What determines the inclination angle of radio pulsars

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Magnetic dipole breaking

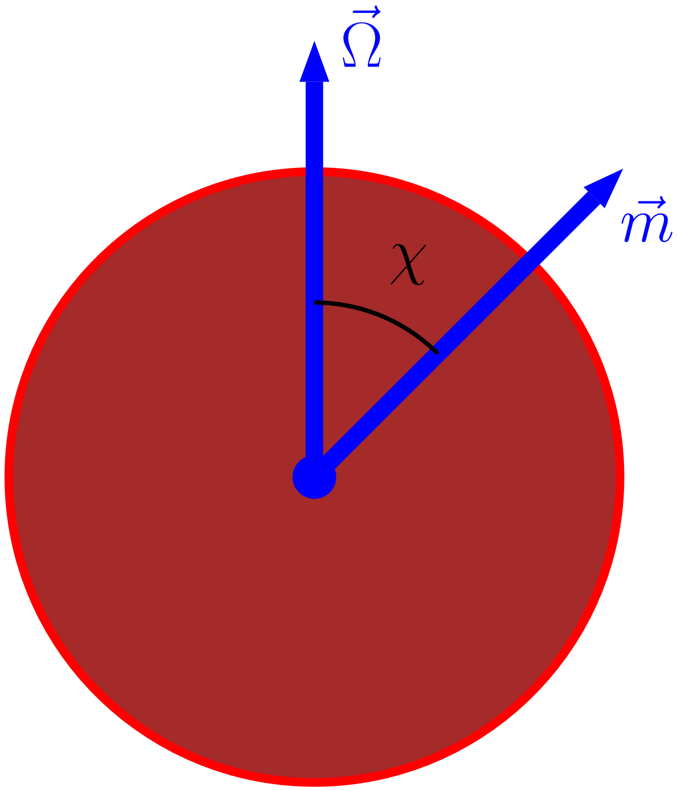
Deutsch 1955, Annales D'Astrophysique, V.18, p.1

Davis и Goldstein, 1970, ApJ, V.159, L81

Melatos, 2000, MNRAS, v.313, p.217

$$\vec{K}_{dip} = \frac{2}{3}\frac{\Omega^2}{c^3}\left[\vec{m} \times [\vec{m} \times \vec{\Omega}]\right] + \frac{3}{5}\frac{1}{ac^2}(\vec{m},\vec{\Omega})[\vec{\Omega} \times \vec{m}] \qquad \frac{\Omega a}{c} \ll 1$$

where $\vec{m}$– magnetic dipole moment, a – radius, $\vec{\Omega}$ – angular velocity



$$I\frac{d\vec{\Omega}}{dt} = \vec{K}_{dip}$$

$$\frac{\cos\chi}{P} = const$$

where $P = \frac{2\pi}{\Omega}$ – period of pulsar

$$\chi \to 0$$

All pulsars must be aligned

Current momentum losses

$$\vec{K}_c = -\frac{2}{3}\alpha\frac{\Omega^3}{c^3}m^2\frac{\vec{m}}{m}\cos\chi$$

$$\alpha = 2\,\frac{3}{4}\frac{j}{j_{GJ}}\left(\frac{R_t(\eta)}{R_0(\eta)}\right)^4$$

where

$$j_{GJ} = \frac{\Omega B}{2\pi}\cos\chi$$

$$R_0(\eta) = a\sqrt{\frac{\Omega a}{c}}\eta^{3/2}$$

$$\eta = \frac{r}{a}$$

$$\vec{K} = \frac{r^3}{4\pi}\int\Big([\vec{n}\times\vec{E}](\vec{n}\vec{E}) + [\vec{n}\times\vec{H}](\vec{n}\vec{H})\Big)\,d\Omega$$

Electron Current with Frame Dragging

$$\vec{j} = -A(\xi) j_{GJ} \frac{\vec{B}}{B}$$

$$A = 1 - \kappa \qquad\qquad \kappa \approx 0.15$$

A.G.Muslimov and A.I.Tsygan (1990, 1992)

V.S.Beskin (1990)

$$\vec{K}_c = -2\frac{\vec{m}}{m}\frac{B_0^2 a^3}{8}\left(\frac{\Omega a}{c}\right)\theta_0^4 A\cos\chi$$

$$\frac{\sin\chi}{P} = const$$

$$\chi \rightarrow \frac{\pi}{2}$$

All pulsars must be orthogonal

V.S.Beskin and E.E.Nokhrina, ApSS v.308 p.569 (2006)

S.A.Eliseeva, S.B.Popov and V.S.Beskin, astro-ph/0611320

$$\vec{K} = \vec{K}_{dip} + \vec{K}_c$$

P.B.Jones (1976) $\alpha \ll 1$

$$\frac{I}{K_0}\frac{d\Omega}{dt} = -(\sin^2\chi + \alpha\cos^2\chi)$$

$$\frac{I}{K_0}\frac{d\cos\chi}{dt} = (1-\alpha)\sin^2\chi\cos\chi$$

$$\frac{I}{K_0}\Omega\frac{d\phi}{dt} = -\frac{9}{10}\frac{c}{\Omega a}\cos\chi$$

$$K_0 = \tfrac{2}{3}\tfrac{\Omega^3 m^2}{c^3}$$

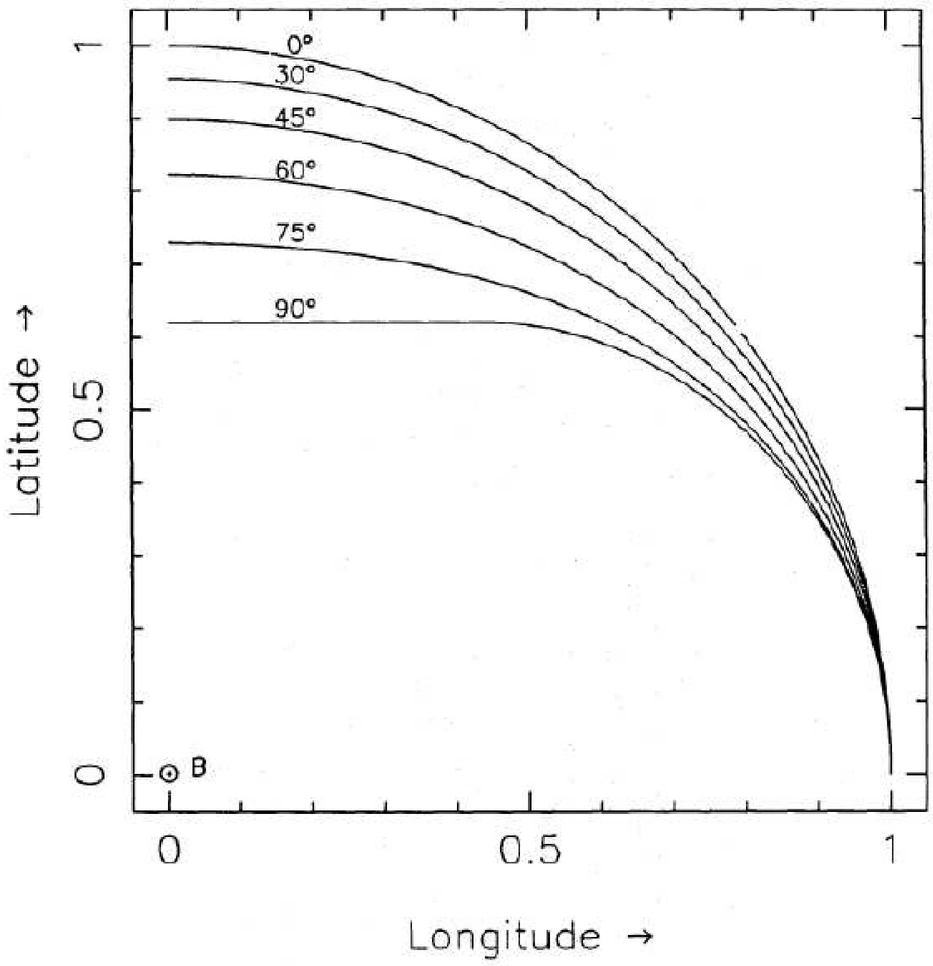
$T_p = \dfrac{2\pi}{\Omega_p}$ where $\Omega_p = -\dfrac{3}{5}\Omega\dfrac{m^2}{ac^2 I}\cos\chi$

$$\frac{T_p}{\tau} = \frac{40\pi}{9}\left(\frac{\Omega a}{c}\right)\frac{\sin^2\chi + \alpha\cos^2\chi}{\cos\chi} \ll 1$$

where

$$\tau = \frac{P}{2\dot{P}} = -\frac{\Omega}{2\dot{\Omega}}$$

Changing form of pulsar tube cross-section



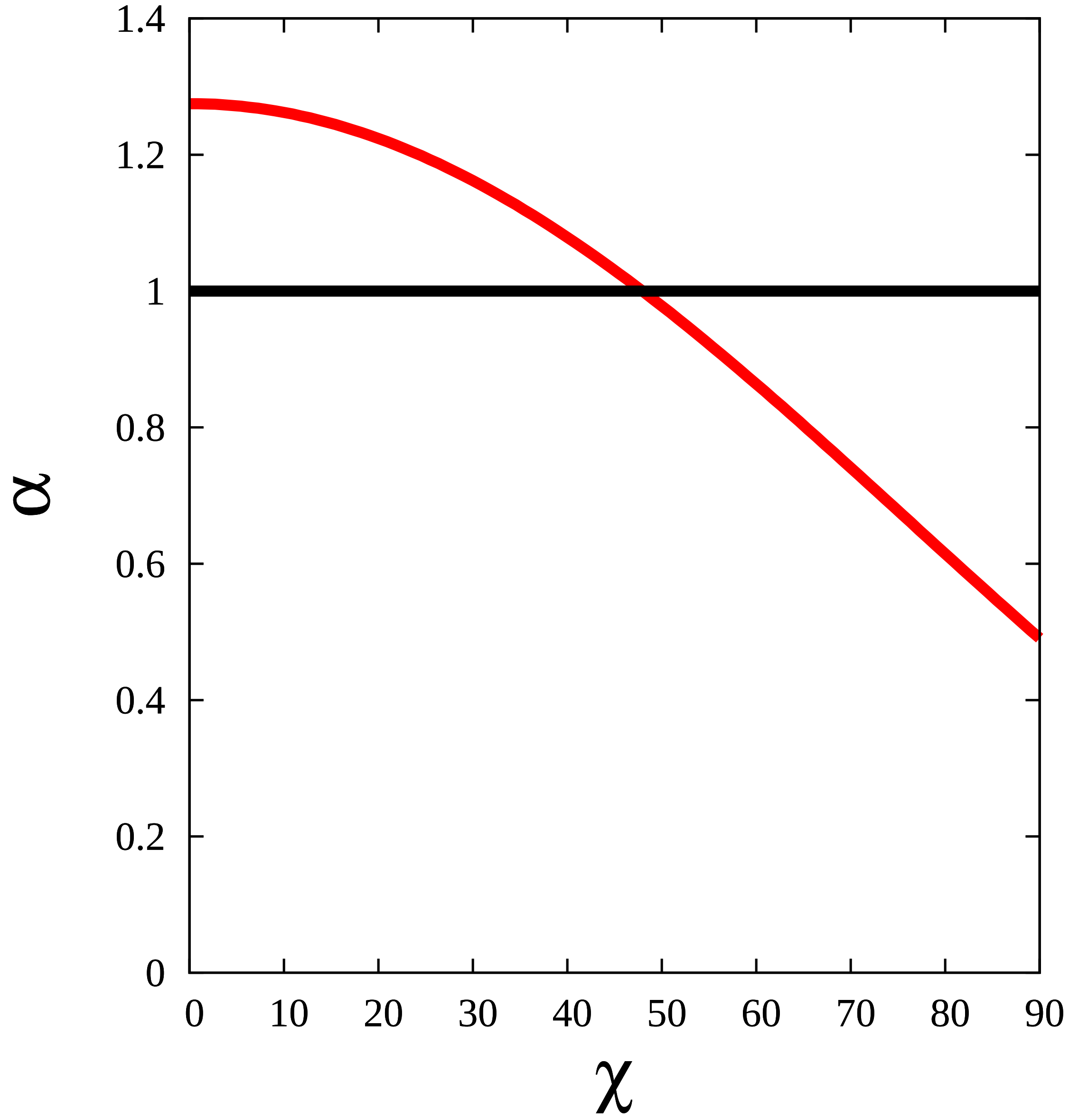
J.D.Biggs (1990)

$$\theta_0^4 \longrightarrow \theta_0^2\theta_1^2$$

$$\theta_1 = \theta_0 g(\chi)$$

$$g(\frac{\pi}{2}) = \left(\frac{4}{27}\right)^{\frac{1}{4}} \approx 0.620$$

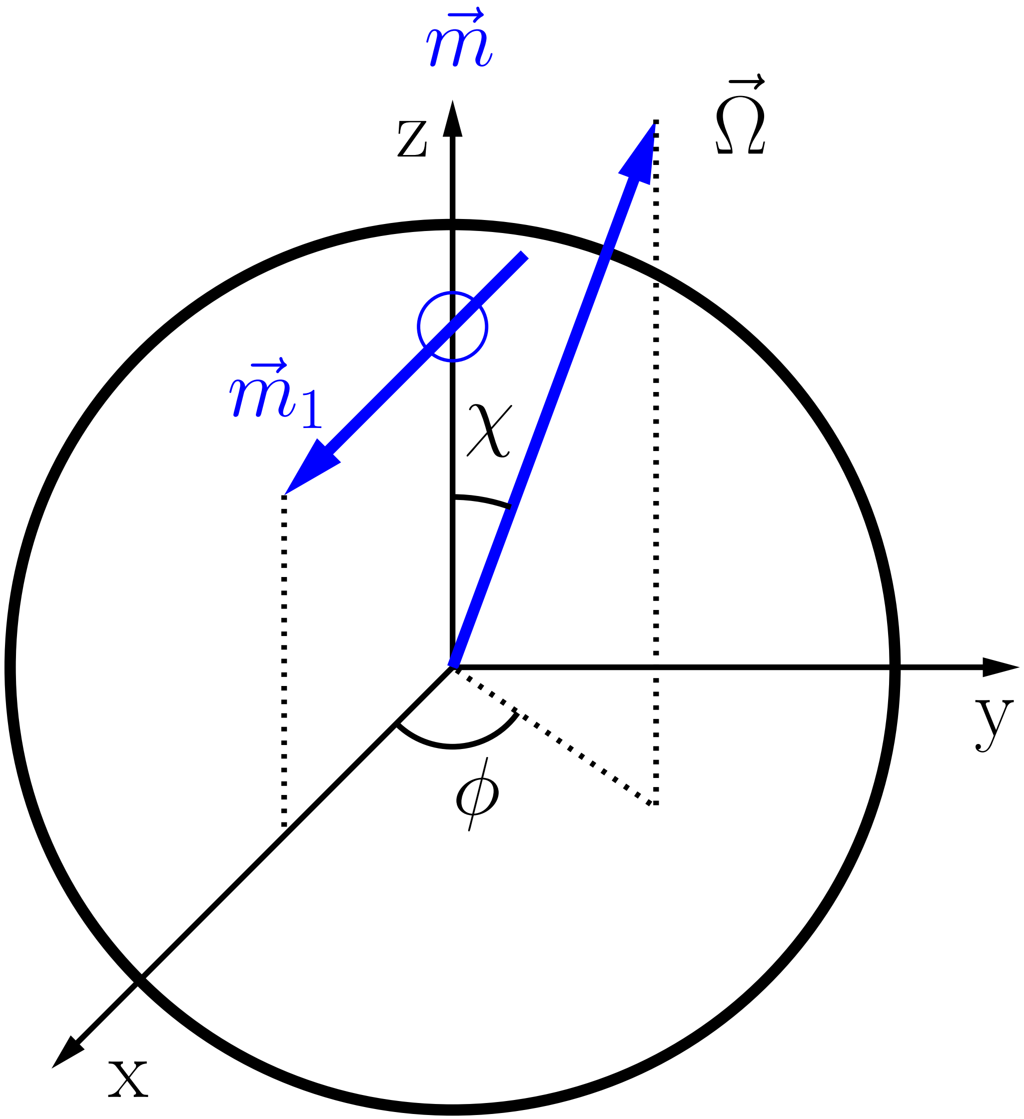
$$g(\chi) = \left(\frac{(1-\mu)^3}{1+3\mu}\right)^{1/4} \quad \text{where} \quad \cos\chi = \frac{1-3\mu}{\sqrt{1+3\mu}}$$



$$\alpha(\chi) = 2 \quad \frac{3}{4} A\, g(\chi)^2$$

$$\chi \approx 48^\circ$$

Nondipolar magnetic field



V.D.Palshin and A.I.Tsygan
(1996)

$$\vec{B} = \vec{B}_0 + \vec{B}_1$$

$$\nu = \frac{B_1}{B_0}$$

Electric Field

$$j = j_{GJ} f(\eta_0)$$

$$f(\eta) = \frac{1}{\sqrt{1+\lambda^2}} \left(\left(1 - \frac{\kappa}{\eta^3}\right) - \lambda \left(1 + \frac{1}{2}\frac{\kappa}{\eta^3}\right) tg\chi \cos\phi \right)$$

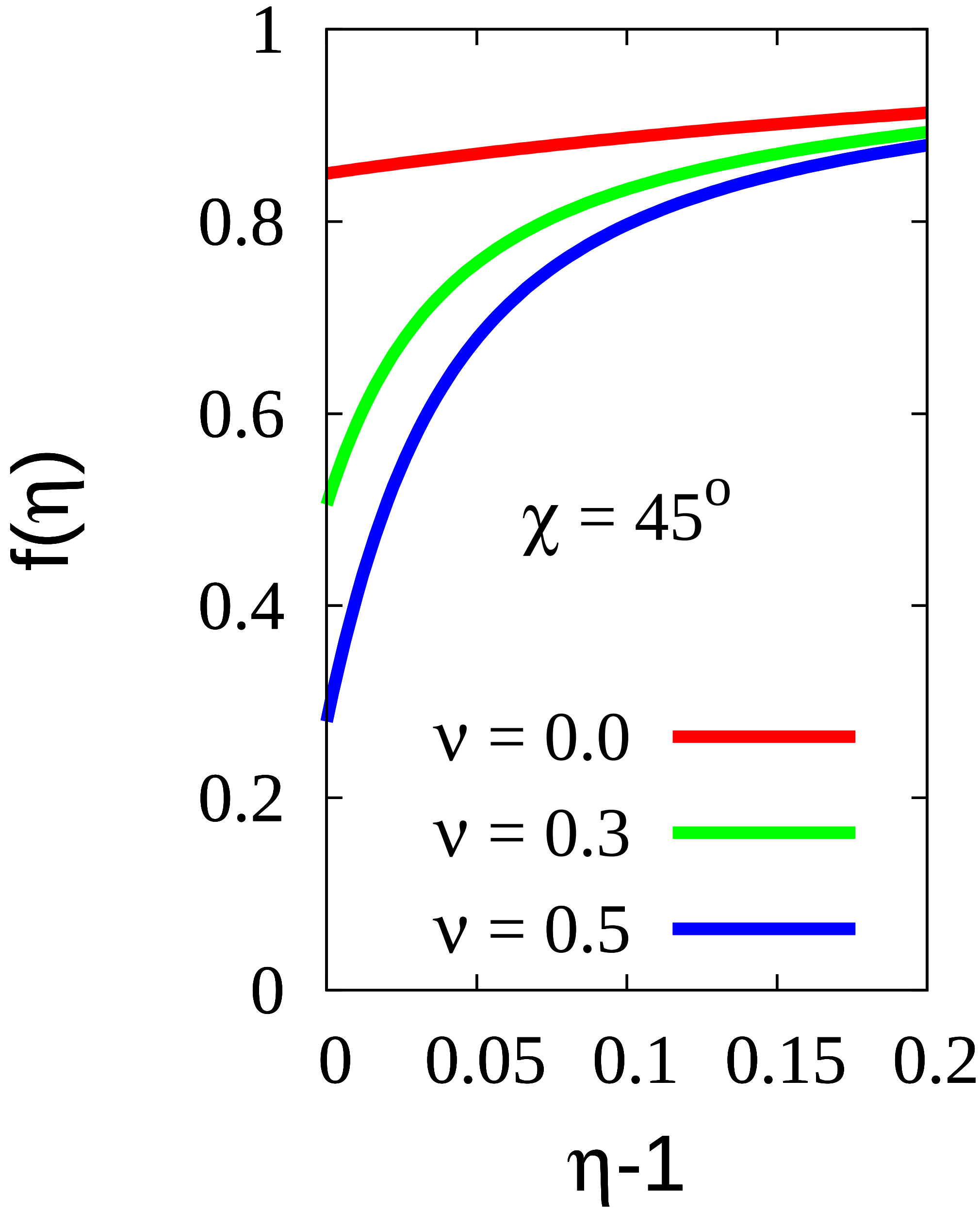
$$\lambda = \nu \left(\frac{\Delta\eta}{\eta - 1 + \Delta} \right)^3$$

$$\Phi = \frac{\Omega F}{2\pi c} \left(f(\eta) - f(\eta_0)\right)\left(1 - \xi^2\right) \cos\chi \qquad \text{at } \eta_0 \le \eta \le \eta_c$$

where $F = \frac{2\pi m \Omega}{c}$

E.M.Kantor and A.I.Tsygan (2003)

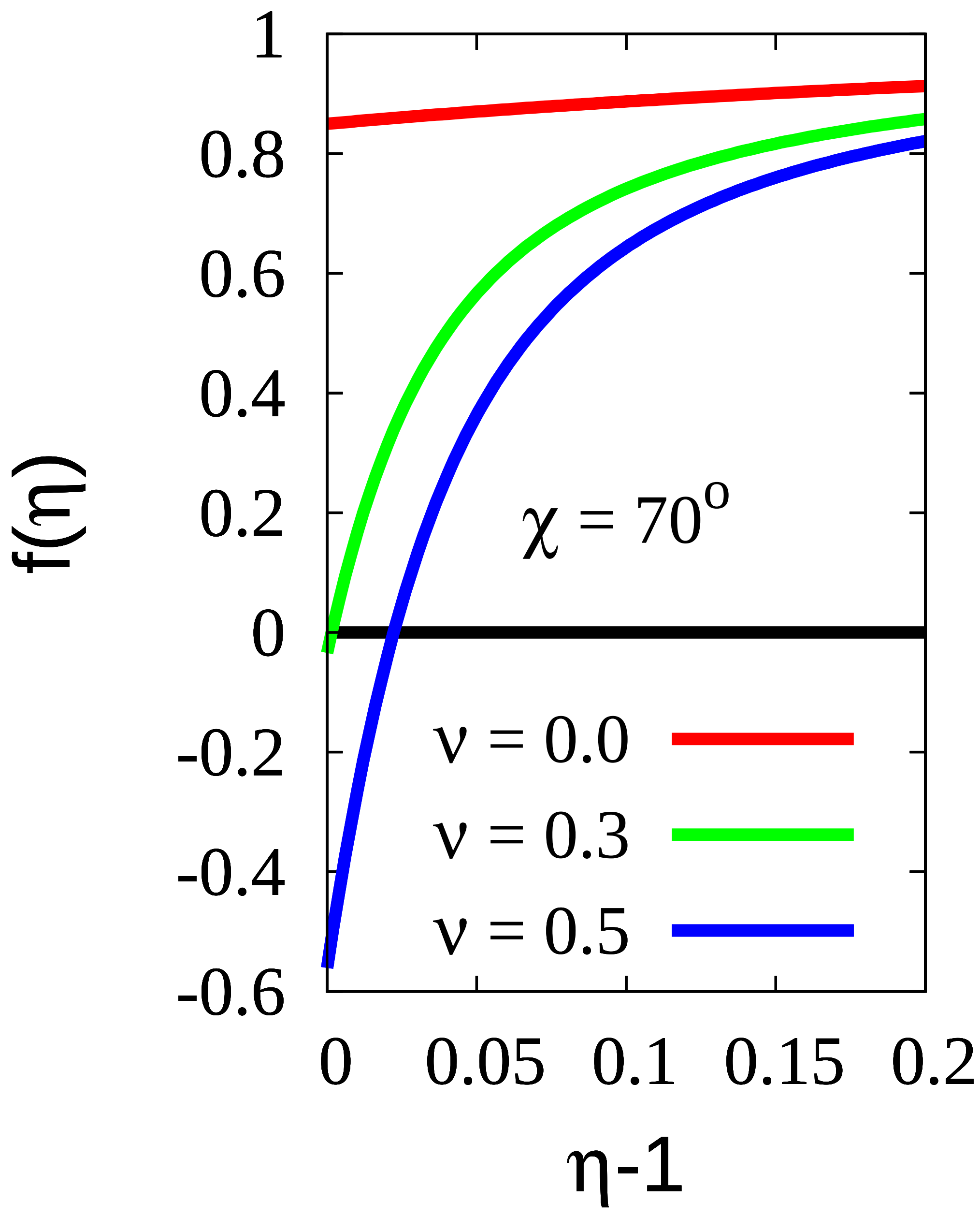
$\cos\phi > 0$



$\eta_0 = 1$ at $f(1) \geq 1$

$j = j_{GJ} f(1)$

$\cos\phi > 0$

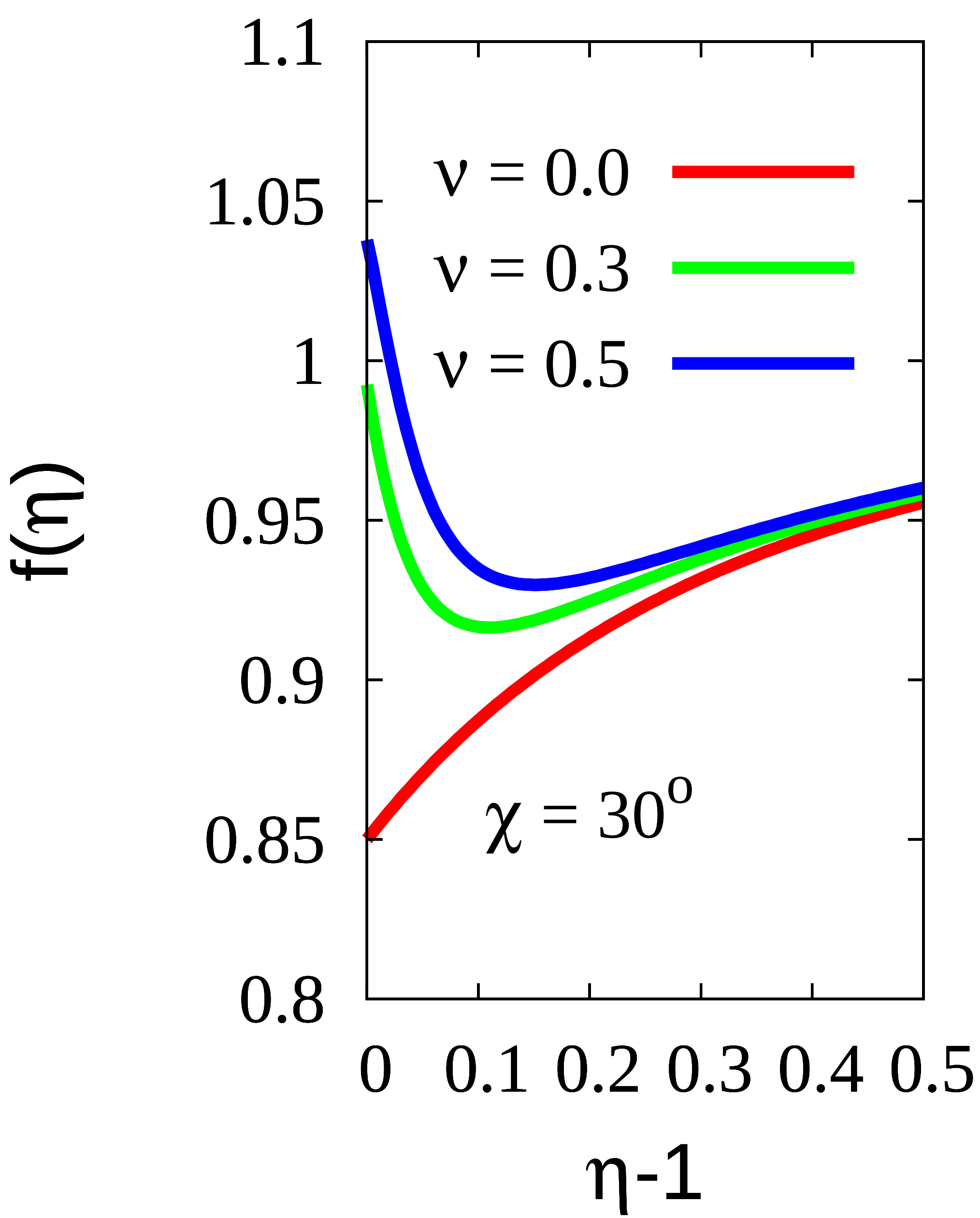


$f(1) < 0$

$f(\eta_0) = 0$

Ruderman
Sutherland
regime

$\cos\phi < 0$



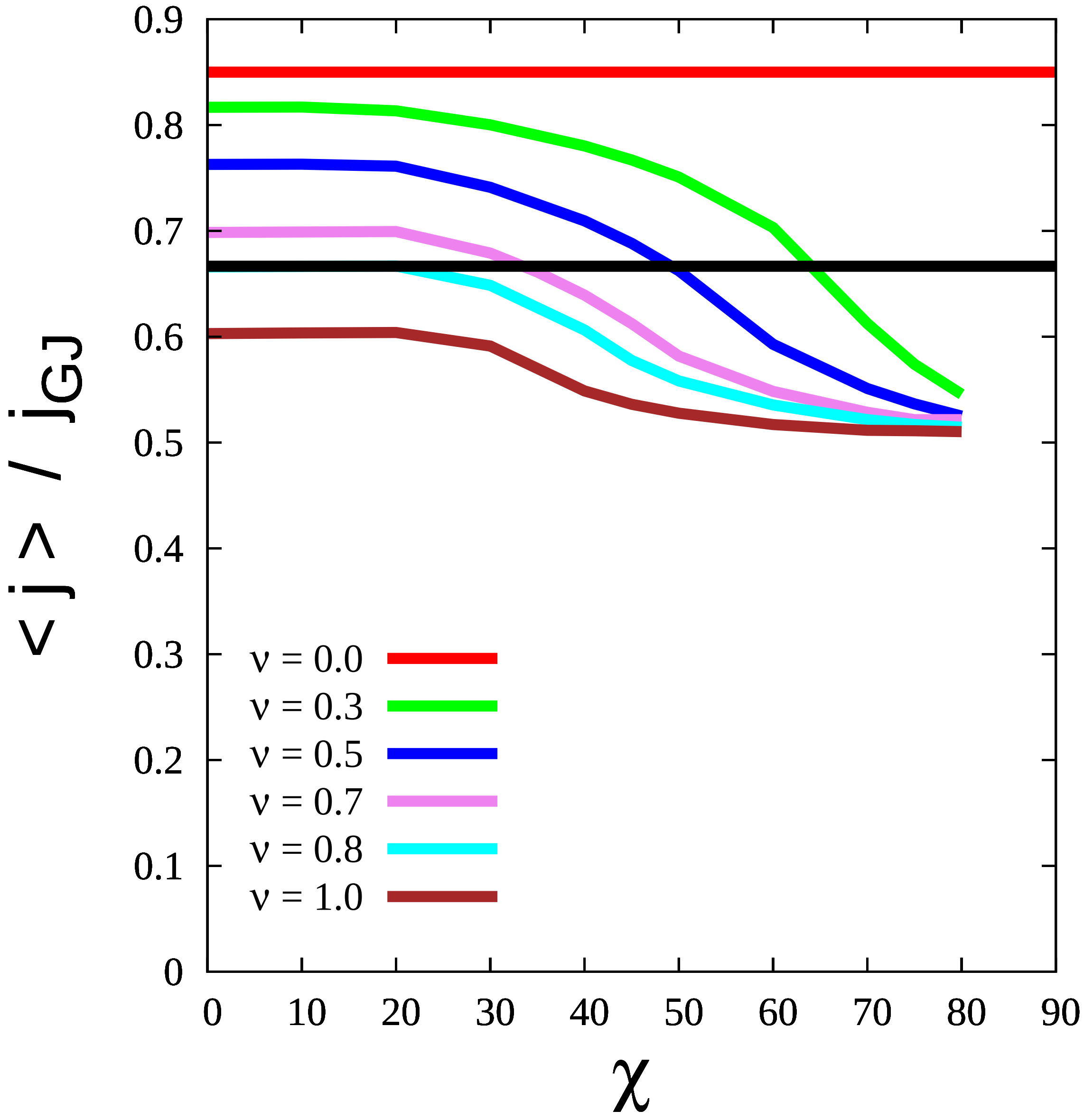
$f(\eta) > 0$

but not monotonic

$f(\eta_0) \approx 1$

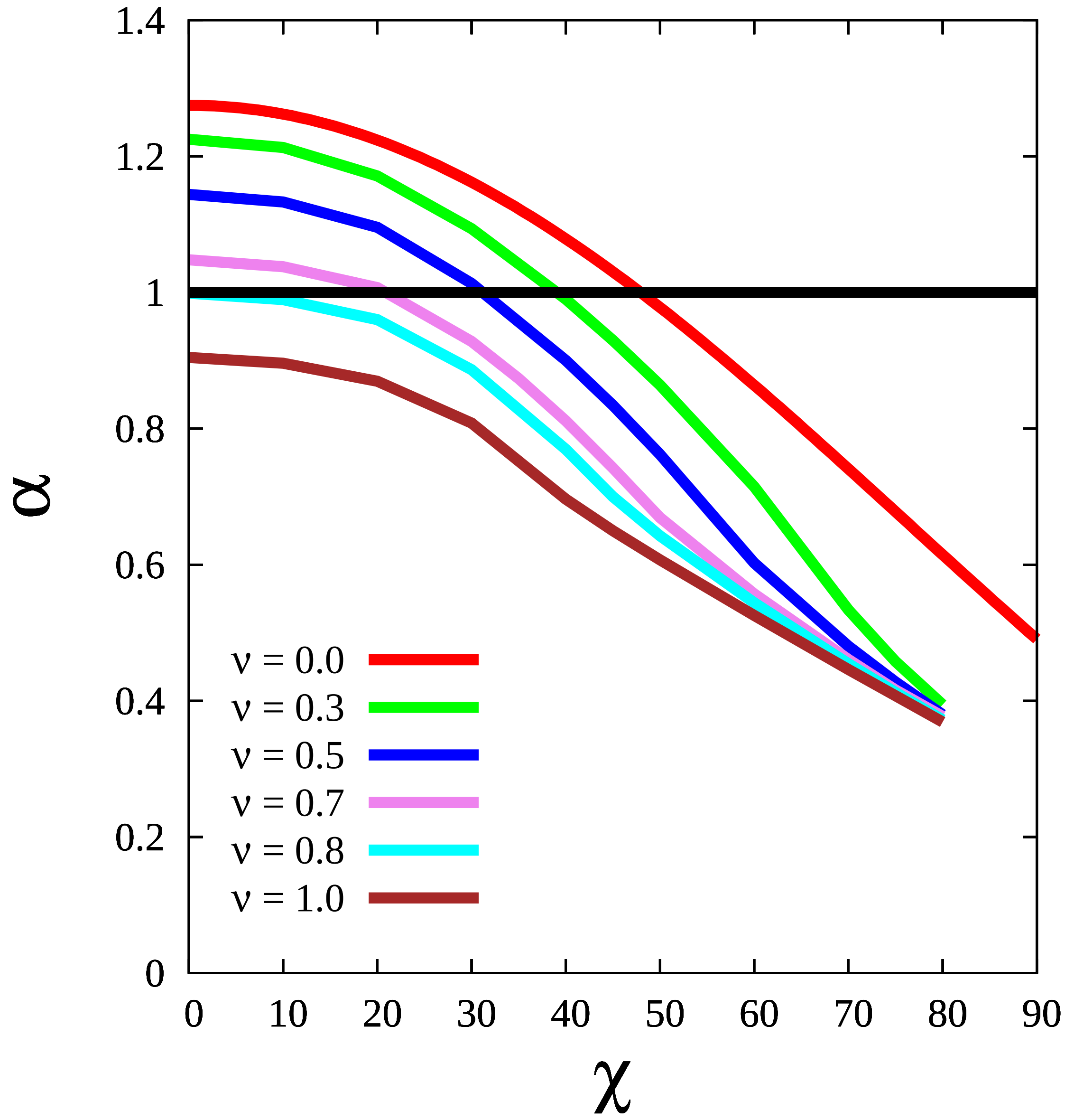
$j \approx j_{GJ}$

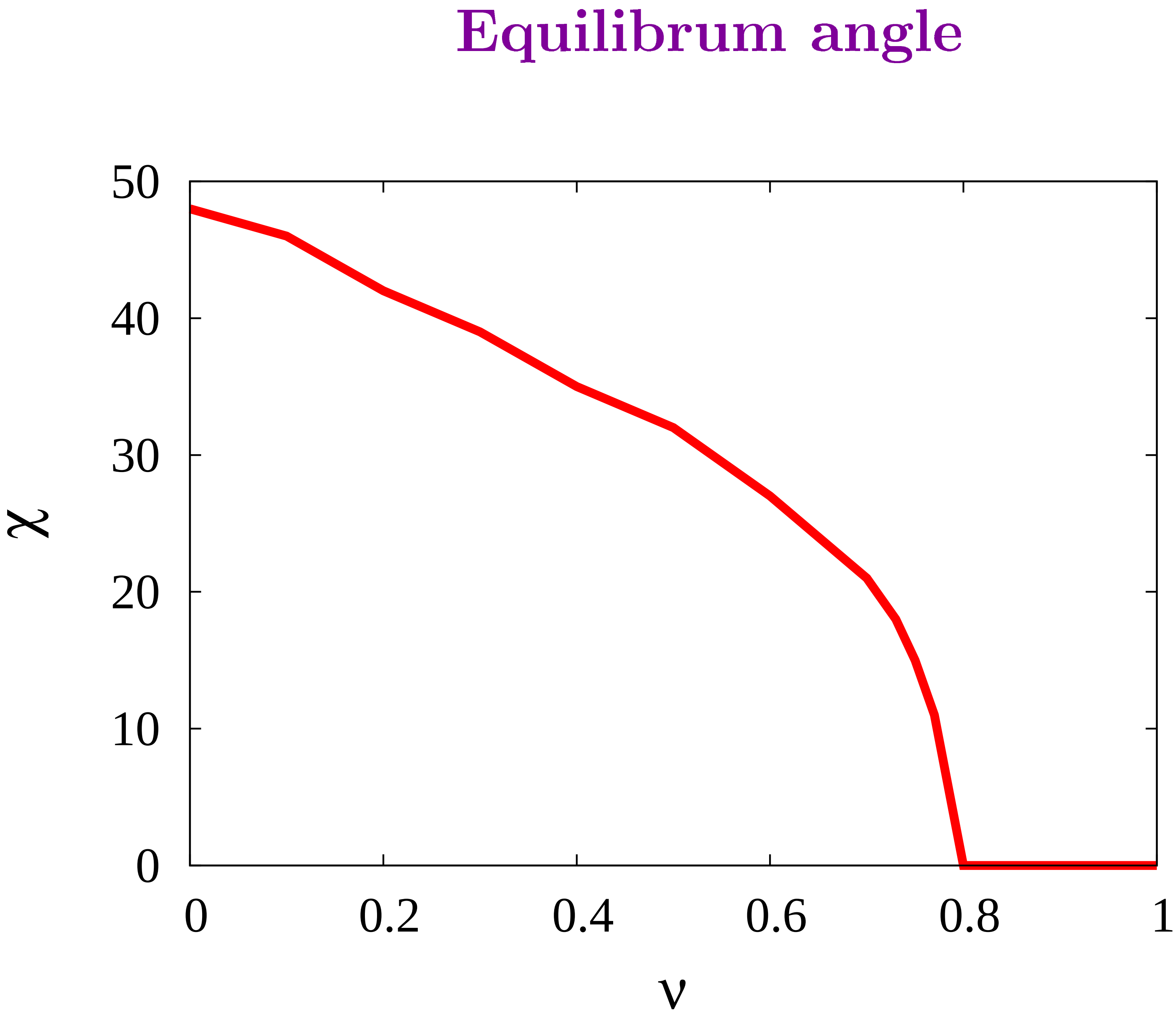
S. Shibata (1997)

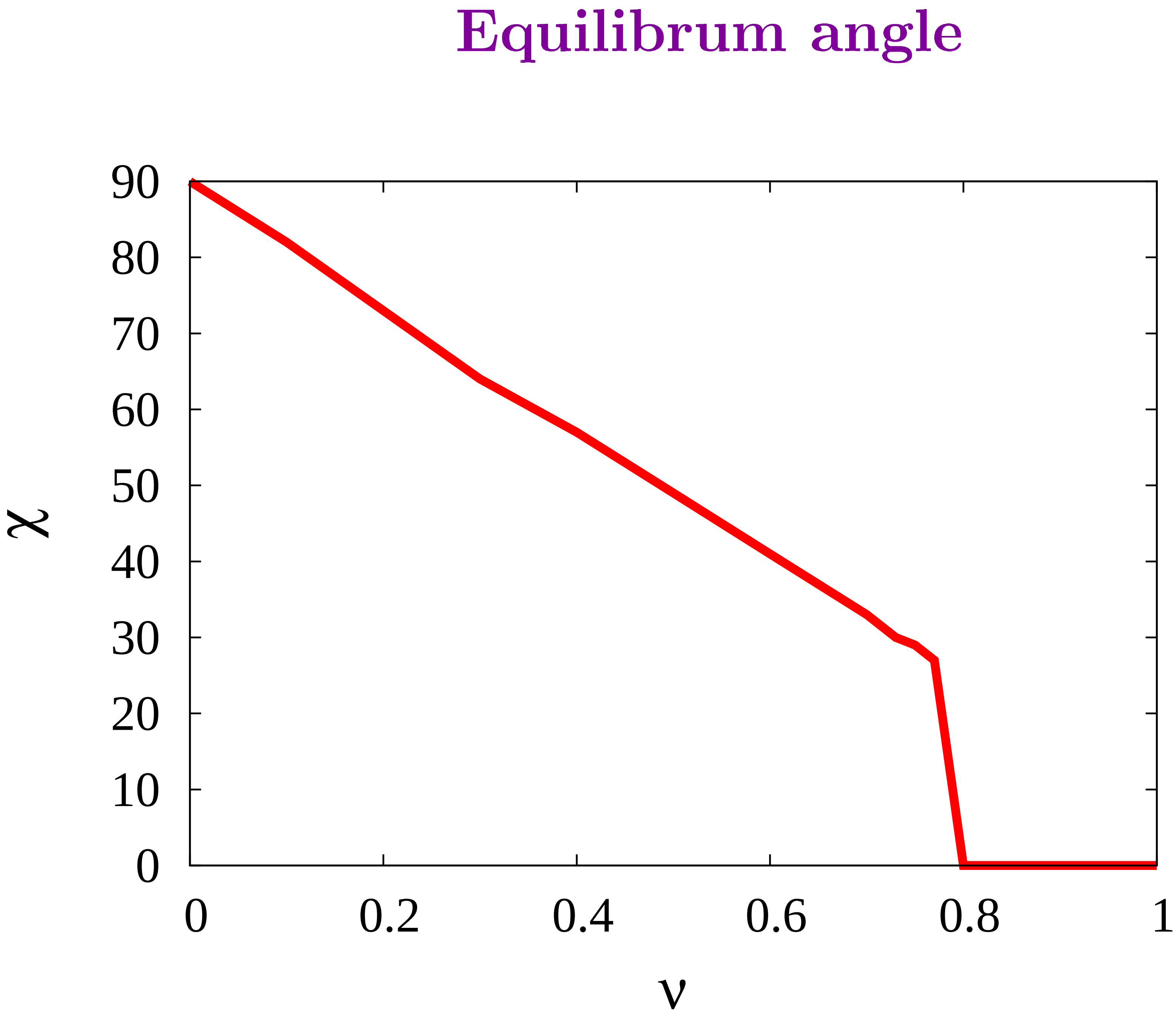


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80
90
χ
< j > / j_GJ
ν = 0.0
ν = 0.3
ν = 0.5
ν = 0.7
ν = 0.8
ν = 1.0

Current momentum losses




Equilibrum angle
χ
ν
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1



Equilibrum angle
90
80
70
60
50
40
30
20
10
0
χ
0
0.2
0.4
0.6
0.8
1
ν