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Neutrino magnetic moment and the shock wave revival in a supernova explosion

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Abstract

The process of the two-step conversion of the neutrino helicity, $\nu_L \to \nu_R \to \nu_L$, is analysed in the supernova conditions, where the first stage is realized due to the interaction of the neutrino magnetic moment with the plasma electrons and protons in the supernova core. The second stage is caused by the neutrino resonant spin-flip in a magnetic field of the supernova envelope. Given the neutrino magnetic moment within the interval $10^{-13}\,\mu_{\rm B} < \mu_\nu < 10^{-12}\,\mu_{\rm B}$, and with the existence of the magnetic field at the scale $\sim 10^{13}$ G between the neutrinosphere and the shock-wave stagnation region, it is shown that an additional energy of the order of 10^{51} erg can be injected into this region during the typical time of the shock-wave stagnation. This energy could be sufficient for stumulation of the damped shock wave.

1 Introduction

In a modelling of the supernova explosion, two main problems arise [1–5]. First, the mechanism of the damped shock wave stimulation has not been developed completely yet. It is believed that the explosion cannot be realized without the shock wave revival. Let us remind, that the main reason of the shock-wave damping is the energy loss by the nuclei dissociation. The second problem is that even in the case of the “successful” theoretical supernova explosion, the energy release turns out to be essentialy less than the observed kinetic energy of the envelope $\sim 10^{51}$ erg. That is known as the FOE problem (ten to the Fifty One Ergs). Thus, it is necessary for the self-consistent description of the explosion dynamics, that the neutrino flux, outgoing from the supernova core, could transfer by some mechanism the energy $\sim 10^{51}$ erg to the supernova envelope.

A possible solution of those problems, first proposed by A. Dar [6], was based on the assumption of the existence of the neutrino magnetic moment being not too small. A huge number of left-handed electron neutrinos ν_e is produced in the collapsing supernova core, and a part of them could convert into right-handed neutrinos due to the interaction of the neutrino magnetic moment with plasma electrons and protons. These right-handed neutrinos, being sterile with respect to the weak interaction, freely escape from the central part of the supernova, if the

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neutrino magnetic moment is not too large, $\mu_\nu < 10^{-11}\,\mu_{\rm B}$, where $\mu_{\rm B}$ is the Bohr magneton. In the supernova envelope, a part of these neutrinos can flip back to the left-handed ones due to the interaction of the neutrino magnetic moment with a magnetic field. It is now believed that the magnetic field strength in the supernova envelope could achieve the critical value $B_e = m_e^2/e \simeq 4.41 \times 10^{13}$ G [1] and even exceed it. The produced left-handed neutrinos, being absorbed in beta-processes, $\nu_e n \to e^- p$, can transfer an additional energy to the supernova envelope.

In our opinion, a reason arises at the present time to reconsider in more detail the Dar's mechanism. In the recent paper [7] we have shown that the evaluations of the right-handed neutrino flux and the luminosity from the supernova core were essentially understated in the previous papers on the subject.

In this paper, we perform an analysis of the two-step conversion of the neutrino helicity, $\nu_L \to \nu_R \to \nu_L$, under the supernova conditions, and of the possibility of the damped shock wave stumulation by this process.

2 The right-handed neutrino luminosity

The process of the neutrino chirality flip $\nu_L \to \nu_R$ under the physical conditions of the supernova core was investigated in the papers [7–10]. The process is possible due to the interaction of the Dirac neutrino magnetic moment with a virtual plasmon, which can be both produced and absorbed:

$$\nu_L \to \nu_R + \gamma^*; \quad \nu_L + \gamma^* \to \nu_R .$$

The detailed calculation of the plasma polarization effect on the photon propagator reveals, in particular, that the contribution of the proton component of plasma is dominant. As a result a new astrophysical bound on the electron-type neutrino magnetic moment was established [7] from the supernova $SN1987A$ data:

$$\mu_\nu < (0.7 - 1.5) \times 10^{-12}\,\mu_{\rm B}\,, \tag{1}$$

which improved the existed constraint by the factor of 2.

Particularly, a function $\Gamma_{\nu_R}(E)$ defining the spectrum of the right-handed neutrino energies, was calculated in Ref. [7]. In other words, this function defines the number of right-handed neutrinos emitted per 1 MeV of the neutrino energy spectrum, per unit time, from the unit volume of the supernova core:

$$\frac{{\rm d}n_{\nu_R}}{{\rm d}E} = \frac{E^2}{2\,\pi^2}\,\Gamma_{\nu_R}(E)\,. \tag{2}$$

The function $\Gamma_{\nu_R}(E)$ also determines the spectral density of the right-handed neutrino luminosity of the supernova core:

$$\frac{{\rm d}L_{\nu_R}}{{\rm d}E} = V\,\frac{{\rm d}n_{\nu_R}}{{\rm d}E}\,E = V\,\frac{E^3}{2\,\pi^2}\,\Gamma_{\nu_R}(E)\,, \tag{3}$$

where V is the volume of the area emitting neutrinos.

The function ${\rm d}L_{\nu_R}/{\rm d}E$ was calculated in Ref. [7], and it is shown in the figure 1 for the typical supernova core parameter values: the temperature $T \simeq 30$ MeV, the electron and neutrino chemical potentials $\tilde\mu_e \simeq 300$ MeV, $\tilde\mu_{\nu_e} \simeq 160$ MeV, the volume $V \simeq 4 \times 10^{18}\,\text{cm}^3$ and for the neutrino magnetic moment $\mu_\nu = 3 \times 10^{-13}\,\mu_{\rm B}$.

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[1] We use the natural system of units $c = \hbar = 1$. $e > 0$ is an elementary charge.

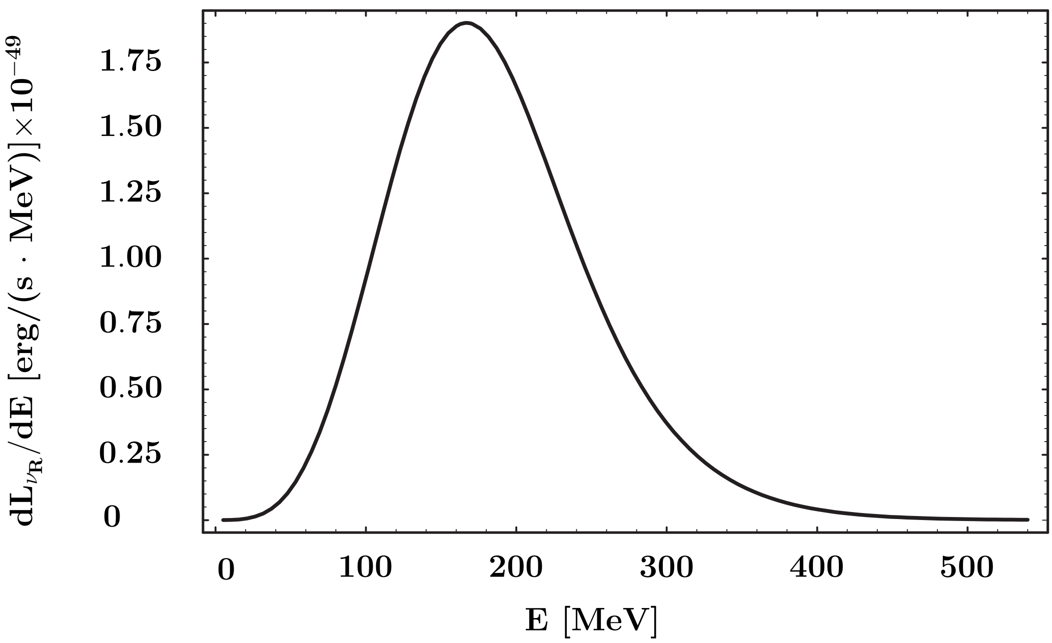

Figure 1: The energy spectrum of the right-handed neutrino luminosity for the plasma temperature $T = 30$ MeV and for $\mu_\nu = 3 \times 10^{-13}\, \mu_{\mathrm{B}}$.

The integral luminosity of the right-handed neutrinos appeared to be the following:

$$L_{\nu_R} = 4 \times 10^{51}\, \frac{\mathrm{erg}}{\mathrm{s}}\,. \tag{4}$$

Hereafter we use for the definiteness the neutrino magnetic moment value $\mu_\nu = 3 \times 10^{-13}\, \mu_{\mathrm{B}}$. On the one hand, this value is sufficiently small to avoid a distortion of the supernova dynamics. On the other hand, it is large enough to provide the required level of the luminosity (4).

If the right-handed neutrino energy was converted into the energy of the left-handed neutrinos, for example due to the well-known mechanism of the spin oscillations, then during the typical stagnation time of the shock wave of the order of some tenths of a second, an additional energy of order 10^{51} erg could be injected into the supernova envelope.

3 The resonant transition $\nu_R \to \nu_L$ in the magnetic field of the supernova envelope

We consider a part of the supernova envelope between the neutrinosphere (of the radius R_ν) and the shock wave stagnation region (of the radius R_s). By the present conceptions, typical values of R_ν and R_s vary rather slightly during the stagnation time. These values could be estimated as $R_\nu \sim$ 20–50 km, $R_s \sim$ 100–200 km. If a sufficiently large magnetic field $\sim 10^{13}$ G exists in this region, then the spin oscillation phenomenon takes place, which can be of the resonant type at certain conditions.

It is convenient to illustrate the magnetic field influence on a neutrino with a magnetic moment by means of the equation of the neutrino helicity evolution in an external uniform magnetic field. Taking into account the additional energy C_L, which the left-handed electron type neutrino ν_e acquires in medium, the equation of the helicity evolution can be written in the form [11–15]

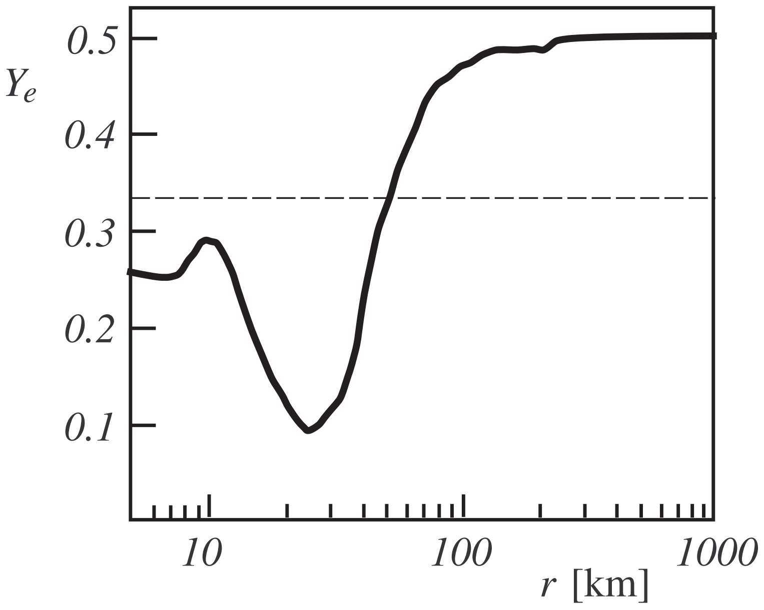

Figure 2: The qualitative behaviour of the dependence $Y_e(r)$ about 0.1 to 0.2 s after the shock formation, with the typical gap caused by the "short" neutrino outburst, see e.g. [4]. The dashed line corresponds to the value $Y_e = 1/3$.

$$\mathrm{i}\,\frac{\partial}{\partial t}\begin{pmatrix}\nu_R\\ \nu_L\end{pmatrix} = \left[\hat{E}_0 + \begin{pmatrix}0 & \mu_\nu B_\perp\\ \mu_\nu B_\perp & C_L\end{pmatrix}\right]\begin{pmatrix}\nu_R\\ \nu_L\end{pmatrix}, \tag{5}$$

where

$$C_L = \frac{3\,G_{\mathrm{F}}}{\sqrt{2}}\,\frac{\rho}{m_N}\left(Y_e + \frac{4}{3}\,Y_{\nu_e} - \frac{1}{3}\right). \tag{6}$$

Here, the ratio $\rho/m_N = n_B$ is the nucleon density, $Y_e = n_e/n_B = n_p/n_B$, $Y_{\nu_e} = n_{\nu_e}/n_B$, n_{e,p,ν_e} are the densities of electrons, protons and neutrino respectively. $B_\perp$ is the transverse component of the magnetic field with respect to the neutrino movement direction, $\hat{E}_0$ is proportional to the unit matrix and is inessential for our analysis.

The expression (6) for the additional energy of left-handed neutrinos C_L deserves a special analysis. It is remarkable that the possibility exists for this value to be zero just in the region of the supernova envelope we are interested in. And in turn this is the condition of the resonant transition $\nu_R \to \nu_L$. Taking into account that the neutrino density in the supernova envelope is sufficiently small, one may neglect the value Y_{ν_e} in the expression (6), that gives the condition of the resonance in the form $Y_e = 1/3$. It should be noted that the values Y_e which are realized in the supernova envelope, typical for the collapsing matter, are: $Y_e \sim$ 0.4–0.5. However, the shock wave causes the nuclei dissociation and makes the substance to be more transparent for neutrinos. This leads to the so-called "short" neutrino outburst and consequently to the significiant matter deleptonization in this region. According to the existing conceptions, a typical gap arises in the radial distribution of the value Y_e, where Y_e may fall down to the value ~ 0.1, see, for example [2,4]. The qualitative behaviour of the dependence $Y_e(r)$ is represented in the figure 2. Thus, a point necessarily exists where Y_e takes the value of 1/3. It is remarkable, that only one such point appears, with $\mathrm{d}Y_e/\mathrm{d}r > 0$, see [2,4].

Notice, that the condition $Y_e = 1/3$ is the necessary but still not the sufficient one for the resonant conversion $\nu_R \to \nu_L$. The realization is also necessary of the so-called adiabatic condition. This means that the diagonal element C_L in the equation (5), at least, should not exceed the nondiagonal element $\mu_\nu B_\perp$, when the shift is made from the resonance point at the distance of the order of the oscillations length. This leads to the condition [16]:

$$\mu_\nu B_\perp \gtrsim \left(\frac{\mathrm{d}C_L}{\mathrm{d}r} \right)^{1/2} \simeq \left(\frac{3\,G_\mathrm{F}}{\sqrt{2}} \, \frac{\rho}{m_N} \, \frac{\mathrm{d}Y_e}{\mathrm{d}r} \right)^{1/2} . \tag{7}$$

The typical parameter values in the considered area are the following, see [2, 4]:

$$\frac{\mathrm{d}Y_e}{\mathrm{d}r} \sim 10^{-8}\,\mathrm{cm}^{-1} , \quad \rho \sim 10^{10}\,\mathrm{g} \cdot \mathrm{cm}^{-3} . \tag{8}$$

For the magnetic field value, providing the realization of the resonance condition, one can find:

$$B_\perp \gtrsim 2.6 \times 10^{13} \mathrm{G} \left(\frac{10^{-13} \mu_\mathrm{B}}{\mu_\nu} \right) \left(\frac{\rho}{10^{10} \mathrm{g} \cdot \mathrm{cm}^{-3}} \right)^{1/2} \left(\frac{\mathrm{d}Y_e}{\mathrm{d}r} \times 10^8\,\mathrm{cm} \right)^{1/2} . \tag{9}$$

Thus, the performed analysis shows that the Dar scenario of the two-step conversion of the neutrino helicity, $\nu_L \to \nu_R \to \nu_L$, can be realized, if the value of the neutrino magnetic moment is in the interval

$$10^{-13}\,\mu_\mathrm{B} < \mu_\nu < 10^{-12}\,\mu_\mathrm{B} , \tag{10}$$

and under the condition that the magnetic field of the scale 10^{13} G exists in the region $R_\nu < R < R_s$. During the shock wave stagnation time $\Delta t \sim 0.2$–0.4 sec the additional energy can be injected into this region, of the order of

$$\Delta E \simeq L_{\nu_R}\,\Delta t \sim 10^{51}\,\mathrm{erg} , \tag{11}$$

which is just enough for the problem solution.

4 Conclusion

We have re-analysed the two-step conversion of the neutrino helicity, $\nu_L \to \nu_R \to \nu_L$, under the supernova conditions. As we have shown, this conversion process could provide an additional energy of the order of 10^{51} erg which can be injected into the region between the neutrinosphere and the shock-wave stagnation area, $R_\nu < R < R_s$, during the typical stagnation time of the order of some tenths of a second. This energy could be sufficient for stumulation of the damped shock wave.

The conditions for the realization of this scenario appear to be not very rigid. The Dirac neutrino magnetic moment should belong to the interval $10^{-13}\,\mu_\mathrm{B} < \mu_\nu < 10^{-12}\,\mu_\mathrm{B}$, and the magnetic field $\sim 10^{13}$ G should exist in the region $R_\nu < R < R_s$.

Acknowledgments

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