

Physics of Neutron Stars

St. Petersburg, June 2008



Plasma Polarization in Massive Astrophysical Objects

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Support: INTAS 93-66 // ISTC 2107 // CRDF № MO-011-0, and by RAS Scientific Programs

"Physics and Chemistry of Extreme States of Matter" and "Physics of Compressed Matter and Interiors of Planets"

Abstract

Macroscopic plasma polarization, which is created in massive astrophysical bodies by gravitation and other inertial forces, is under discussion. New source of such polarization is introduced into consideration. It is non-ideality effects due to strong Coulomb interaction of charged particles. This 'non-ideality' polarization effect may be significant in comparison with the well-known gravitational polarization. The latter effect was established long ago in papers of Pannekoek, Rosseland and others for the case of ideal, isothermal and nondegenerated plasma in outer layers of a star. Their approach was extended (for example, Bilsten et al.) later on conditions of dense and degenerated interiors of compact stars. Present work presumes non-correctness of this extension because it based on partial pressures and 'partial' hydrostatic equilibrium equations separately for each species of particles. Present consideration is based on the density functional approach combined with 'local densities approximation'. We study simplified situation of totally equilibrium isothermal star without influence of magnetic field and relativistic effects. The extremum condition for thermodynamic potential results in two set of equivalent conditions: constancy for generalized partial (electro-) chemical potentials and/or equilibrium for the forces acting on each charged particle. In this latter form new 'Coulomb non-ideality' force comes into equilibrium equation in addition to two traditionally studied: gravitational and electrostatic ones. In most cases this new 'force' increases final electrostatic field in comparison with that of standard ideal-gas solution. Our resulting formula reproduces two known limiting cases for degenerated and non-degenerated ideal gas and leads to some additional effects. The hypothetical sequences of these effects on structure, thermo- and hydrodynamics of neutron star are under discussion.

Introduction

Long-range nature of Coulomb and gravitational interactions leads to specific manifestation of their joint action in massive astrophysical objects (MAO). The main of them is polarization of plasmas under gravitational attraction of ions. Extraordinary smallness of gravitational field in comparison with electric one (the ratio of gravitational to electric forces for two protons is ~ 10^{-36}) leads to the fact that extremely small and *thermodynamically* (energetically) *negligible* deviation from electroneutrality can provide to *thermodynamically noticeable* (even significant) consequences at the level of first (thermodynamic) derivatives. This is the main topic of present contribution (*).

(*) We neglect relativistic effects and influence of magnetic field

Milestones

1903 // W. Sutherland – Discussed basic idea of gravitational polarization in MAO

1922 // A. Pannekoek 1924 // S. Rosseland Obtained the key relation of proportionality for average gravitational and electrostatic fields (counting per proton) for the case of ideal non-degenerated plasma of the Sun { $F_{\rm E} = \frac{1}{2} F_{\rm G}$ }

1924 // E. Milne – Net charge on the star // Discussed basic idea of non-electroneutrality of stars 1926 // A. Eddington – Respected these ideas in his book 1968 // L. Rosen – Discussed gravitational polarization in the stars as a standard (to be continued)

Milestones

(continued)

1976 // T.Montmerle & A.Mishaud	Idea that protons are "repelled out" by electrostatic field from
1979 // A.Mishaud & G.Fontain	helium star envelope due to the gravitational polarization

1978 // J. Bally & E. Harrison – *The Electrically Polarized Universe* // Idea of non-electronuetrality of all gravitational objects in the Universe, including stars, galaxies and their clusters

1980 // C.Alkock – *Electric field of a chemically inhomogeneous star* /Electrostatic pollution of hydrogen from helium envelope of white dwarfs

1986 // C.Alkock, Fachri, Olinto – *Electric field on the Strange Star Surface* / Idea of huge local charge densities and average electrostatic field at the surface of the "strange" star 1992 // N.Glendenning / Introduced concept of «Structured Mixed Phase» for quark-hadron phase transition / *Compact Stars: Springer, 2000*.

1996 // D. Kirzhnits – Gravitational polarization give no noticeable observable effects 2001-2005 // L.Bildsten *et al* – Extended the idea of influence of gravitational polarization on diffusion of heavy ions in interiors of white dwarfs. Influence on star cooling and evolution.

2003-2005 // S.Ray et al. 2005 // A.Mattei 2007 // A/Di Prisco et al.	<i>Exotics</i> : Ideas of ultra high charges and fields, charged black holes, charged gravitational collapse <i>etc</i> .
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And many other papers probably missed by this list . . .

Electrostatics of equilibrium Coulomb systems

(Basic points)

W. Sutherland, 1903:

Gravitational field always polarizes ion-electronic plasma of a star due to low ratio of electronic and ionic masses.

Gravitational polarization is part of more general phenomenon:

Plasma polarization in MAO occurs under any inertial forces: rotation, vibration, inertial expansion and compression etc. due to the same reason: low mass of electrons (in comparison with ionic one).

Mass-dependent polarization is part of more general phenomenon:

Any non-uniformity in thermodynamically equilibrium system of Coulomb particles is accompanied by its polarization and existence of stationary profile of average electrostatic potential.

This average electrostatic potential is thermodynamic quantity. It depends significantly on thermodynamic parameters.

Important particular case:

Any two-phase interface in thermodynamically equilibrium system of Coulomb particles is accompanied by existence of stationary drop of electrostatic potential at this interface. It is valid for terrestrial applications: plasma, ionic liquids and molten salts (*potential Galvani*), as well as in astrophysical applications.

Galvani potential in Coulomb systems

Equilibrium potential of two-phase interface is thermodynamic quantity: - it depends on thermodynamic parameters of coexisting phases (bulk properties).

Interrelation with work function:

In contrast to the work function Galvani potential does not depend on properties of two-phase interface itself: its purity, form, dimensionality etc.

EXAMPLES:

- Electrostatic "portrait" of Wigner crystallization in one-component plasma model (OCP), in crystallization and demixing transitions in the model of binary ionic mixture (BIM) and in real crystallization in interiors of white dwarfs.
- Galvani potential of gas-liquid interface in metals, ionic liquids and molten salts
- Electrostatic "portrait" of "plasma" phase transition in interiors of Giant planets
- Electrostatic potential of mean-phase interface for hypothetical quark-hadron phase transition in interiors of neutron ("strange") stars.

Electrostatic potential of two-phase interface in equilibrium Coulomb systems



Electrostatics of massive astrophysical bodies

(Basic points)

Gravitational attraction always <u>polarizes</u> plasma of massive astrophysical bodies due to smallness of electronic mass in comparison with ionic one (small parameter - m_e/m_i).

Resulting average electrostatic field <u>must be</u> of the <u>same order</u> as gravitational field (counting per one proton)

<u>Comment</u>: Ions in thermodynamic equilibrium are suspended, figuratively speaking, in the electrostatic field of strongly degenerated and weakly compressed electrons.

This proportionality (*congruence*) of gravitational and average electrostatic fields does not restricted by condition of strong ionization. The same is true in weakly ionized plasmas.

The key (dominating) factors for this ratio are non-ideality degree and degree of electronic degeneracy.

Average electrostatic field is equal to the <u>one half</u> of gravitational field (counting per one proton) in hypothetical isothermal ideal and non-degenerated electron-proton plasma of the Sun.

Electrostatics of massive astrophysical bodies

(Continued)

Average electrostatic field is supposed to be equal to *twice* of gravitational field (counting per one proton) in opposite case of hypothetical isothermal ideal but strongly degenerated plasma of compact stars (WD and NS) /Bilsten *et al*/.

Exact equality ($F_E = 2F_G$) corresponds to the zero order approximation in expansion on the small parameter $x_m \equiv m_e/m_i$

Influence of non-isothermal conditions

Real plasmas of compact stars (white dwarfs and neutron stars) are close to isothermal conditions due to high thermal conductivity of degenerated electrons. At the same time plasma of the stars of main branch, for example, of the Sun, is not isothermal and therefore globally thermodynamically equilibrium. There exists temperature profile and thermo-diffusion in such plasmas. Nevertheless, in most cases temperature decreases toward the surface of a star, so that one may expect that this thermo-diffusion *increases* resulting electrostatic field in comparison with the isothermal conditions. Thermo-diffusion "repels" light electrons toward the periphery of a star and therefore increases resulting plasma polarization.

Historical comments

<u>Plasma polarization at r</u>	micro-level – Debye and H	lückel, Phys. Zeitschr., 24, 8, 1923.	
Plasma polarization at r	macro-level – Pannekoek	A., Bull. Astron. Inst. Neth., 1 (1922)	
= = «» = =	Rosseland S	. Mon. Roy. Astron. Soc., 84, (1924)	
		<i>,</i> ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	
Mr. S. Rosseland.	Electrical State of a Star.	© Royal Astronomical Society	
		(Communicated by Prof. A. S. Laaington.	
Application to plasma:			
1) - ideal	$dP_{o}/dr = -GMm_{o}n_{o}/r^{2} -$	$m_e E$ M – mass of the Sun,	
2) - non-degenerate		G – gravitational constant,	
3) - isothermal ($T = const$)	$dP_{p}/dr = -G/VIm_{p}n_{p}/r^{2} +$	$m_{\rm p}qE$ $m_{\rm e}, m_{\rm p}$ – electronic & ionic masses	
4) - electroneutral	Pannekoek - Posseland		
$\{ n_{+}(r) = n_{-}(r) \}$	Familekov		
5) - equilibrium	electrostatic field		
<i>c)</i> equinerient	$\mathbf{T}(n)$ (1 (\mathbf{a}) $\mathbf{T}(n)$	$\mathbf{\Gamma}(e) = (1 \mid 2) \mathbf{\Gamma}(p) \qquad (\mathbf{*})$	
(-md+nk)	$F_E^{(p)} = -(1/2)F_G^{(p)}$	$F_E^{(0)} = +(1/2)F_G^{(p)} $ ()	
$N_e = N_e exp \frac{(-m\phi + e\phi)}{kT}$	Conoralizatio	n to ideal places of	
	Generalization to ideal plasma of		
$N_p = N_p^{\circ} exp \frac{(-M\phi - Ze\psi)}{kT}$	ions (A,Z) and electrons		
	$F_E^{(p)} = -\frac{A}{\sqrt{2}}$	$F_{G}^{(p)} = -\frac{Z}{F_{G}^{(2)}} F_{G}^{(2)}$	
$\psi = -\frac{M-m}{m}$	Z = (Z +	1) $(Z+1)$	
$\phi = e(Z+I)$		(7) $\mathbf{\Gamma}$ (7) all alterates the set of second sec	
dur gravitational and electrost	(") $F_E^{\psi}, F_G^{\psi}, F_H$	$F_{G}^{(2)}$, $F_{G}^{(2)}$, - electrostatic and gravitational	
$\psi, \psi' = y_i a v_i a i 0 i a i 0 i e c i 0 s i$	torces actin	ng on one proton (p) and ion (A,Z)	

Extension to the strongly degenerate plasma

The model of **L**. **Bildsten** *et al.* (2001 – 2007)

L. Bildsten & D. Hall //*Ap.J.*, 549: (2001) *Gravitational settling of*²²*Ne in liquid white dwarf interior* P. Chang & L. Bildsten // *Ap.J.*, 585 (2003) *Diffusive nuclear burning in neutron star envelopes* // <u>arXiv:astro-ph/0211028v1</u> (last correction - 2007)

$$\frac{dP_e}{dr} = -n_e \{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i \{A_i m_p g(r) - Z_i eE\}$$

L. Bildsten et al. (2000-2007)



Expansion on small parameter
$$x_c$$

$$F_E^{(p)} = -\frac{A}{Z}F_G^{(p)} + x_cB_1 + x_c^2B_2 + \dots$$

1) - ideal
2) - non-degenerate
3) - isothermal
$$(T = const)$$

4) - electroneutral
 $\{n_+(r) = n_-(r)\}$
5) - equilibrium

Pannekoek – Rosseland (1924)

 $\frac{\text{Zero-approximation}}{F_E^{(p)} \approx -\frac{A}{Z} F_G^{(p)}}$

$$\frac{\text{The SUN:}}{(p^+ + e^-)}$$

$$F_E^{(p)} \approx -(1/2)F_G^{(p)}$$

$$\frac{\text{White Dwarfs:}}{P_E^{(p)} \approx -2F_G^{(p)}}$$

$$F_E^{(p)} \approx -2F_G^{(p)}$$

NB!

- Average electrostatic field must be of the *same order* as gravitational one*

(* - counting on one proton)

 $|F_{\scriptscriptstyle E}^{(Z)}/F_{\scriptscriptstyle C}^{(Z)}| \geq 1$

Question:

? - Do both limiting cases (*ideal non-degenerate and degenerate electrons*) restrict interval of possible <u>ratio</u> of <u>gravitational</u> and <u>electrostatic</u> forces - ?



! Yes : - if one takes into account the electron degeneracy only !

! No : - if one takes into account <u>non-ideality</u> effects additionally ! (see below)

I.e.

 $|F_{E}^{(p)}/F_{G}^{(p)}| \ge 2$

It may be

Widely used approach (standard)

From unique equation of hydrostatic (i.e. mechanical) equilibrium of electro-neutral matter in gravitational field . . .

$$\frac{dP_{\Sigma}}{dr} = -\{n_e(r)m_e + n_i(r)m_i\}g(r) = -\rho(r)g(r)$$

... to the set of separate equations of hydrostatic equilibrium for each charged specie (in terms of partial pressures)

$$\frac{dP_e}{dr} = -n_e \{m_e g(r) + eE\}$$

$$\frac{dP_i}{dr} = -n_i \{A_i m_p g(r) - Z_i eE\}$$

What is non-correct ?

<u>NB</u>!

 partial pressures and separate equations of "hydrostatic" equilibrium are not well-defined quantities in non-ideal plasmas of compact stars

What should be done instead ?

Integral Form of Thermodynamic Equilibrium Conditions

Variational formulation of equilibrium statistical mechanics

Multi-component version of the "density functional theory" (DFT) *

$$F = \min_{\{n_j(x)\}:\{n_{jk}(x,y)\}} F \Big[T, \{V_{jk}(\cdot)\} \mid \{n_j(x)\}:\{n_{jk}(x,y)\} \Big]_{\{T = const, V(r) = const\}}$$

Standard: separation of non-local parts.

$$F\{T, V(r) / \left[\{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\} \right] \equiv \sum_{jk} \frac{Gm_j m_k}{2} \int \frac{n_j(\overline{x}) \cdot n_k(\overline{y})}{\left|\overline{x} - \overline{y}\right|} d\overline{x} d\overline{y} + \sum_{jk} \frac{Z_j Z_k e^2}{2} \int \frac{n_j(\overline{x}) \cdot n_k(\overline{y})}{\left|\overline{x} - \overline{y}\right|} d\overline{x} d\overline{y} + F^* \left[\{n_i(\cdot)\} / \{n_{ij}(\cdot, \cdot)\} \right]$$

NB! The rest *F**{...} is the free energy of new system *on compensating background(s)* **Local Densities Approximation**

$$F^*\left[\{n(\cdot)\}\right] = \int f\left(\{n_i(\mathbf{r}): n_k(\mathbf{r})...\}\right) d\mathbf{r} \qquad f\left(\{n\}\right) \equiv \lim\left\{\frac{F\left(N_i, N_k, ..., V, T\right)}{V}\right\}_{\{N_k\}, V \to \infty}^{N_k/V \to n_k}$$

NB! The *local* free energy density $f({n})$ must be defined for *non-electroneutral* densities ${n_k}$

(*) C. De Dominicis,1962 // Hohenberg & Kohn,1964 // Kohn & Sham,1965 etc..

Local Form of Thermodynamic Equilibrium Conditions

Heat exchange:

Impulse exchange:

$$T(\mathbf{r}) = \mathrm{const}$$

$$\nabla P_{\Sigma} = -\rho(\mathbf{r})\nabla\varphi_{G}(\mathbf{r})$$

Equilibrium for particle exchange:

Constance of total (generalized) electro-chemical potential

 $m_{jj} \varphi_{\rm G}(\mathbf{r}) + q_j \varphi_{\rm E}(\mathbf{r}) + \mu_j^{(\rm chem)} \{ n_{\rm i}(\mathbf{r}), n_{\rm e}(\mathbf{r}), \{ n_{jk}(\mathbf{x}, \mathbf{y}) \} T \} = \text{const}$ *in terms of potentials* (j,k = electrons, ions)

Balance of all forces including generalized "non-ideality" force

$$m_{j}\nabla\varphi_{\rm G}(\mathbf{r}) + q_{j}\nabla\varphi_{\rm E}(\mathbf{r}) + \nabla\mu_{j}^{(\rm chem)} \{n_{\rm i}(\mathbf{r}), n_{\rm e}(\mathbf{r}), \{n_{\rm jk}(\mathbf{x},\mathbf{y})\} T\} = 0$$
(*j*,*k* = electrons, ions)

<u>in terms of forces</u>

 $\varphi_{\rm G}({f r})$ И $\varphi_{\rm E}({f r})$ – gravitational and electrostatic potentials

<u>NB</u> !

The set of equations for <u>electro-chemical potentials</u> instead of the set of separate equations of "hydrostatic" equilibrium for partial pressures !

Non-ideality effects

(in the local density approximation)

Equilibrium condition with "thermodynamic" (non-ideality) force

 $m_j \nabla \varphi_{\rm G}(\mathbf{r}) + Z_j e \nabla \varphi_{\rm E}(\mathbf{r}) + \nabla \mu_j^{(\rm chem)} \{ n_{\rm i}(\mathbf{r}), n_{\rm e}(\mathbf{r}), T \} = 0$ (*j* = электроны, ионы)

Final equation for average electrostatic field

(with taking into account non-ideality and degeneracy effects)

$$m_{j}\nabla\varphi_{G}(\mathbf{r}) + Z_{i}e\nabla\varphi_{E}(\mathbf{r})\left[1 + \frac{(\mu_{ii}^{0} + \Delta_{i}^{i} + Z\Delta_{i}^{i})}{Z(\mu_{ee}^{0} + \Delta_{i}^{e} + Z\Delta_{e}^{e})}\right] = 0$$

 $\mu_j^0(n_j,T)$ – ideal-gas part of (*local*) chemical potential of specie j $\Delta \mu_j^{(chem)}(n_j,n_i...,n_k,T)$ – non-ideal-gas part of (*local*) chemical potential of specie j

$$\mu_{jj}^{0} \equiv \left(\frac{\partial \mu_{j}^{0}}{\partial n_{j}}\right) \qquad \Delta_{k}^{j} \equiv \left(\frac{\partial \Delta \mu_{j}}{\partial n_{k}}\right)$$

Non-ideality effects in local density approximation

(continued)

1) Ideal and **non-degenerate gas** $(n\lambda_e^3 \ll 1)$ $F_G^{(Z)} + 2F_E^{(Z)} = 0$

Polarization compensates just <u>one half</u> of gravitational attraction (*for symmetric ion A=2Z*)

2) Ideal and highly-degenerate gas $(n\lambda_e^3 >> 1)$ $F_G^{(Z)} + F_E^{(Z)} = 0$

Polarization compensates almost *totally* gravitational attraction of ions

3) Non-ideal and non-degenerate gas $(n\lambda_e^3 \ll 1)$

Polarization compensates *more than one half* of gravitational attraction (*for symmetric ion*)

4) Non-ideal and highly-degenerate gas $(n\lambda_e^3 \gg 1)$

$$F_G^{(Z)} + F_E^{(Z)}[1 - \varepsilon(\Gamma, n\lambda^3)] = 0$$

Polarization compensates <u>not only</u> gravitational attraction but additional "non-ideality force" directed towards the center of a star

«Global» non-ideality effect !

<u>Plasma polarization in massive astrophysical objects</u> <u>Application - I</u> Electrostatics of a star

Proportionality (congruence) of average electrostatic and gravitational potentials

Excess charge profile in a star is similar (proportional) to their density profile

$$Q(\mathbf{r}) \sim \rho(\mathbf{r})$$

Primitive estimation:

- Maximal value of electrostatic field (at the surface) $E_{max}(r = R)$
- Maximal value of electrostatic potential (in the centre) $U_{max}(r=0)$

$$E_{max} \cong gm_{n}/e = (GMm_{n}/R^{2}e) \approx 2.85 \cdot 10^{-8} \cdot [M^{*}/(R^{*})^{2}] \text{ V/cm}$$

 $U_{max} \cong gR/2 = (GMm_p/2R) \approx 1.10^3 (M^*/R^*) \text{ eV}$

$$M^* \equiv M/M_{cp}$$
; $R^* \equiv R/R_{cp}$
 $M_{cp} \cong 1.99 \cdot 10^{33}$ g. $R_{cp} \cong 6.96 \cdot 10^{10}$ cm
-- mass and radius of the Sun

Electrostatic potential parameters:

	SUN $M \equiv M_{c}$ $R \equiv R_{c}$	White Dwarf $M_{WD} = M_{c}$ $R_{WD} = R_{Earth}$	Neutron Star $M_{\rm NS} = M_{\Xi}$ $R_{\rm NS} = 10$ km
U _{max} [eV]	1 keV	1 MeV	70 MeV
E _{max} [V/cm]	3.10-8	0.03	150

Macroscopic charge on phase boundaries in massive astrophysical objects /MAO/

Basic property of phase boundary in equilibrium Coulomb system:

Any two-phase interface in equilibrium system of Coulomb particles is accompanied by existence of stationary drop of electrostatic potential at this interface.

Additionally in massive astrophysical objects /MAO/:

Any two-phase interface in MAO is accompanied generally by additional existence of *discontinuity* in *average electrostatic field* at this interface.

It means existence of average *macroscopic charge* at this interface.

Three sources leading to such discontinuity

- 1. Drop in dominating ion's ratio (A/Z) at the surface between layers.
- 2. Differential rotation of the layers.
- 3. <u>Discontinuity</u> in <u>non-ideality</u> of charged subsystem at the <u>phase interface</u>.

Comments:

- The first mechanism is valid even for ideal plasmas with degenerated electrons:

 $e\Delta E = m_p \Delta(A/Z)g(r^*)$ (r^{*} - position of two-phase interface)

- The second mechanism could be discussed, for example, for differential rotation in strange star

- The third way is valid due to non-ideality effects even in the case of constant ratio (A/Z) for example, in the case of "non-congruent" crystallization or demixing fluid-fluid in combination of ions { $_{12}C^{6+} + {}_{16}O^{8+}$ } in White Dwarfs of upper layers of neutral stars.

Macroscopic charge on phase boundaries in MAO

Typically – ratio *A*/*Z increases* when we cross the interface to inner layer. It means that *decreasing of* electrostatic field, i.e. *negative* charge on two-layer interface.



Conclusions and perspectives

<u>Long-range nature</u> of Coulomb and gravitational forces leads to specific consequences of their <u>joint action</u> in massive astrophysical objects.

Cassini-Huygens

- The most important is *plasma polarization* under gravitational attraction of ions.
- Small and <u>thermodynamically</u> (energetically) <u>negligible</u> deviation from electroneutrality can provide to <u>thermodynamically</u> <u>noticeable</u> consequences at the level of <u>thermodynamic</u> <u>derivatives</u>.
- Gravitational plasma polarization in compact stars can manifest itself not only in thermodynamics but also at <u>features</u> of <u>hydrodynamics</u> of a star.
- Plasma polarization in a star could be response not only on gravitational attraction, but on *any mass-acting inertial forces*.



There will be enough challenges

to keep us all happily occupied for years to come.

Hugh Van Horn (1990) (Phase Transitions in Dense Astrophysical Plasmas)





Support: INTAS 93-66 // ISTC 2107 // CRDF № MO-011-0, and by RAS Scientific Programs "Physics and Chemistry of Extreme States of Matter" and "Physics of Compressed Matter and Interiors of Planets"