On the mean profiles of radio pulsars

V.S. Beskin [1], A.A.Philippov [2]

[1] *P.N.Lebedev Physical Institute*

[2] *Moscow Institute of Physics and Technology*

The paradigme

Radio pulsars – rotating solitary* neutron stars

Radio pulsars – rotating solitary* neutron stars

- Mass $M \sim 1.4\ M$
- Radius $R \sim (10\text{-}\ 15)$ km
- Rotating period $P \sim 1$ s
- Magnetic field $B0 \sim 10$ G
- Radio luminosity $Lr \sim 10$ erg/s (~10 - 10)
- Coherent mechanism: $T \sim 10$ K (~10 ???)

The key electrodynamic idea

(Kardashev,1964; Pacini, 1967)

Magneto-dipole (vacuum) radiation

$$W_{\rm tot} = -J_{\rm r}\Omega\dot{\Omega} \approx \frac{1}{6}\frac{B_0^2\Omega^4R^6}{c^3}\sin^2\chi$$

*W*tot ~ (10 - 10) erg/s

In reality is it not so (magnetosphere is filled with plasma), but is enough for evaluation

Strong magnetic field

B ~ 10 G ~ Bcrit = me c̄ /eh = 4.4 10 G

- Pair creation

$$\gamma + B \rightarrow e^+ + e^- + B$$

$$w = \frac{3\sqrt{3}}{16\sqrt{2}} \frac{e^3 B \sin\theta}{\hbar m_e c^3} \exp\left(-\frac{8}{3} \frac{B\hbar}{B \sin\theta} \frac{m_e c^2}{\mathscr{E}_{ph}}\right)$$

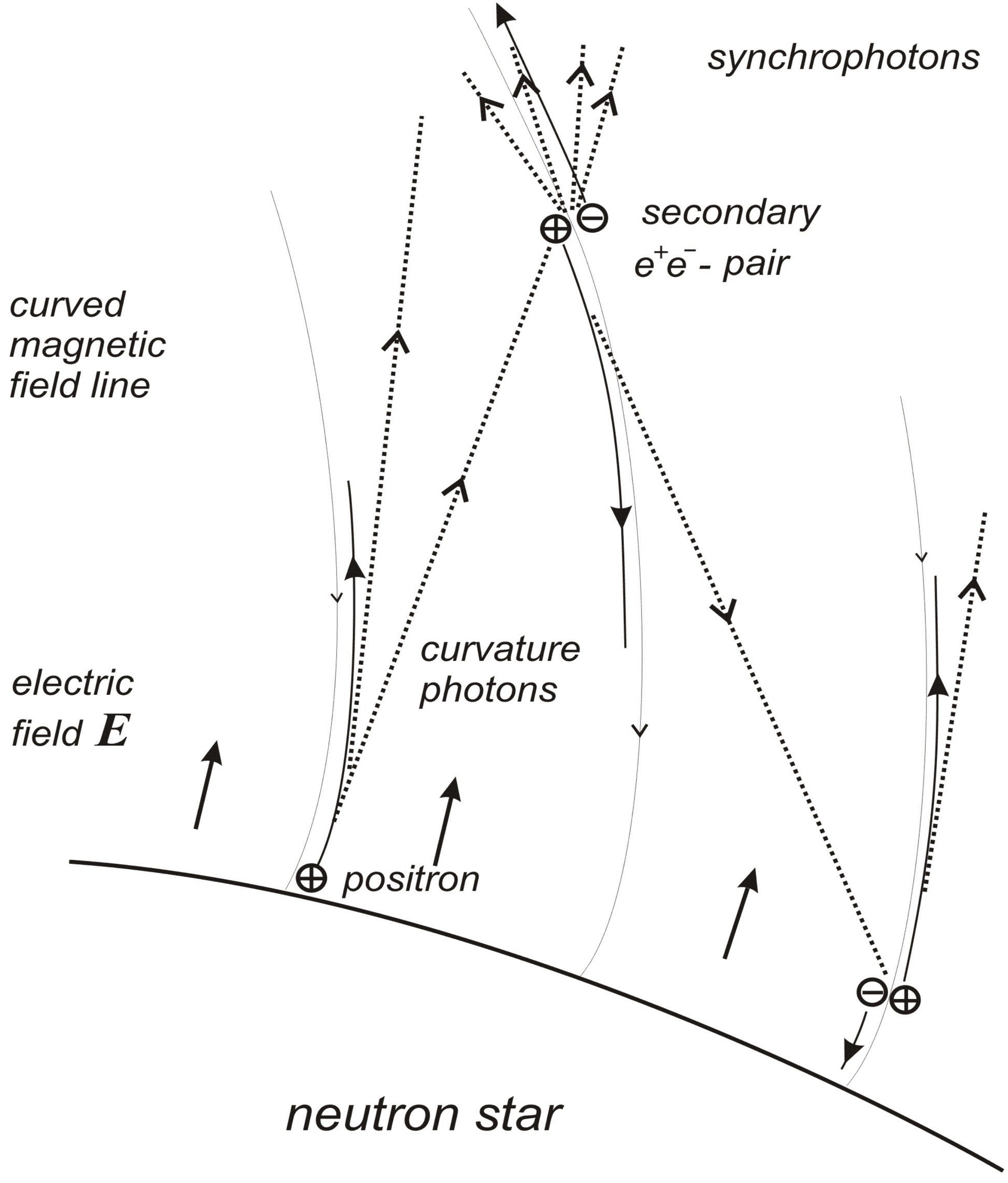
- 1D motion

$$\tau_s \approx \frac{1}{\omega_B} \left(\frac{c}{\omega_B r_e}\right) \sim 10^{-15}\ \mathrm{s}$$

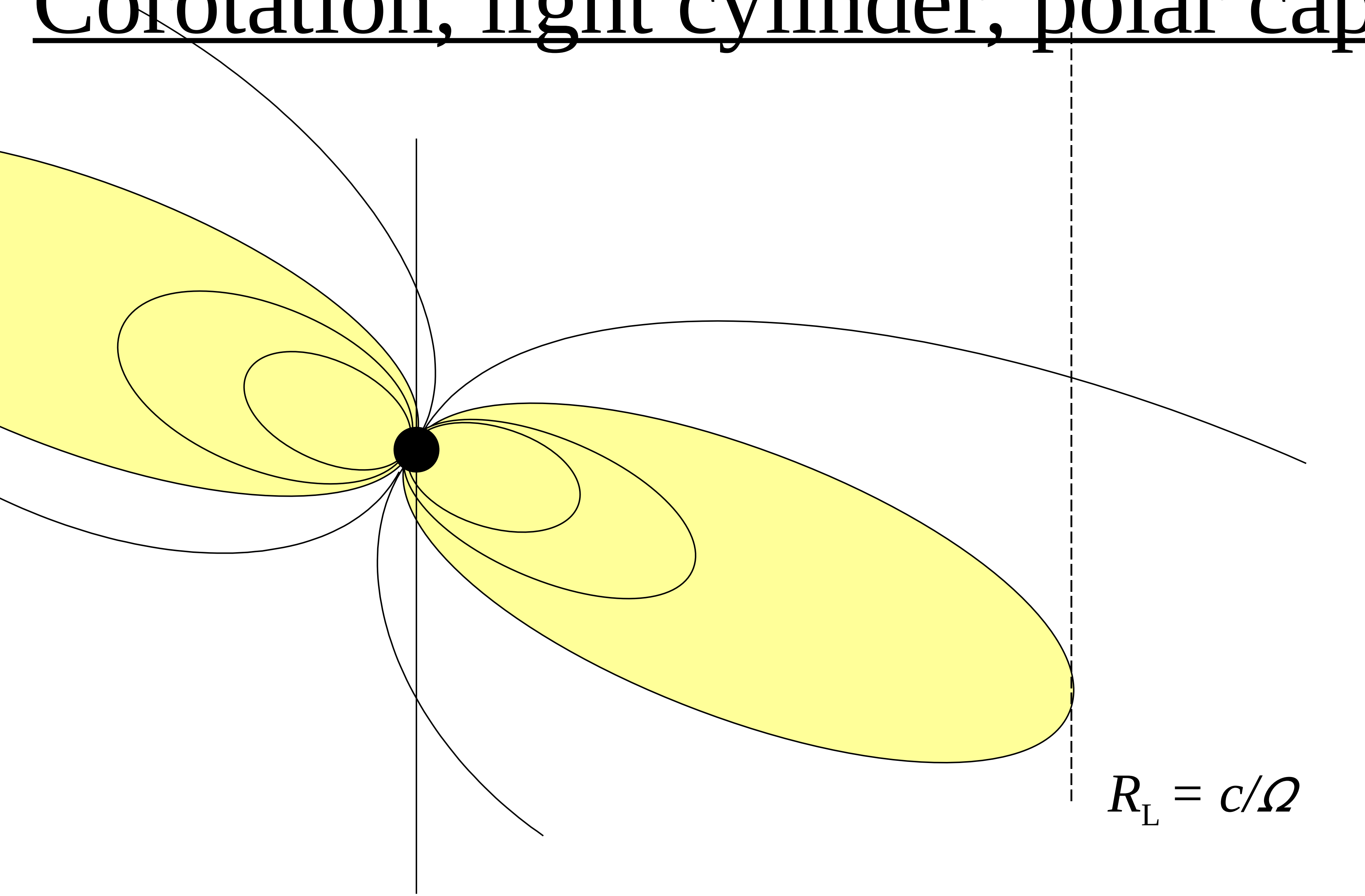
- Electric field

$$E_{\parallel} \sim \frac{\Omega R}{c} B_0$$

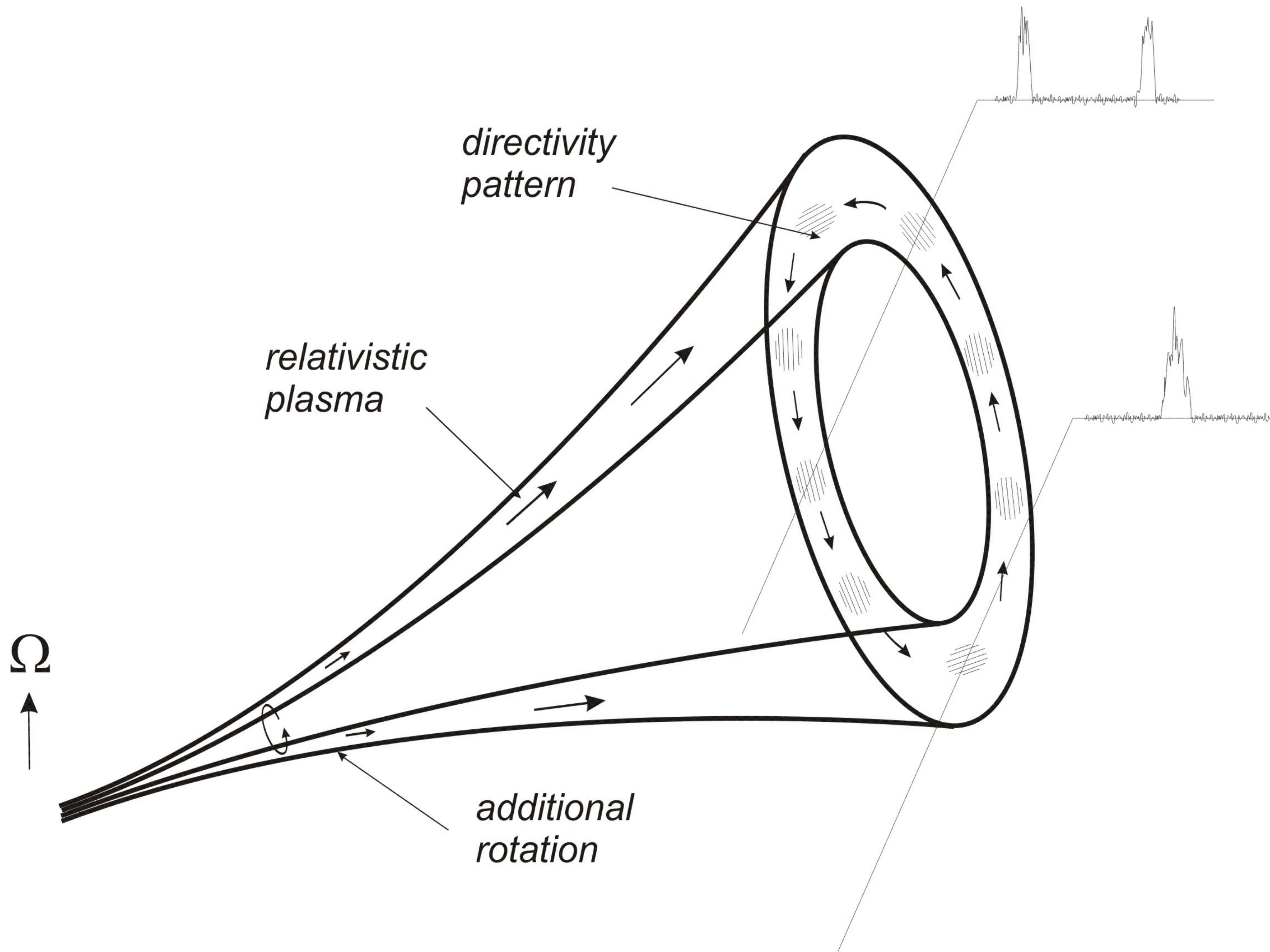
Particle creation



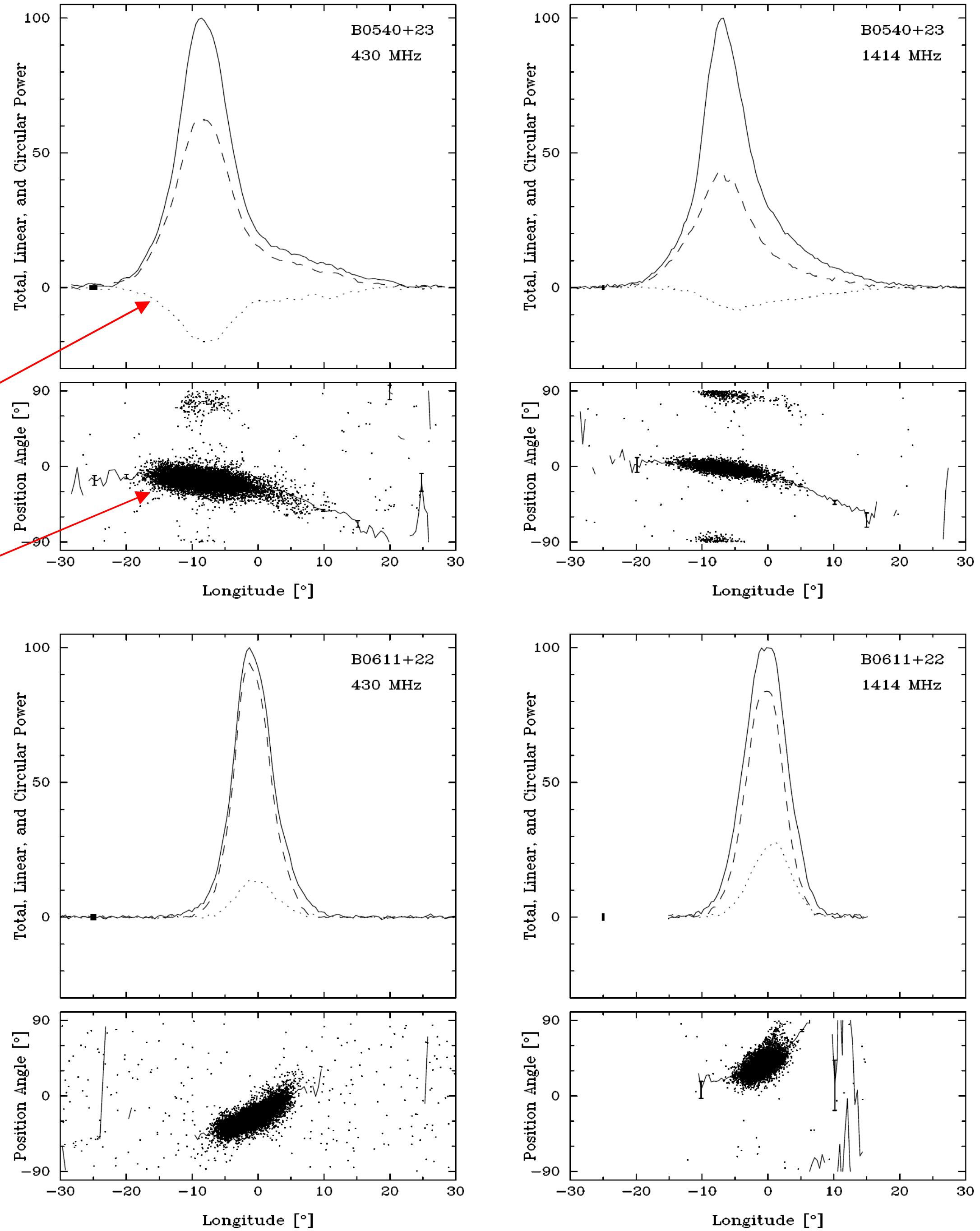
Corotation, light cylinder, polar cap



“Hollow cone” model



T.Hankins, J.Rankin. Astron. J., **139**, 168 (2010)



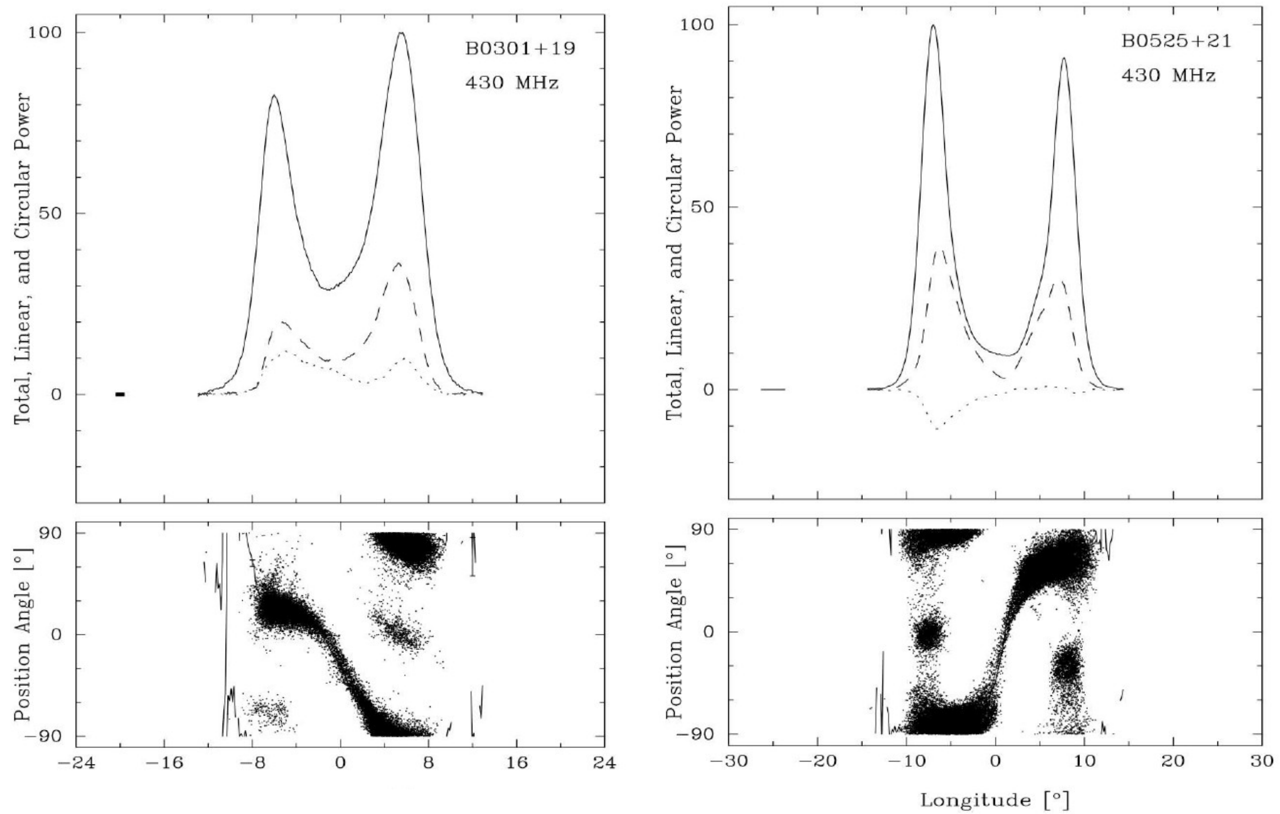
signs V vs d$p.a.$/dϕ correlation

Single – both up or down

T.Hankins, J.Rankin. Astron. J., **139**, 168 (2010)

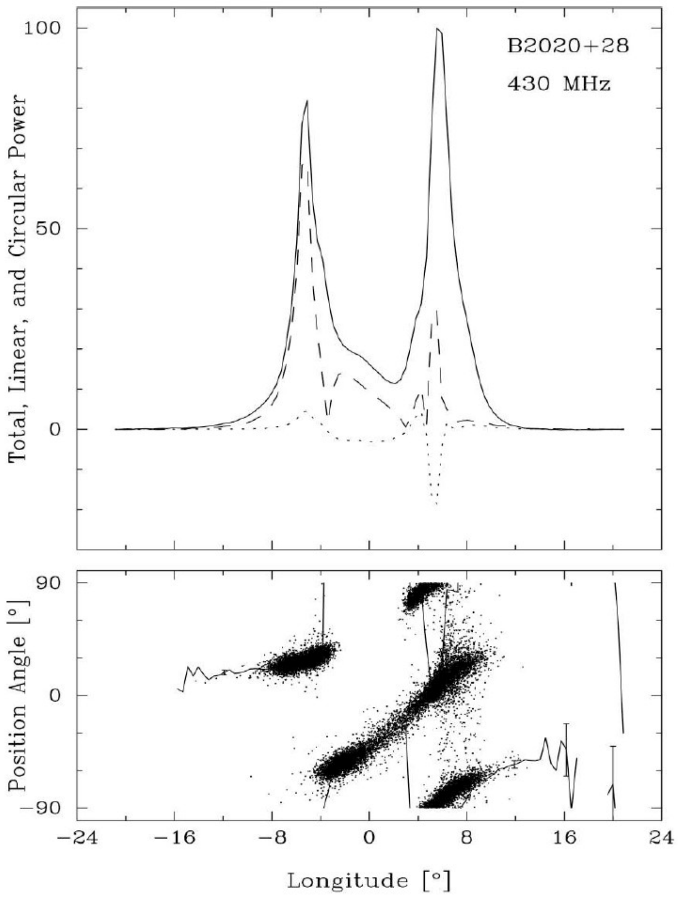
signs V vs d$p.a.$/dϕ correlation

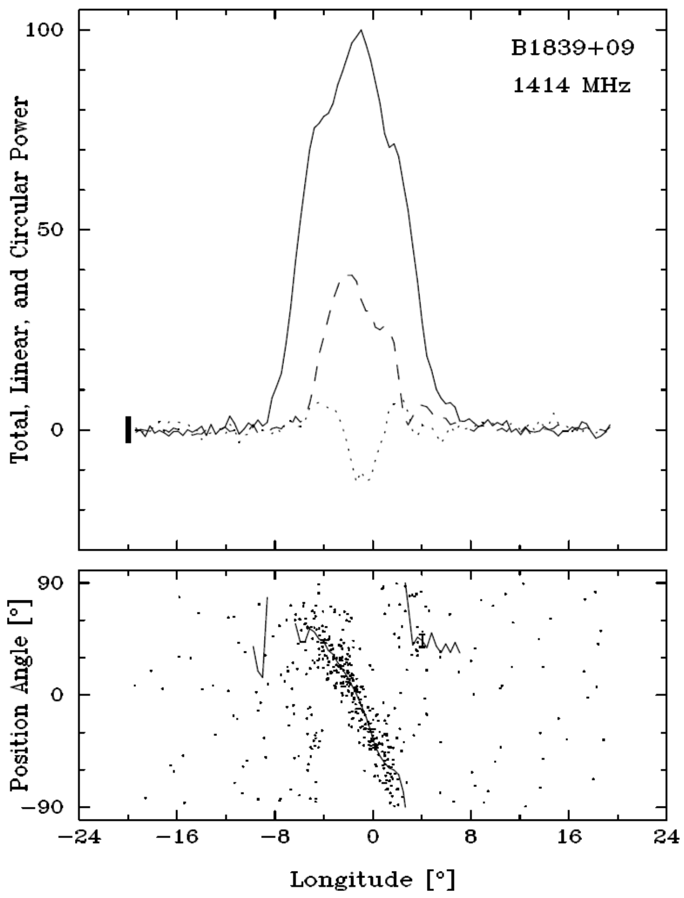
Double – the opposite signs



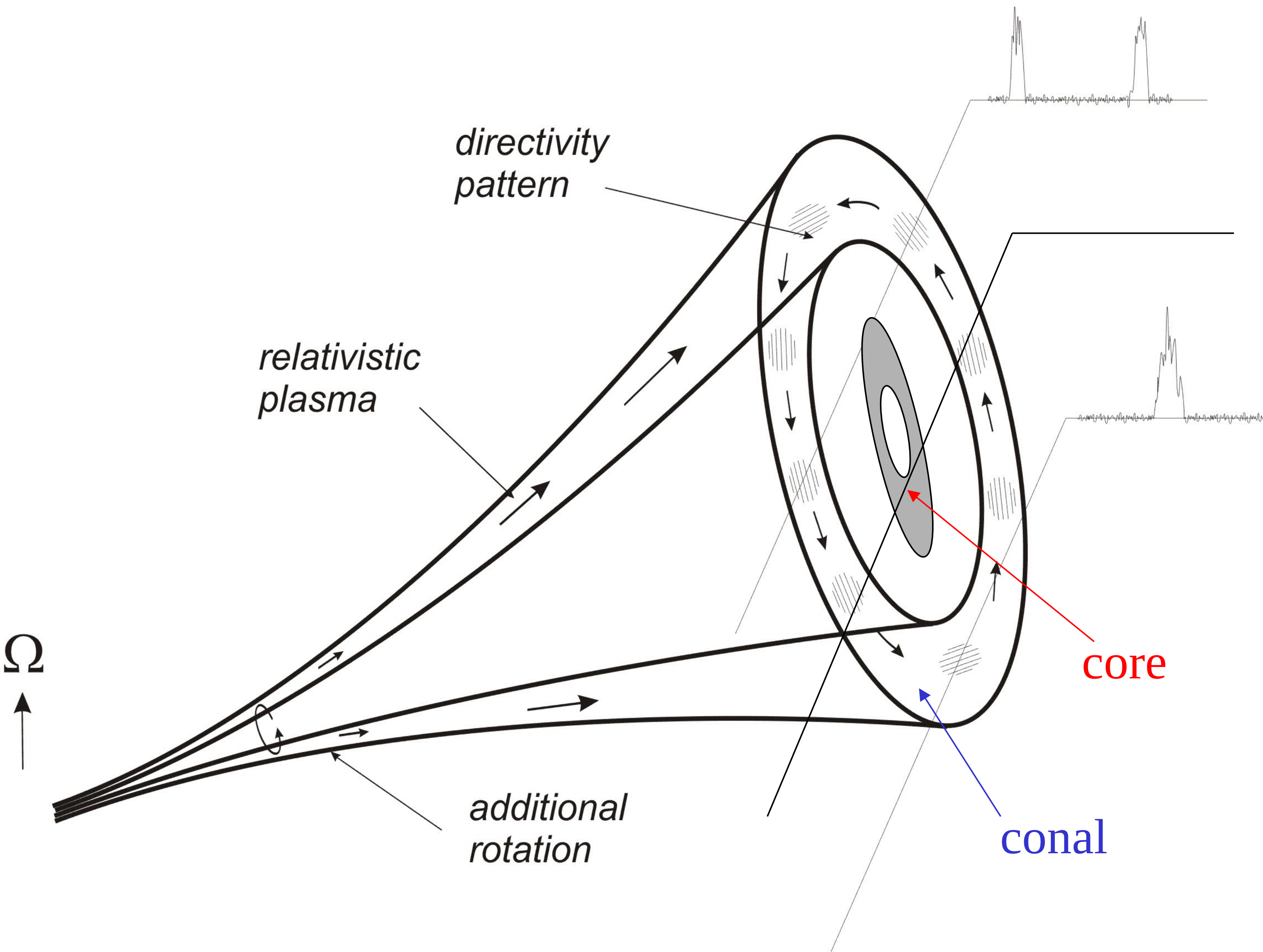
J.Han, R.Manchester, R.Xu, G.Qiao. MNRAS, **300**, 373 (1998)

T.Hankins, J.Rankin. Astron. J., **139**, 168 (2010)

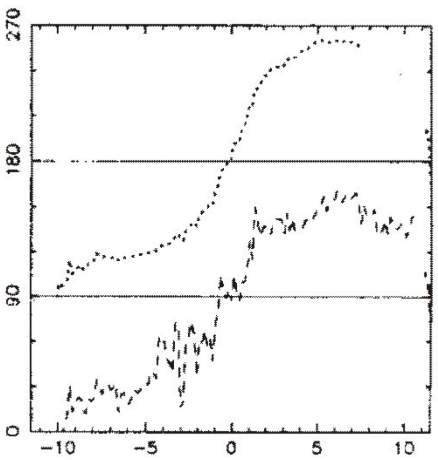


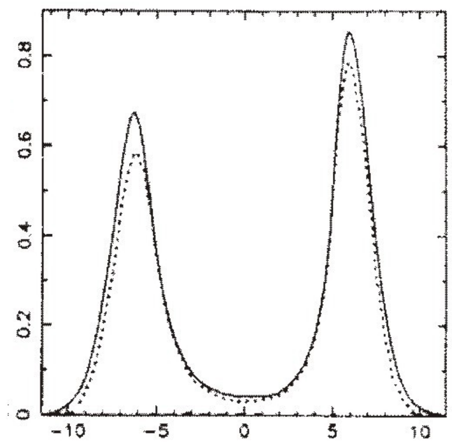


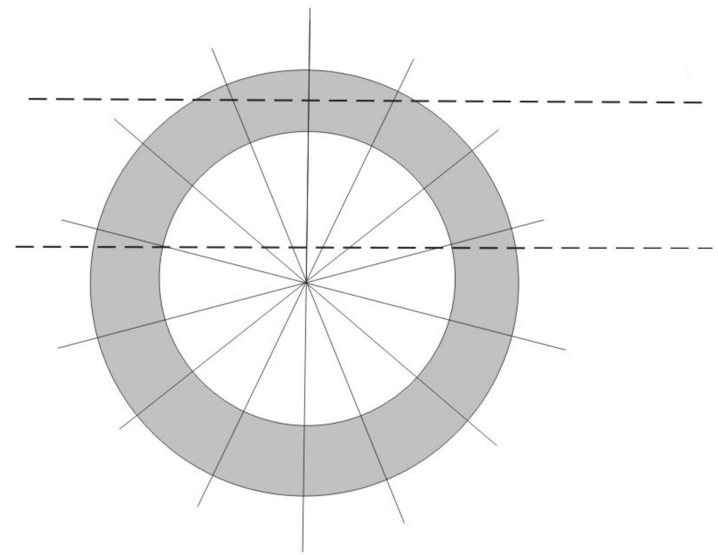
“Core-conal” model



Correlation, orthogonal modes







Periphery passage

– single profiles, small change of the *p.a.*

Central passage

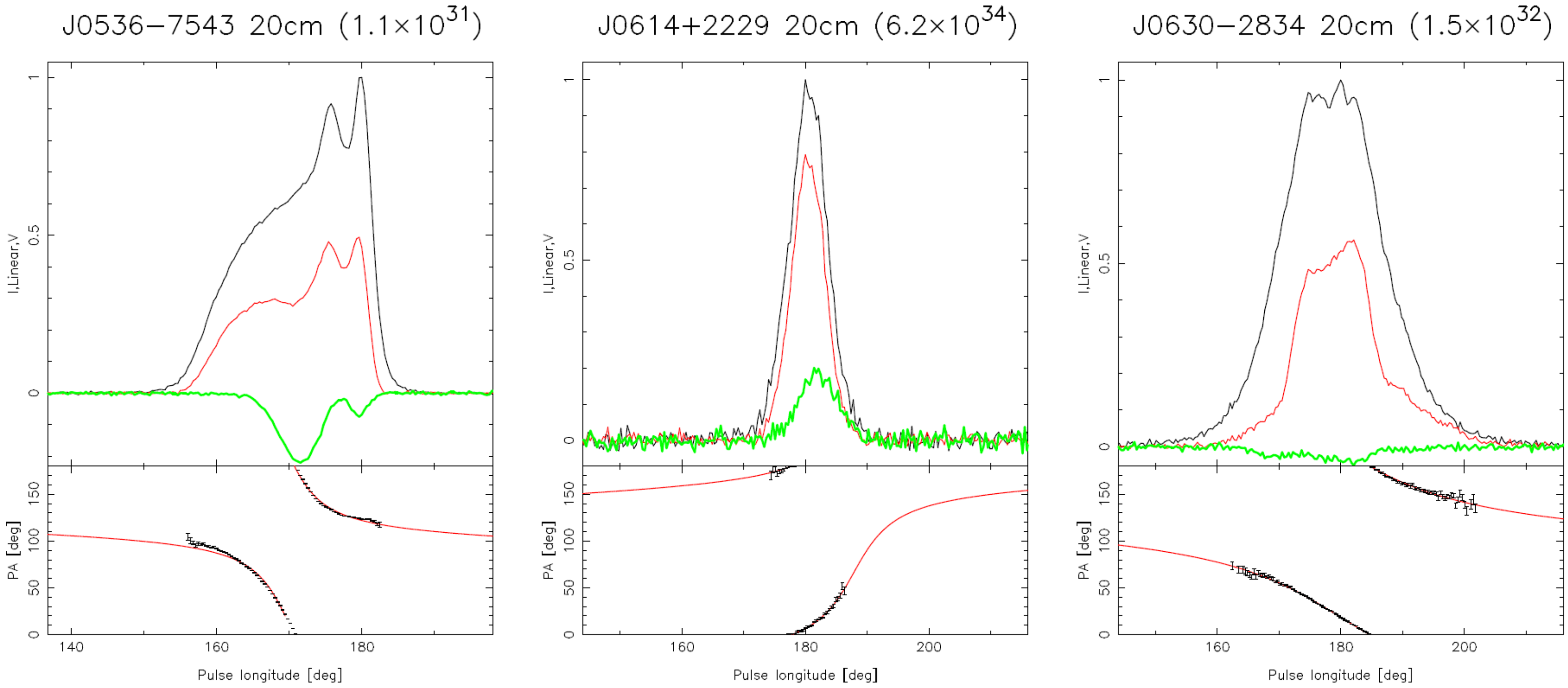
– double profiles, *p.a.* changes up to 180^{o}.

Position angle *p.a.*

Rotation Vector Model (RVM)

$$p.a. = \arctan\left(\frac{\sin\chi\sin\phi}{\sin\chi\cos\zeta\cos\phi - \sin\zeta\cos\chi}\right)$$

V. Radhakrishnan, D. J. Cooke. Ap Lett., **3**, 225 (1969)



P.Weltevrede, S.Johnston. MNRAS, **391**, 1210 (2008)

Everything is clear

- Stability of pulsation – neutron star rotation
- Energy source – kinetic energy of rotation
- Mechanism of energy loss – electrodynamics
- Neutron star is a radio pulsar if there is secondary electron-positron generation near magnetic poles

Everything is clear?

- Stability of pulsation – neutron star rotation
- Energy source – kinetic energy of rotation
- Mechanism of energy loss – electrodynamics
- Neutron star is a radio pulsar if there is secondary electron-positron generation near magnetic poles
- Radio emission – ????

Several private remarks

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Magnetosphere

Screening of the magnetodipole radiation for zero longitudinal current

Determination of the current losses for arbitrary inclination angle

Determination of the current flowing in the magnetosphere

Radio Emission

Dielectric tensor in the inhomogeneous media

Instability of the curvature-plasma waves

Saturation due to nonlinear interaction of waves

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Magnetosphere

Screening of the magnetodipole radiation for zero longitudinal current

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

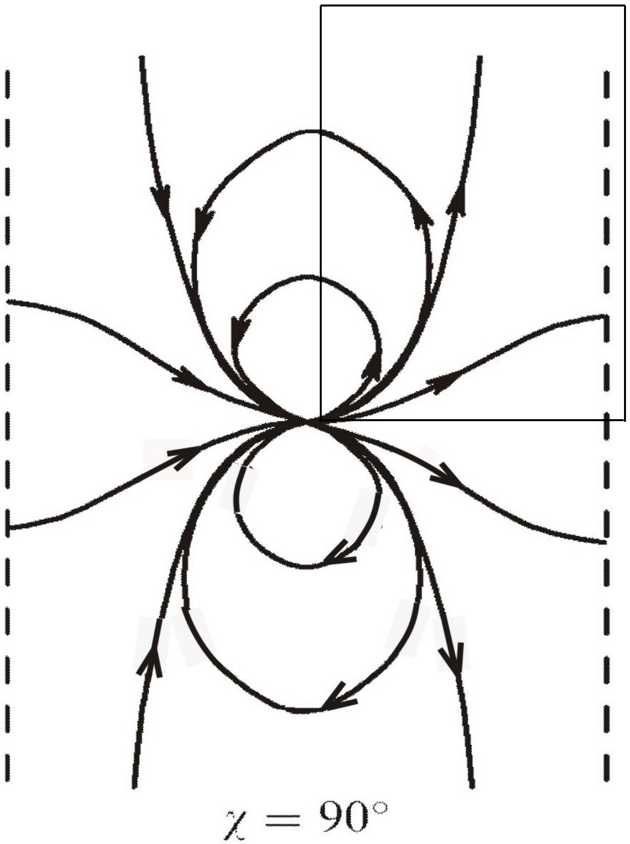
(1983, 1984, 1988, 1993)

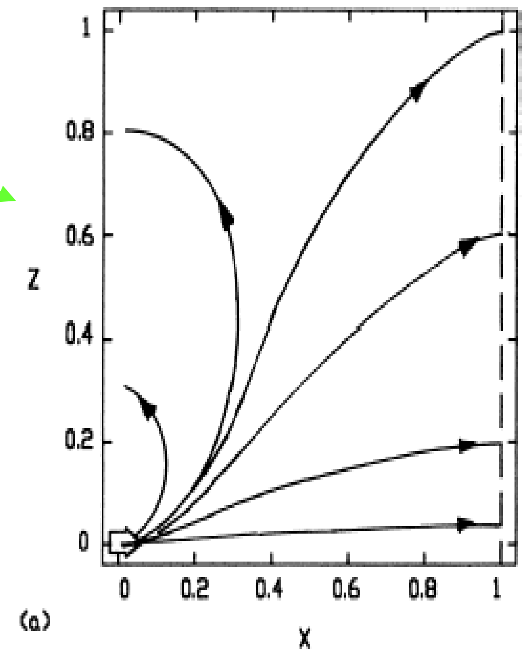
Magnetosphere

Screening of the magnetodipole radiation for zero longitudinal current

- Independent confirmation by L.Mestel, P.Panagi, S.Shibata. MNRAS, **309**, 388 (1999)
- The absence of the *1/r* alternative electromagnetic fields in Spitkovsky incline solution

Orthogonal rotator, $I = 0$

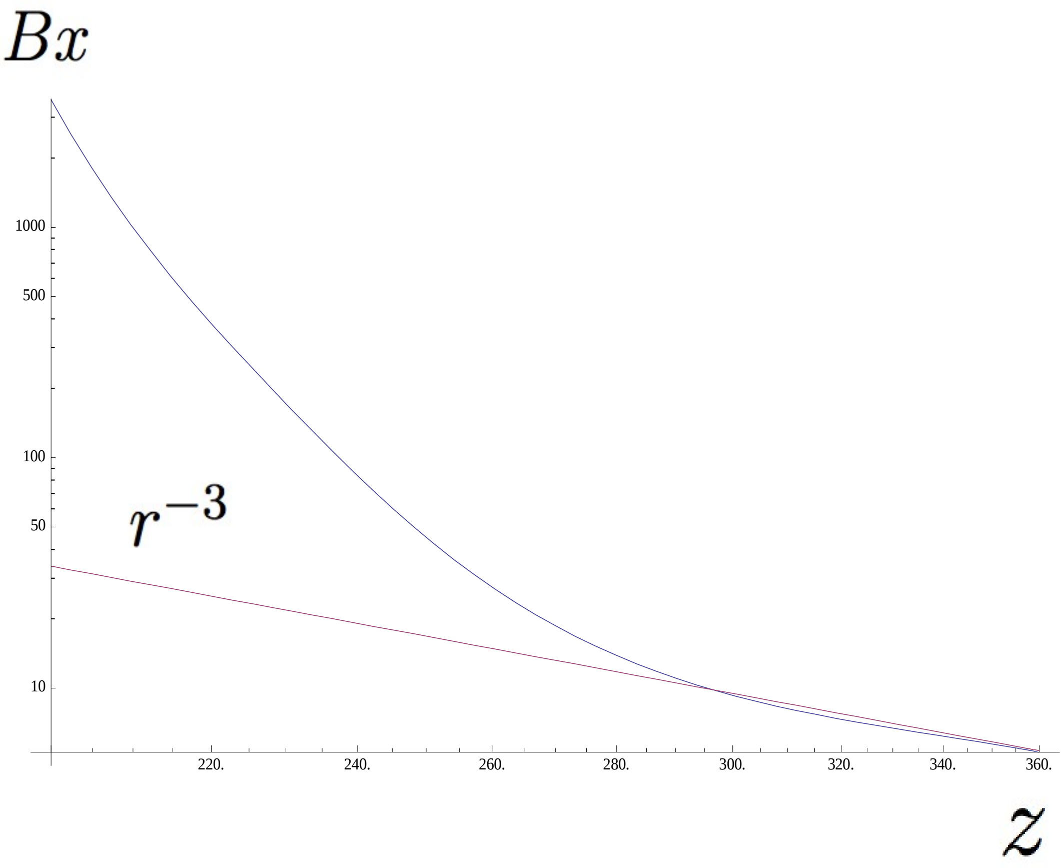


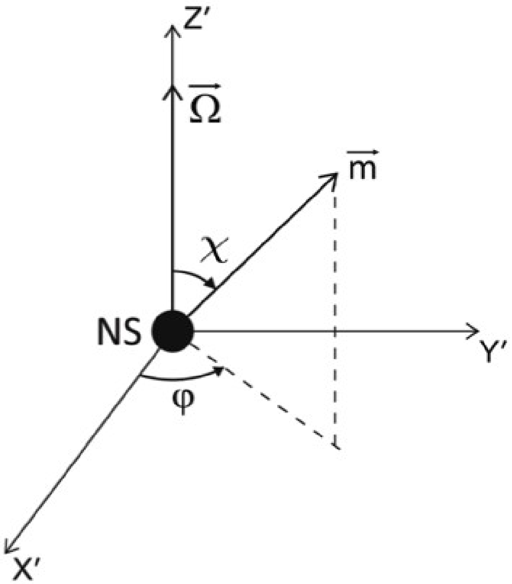


V.S.Beskin, A.V.Gurevich, Ya.N.Istomin. Sov. Phys. JETP, **58**, 235 (1983)

L.Mestel, P.Panagi, S.Shibata. MNRAS, **309**, 388 (1999)

Spitkovsky solution, $\chi = 60$





In vacuum $B_x = \frac{\ddot{d}}{cr}$

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

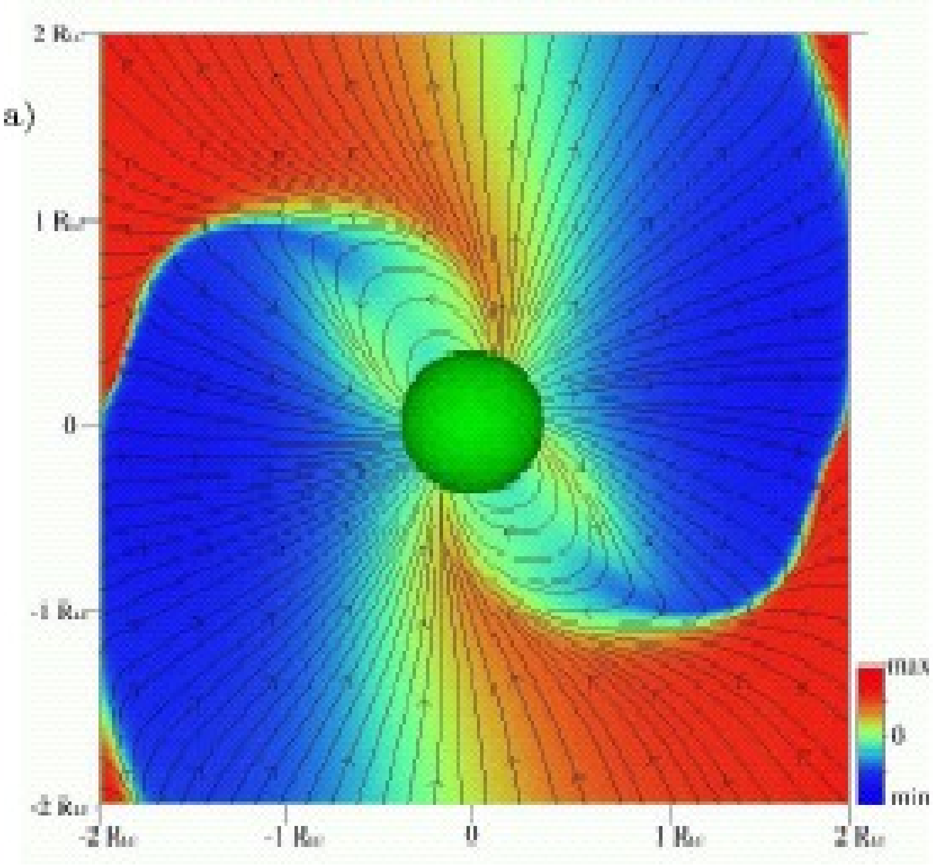
Magnetosphere

Determination of the current losses for arbitrary inclination angle

Orthogonal rotator – energy loss

A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

$$W_{\rm tot} \approx \frac{1}{4}\frac{B_0^2\Omega^4R^6}{c^3}\left(1+\sin^2\chi\right)$$

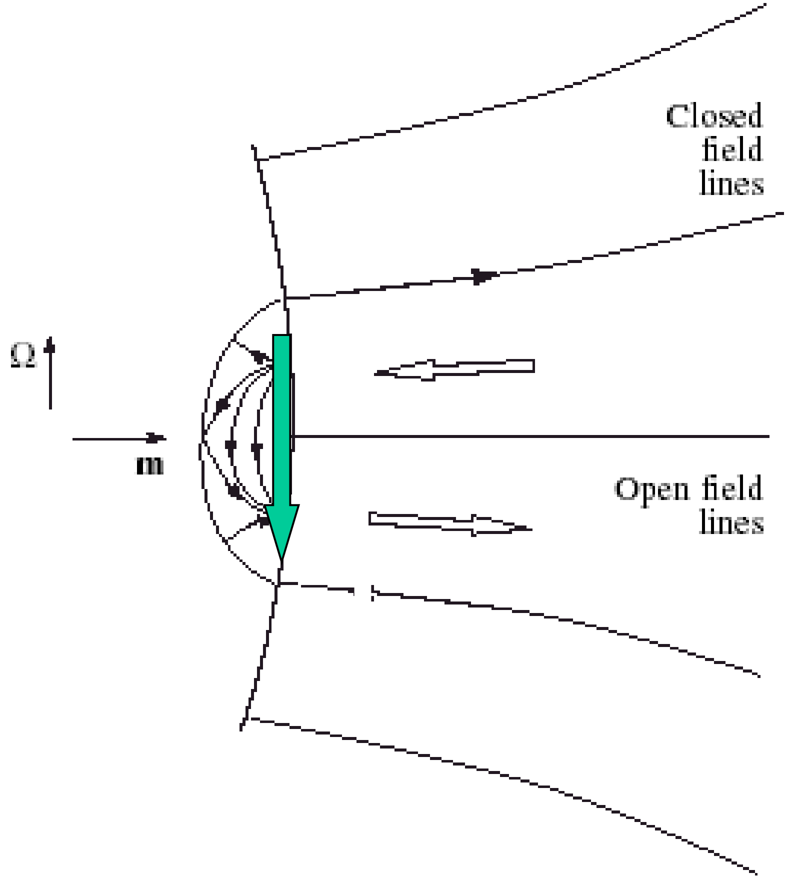


Orthogonal rotator – energy loss

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)

$$j_{\rm GJ} \approx \frac{\Omega B}{2\pi} \cos\theta$$

$$\mathbf{K} = \frac{1}{c}\int [\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]]\mathrm{d}S$$



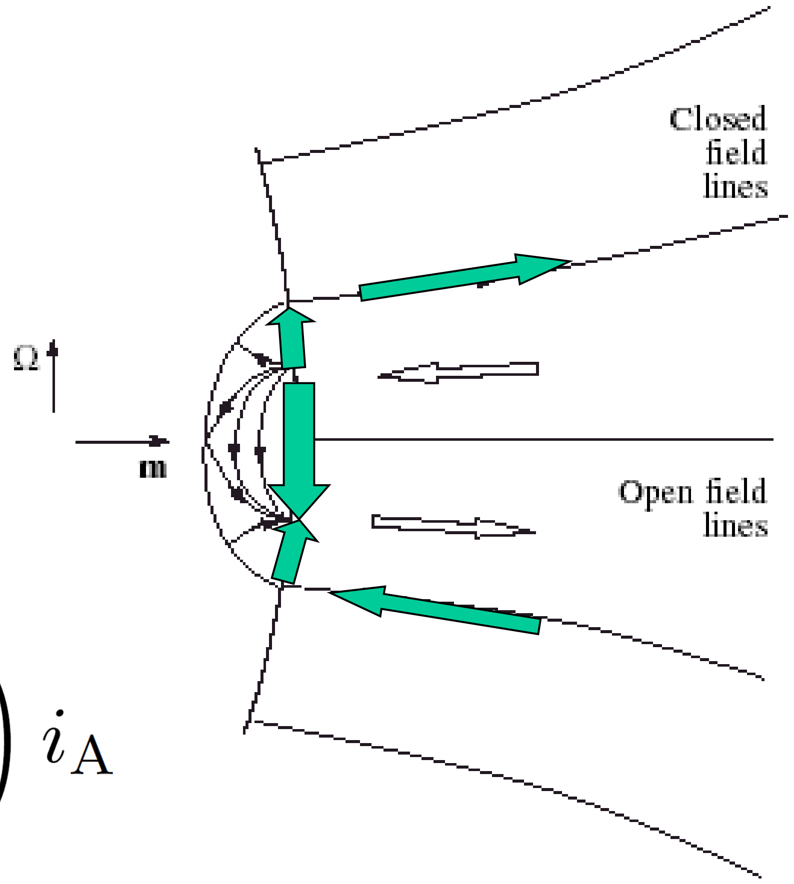
Orthogonal rotator – energy loss

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)

$$j_{\mathrm{GJ}} \approx \frac{\Omega B}{2\pi} \cos \theta$$

$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]] \mathrm{d}S$$

$$W_{\mathrm{tot}} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c} \right) i_{\mathrm{A}}$$

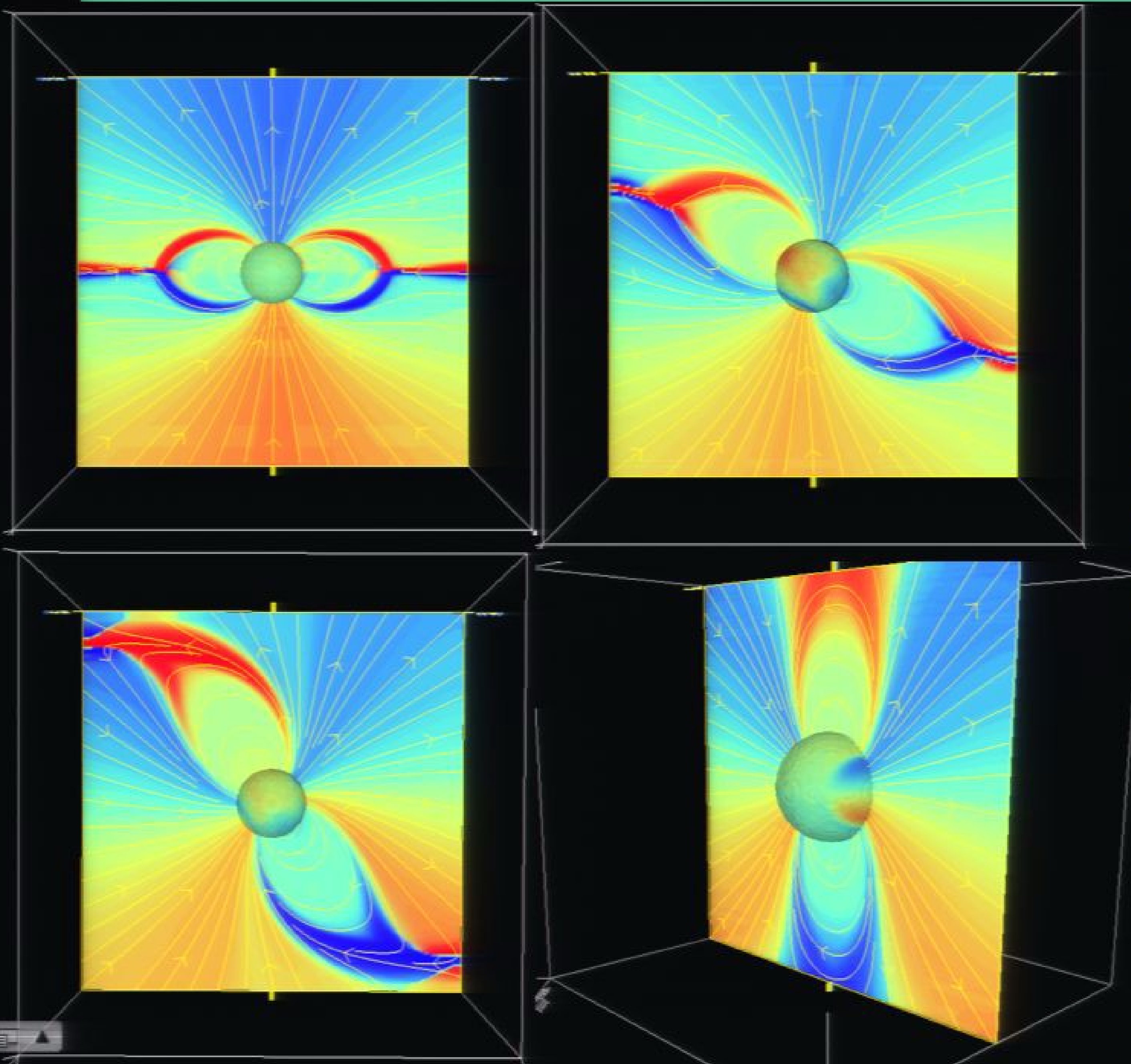


$i_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$

No contradiction!

In Spitkovsky solution the electric current is larger than local GJ one ($i_A >> 1$).

Magnetospheric currents



Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Magnetosphere

Determination of the current flowing in the magnetosphere

THE MAIN QUESTION UP TO NOW

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Radio Emission

Dielectric tensor in the inhomogeneous media

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin (1988,1993)

In our theory we have included into consideration the curvature of magnetic field lines

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_{zz} \end{pmatrix}$$

$$= \delta_{ij} - 2\pi \mathrm{i} \, \frac{R_\mathrm{c}^{2/3}}{k_\parallel^{1/3}} \int \mathrm{d}p_\varphi \, \frac{\omega_\mathrm{p}^2}{\omega} \, \frac{\partial f^{(0)}}{\partial p_\varphi} \begin{pmatrix} \dfrac{\mathcal{F}''(\zeta)}{(k_\parallel R_\mathrm{c})^{2/3}} & -\mathrm{i} \, \dfrac{\mathcal{F}'(\zeta)}{(k_\parallel R_\mathrm{c})^{1/3}} \\ \mathrm{i} \, \dfrac{\mathcal{F}'(\zeta)}{(k_\parallel R_\mathrm{c})^{1/3}} & \mathcal{F}(\zeta) \end{pmatrix}$$

$$\mathcal{F}(\zeta) = \mathrm{Ai}(\zeta) + \mathrm{iGi}(\zeta) = \frac{1}{\pi} \int_0^\infty \mathrm{d}\tau \, \exp\left(\mathrm{i}\tau\zeta + \mathrm{i} \, \frac{\tau^3}{3} \right)$$

$$\zeta = 2(\omega - k_\parallel v_\varphi) \, \frac{R_\mathrm{c}^{2/3}}{k_\parallel^{1/3} v_\varphi}$$

Curvature Instability of Relativistic Particle Beams

O. Larroche
Commissariat à l'Energie Atomique, Centre d'Etudes de Limeil–Valenton, 94190 Villeneuve–Saint Georges, France

and

R. Pellat
Centre de Physique Théorique, Ecole Polytechnique, 91128 Palaiseau Cedex, France
(Received 1 June 1987)

Let us notice that our non-WKB condition (14) is opposite to that found by Beskin, Gurevich, and Istomin[7]; we conclude that their computation of the dielectric tensor of a plasma in a strong curved magnetic field is wrong.

Diamond Jubilee Symposium on Pulsars
14–17 March 1994
Raman Research Institute, Bangalore

J. Astrophys. Astr. (1995) **16,** 137–164

The Models for Radio Emission from Pulsars – The Outstanding issues

D.B. Melrose *Research Centre for Theoretical Astrophysics, School of Physics, University of Sydney, NSW 2006, Australia*

Another (reactive) form of curvature-drift induced instability was proposed by Beskin, Gurevich & Istomin (1993), who considered the limiting case $B \to \infty$. The curvature drift speed (4.1) vanishes in this limit, and the nature of this instability is unclear. It has been claimed that this instability is spurious (Nambu 1989; Machabeli 1991).

JONATHAN ARONS
University of California, Berkeley

Physics Today, May, (1995)

Finally, they present their theory of pulsar radio emission based on their calculations of the properties of small-amplitude waves in the outflowing relativistic pair plasma. This theory is known to be incorrect; M. Nambu, G. Machabeli, and Q. Luo and D. B. Melrose have identified and published on several fatal flaws.

Comparative analysis of two formulations of geometrical optics. The effective dielectric tensor

M Bornatici† and Yu A Kravtsov‡§

† INFM, Department of Physics A. Volta, University of Pavia, Pavia I-27100, Italy
‡ Space Research Institute, Russian Academy of Sciences, Moscow 117810, Russia
§ Center of Space Researches, Polish Academy of Science, Warsaw 00716, Poland

Plasma Phys. Control Fusion, **42**, 255 (2000)

[19] Beskin V S, Gurevich A V and Istomin Ya N 1987 *Sov. Phys–JETP* **65** 715
[20] Istomin Ya N 1988 *Sov. Phys.–JETP* **68** 1380
[21] Istomin Ya N 1994 *Plasma Phys. Control Fusion* **36** 1081
[22] Nambu M 1989 *Plasma Phys. Control Fusion* **31** 143
[23] Machabeli G Z 1991 *Plasma Phys. Control Fusion* **33** 1227
Machabeli G Z 1995 *Plasma Phys. Control Fusion* **37** 177
[24] Nambu M 1996 *Phys. Plasmas* **3** 4325

$$\varepsilon_{ij}^{(\mathrm{eff})}(\boldsymbol{k},\omega;\boldsymbol{r},t) = \left[1 + \frac{\mathrm{i}}{2}\left(\frac{\partial^2}{\partial k_i \partial x_i} - \frac{\partial^2}{\partial \omega \partial t}\right)\right] \varepsilon_{ij}^{(0)}(\boldsymbol{k},\omega;\boldsymbol{r},t). \tag{A.8}$$

The derivation of result (A.8) relies on the comparison between the symmetrized and non-symmetrized constitutive relation, in accordance with previous work for the case of a stationary medium [19–21], for which the third term on the right-hand side of (A.8) is absent. On the other hand, the same result can be obtained from using only the non-symmetrized constitutive relation along with either the consistency condition for the first-order geometrical optics approximation [1,3], or the conservation of the wave energy [19]; the latter treatment being carried out for a stationary medium only.

The result (A.8) has caused some confusion and controversy in the literature [22,23], which has apparently been the result of not appreciating the distinction between symmetrized and non-symmetrized constitutive relations, on the one hand, and between the effective dielectric tensor and the plane-wave dielectric tensor, on the other hand. More explicitly, in [22] and [23] only the symmetrized form of the constitutive relation is considered, with the consequence that the dielectric tensor inherent in those approaches is the effective dielectric tensor, defined in (A.5*b*). In particular, in contrast with the claim of [23], result (A.8), which requires the non-symmetrized form of the constitutive relation to be considered along with the corresponding symmetrized form, cannot be obtained on the basis of the sole symmetrized constitutive relation.

The issue of the effective dielectric tensor (for a stationary medium) has recently been reconsidered [24], with the conclusion that result (A.8) is recognized to be correct, thus refuting somehow the earlier analysis [22] by the same author. On the other hand, in [24] the proper significance of the plane-wave dielectric tensor $\varepsilon_{ij}^{(0)}$, entering (A.8), is not yet fully appreciated.

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Magnetosphere

Screening of the magnetodipole radiation for zero longitudinal current

Determination of the current losses for arbitrary inclination angle

Determination of the current flowing in the magnetosphere

Radio Emission

Dielectric tensor in the inhomogeneous media

Instability of the curvature-plasma waves

Saturation due to nonlinear interaction of waves

Several private remarks

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(1983, 1984, 1988, 1993)

Magnetosphere

Screening of the magnetodipole radiation for zero longitudinal current

Determination of the current losses for arbitrary inclination angle

Determination of the current flowing in the magnetosphere

Radio Emission

Dielectric tensor in the inhomogeneous media

Instability of the curvature-plasma waves

Saturation due to nonlinear interaction of waves

But let’s return …

Theory of the Radio Emission

- Properties of the outgoing plasma

(consensus)

- Coherent mechanism

    Base instability

    Saturation (nonlinear effects)

(no common point of view)

- Propagation effects

(there was the missing link)

Theory of the Radio Emission

- Properties of the outgoing plasma

(consensus)

Concentration of the electon-positron plasma
$n = \lambda\, n\mathrm{GJ}$ (primary beam $n \sim n\mathrm{GJ}$)

Multiplicity parameter

$$\lambda \sim 10$$

Particle energy: beam – $\gamma \sim 10$, main flow

$$\gamma \sim 100$$

Ejection rate 10 pairs/s (Crab – 10 pairs/s)

‘Hollow cone’ – implicit assumptions

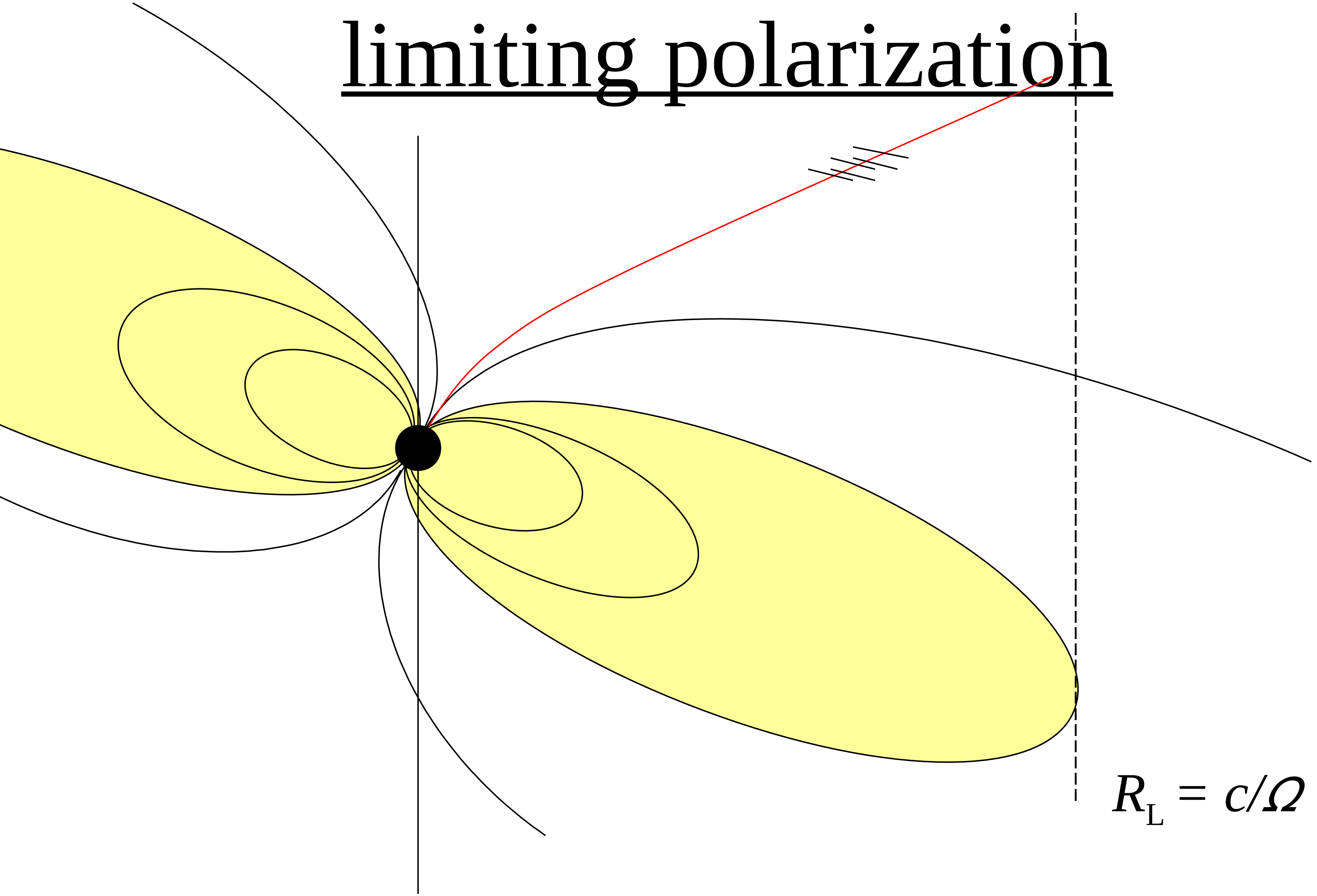
- Rectilinear propagation of radio waves
- Cyclotron absorption is not important
- Polarization is formed in the region of radiation

‘Hollow cone’ – implicit assumptions

- Rectilinear propagation of radio waves
- Cyclotron absorption is not important
- Polarization is formed in the region of radiation

All these points are incorrect

Refraction, cyclotron absorption, limiting polarization



Main parameters

$$\Delta n = -\frac{1}{2} < \frac{\omega_p^2 \omega_B^2}{\gamma^3 \varpi^2 (\omega_B^2 - \gamma^2 \varpi^2)} > \frac{\sqrt{q^2+1}}{q} \sin^2\theta$$

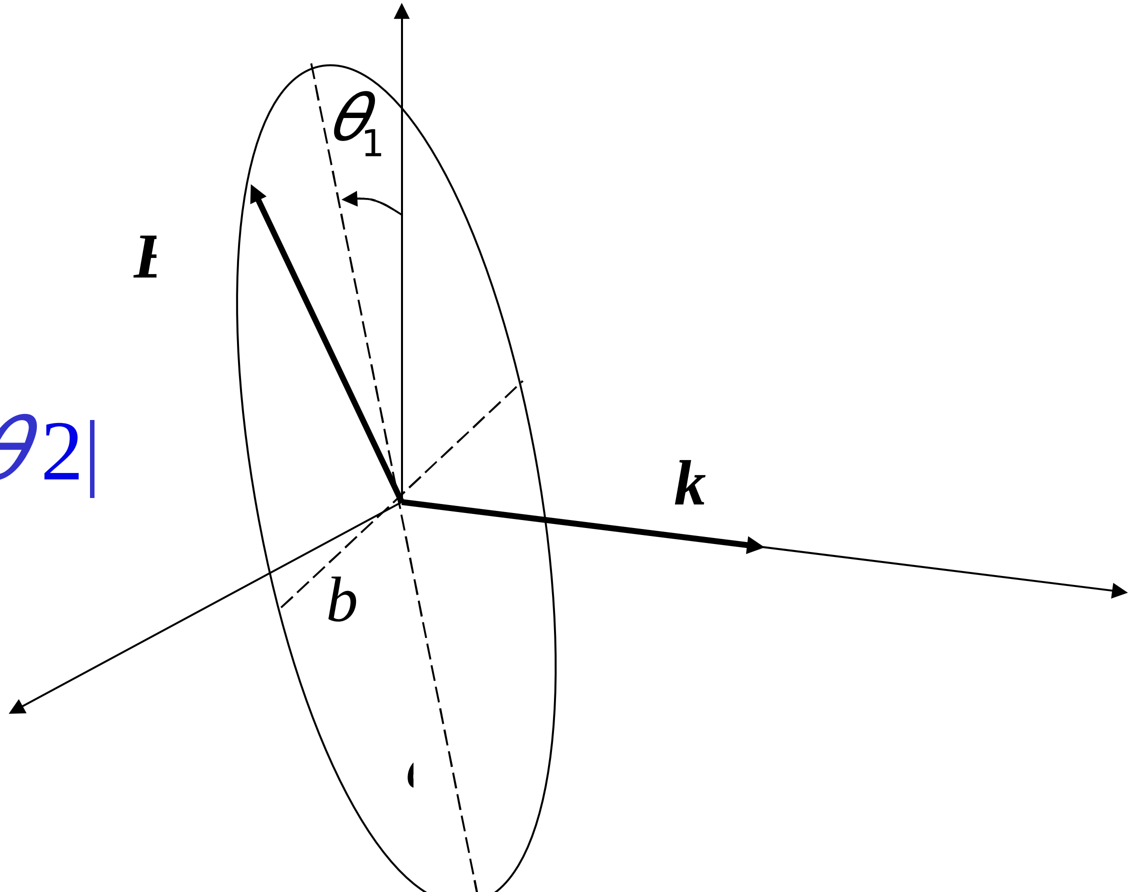
$$q = \frac{\omega_B \lambda \sin^2\theta}{2\omega\gamma^3 (1 - \cos\theta v_{\|}/c)^2 (\cos\theta - v_{\|}/c)}$$

$$K_i^{-1} = i\frac{E_x}{E_y} = q \pm \sqrt{1+q^2}$$

$q >> 1$ ($K = 2q$, $1/2q$) – linear polarization

$q << 1$ ($K = +1$, -1) – circular polarization

Polarization



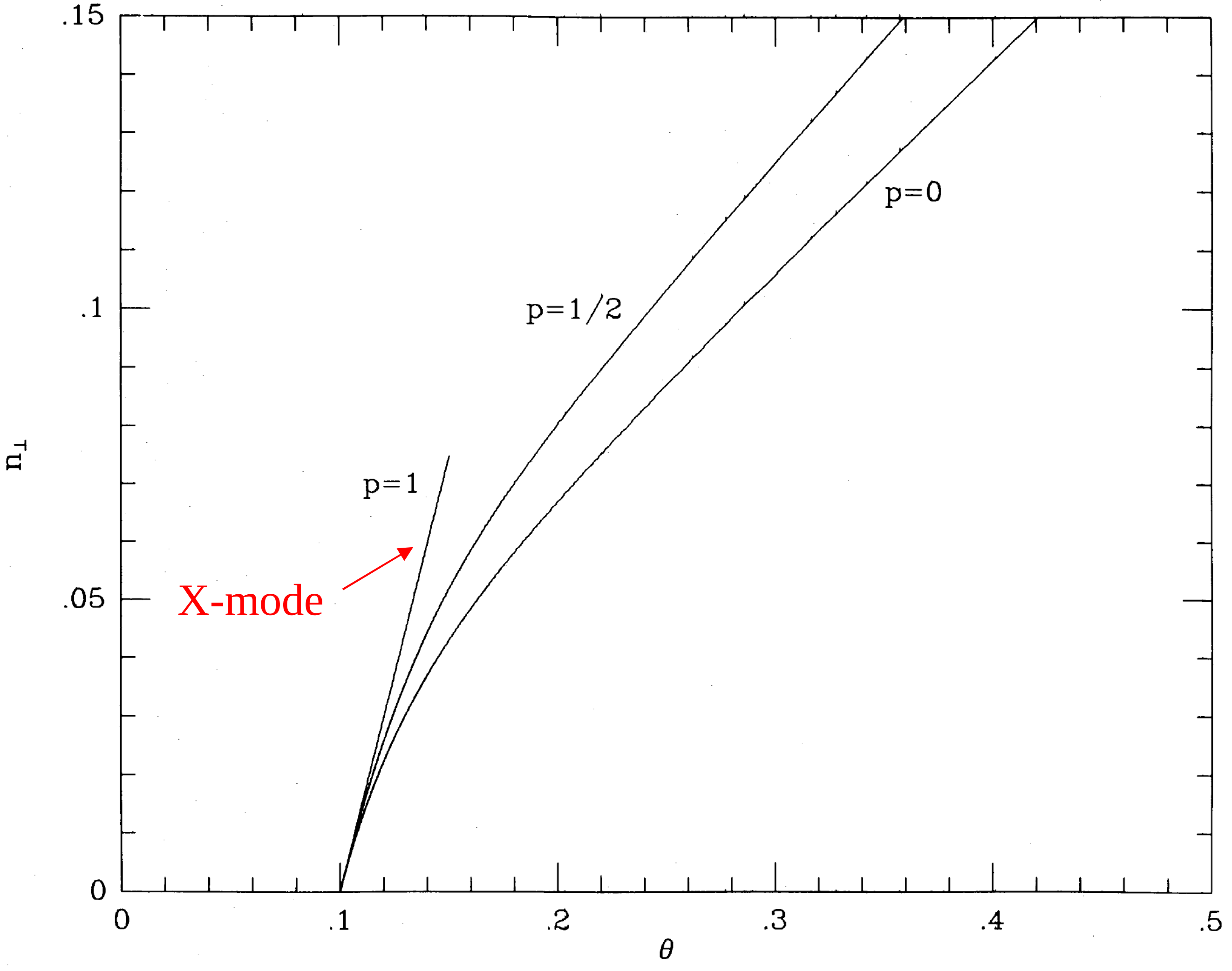
$K = a/b = |\tanh\theta 2|$

$V = I \tanh 2\theta 2$

Refraction

J.Arons, J.Barnard. ApJ, **302**, 120 (1986)

WAVE PROPAGATION IN PULSAR MAGNETOSPHERES



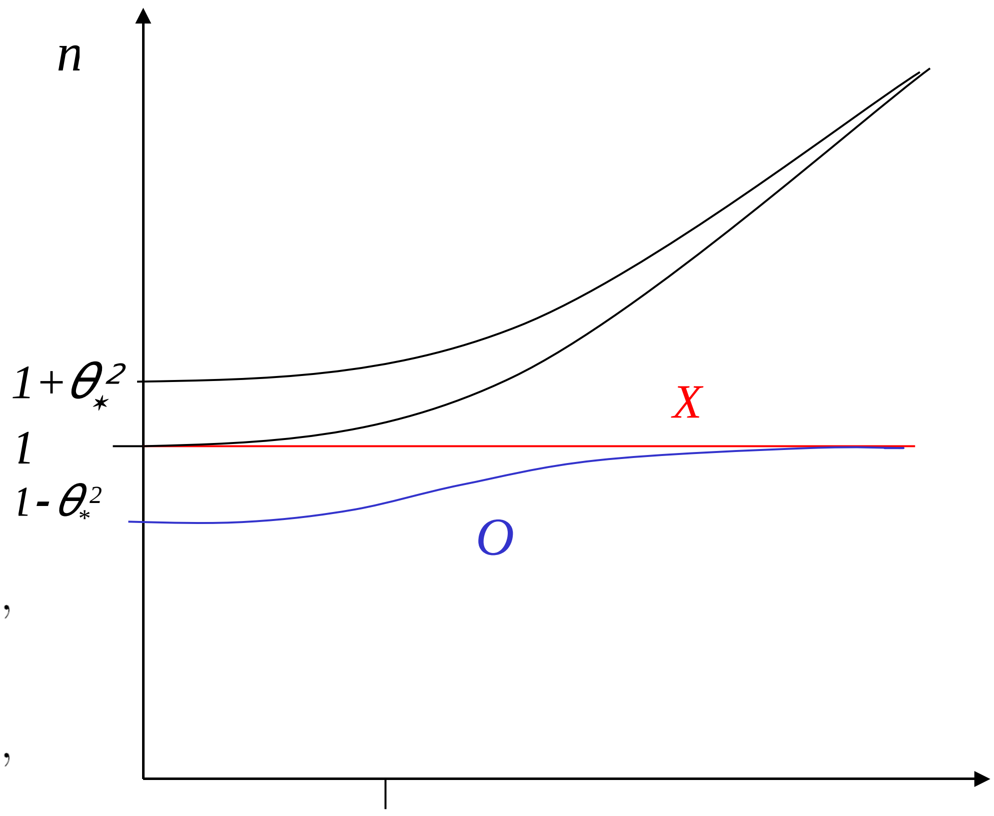
Four, not three waves!

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin, ApSS. **146,** 205 (1988)

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - < \frac{\omega_p^2}{\varpi^2 \gamma^3} > \end{pmatrix}$$

If $$A_{\rm p} = \frac{\omega_p^2}{\omega^2} < \gamma > \gg 1$$

$$\begin{aligned} n_1 &= 1, \\ n_2 &= 1 + \frac{\theta^2}{4} - \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2}, \\ n_3 &= 1 + \frac{\theta^2}{4} + \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2}, \\ n_4 &= \frac{1}{\cos \theta}. \end{aligned}$$



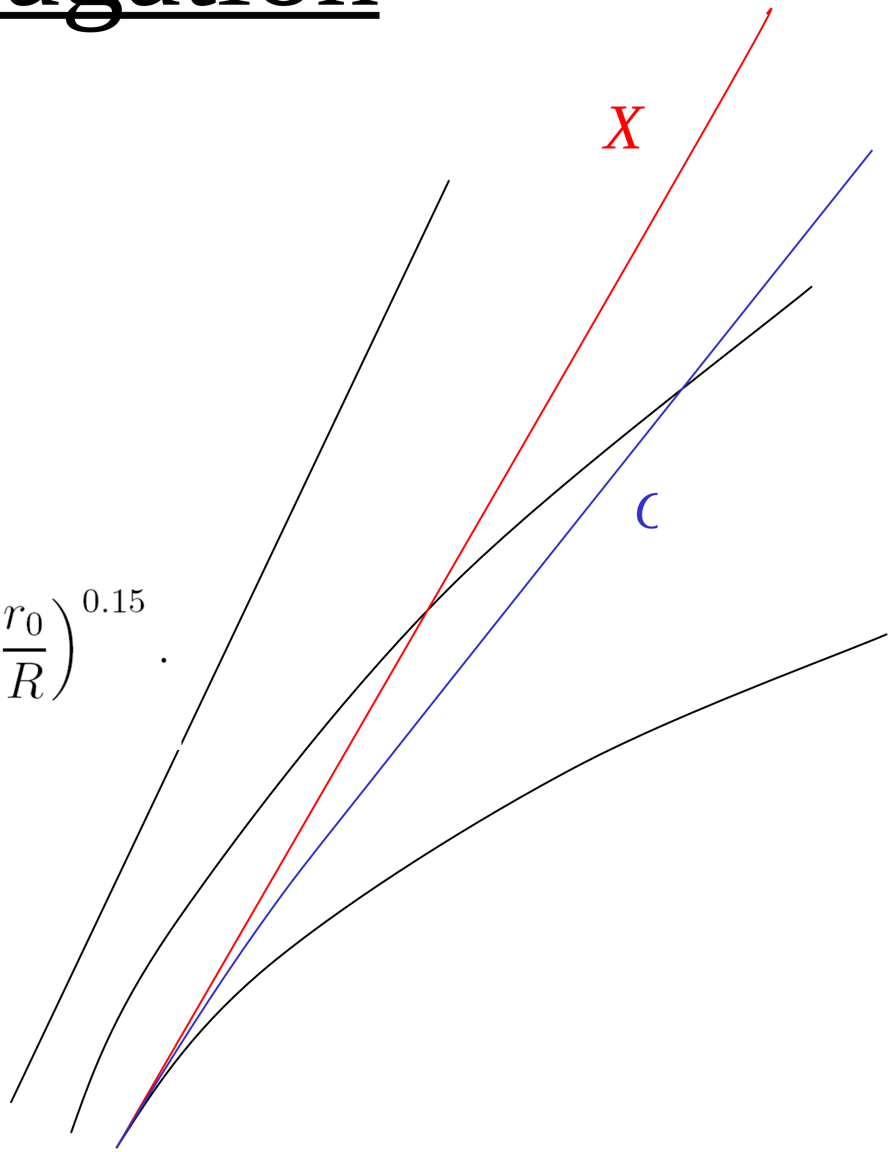
$$\theta_* = < \left(\frac{\omega_p^2}{\omega^2 \gamma^3} \right)^{1/4} >$$

Propagation

$$\frac{\mathrm{d}r_\perp}{\mathrm{d}l} = \frac{\partial}{\partial k_\perp}\left(\frac{k}{n_j}\right),$$

$$\frac{\mathrm{d}k_\perp}{\mathrm{d}l} = -\frac{\partial}{\partial r_\perp}\left(\frac{k}{n_j}\right)$$

$$W \approx \left(\frac{\Omega R}{c}\right)^{0.36}\left(\frac{\omega_{p0}}{\omega}\right)^{0.14} < \gamma^{-3} >^{0.07} \left(\frac{r_0}{R}\right)^{0.15}.$$



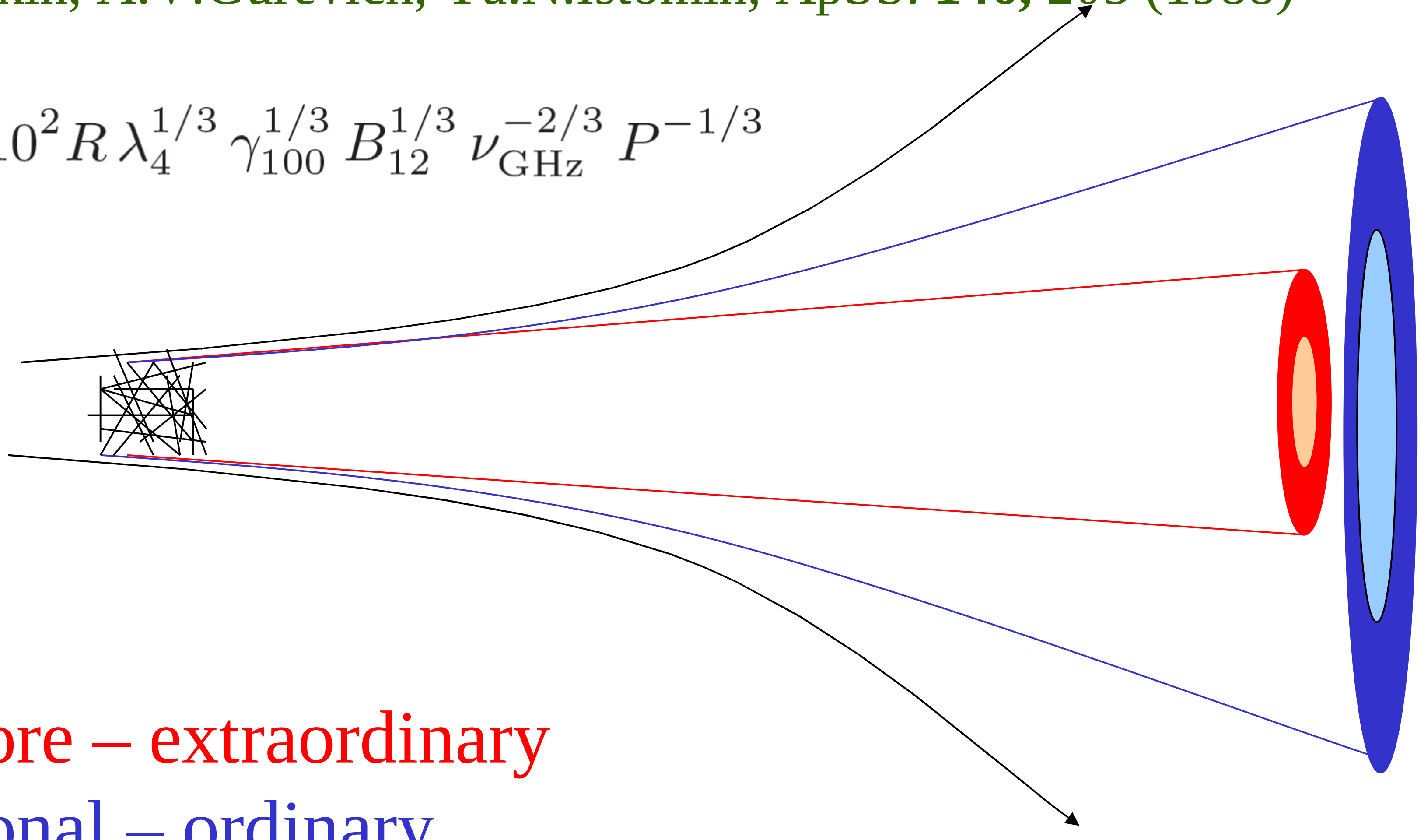
On can know $r_0(\nu)$

Yu. Lyubarsky, S.Petrova

Core & Conal

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin, ApSS. **146,** 205 (1988)

$$r_{\mathrm{A}} \approx 10^2 R\, \lambda_4^{1/3}\, \gamma_{100}^{1/3}\, B_{12}^{1/3}\, \nu_{\mathrm{GHz}}^{-2/3}\, P^{-1/3}$$



Core – extraordinary

Conal – ordinary

Cyclotron absorption

A.B.Mikhailovsky, O.G.Onishchenko, G.I.Suramlishvli, S.E.Sharapov. Sov. Astron. Lett., **8**, 685 (1982)

$$\varepsilon \approx 1 + \frac{\omega_p^2}{\omega^2} < \frac{\varpi}{(\omega_B - \gamma\varpi)} >$$

$$\mathrm{Im}\, k \approx -\ \pi \frac{\omega_p^2}{2\omega c} < \varpi\delta(\omega_B - \gamma\varpi) >$$

$$\kappa \approx \lambda(1 - \cos\theta_{\mathrm{res}})\frac{r_{\mathrm{res}}}{R_{\mathrm{L}}}$$

Cyclotron absorption

A.B.Mikhailovsky, O.G.Onishchenko, G.I.Suramlishvli, S.E.Sharapov. Sov. Astron. Lett., **8**, 685 (1982)

If $\lambda \sim 10$, then cyclotron absorption is too large…

Limiting polarization

Escaping into vacuum, where $\Delta n = 0$, and, hence, the geometric optics approximation becomes invalid, the polarizations of normal modes do not follow the orientation of the magnetic field in the picture plane.

$$c/\omega\Delta n > r$$

Limiting polarization

- **Four equtions** (four Stokes parameter)

V.N.Sazonov. Sov. Phys. JETP, **29**, 578 (1969)

S.A.Petrova, Yu.E.Lyubarskii. Astron. Ap., **355**, 1168 (2000)

A.E.Broderick, R.D.Blandford. ApJ, **718**, 1085 (2010)

Z.Wang, D.Lai, J.Han. MNRAS, **403**, 2 (2010)

R.V.Shcherbakov, L.Huang. MNRAS, **410**, 1052 (2011)

- **Budden equation**

V.V.Zheleznyakov. 1970, Radio Emission of the Sun and Planets, Pergamon, Oxford

- **Kravtsov-Orlov approach**

Yu.A.Kravtsov, Yu.I.Orlov. 1990, Geometrical Optics of Inhomogeneous Media, Springer, Berlin

Limiting polarization

Location of the region where the polarization of the outgoing radiation is formed

e.g., A.F.Cheng, M.A.Ruderman. ApJ, **229**, 348, (1979)
J.J.Barnard. Ap. J., **303**, 280 (1986)
Z.Wang, D.Lai, J.Han. MNRAS, **403**, 2 (2000)

$$r_{\rm esc} \approx 10^3 R\, \lambda_4^{2/5}\, \gamma_{100}^{-6/5}\, B_{12}^{2/5}\, \nu_{\rm GHz}^{-2/5}\, P^{-1/5}$$

q ~ 10 - 100,

K ~ 1 - 10 %

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov (1980)

$$\varepsilon_{ij} = \varepsilon\delta_{ij} + \chi_{ij}$$

$$\frac{\mathrm{d}\Theta}{\mathrm{d}\sigma} = \kappa + \frac{i\omega}{4c}[(\chi_{b\nu} - \chi_{\nu b}) + (\chi_{b\nu} + \chi_{\nu b})\cos 2\Theta - (\chi_{\nu\nu} - \chi_{bb})\sin 2\Theta]$$

$$\Theta = \theta_1 + i\theta_2$$

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta nq}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{\mathrm{d}\theta_2}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta nq}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{\mathrm{d}\theta_2}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Gives the direct information about the polarization of outgoing radiation

$$\theta_1 = \beta, \quad \beta + \pi/2,$$

$$\mathrm{sh}2\theta_2 = \pm\frac{1}{q}, \quad |\mathrm{th}\theta_2| = K.$$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Ordinary wave – $\theta_1 = \beta(r)$

Extraordinary wave – $\theta_1 = \beta(r) + \pi/2$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

If the shear of the magnetic field is large, and $\omega_B > \gamma\varpi$

$$\theta_2 \approx -\frac{1}{2|q|}\cdot\frac{d\beta/dx}{|v_{\|}/c - \cos\theta|}\cos[2\theta_1 - 2\beta(r)], \qquad x = \Omega r/c$$

The sign of the circular polarization is determined by $d\beta/dx$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

The Key point

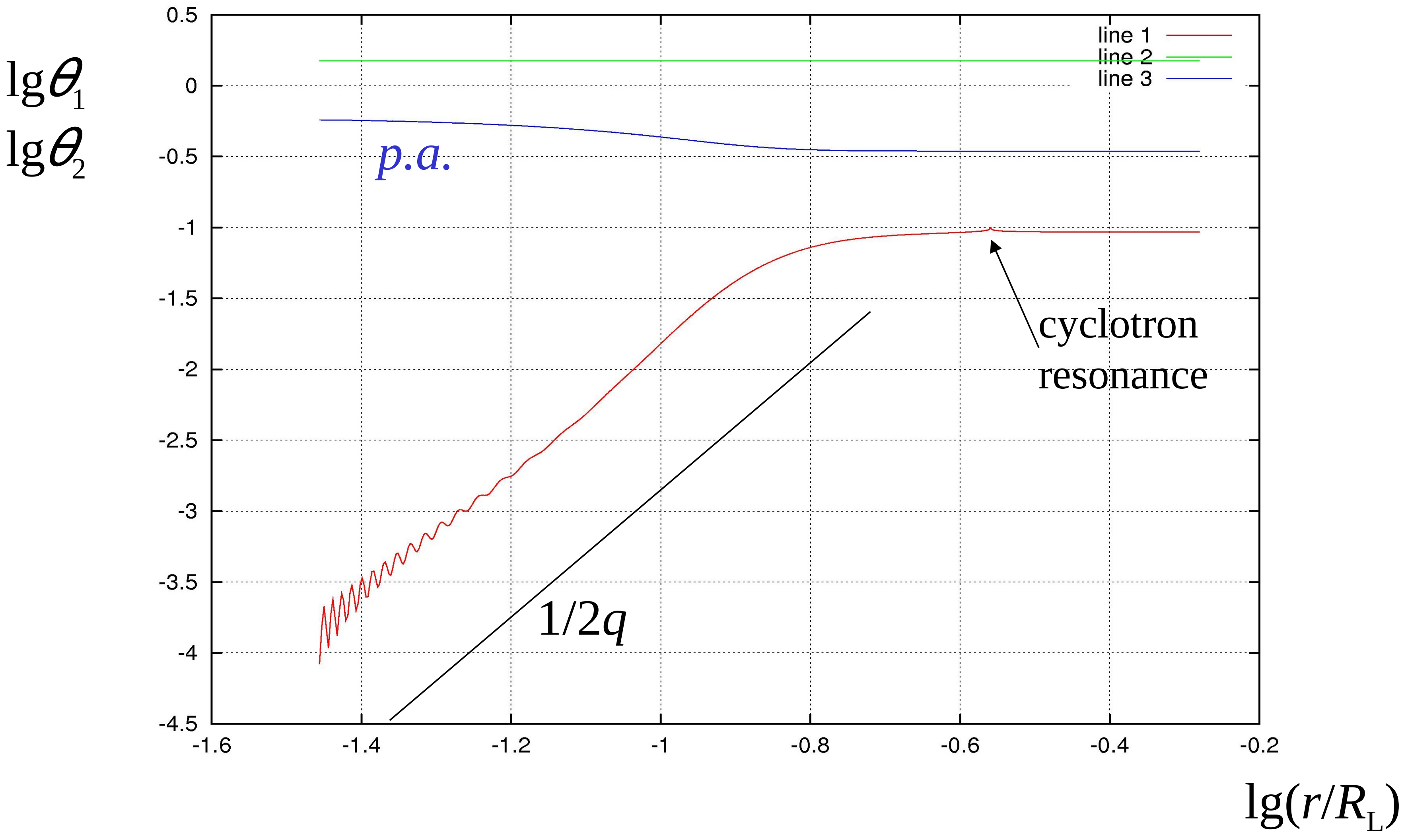
If the shear of the magnetic field is large, and $\omega_B > \gamma\varpi$

$$\theta_2 \approx -\frac{1}{2|q|} \cdot \frac{\mathrm{d}\beta/\mathrm{d}x}{|v_{\parallel}/c - \cos\theta|} \cos[2\theta_1 - 2\beta(r)], \qquad x = \Omega r/c$$

The sign of the circular polarization is determined by $\mathrm{d}\beta/\mathrm{d}x$

P = 1.5 s, B0 = 0.6 10   G, ν = 1 GHz,

*r*0 = 10R, γ= 100, λ = 5 10 , χ =45 , ξ = 47 , *r*in=0.5



Two main theoretical results

- For originally fully polarized mode it is enough to solve TWO equations, not FOUR.
- For high enough shear of the external magnetic field the circular polarization is given by well-determined diagonal components of the dielectric tensor.
- As a result, we can recognize the mode!

Main result

- For ordinary wave (conal) the signs $\mathrm{d}p.a./\mathrm{d}\phi$ and V are to be opposite, and for the extraordinary wave (core) are to be the same.
- This property depends neither on the sign $\boldsymbol{\Omega m}$, nor on the pole of the neutron star.

Core & Conal

Profile	O_S	O_D	X_S	X_D
Number	6	23	45	6
$\sqrt{P}W_{50}$	6.8 ± 3.1	10.7 ± 4.5	6.5 ± 2.9	5.3 ± 3.0

P.Weltevrede, S.Johnston. MNRAS, **391**, 1210 (2008)
T.Hankins, J.Rankin. Astron. J., **139**, 168 (2010)

First step

A.S.Andrianov, V.S.Beskin. Astron. Lett. **36**, 248 (2010)

- Simple model of the dielectric tensor
- Simple model of magnetic field
- No refraction
- No electric drift

At present

We include into consideration

- The electric drift
- Arbitrary magnetic field structure
- Arbitrary number density cross section
- Arbitrary particle energy distribution function

Equations – no drift

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{\mathrm{d}\theta_2}{\mathrm{d}r} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Equations with drift

$$\frac{\mathrm{d}\Theta_1}{\mathrm{d}l} = \frac{\omega}{2c}\mathrm{Im}[\varepsilon_{x'y'}] - \frac{1}{2}\frac{\omega}{c}\Lambda\cos[2\Theta_1 - 2\beta(l) - 2\delta(l)]\sinh 2\Theta_2,$$

$$\frac{\mathrm{d}\Theta_2}{\mathrm{d}l} = \frac{1}{2}\frac{\omega}{c}\Lambda\sin[2\Theta_1 - 2\beta(l) - 2\delta(l)]\cosh 2\Theta_2.$$

$$\Lambda = \mp\sqrt{(\mathrm{Re}[\varepsilon_{x'y'}])^2 + \left(\frac{\varepsilon_{x'x'} - \varepsilon_{y'y'}}{2}\right)^2}$$

$$\tan(2\delta) = -\frac{2\mathrm{Re}[\varepsilon_{x'y'}]}{\varepsilon_{y'y'} - \varepsilon_{x'x'}}$$

Dielectric tensor – no drift

$$\varepsilon_{ij} = \begin{pmatrix} 1+ < \frac{\omega_p^2 \varpi^2 \gamma}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & i < \frac{\omega_p^2 \omega_B \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & < \frac{\omega_p^2 \gamma k_x v_\parallel \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \\ -i < \frac{\omega_p^2 \omega_B \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & 1+ < \frac{\omega_p^2 \varpi^2 \gamma}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & -i < \frac{\omega_p^2 \omega_B k_x v_\parallel}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \\ < \frac{\omega_p^2 \gamma k_x v_\parallel \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & i < \frac{\omega_p^2 \omega_B k_x v_\parallel}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & 1- < \frac{\omega_p^2}{\varpi^2 \gamma^3} > + < \frac{\omega_p^2 \gamma k_x^2 v_\parallel^2}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \end{pmatrix}$$

$$\varpi = \omega - k_x v_\parallel$$

Dielectric tensor with drift

$$\varepsilon_{xx} = 1 - < \frac{k_z^2 U_x^2 \omega_p^2}{\tilde{\omega}^2 \gamma^3 \omega^2 (1 - U^2/c^2)} > + < \frac{\omega_{\mathrm{p}}^2 (\tilde{\omega}_0^2 + \frac{U_x^2}{c^2} (\frac{(k_z v_\parallel)^2}{1 - U^2/c^2} - \omega^2)) \gamma}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} >,$$

$$\varepsilon_{xy} = - < \frac{k_z^2 U_x U_y \omega_p^2}{\tilde{\omega}^2 \gamma^3 \omega^2 (1 - U^2/c^2)} > + i < \frac{\omega_{\mathrm{p}}^2 \omega_B (\tilde{\omega}_0 - \omega U^2/c^2)}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} > + < \frac{\omega_{\mathrm{p}}^2 (\tilde{\omega}_0 k_x U_y + \frac{U_x U_y}{c^2} (\frac{(k_z v_\parallel)^2}{1 - U^2/c^2} - \omega^2)) \gamma}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} >,$$

$$\varepsilon_{xz} = - < \frac{k_z U_x \omega_p^2 (\omega - k_x U_x)}{\tilde{\omega}^2 \gamma^3 \omega^2 (1 - U^2/c^2)} > + < \frac{\omega_{\mathrm{p}}^2 ((\tilde{\omega}_0 - \omega U^2/c^2) v_\parallel (k_x - \omega U_x/c^2) + \frac{U_y^2}{c^2} k_z k_x v_\parallel^2) \gamma}{(1 - U^2/c^2) \omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} > - i < \frac{\omega_B \omega_p^2}{\omega (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} \frac{U_x v_\parallel}{c^2} >,$$

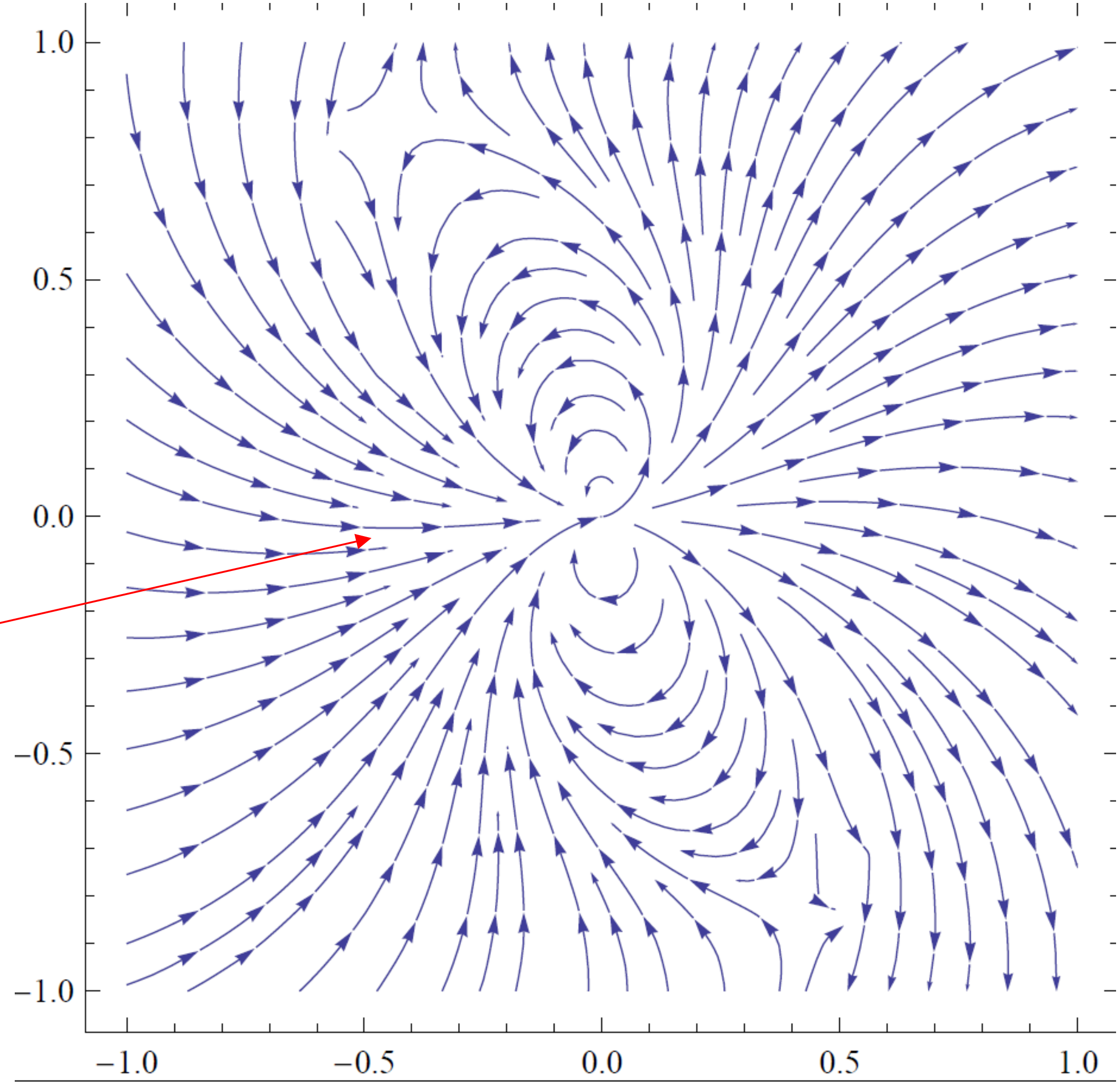
$$\varepsilon_{yy} = 1 - < \frac{k_z^2 U_y^2 \omega_p^2}{\tilde{\omega}^2 \gamma^3 \omega^2 (1 - U^2/c^2)} > + < \frac{\omega_{\mathrm{p}}^2 (\tilde{\omega}^2 + \frac{U_y^2}{c^2} (\frac{(k_z v_\parallel)^2}{1 - U^2/c^2} - \omega^2) + (k_x^2 U_y^2)) \gamma}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} >,$$

$$\varepsilon_{zz} = 1 - < \frac{\omega_p^2 (1 - \frac{k_x U_x}{\omega})^2}{\tilde{\omega}^2 \gamma^3 (1 - U^2/c^2)} > + < \frac{\omega_{\mathrm{p}}^2 ((k_x c - \omega U_x/c)^2 + \frac{U_y^2}{c^2} (\omega^2 - (k_x c)^2)) \gamma}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} \frac{v_\parallel^2/c^2}{1 - U^2/c^2} >,$$

$$\varepsilon_{yz} = - < \frac{k_z U_y \omega_p^2 (\omega - k_x U_x)}{\tilde{\omega}^2 \gamma^3 \omega^2 (1 - U^2/c^2)} > + < \frac{\omega_{\mathrm{p}}^2 ((k_x^2 c^2 - \omega^2)(1 - U^2/c^2) + k_z v_\parallel (\omega - k_x U_x)) \gamma}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} \frac{U_y v_\parallel / c^2}{1 - U^2/c^2} > - i <$$
$$\frac{\omega_B \omega_p^2}{\omega^2 (\omega_B^2 (1 - U^2/c^2) - \gamma^2 \tilde{\omega}^2)} v_\parallel (k_x - \frac{\omega U_x}{c^2}) >,$$

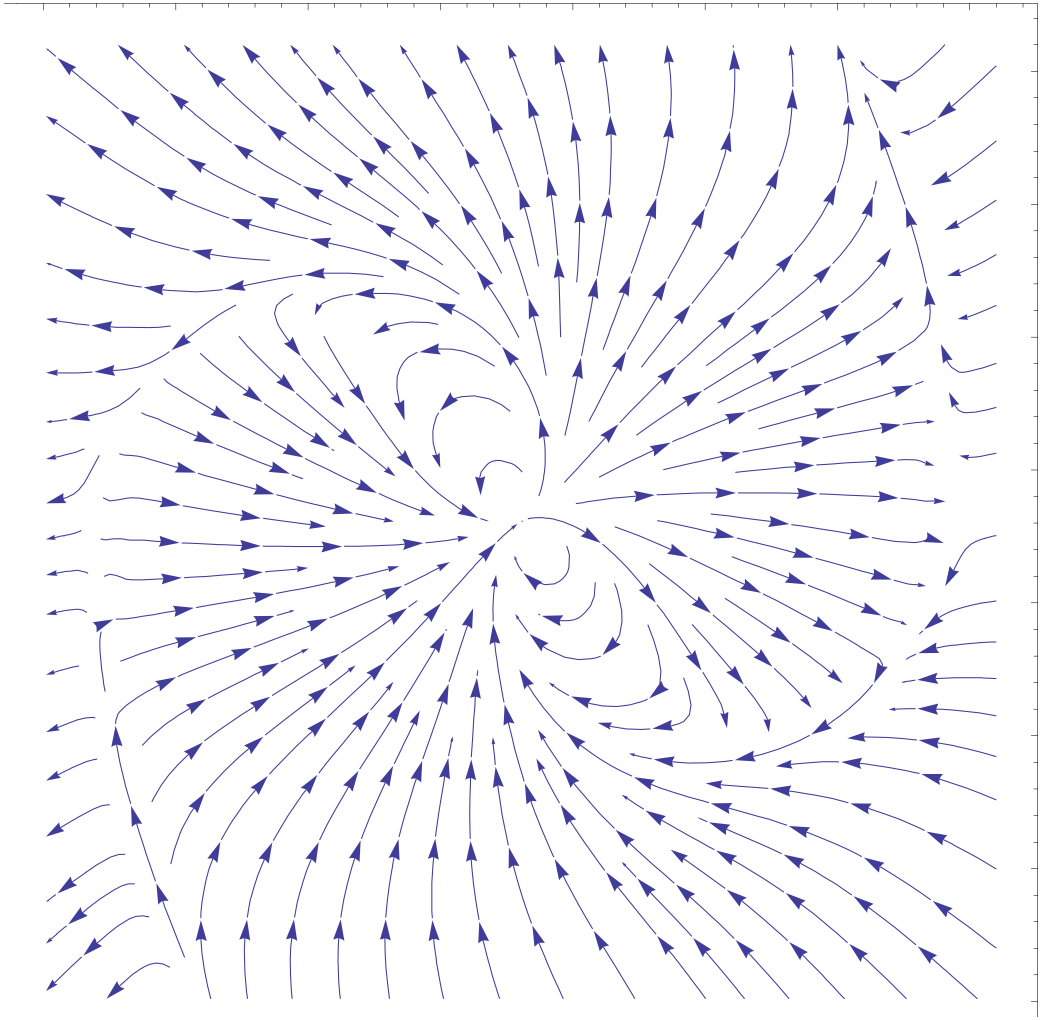
Magnetic field structure

Rotating dipole
(with or without
radiative terms)
+
Michel monopole



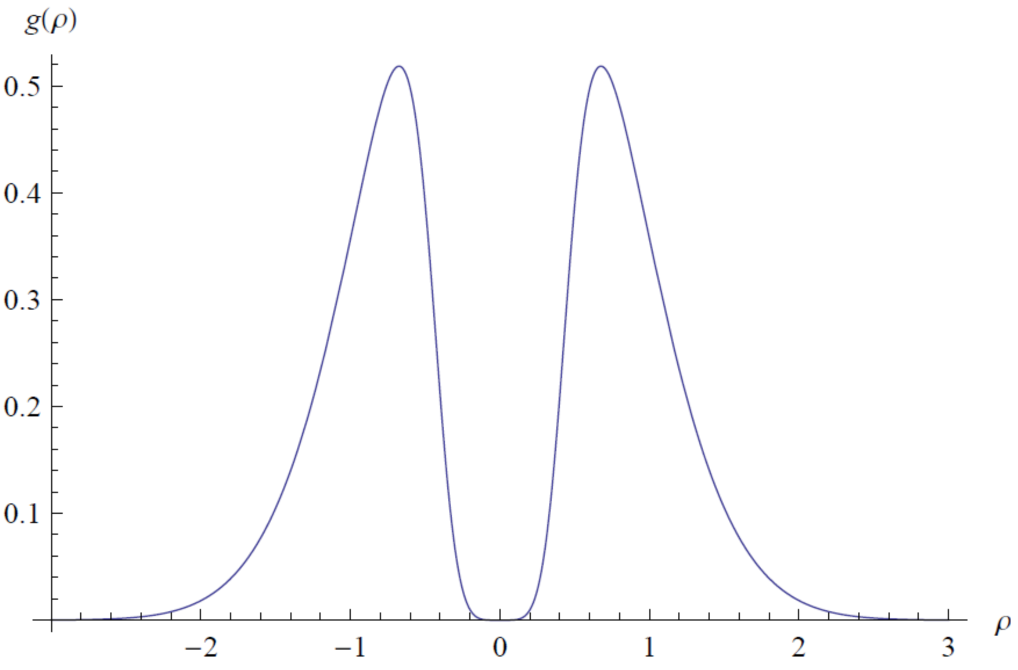
Here $\chi = 90$,
no radiative terms

Spitkovsky, poloidal field, $\chi = 60$



Plasma profile

Arbitrary 2D profile



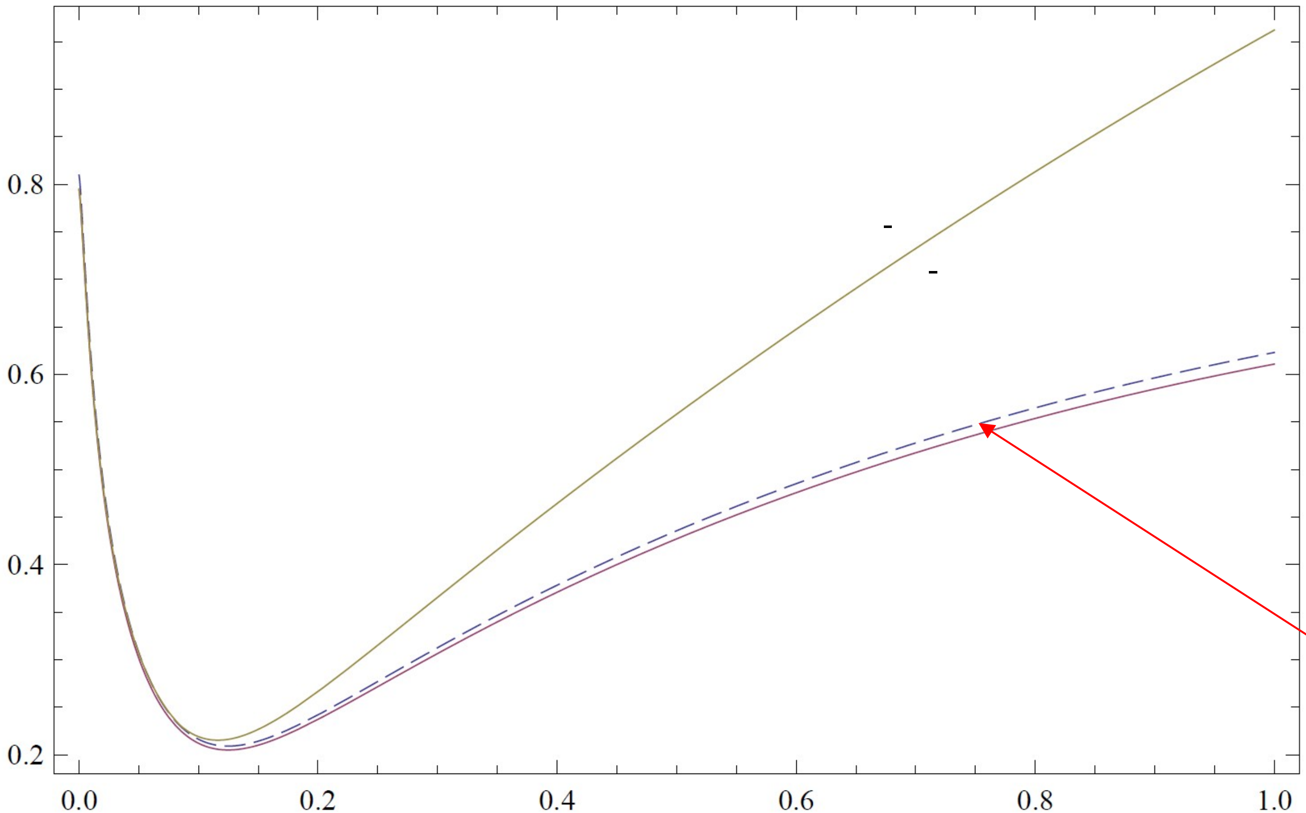
$$n_{\mathrm{e}}(\theta_{\mathrm{m}}, \varphi_{\mathrm{m}}) = \lambda g(\theta_{\mathrm{m}}, \varphi_{\mathrm{m}}) n_{\mathrm{GJ}}$$

Back integration

ρ/R_0

$r0 = 30R$, $\chi = 45$, $\xi = 48$, $\psi = 5$

"rotating dipole"

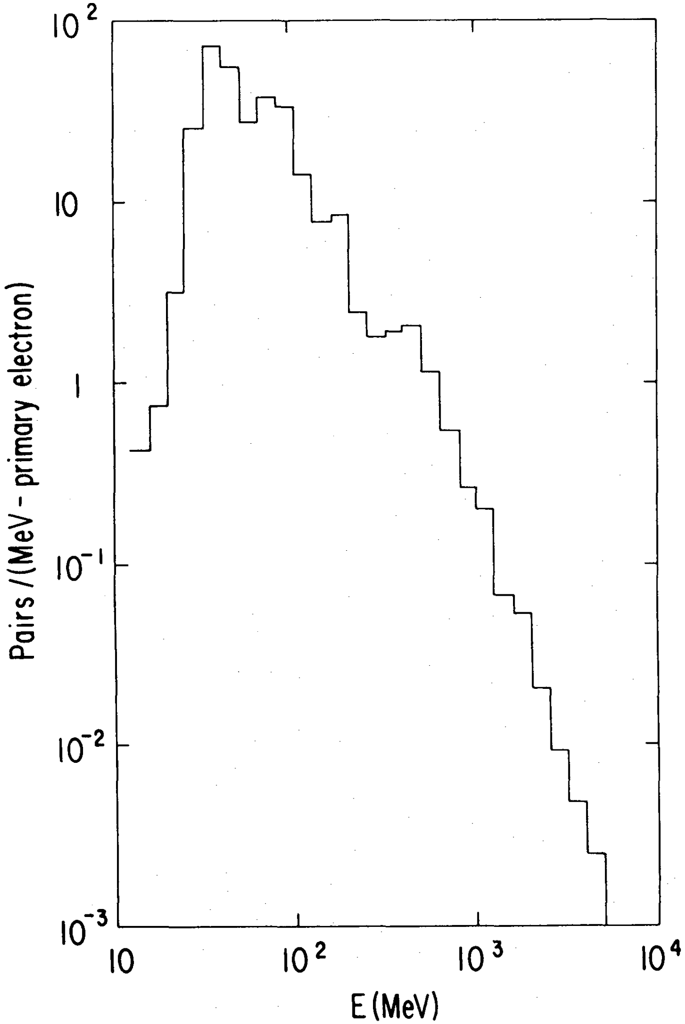


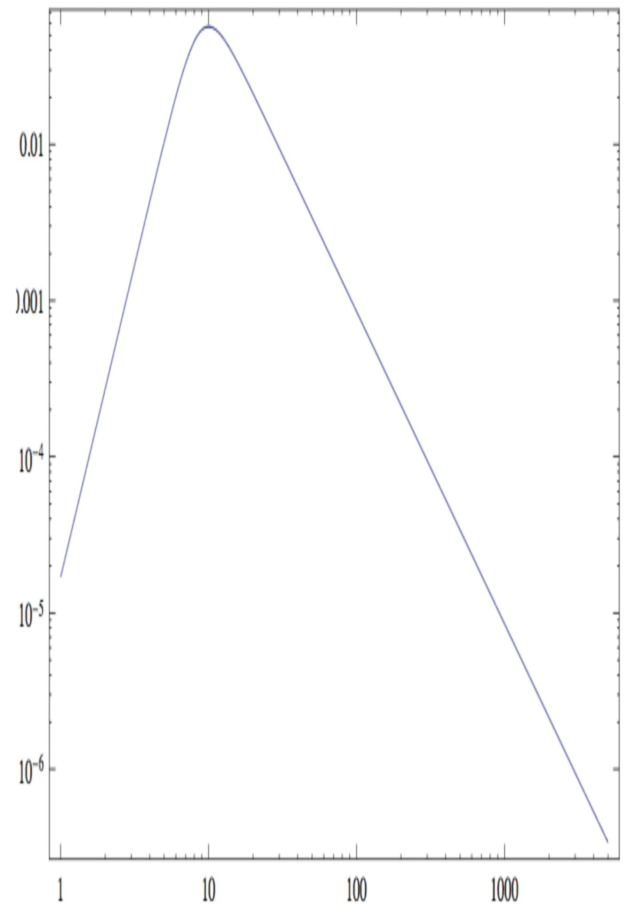
r/R_L

Good agreement between numerical simulation and analytic result!

$$\rho = \frac{1}{R_0/R}\frac{\sin\theta_m}{\sqrt{r/R}}$$

Energy distribution function





J. K. Daugherty, A. K.Harding.
ApJ, **252,** 337 (1982)

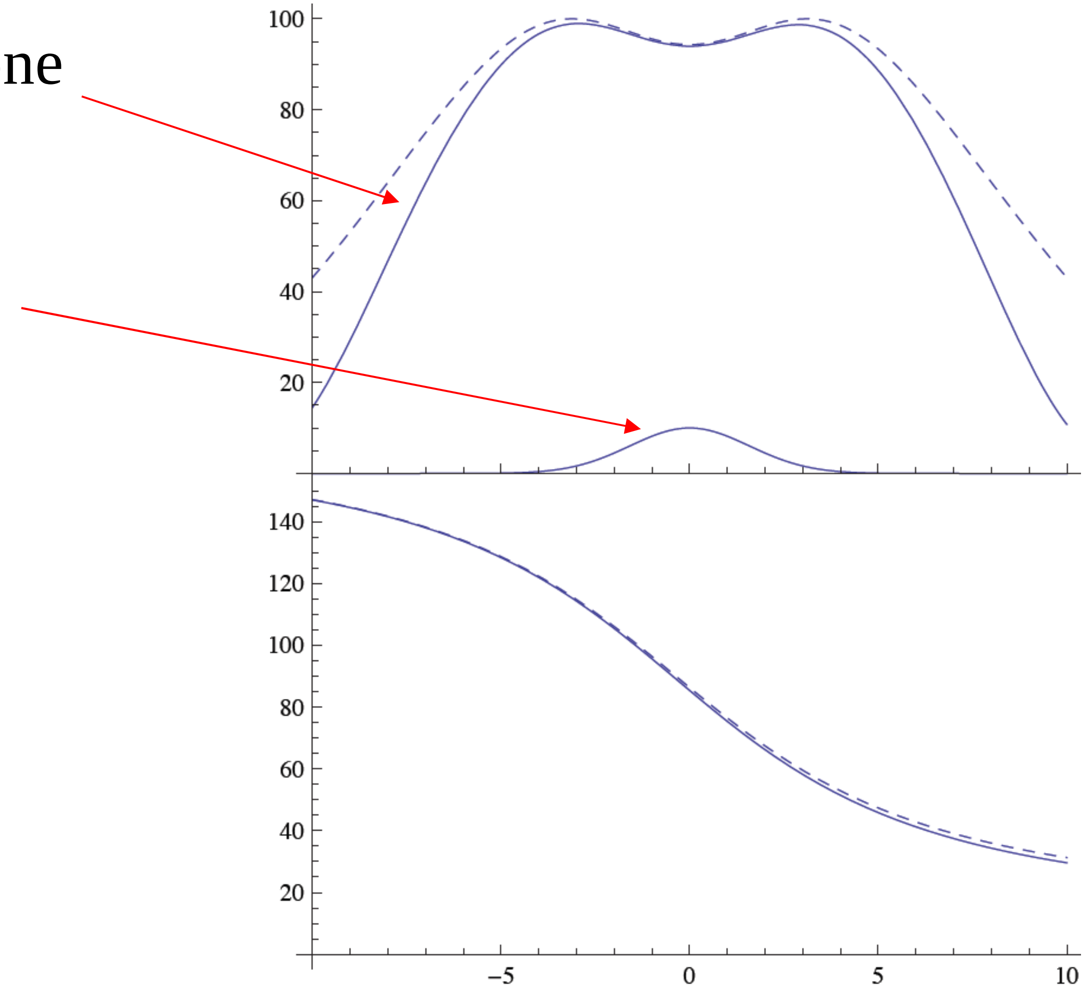
Cyclotron absorption

$$\begin{aligned}\tau &\approx \frac{4\pi^2 e^2}{m_{\mathrm{e}} c} \int\limits_0^\infty \int\limits_0^\infty n_{\mathrm{e}}(l) \frac{\tilde{\omega}}{\omega} f(\gamma) \delta \left(|\omega_B| \sqrt{1 - \frac{U^2}{c^2}} - \gamma \tilde{\omega} \right) \mathrm{d}\gamma \mathrm{d}l \\ &= \frac{4\pi^2 e^2}{mc} \int\limits_0^\infty n(l) \frac{1}{\omega} f \left(\frac{|\omega_B| \sqrt{1 - U^2/c^2}}{\tilde{\omega}} \right) \mathrm{d}l.\end{aligned}$$

Cyclotron absorption

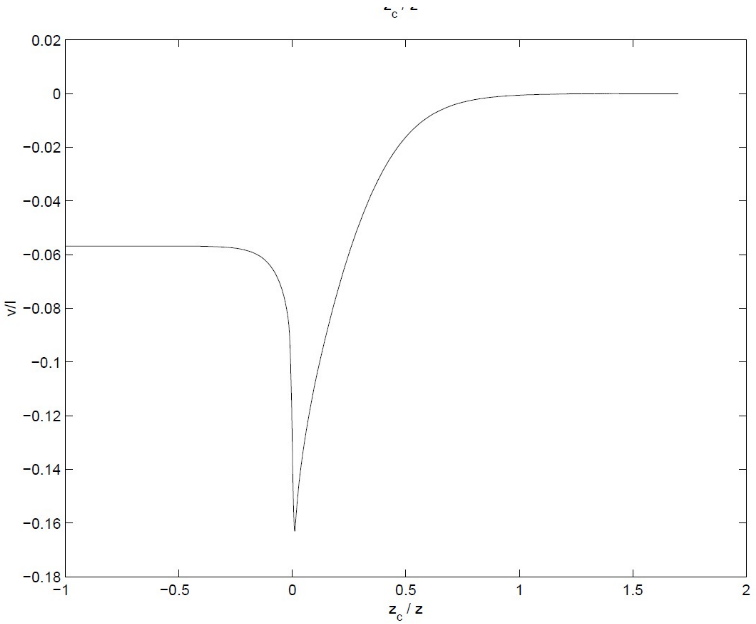
Hollow cone

No hole

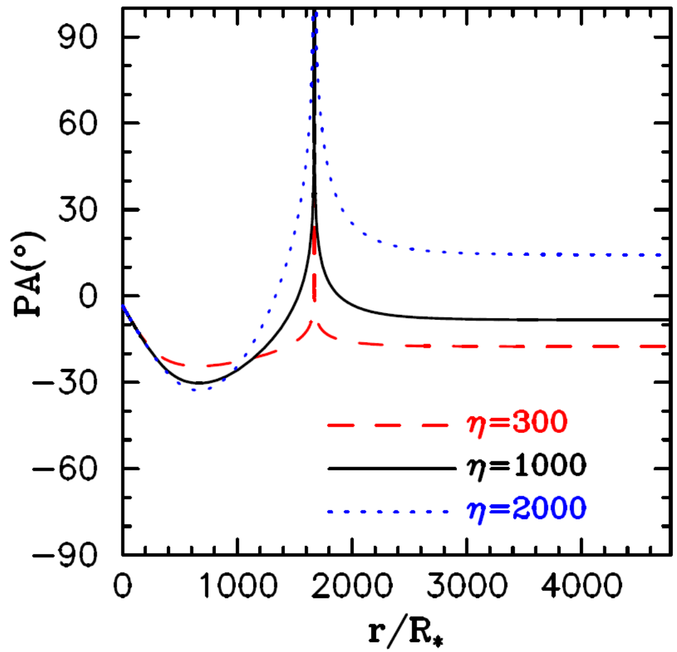


Through the cyclotron resonance

V/I



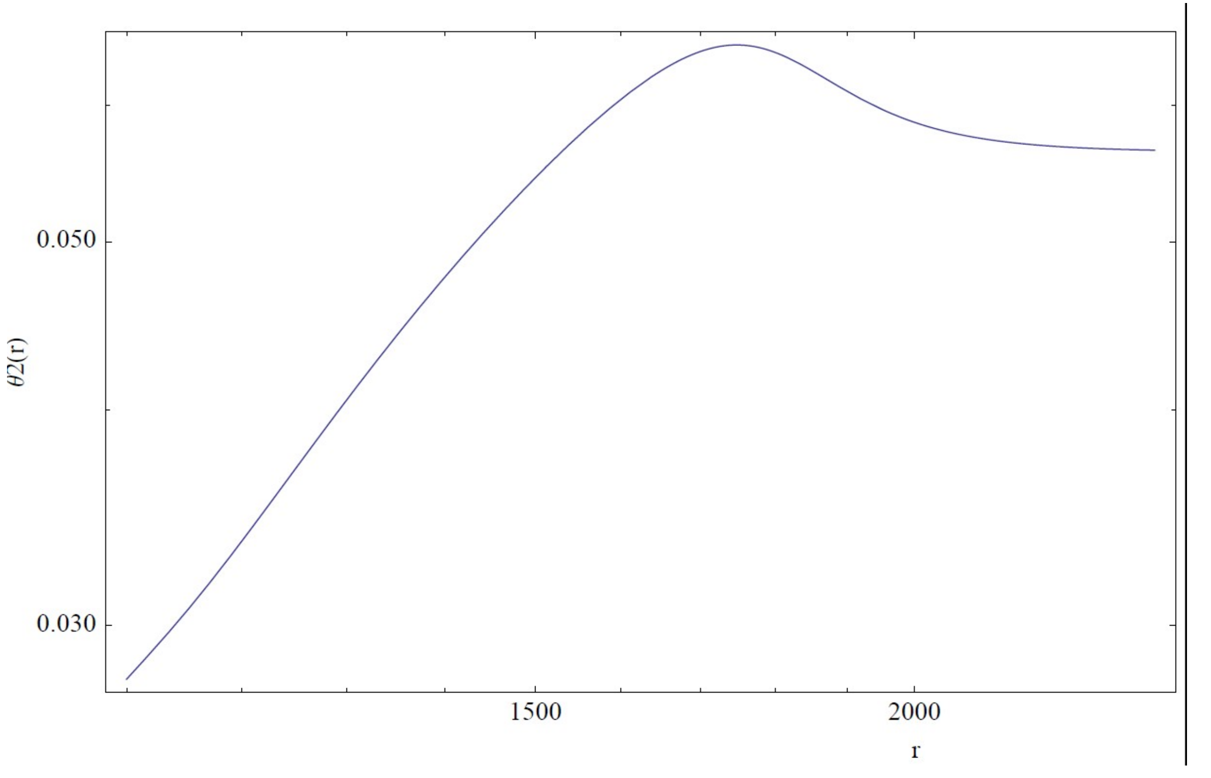
S.Petrova. MNRAS, **366**, 1539 (2006)



C.Wang, J.Han, D.Lai arXiv 1105.2602 (2011)

Through the cyclotron resonance

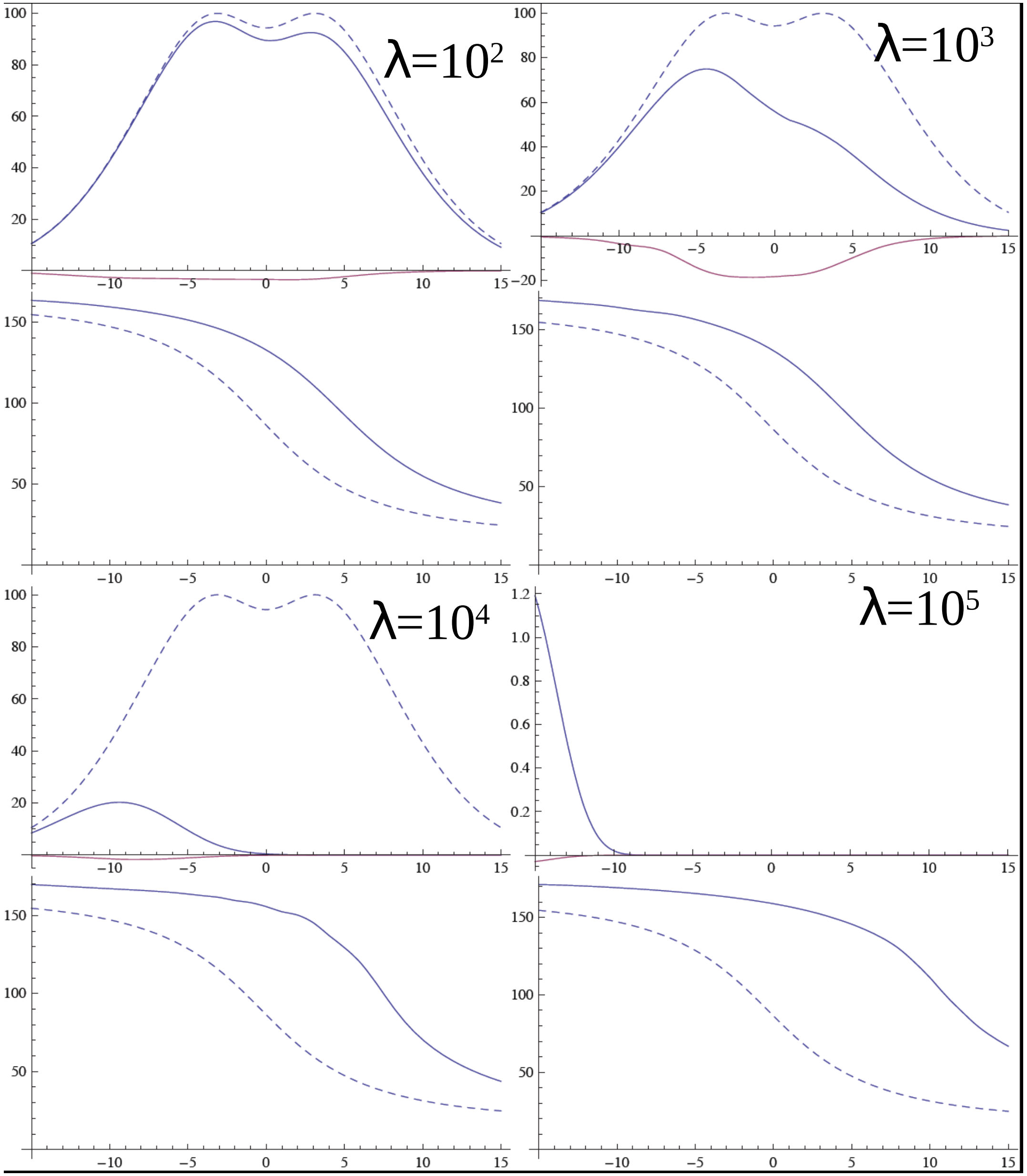
For real wide distribution function



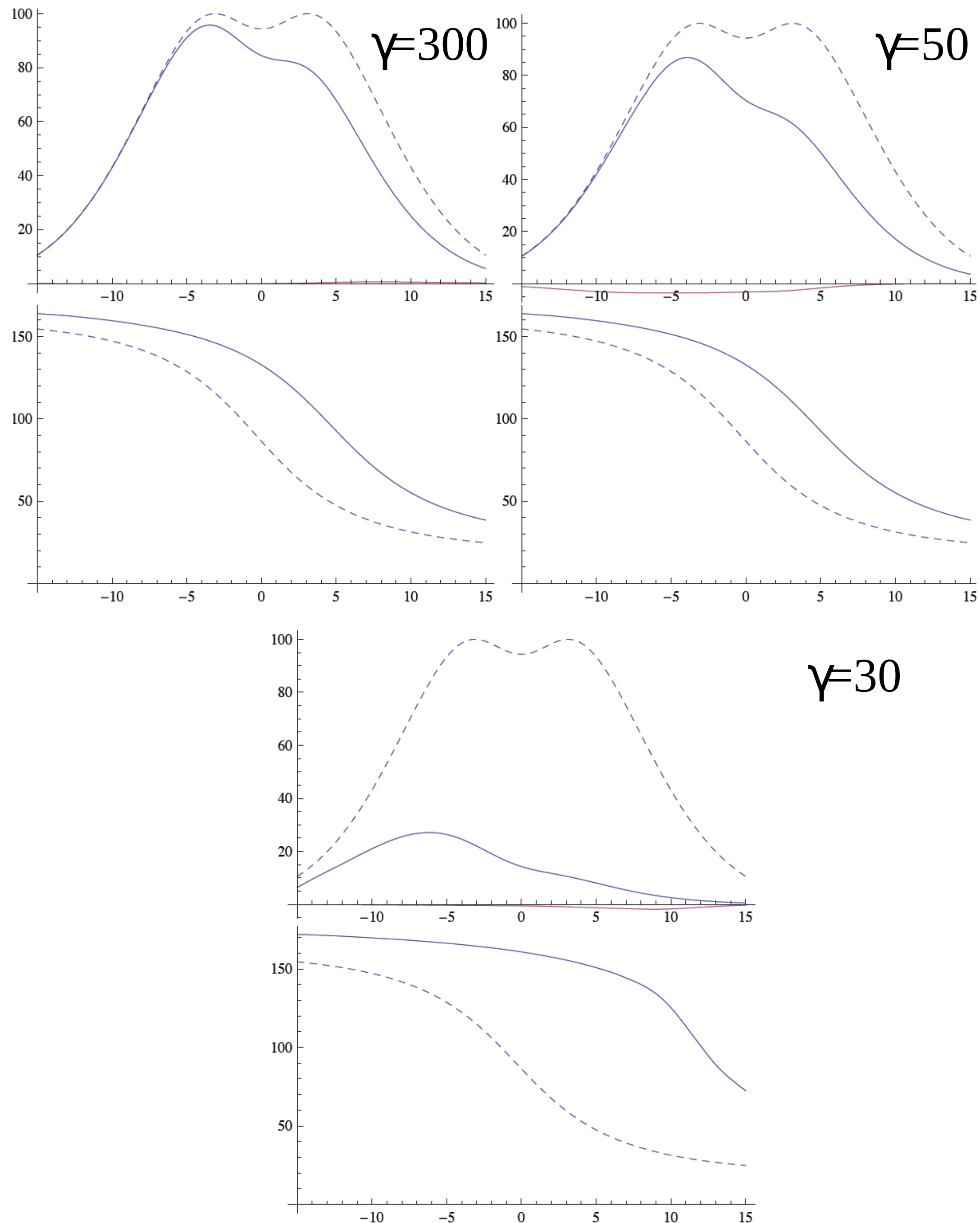
Results

P	B_0	χ	ζ	ρ_0	$r_{\rm rad}$	γ	λ
1 s	10^{12} G	45°	48°	0.5	$30R$	50	10^3

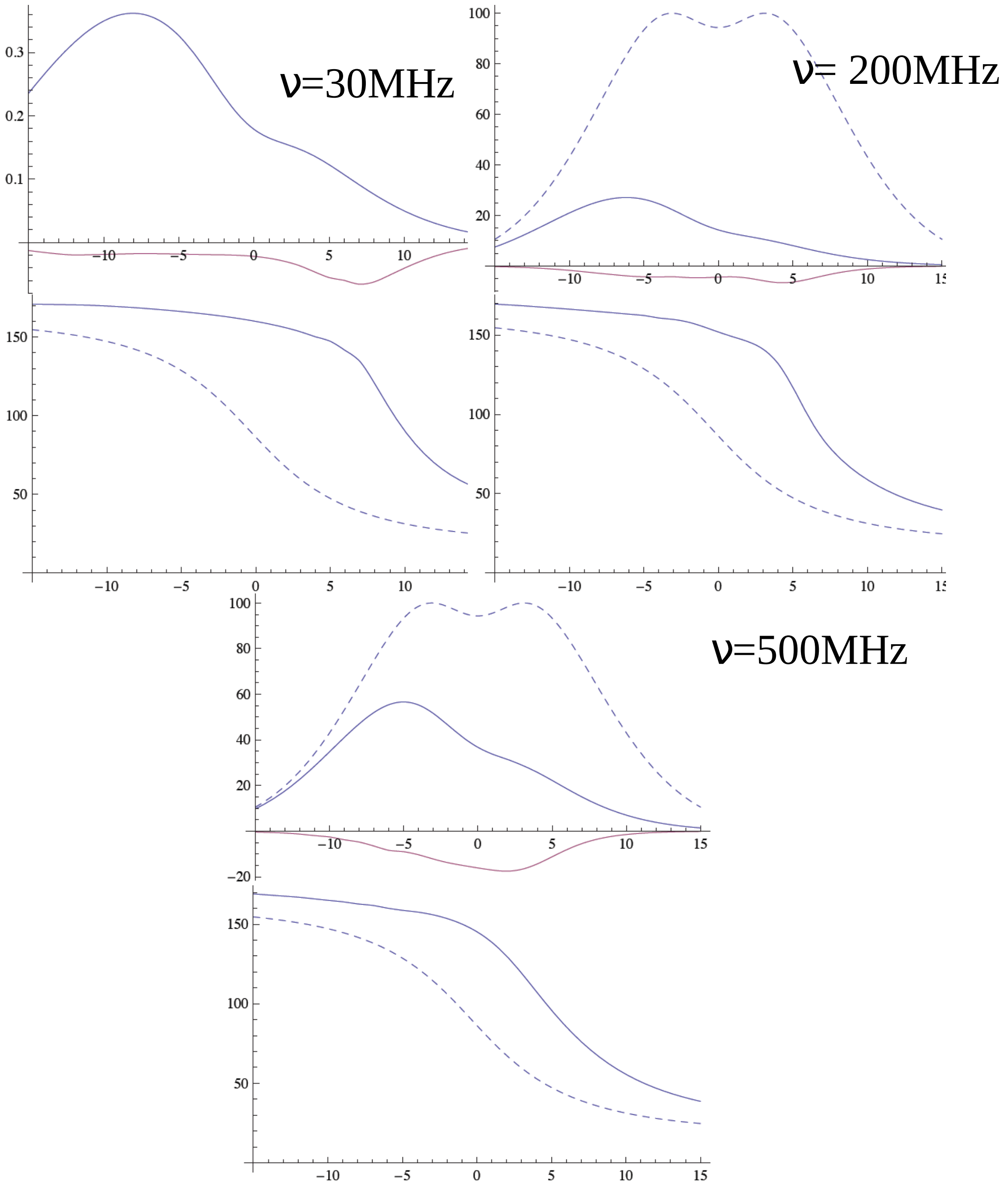
Multiplicity λ



Beam energy γ



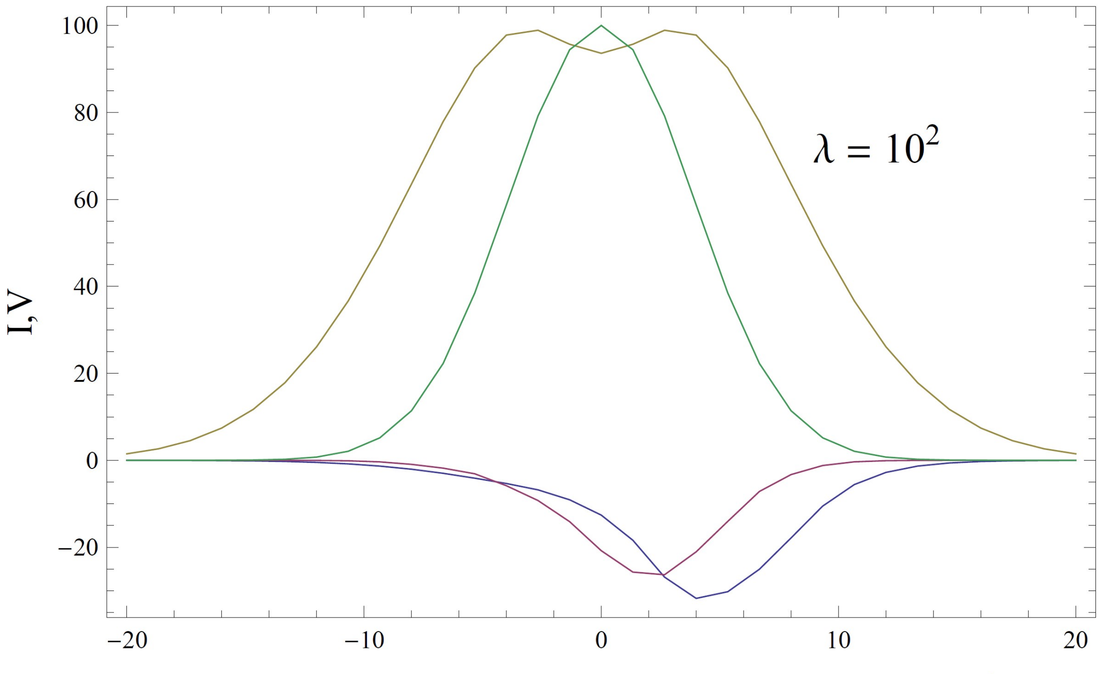
Frequency ν

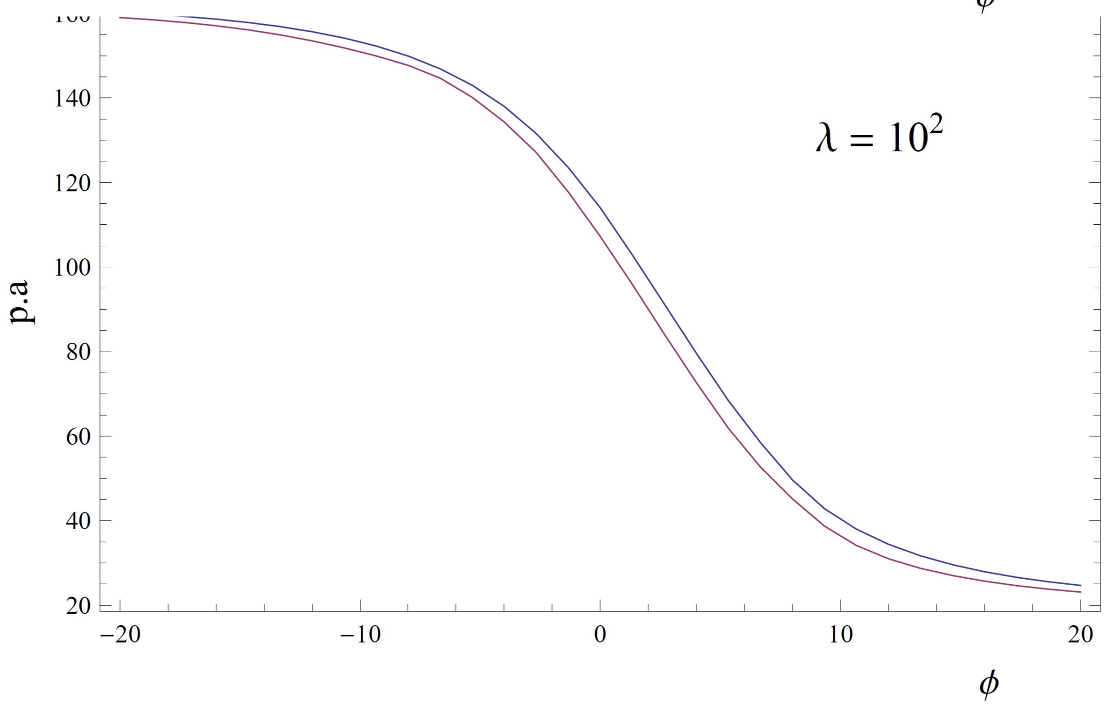


Different levels of radiation, X-mode

$r$rad = 10R and

$r$rad = 30R

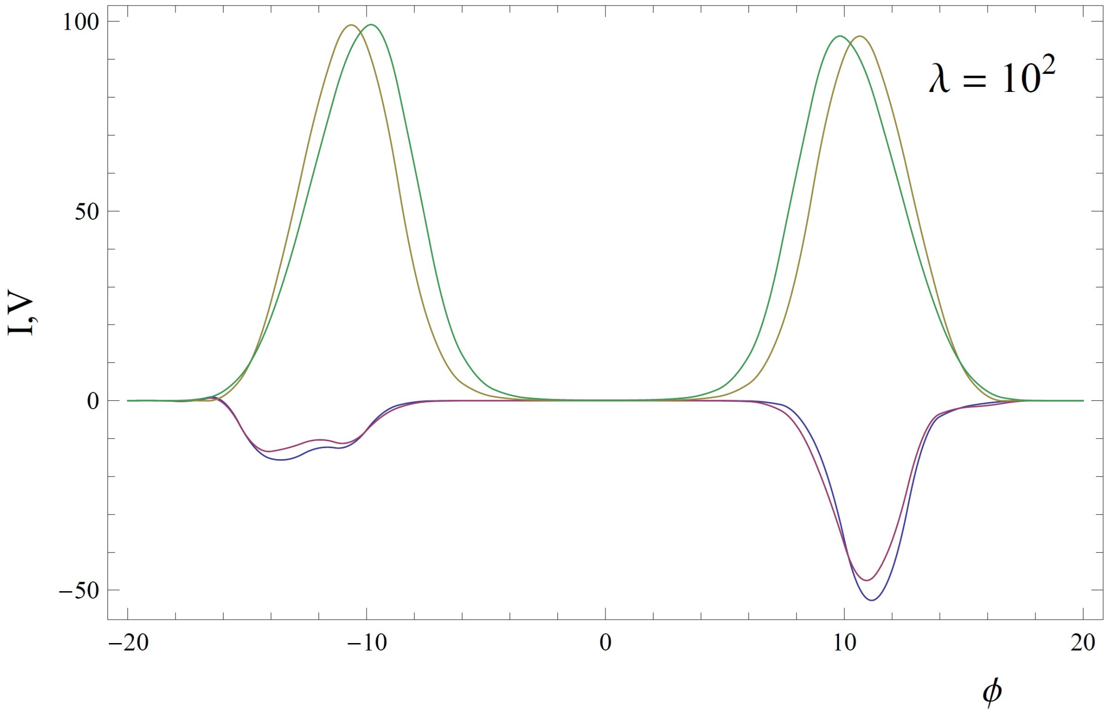


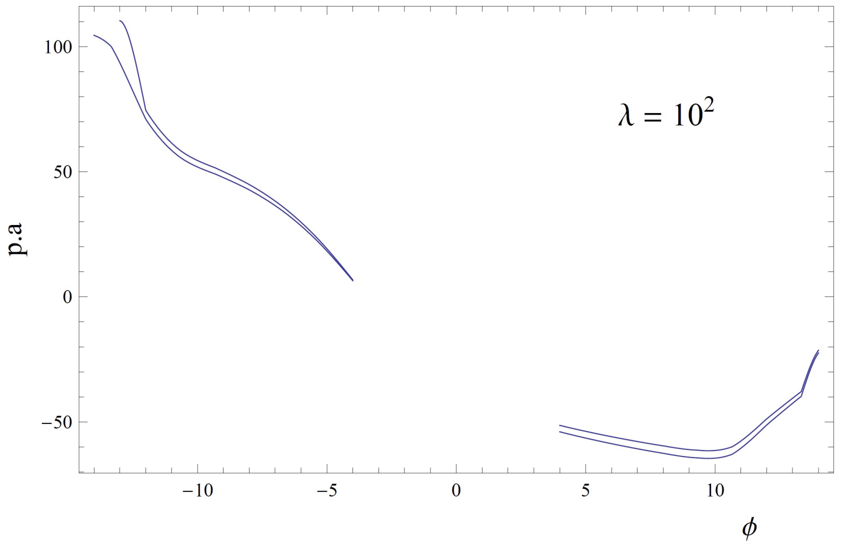


Different levels of radiation, O-mode

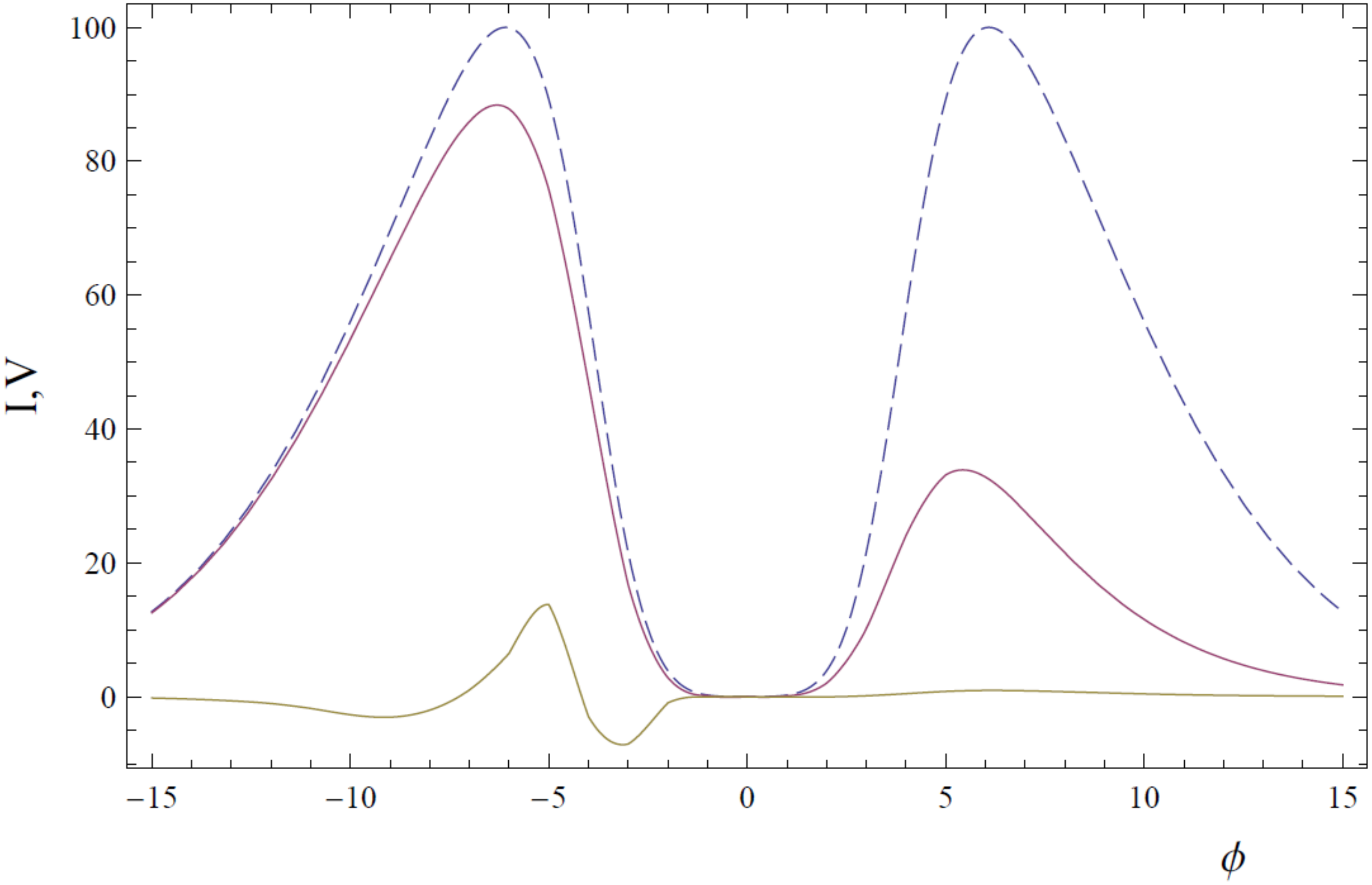
$r$rad = 10R and

$r$rad = 30R

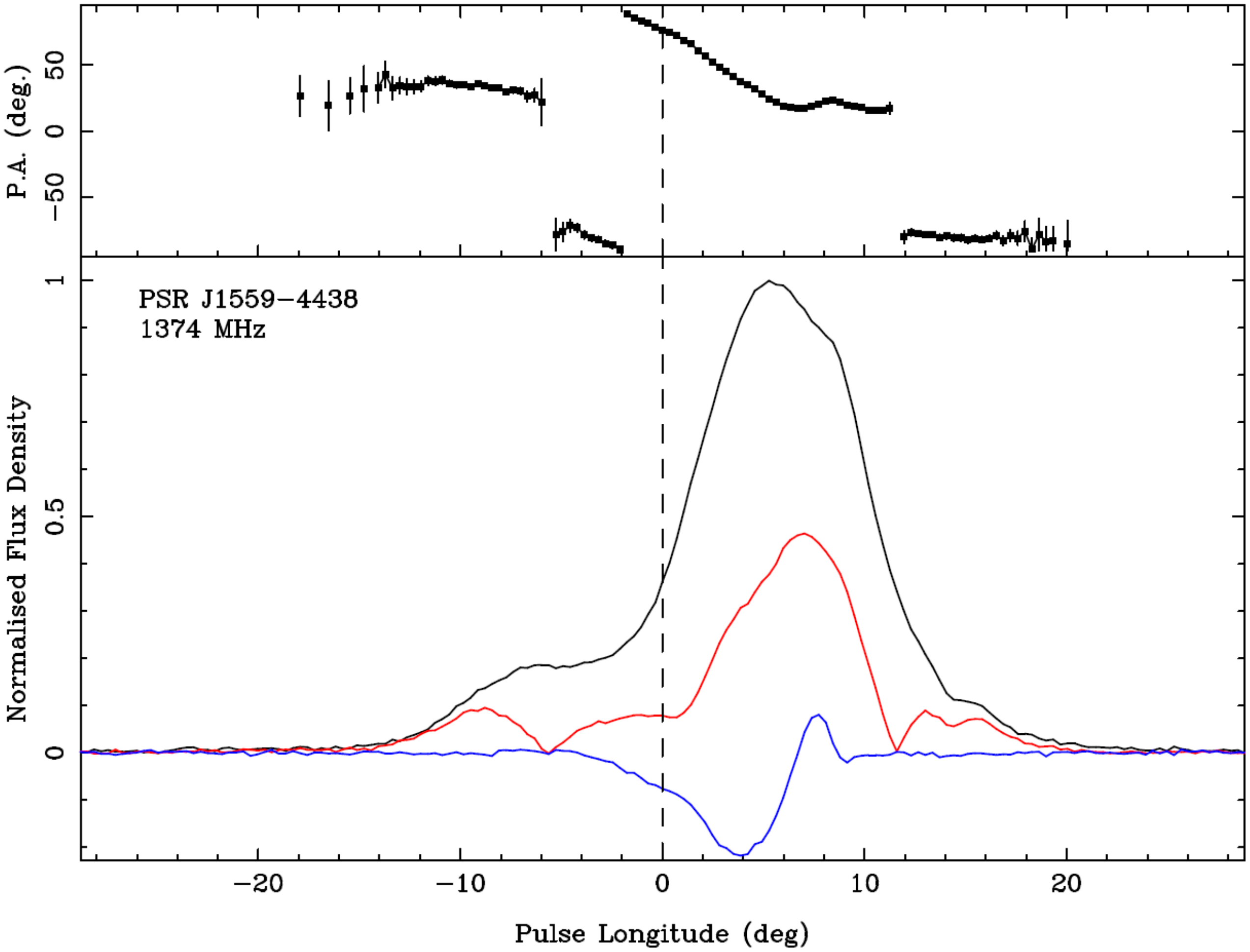




Sign V
switch



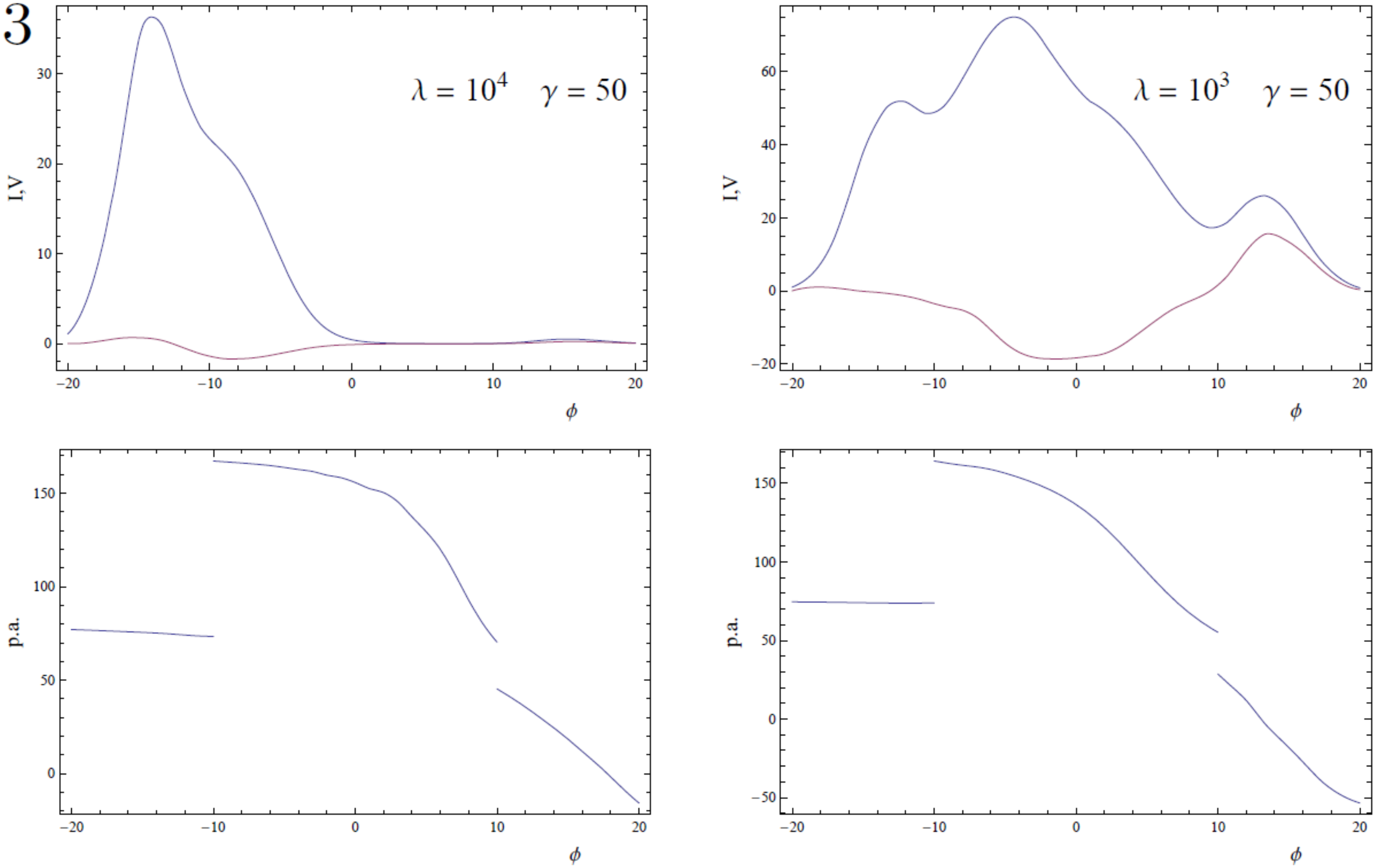
Sign V switch



S.Johnston, M.Kramer, et al. MNRAS, **381**, 1625 (2007)

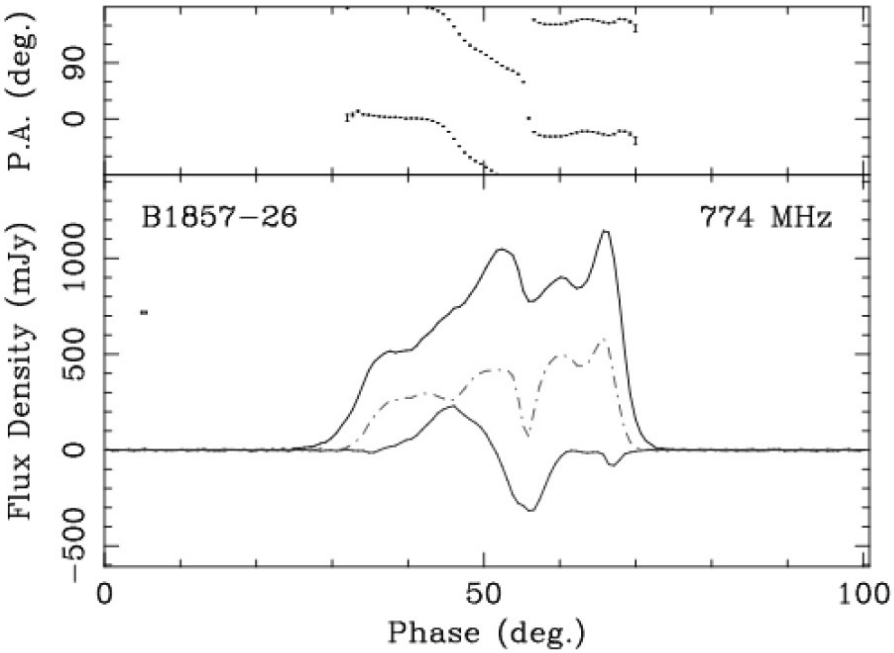
Two modes,

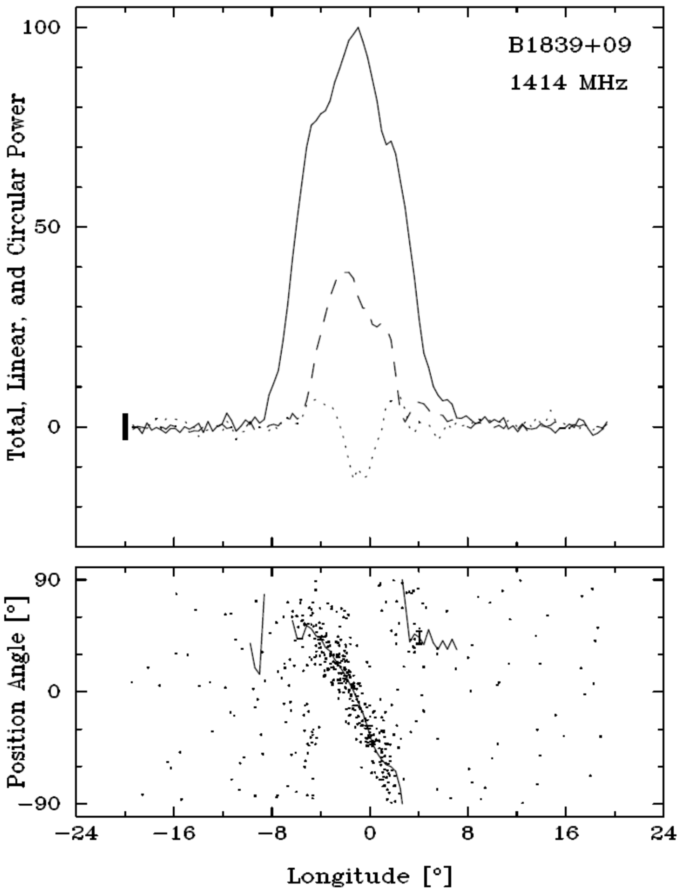
$I_O/I_X = 1/3$



In general, O-X-O for triple pulsars

Two modes

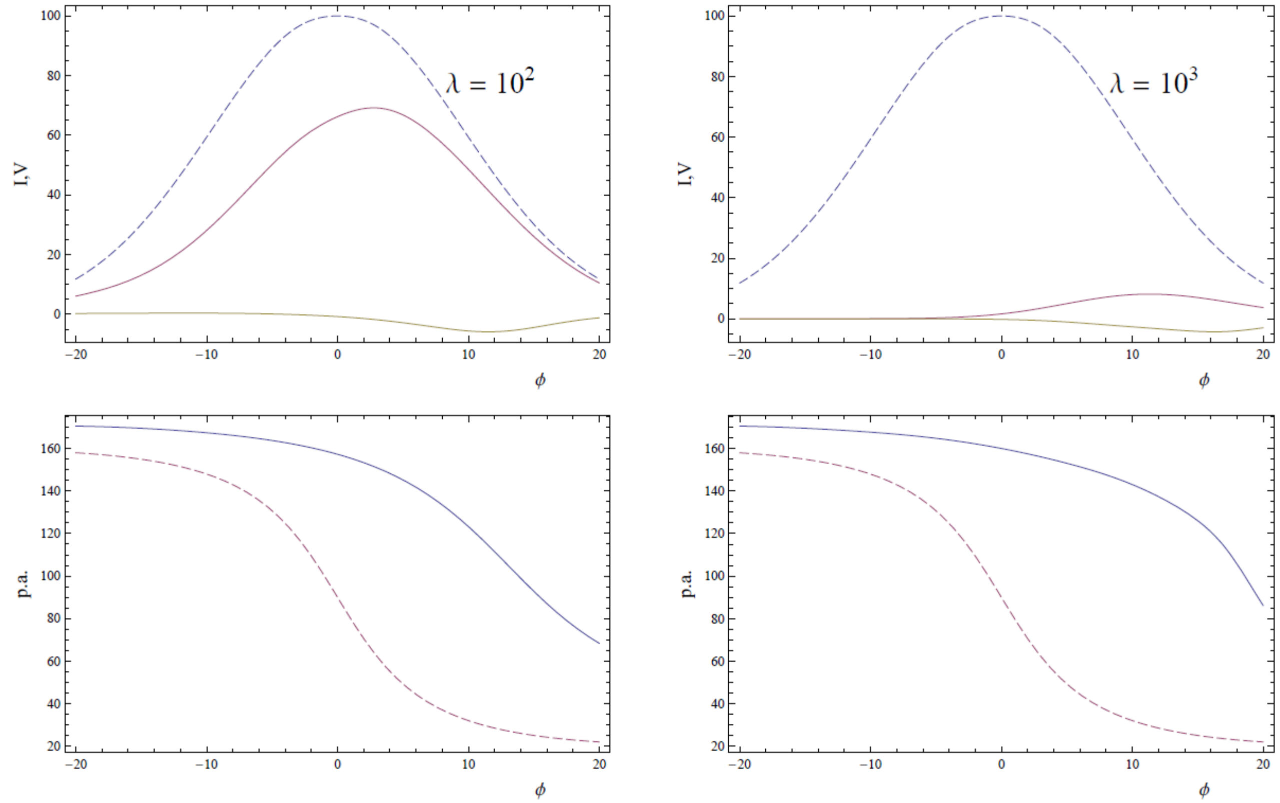




S.Johnston, M.Kramer, et al. MNRAS, **381**, 1625 (2007)

Millisecond pulsar

P	B_0	χ	ζ	ρ_0	$r_{\rm rad}$	γ	λ
20 ms	10^8 G	45°	48°	0.2	$1.25R$	50	10^3



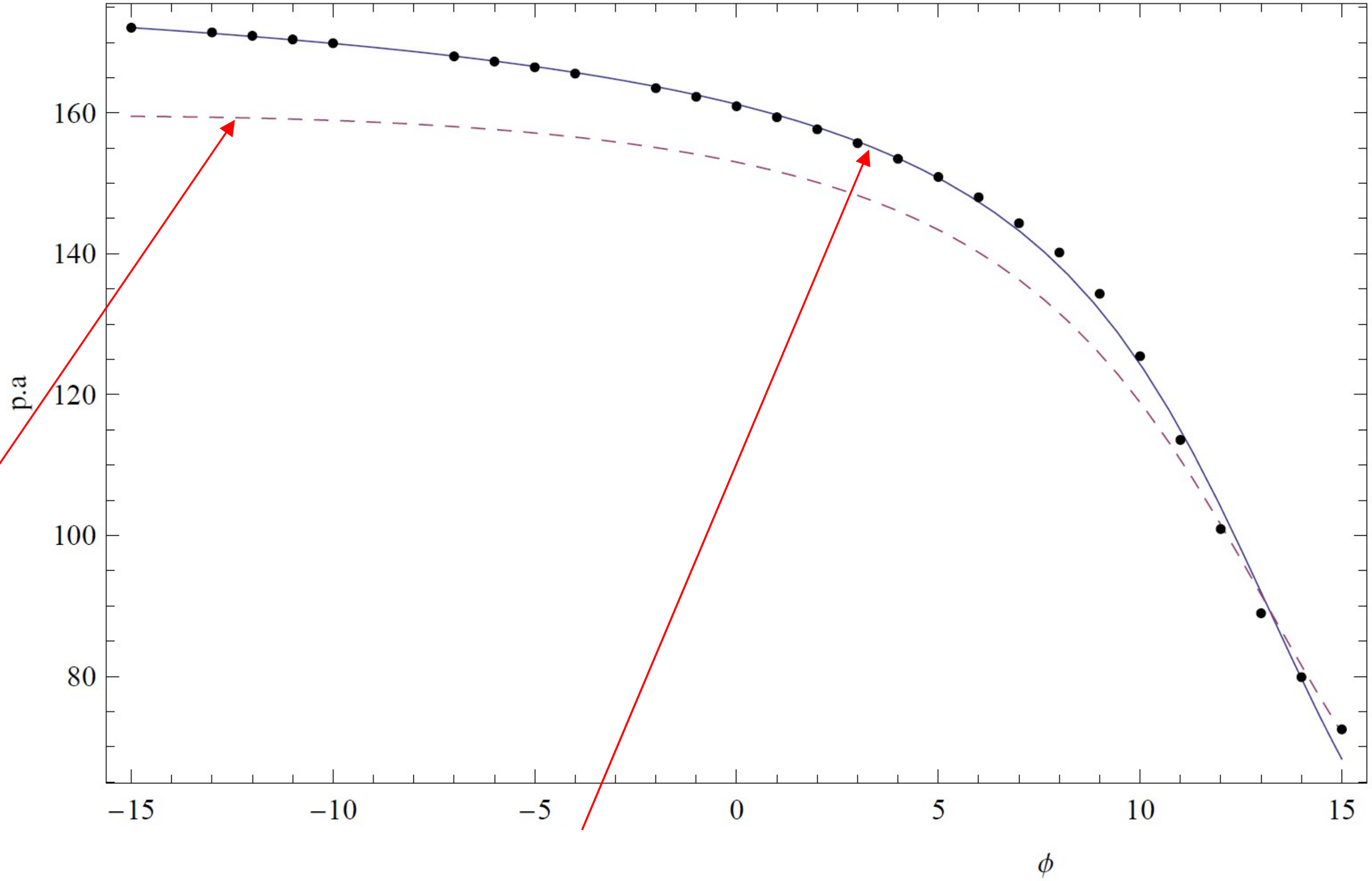
One can have the absorption in the leading part.

$(\mathrm{d}\,p.a./\mathrm{d}\phi)_{max}$

λ	γ	ν (GHz)	Value	SV	RVM value
10^2	50	1	-9.47	4.7	-10.14
10^4	50	1	-14.47	7.44	-10.14
10^5	50	1	-11.70	10.5	-10.14
10^3	10	1	-12.72	11.5	-10.14
10^3	50	1	-9.86	4.3	-10.14
10^3	100	1	-9.47	4.7	-10.14
10^3	300	1	-9.46	4.7	-10.14
10^3	50	0.03	-15.92	8.4	-10.14
10^3	50	0.5	-11.82	3.85	-10.14
10^3	50	0.2	-18.02	5.48	-10.14

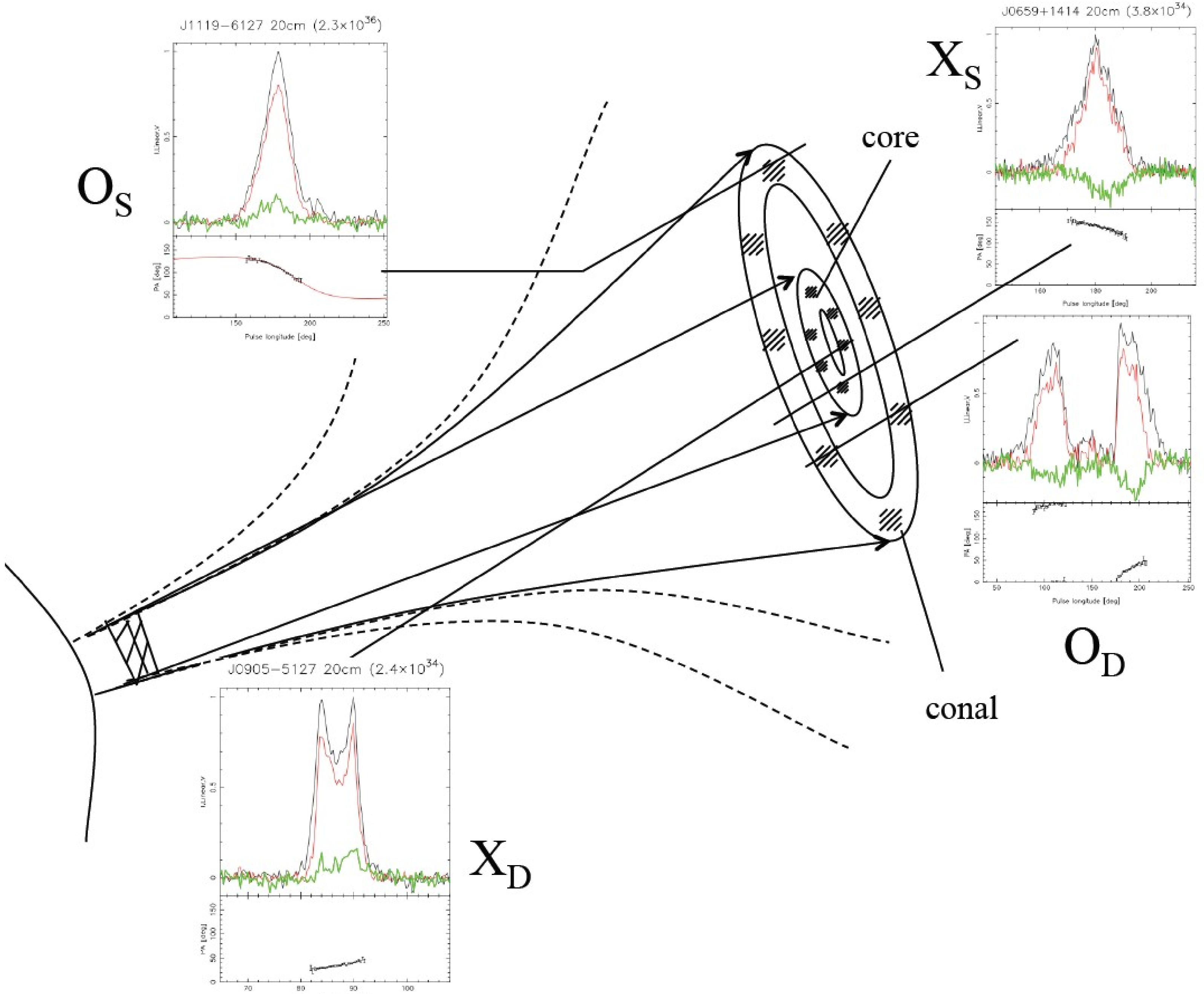
$(\,\mathrm{d}\,p.a./\mathrm{d}\phi)_{\max}$

$\chi = 45^\circ$ and $\zeta = 48^\circ$



RVM curve

Wrong fit $\chi = 93^\circ$ and $\zeta = 97^\circ$



J1119−6127 20cm (2.3×10^36)
O_S
J0659+1414 20cm (3.8×10^34)
X_S
core
O_D
conal
J0905−5127 20cm (2.4×10^34)
X_D
I,Linear,V
PA [deg]
Pulse longitude [deg]

Core & Conal

Profile	O_S	O_D	X_S	X_D
Number	6	23	45	6
$\sqrt{P}W_{50}$	6.8 ± 3.1	10.7 ± 4.5	6.5 ± 2.9	5.3 ± 3.0

P.Weltevrede, S.Johnston. MNRAS **391**, 1210 (2008)
T.Hankins, J.Rankin. Astron. J., **139**, 168 (2010)

Main results

- One-to-one correspondence between the signs *V* and d*p.a.*/dϕ.

For the X-mode the signs should be the SAME, and the OPPOSITE for the O-mode.

- For central passage the *V* sign reversal in the core can occur.

- The standard *S*-shape form of the *p.a.* swing can be realized for small enough multiplicity and large enough bulk Lorentz-factor only.

- In general, the trailing side of the emission beam is absorbed.

Leading part can be absorbed when the polarization forms close to the light cylinder.

- Electric drift changes the sign of the effect.

Conclusion

Only now it's possible to compare the theory with the observational data quantitatively.

Two extra remarks

Acknowledgements

Anatoly, thanks a lot!

You’re welcome!

(*Mathematica 7 or C*)