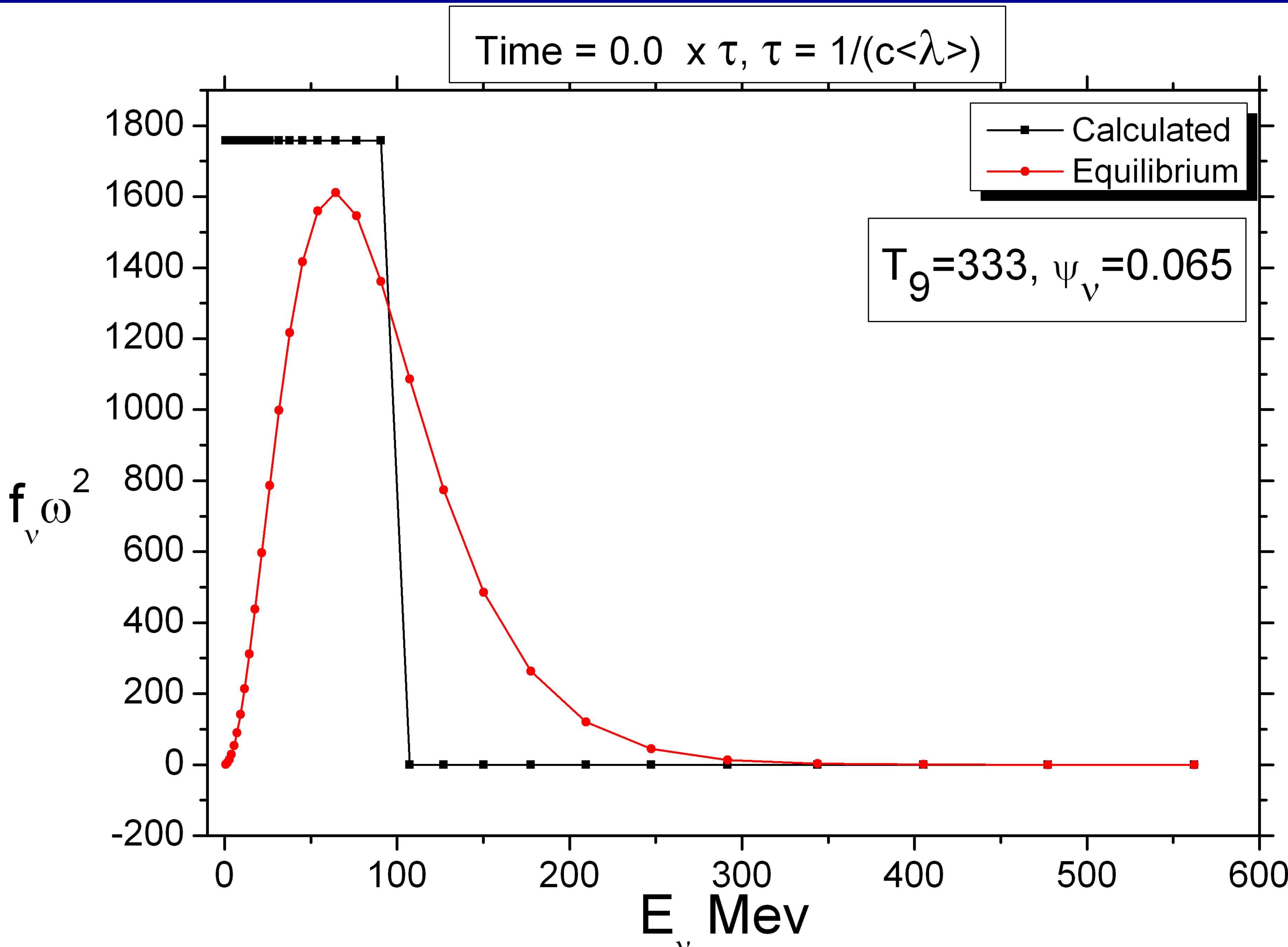

Time = 0.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
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-200
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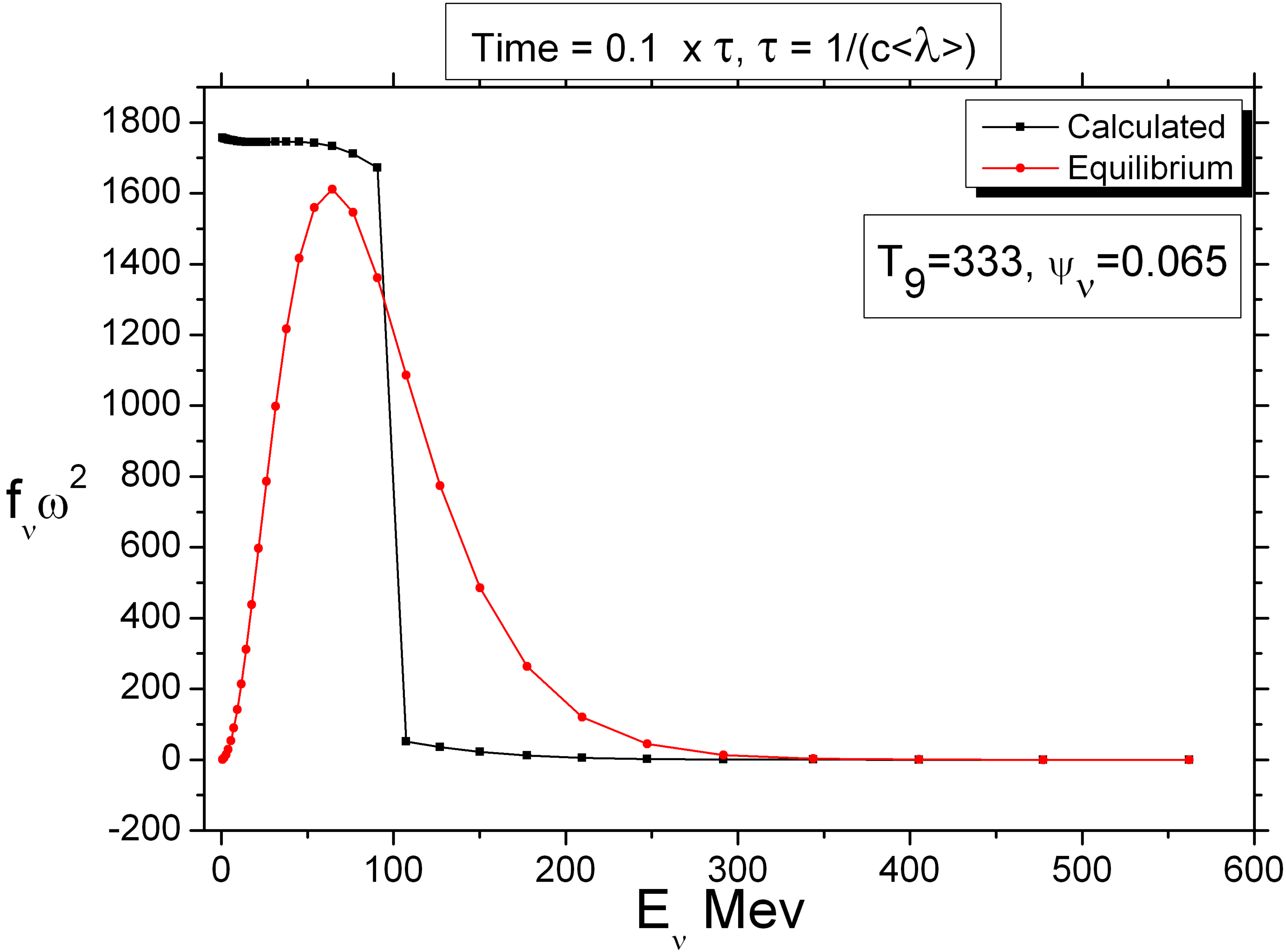

Time = 0.1 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
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-200
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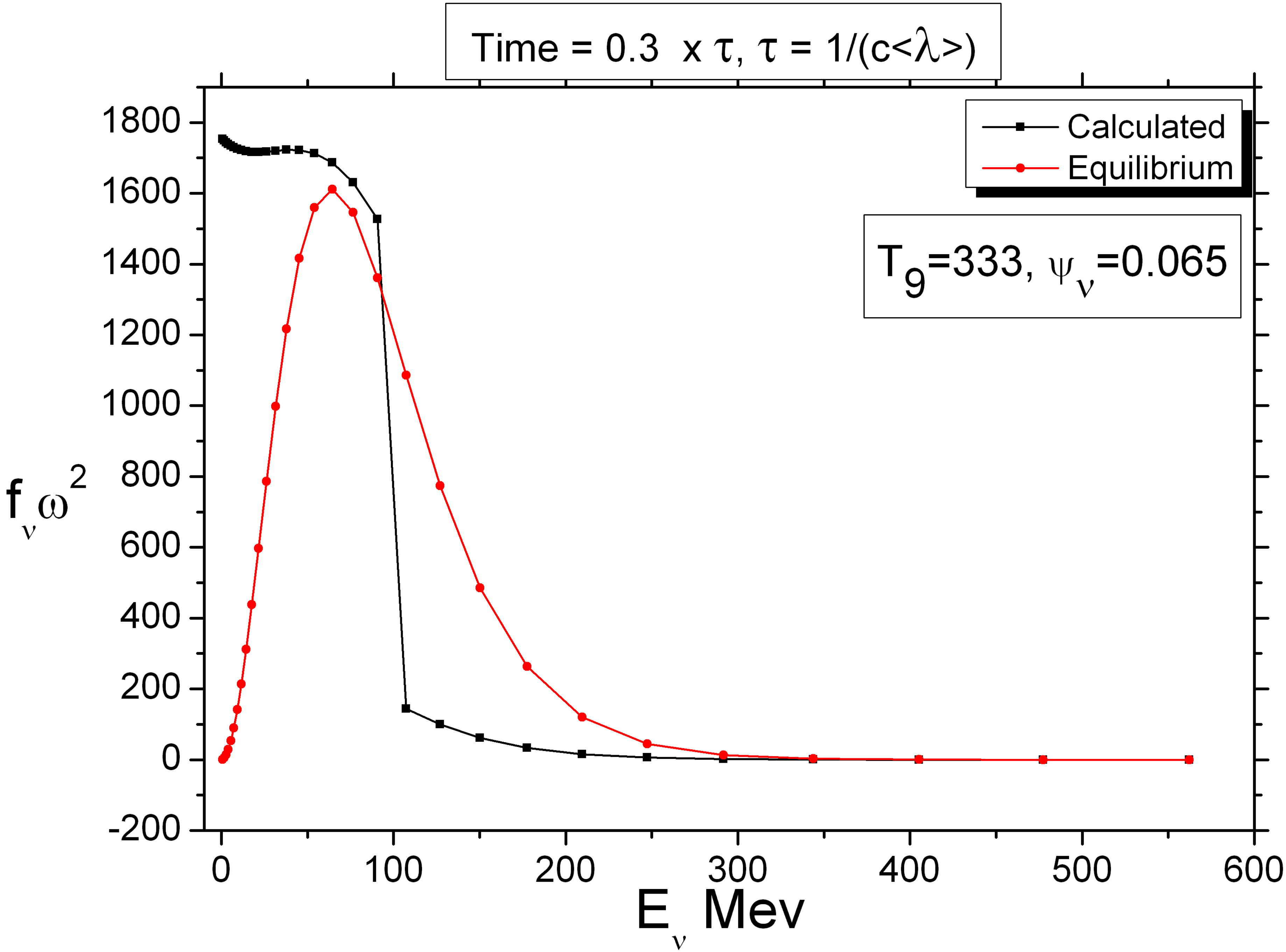
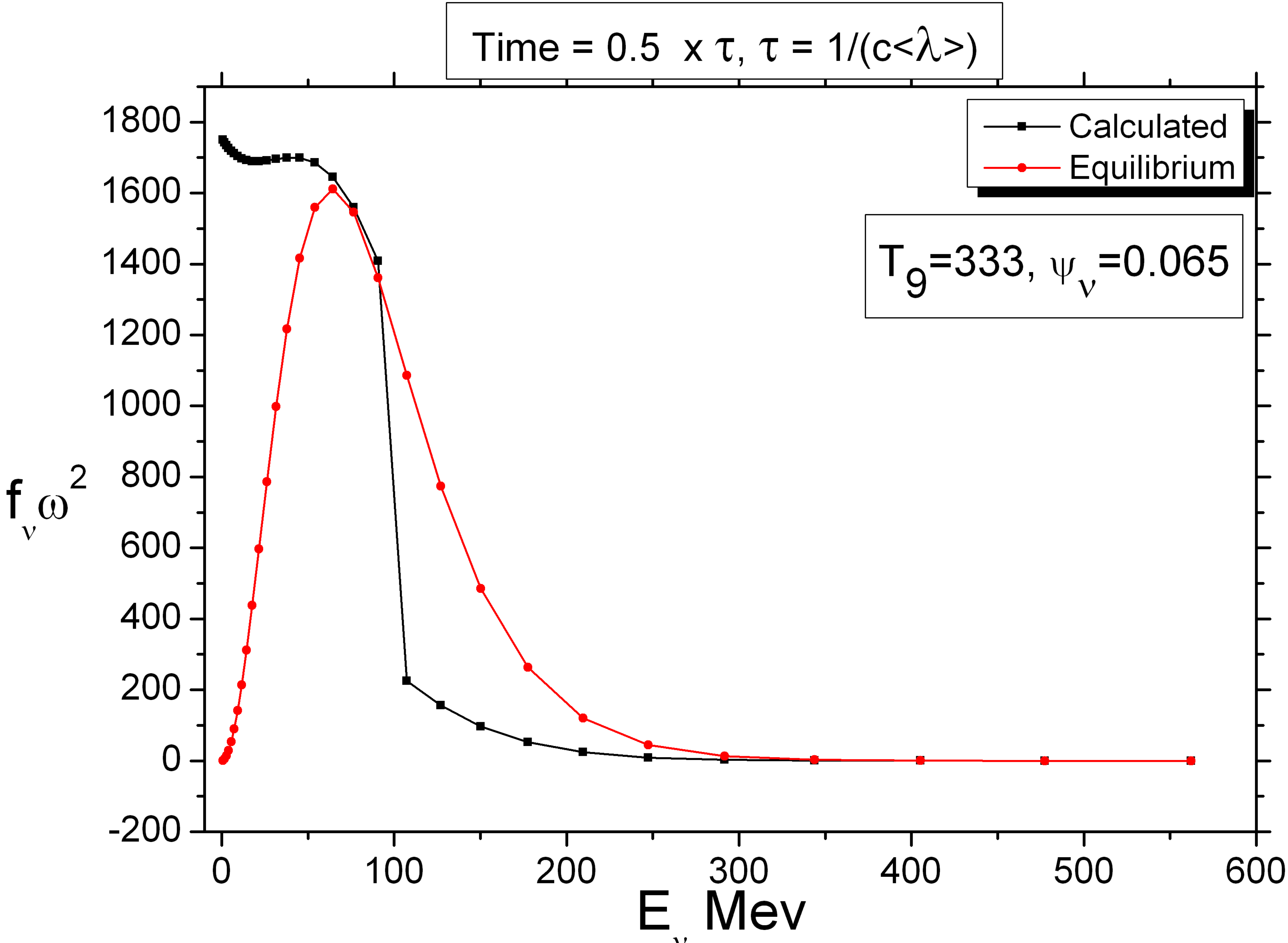
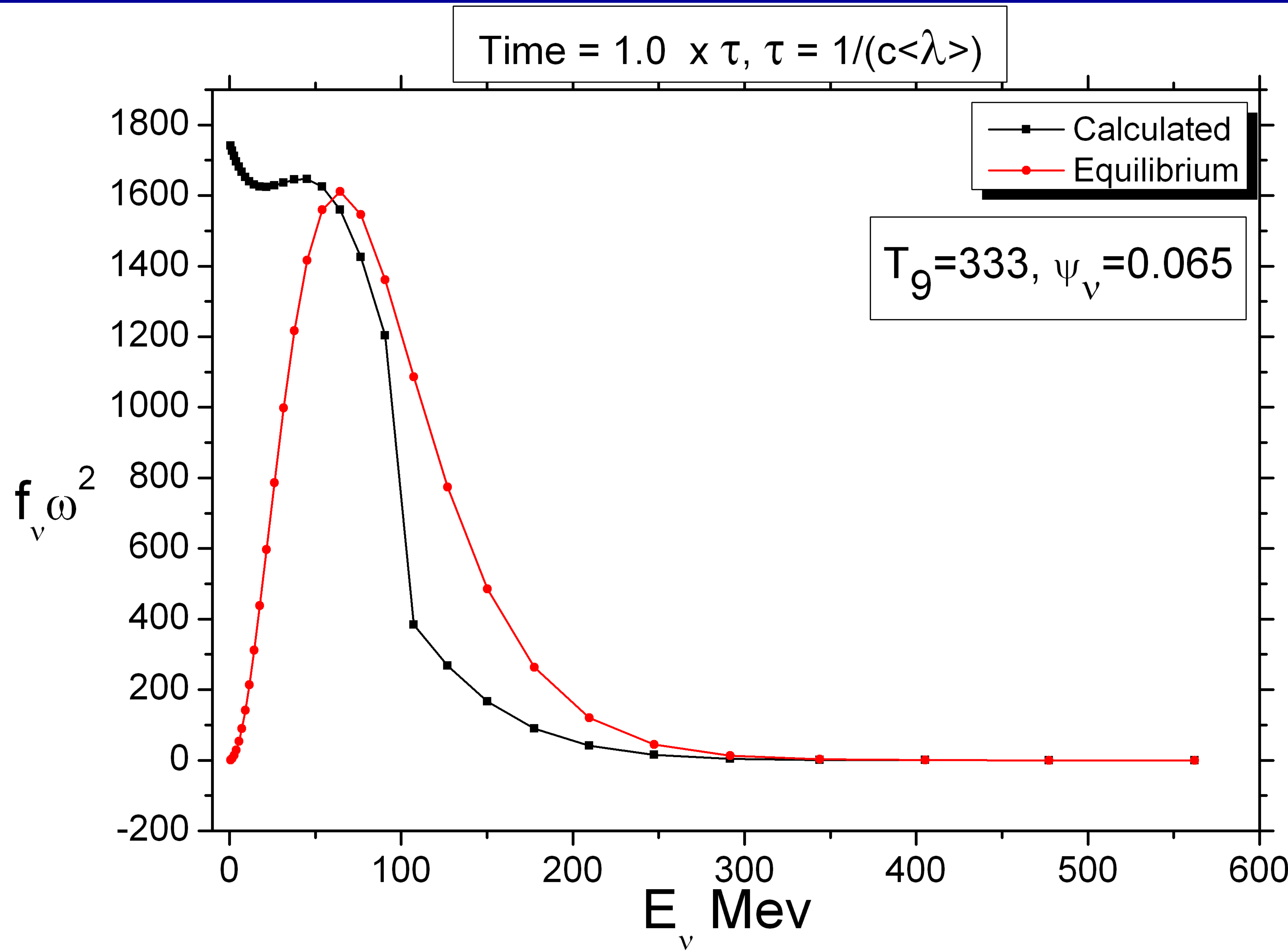


Time = 0.3 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600
Eν Mev



Time = 0.5 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600
Eν Mev



Time = 1.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
1800
1600
1400
1200
1000
800
600
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200
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-200
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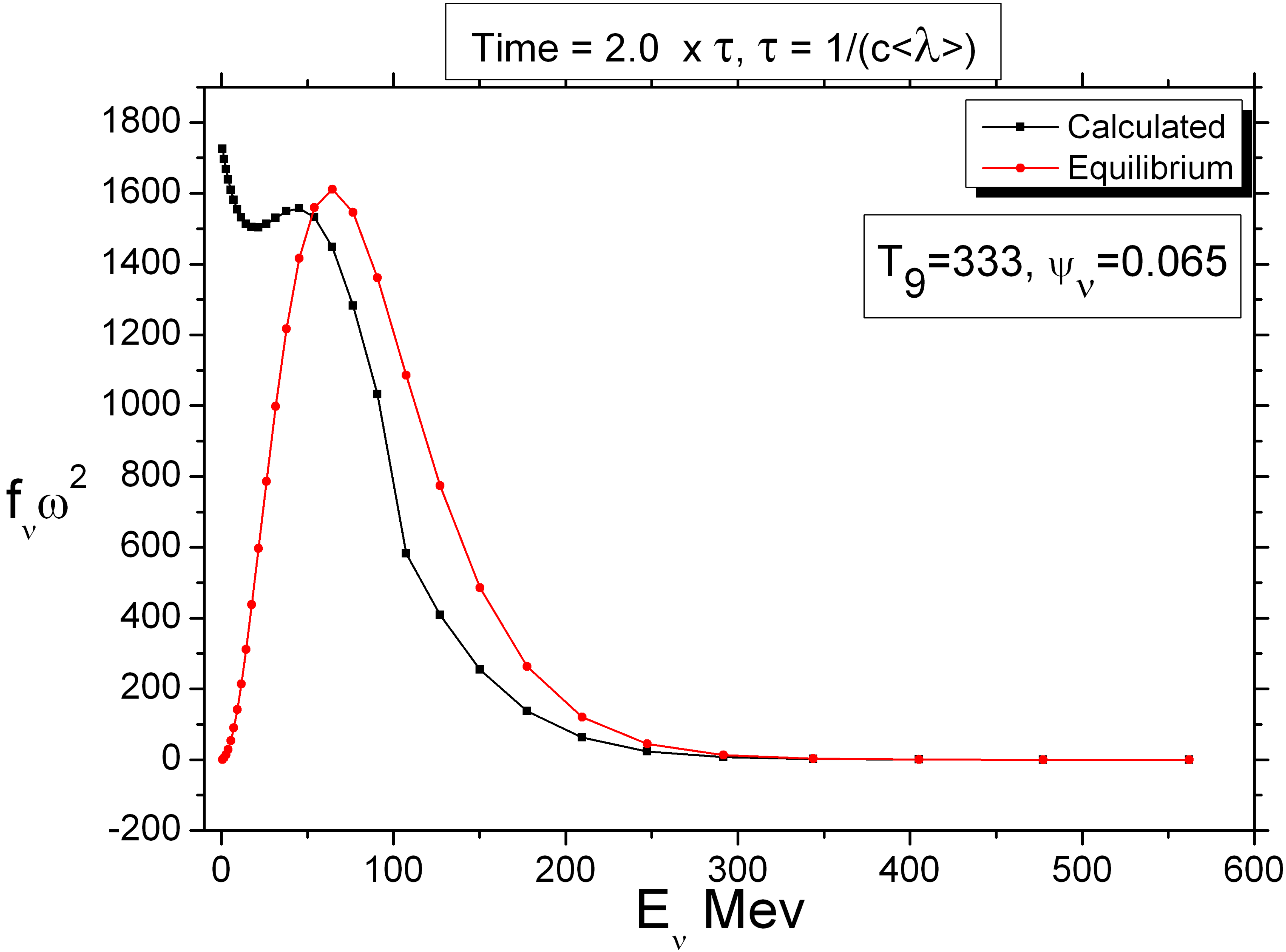
500
600
Eν Mev



Time = 2.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
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200
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-200
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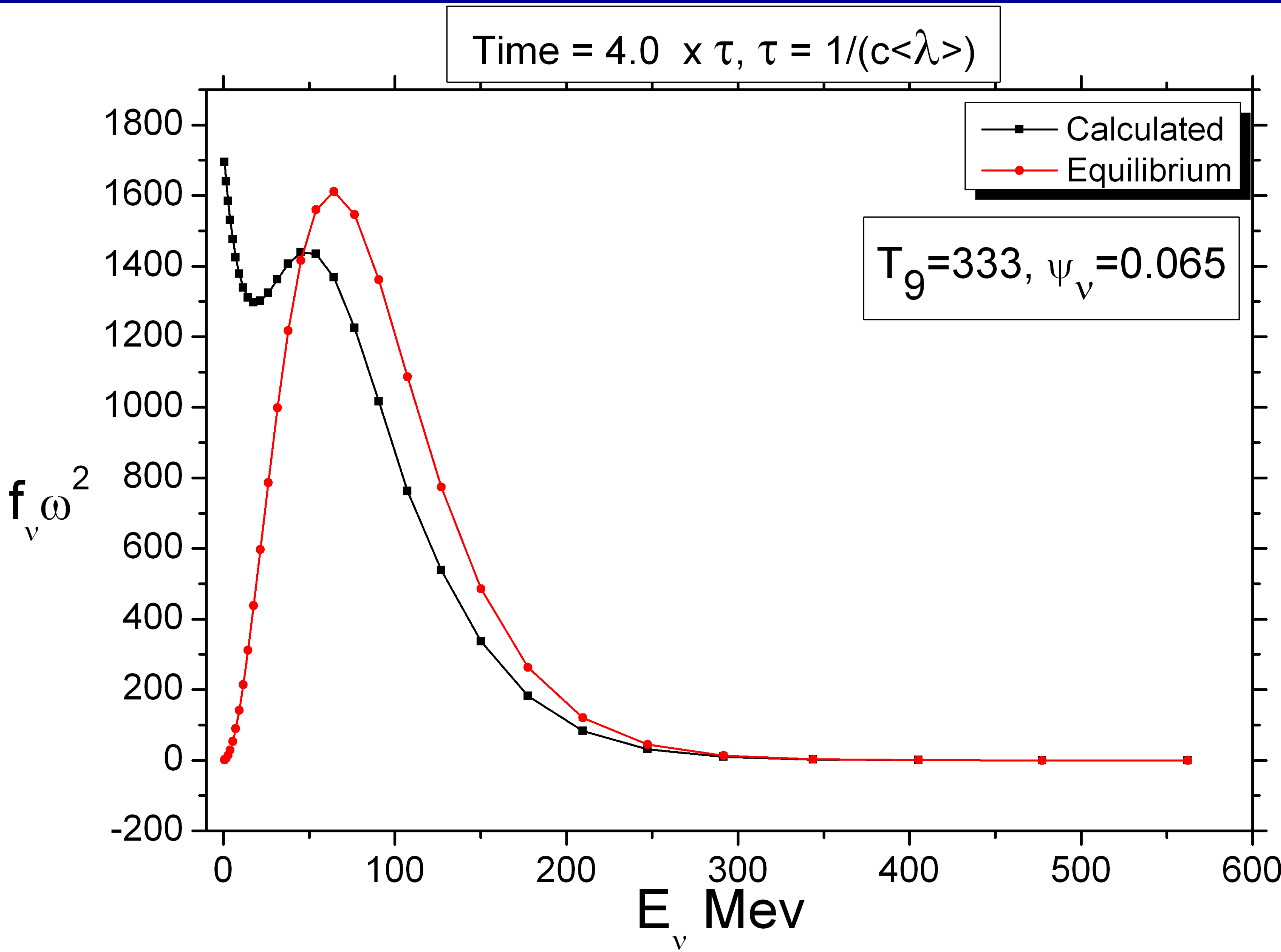
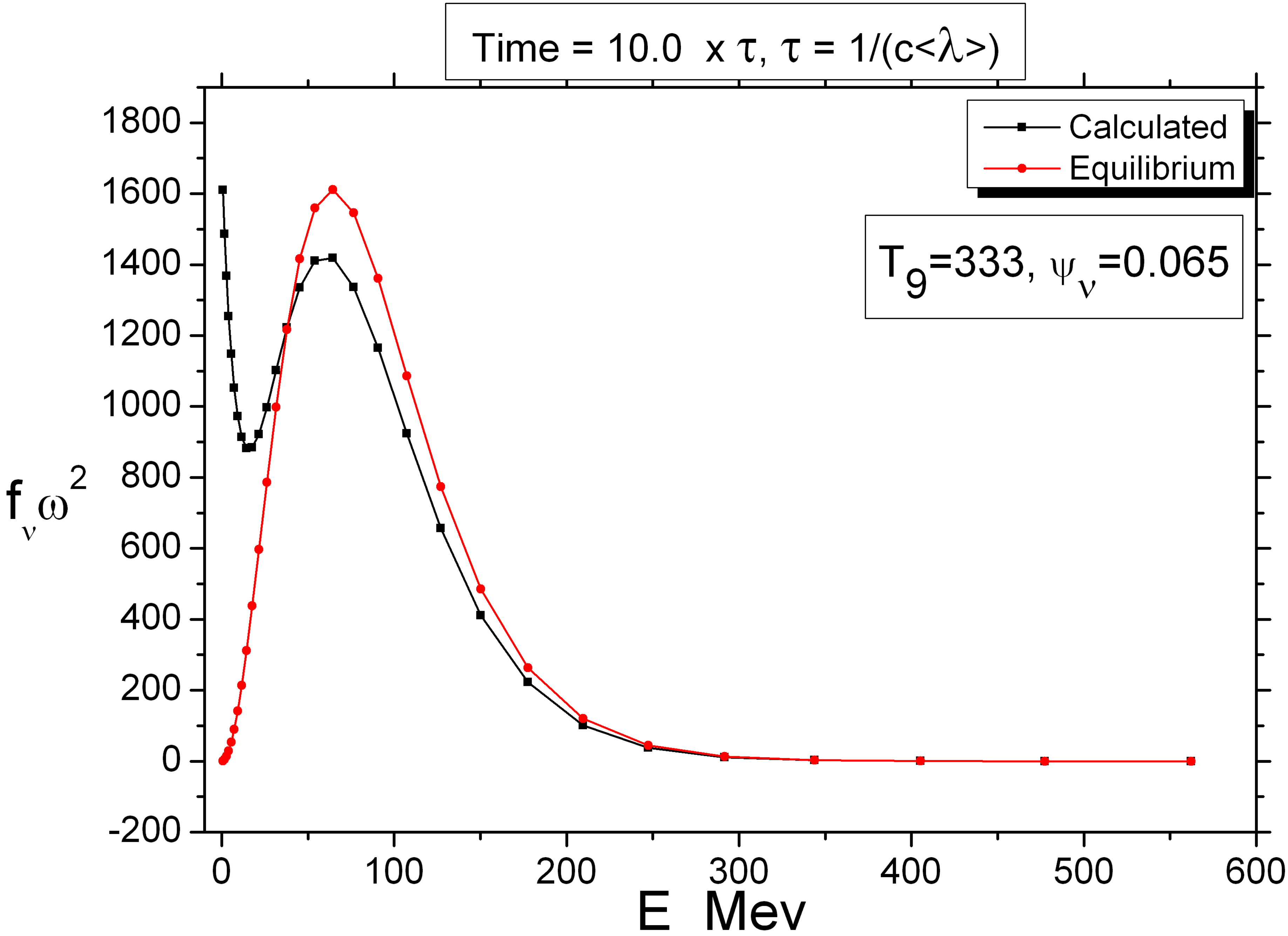
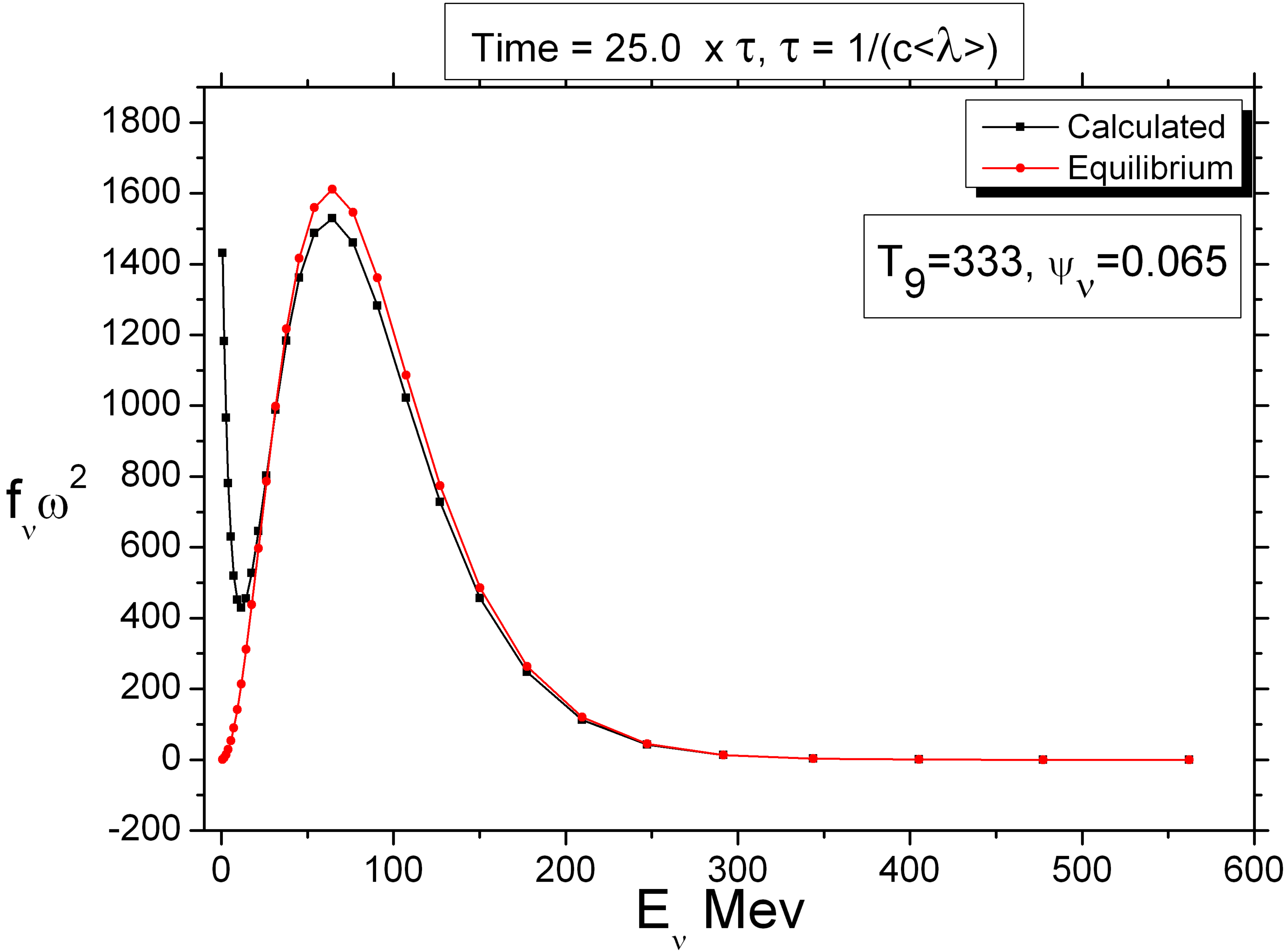
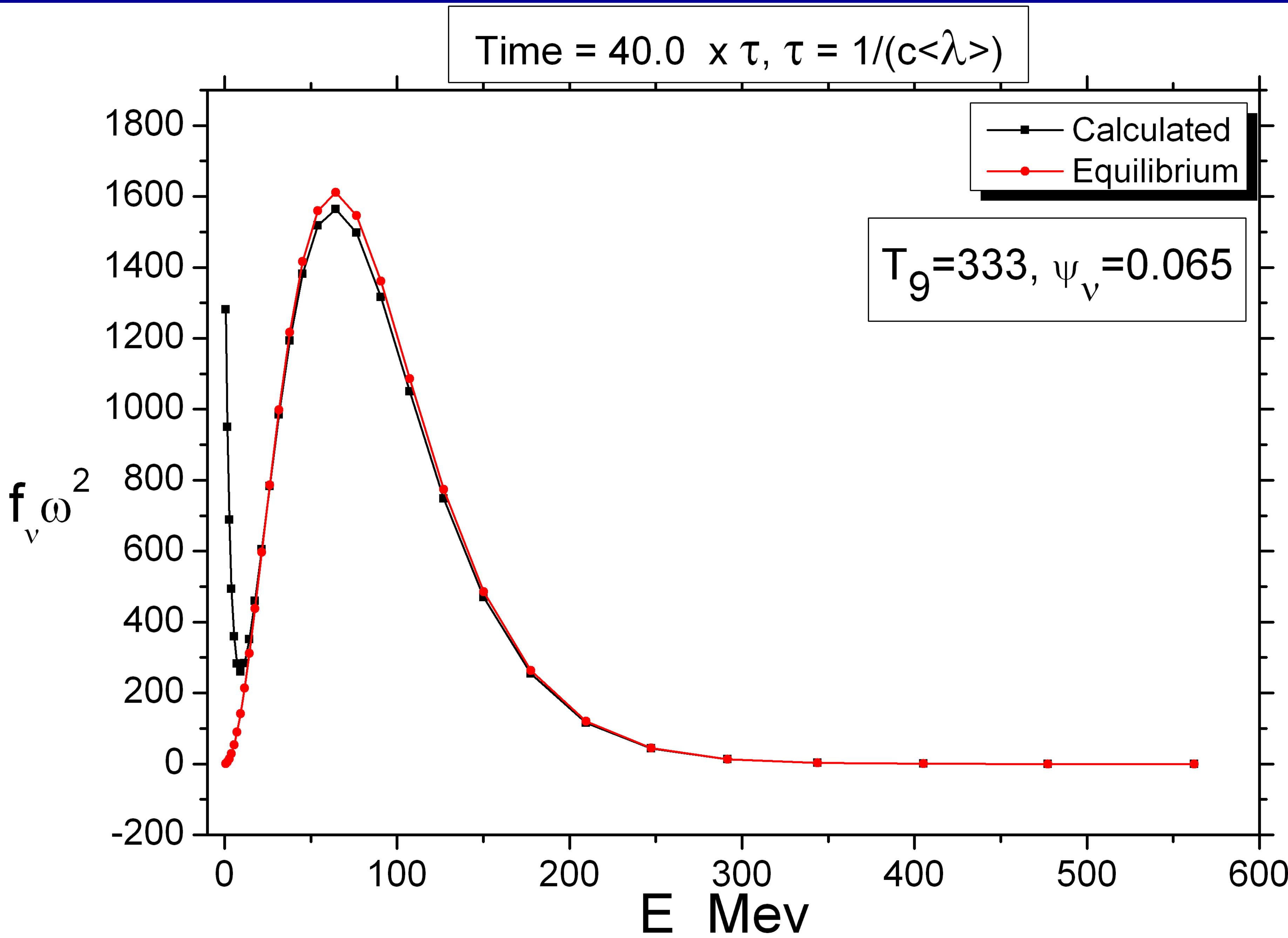

Time = 4.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600

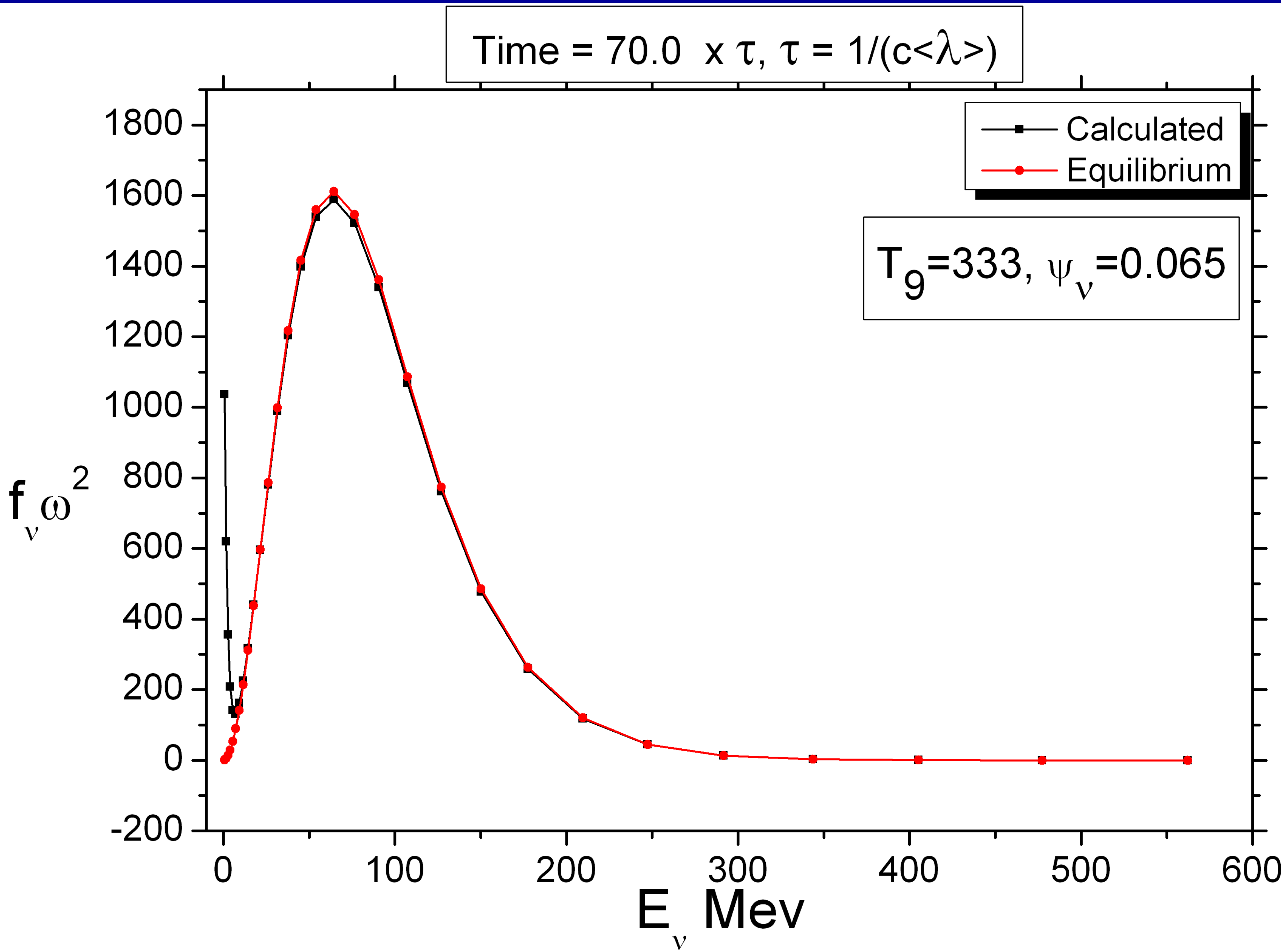


Time = 10.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600


Time = 25.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T_9=333, ψ_ν=0.065
$f_\nu \omega^2$
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600
E_ν Mev



Time = 40.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600



Time = 70.0 x τ, τ = 1/(c<λ>)
Calculated
Equilibrium
T9=333, ψν=0.065
fνω2
Eν Mev
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
100
200
300
400
500
600

Полная система уравнений нейтринной теплопроводности в случае цилиндрической симметрии

$$\frac{dr}{dt} = V_r, \quad \frac{dz}{dt} = V_z, \quad \varpi r = V_\varpi,$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \rho V_r\right) + \frac{\partial}{\partial z}\left(\rho V_z\right) = 0$$

$$\frac{dV_r}{dt} - V_\varpi \varpi = -\frac{1}{\rho}\frac{\partial}{\partial r}\left(P + P_\nu\right) - \frac{\partial \Phi}{\partial r},$$

$$\frac{dV_\varpi}{dt} + V_r \varpi = 0,$$

$$\frac{dV_z}{dt} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(P + P_\nu\right) - \frac{\partial \Phi}{\partial z}$$

Здесь ϖ – угловая скорость а, Vr, Vz и Vw – компоненты скорости

Φ – гравитационный потенциал

Полная система уравнений нейтринной теплопроводности в случае цилиндрической симметрии

$$\frac{d}{dt}\left(E + \frac{U_\nu}{\rho}\right) + (P + P_\nu)\frac{d}{dt}\left(\frac{1}{\rho}\right) = -\frac{1}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rH_r\right) + \frac{\partial H_z}{\partial z}\right],$$

$$\frac{d}{dt}\left(Y_e + Y_\nu\right) + \frac{m_u}{\rho}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rF_r\right) + \frac{\partial F_z}{\partial z}\right] = 0$$

Данная система уравнений описывает приближение нейтринной теплопроводности в случае цилиндрической симметрии и может быть применима, в частности, для описания вращающихся звёздных конфигураций.

Определение потоков

$$f = f_{eq} + \frac{1}{\lambda}\left(\mu f_r + \sqrt{1-\mu^2}\cos\varphi\, f_z\right)$$

Функции g_T и g_ψ те же самые!

$$f_{\xi=r,z} = -f_{eq}\left(g_T \frac{1}{T^2}\frac{\partial T}{\partial \xi} + g_\psi \frac{\partial \psi_\nu}{\partial \xi}\right)$$

$$F_{\xi=r,z} = \frac{4\pi}{3h^3c^2}\int \frac{f_\xi}{\lambda}\omega^2 d\omega, \qquad F_{\xi=r,z} = -G_{11}\frac{\partial \psi_\nu}{\partial \xi} - G_{12}\frac{1}{T^2}\frac{\partial T}{\partial \xi}$$

$$H_{\xi=r,z} = \frac{4\pi}{3h^3c^2}\int \frac{f_\xi}{\lambda}\omega^3 d\omega \qquad H_{\xi=r,z} = -G_{21}\frac{\partial \psi_\nu}{\partial \xi} - G_{22}\frac{1}{T^2}\frac{\partial T}{\partial \xi}$$

Conclusion

The neutrino heat conduction theory (NHC) consistently describes fluxes of energy and lepton charge emerging from the neutrino opaque core.

The fluxes are proportional to the gradients of temperature and neutrino chemical potential.

Incoherent neutrino scattering enters the NHC through 0-th and 1-st moments of the Legendre expansion of scattering kernel.

Coherent scattering is described by the transport cross-section algorithm.

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