Gravitational collapse to third family compact stars

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Motivation

Compact stars offer a unique opportunity to study the QCD phase diagram at high densities and low temperatures. Einstein's field equations of classical gravity provide a link between the equation of state of superdense matter and the structural properties of stellar objects. Our goal is to study the behaviour of superdense matter encountered in the interiors of compact stars and find its observable signatures. Of particular importance is the dynamical rearrangement of the star due to a phase transition [1]. This can be the collapse to a black hole or to a twin star belonging to the third family [2].

Equation of State of superdense matter

The composition and equation of state of the inner regions of compact stars remains largely unknown. Possible candidates are meson condensates, hyperons and deconfined quark matter. A perturbative treatment of QCD at the density regime of compact stars is not sufficient due to a large value of the strong coupling constant. Thus effective models for the different phases are required. To describe the hadronic part we use the relativistic mean-field model with the parameter set TM1, which was fitted to the properties of neutron-rich nuclei. The deconfined phase is described with an effective MIT-like model [3] with finite strange quark mass m_s and superconducting gap Δ. Both phases are matched by a Maxwell construction, selecting the phase with the higher pressure at given baryon chemical potential (see Fig. 1). The free parameters of the effective quark model are B, m_s and Δ. We study the influence off the parameter choice on the equilibrium states and transition processes. Such a transition occurs, when the central density exceeds a certain threshold density triggering the star's instablity with respect to the quark core formation and leading to radial oscillations. The deconfinement phase transition yields a softer equation of state with lower maximum mass, as shown in Fig. 2. Such a transition may be interesting for the prospects of detecting gravitational waves. Indeed, when a superconducting quark phase is formed, the dominant damping mechanism will be GW-emission due to low viscosity.

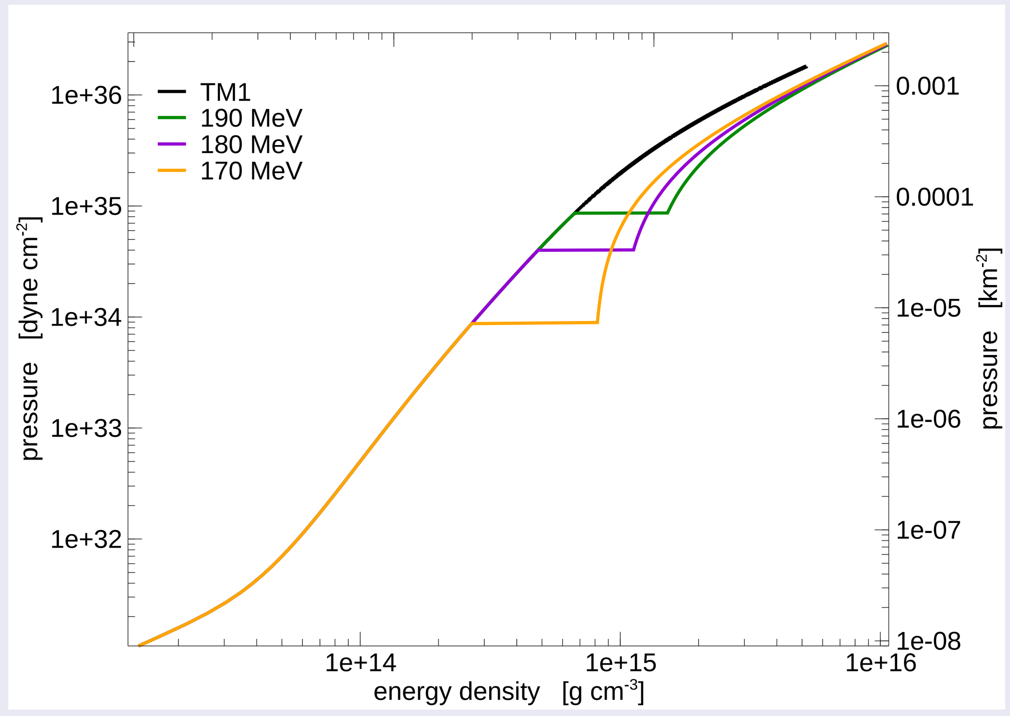

FIG. 1: Hybrid equation of state for different parameter set choices, varying the Bag constant only. The value of $B^{\frac{1}{4}}$ is given in units of MeV. Increasing Bag constant shifts the onset of the first order phase transition to higher energies.

Simplified dynamical model

The dynamical process of quark core formation is studied assuming two constant baryon density phases in Newtonian hydrodynamics [1]. The dynamics is governed by Euler's equation of an ideal liquid in its own gravitational field $\frac{\partial v}{\partial t}+\frac{1}{2}\frac{\partial v^2}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{\partial \Phi}{\partial r}=0$. The discontinous velocity profile is obtained from the continuity equation for baryon current.

$$v(r)=\begin{cases}0 & \text{if } r<R_c(t)\\ -\frac{(\lambda-1)R_c^2\dot{R}_c}{r^2} & \text{if } r>R_c(t)\end{cases}$$

The differential equation for the core radius R_c follows from integrating Euler's equation from the phase boundary R_c to the surface R_s.

$$0=A+B+C+D$$

$$A=(1-\lambda)\left(2\dot{R}_c^{\,2}+\ddot{R}_c\right)\left(1-\frac{R_c}{\left(R_i^3-(\lambda-1)\,R_c^3\right)^{\frac{1}{3}}}\right)$$

$$B=\frac{1}{2}(\lambda-1)^2\,R_c^4\dot{R}_c^{\,2}\left(\frac{1}{\left(R_i^3-(\lambda-1)\,R_c^3\right)^{\frac{4}{3}}}-\frac{1}{R_c^4}\right)$$

$$C=\begin{cases}-\frac{2}{3}\pi\rho_b R_i^2+\lambda\,(\lambda-1)\,\dot{R}_c^{\,2} & \text{if } \dot{R}_c<0\\ -\frac{2}{3}\pi\rho_b R_i^2 & \text{if } \dot{R}_c>0\end{cases}$$

$$D=\frac{4}{3}\pi\left(\rho_b-\rho_a\right)R_c^2\left(\frac{R_c}{\left(R_i^3-(\lambda-1)\,R_c^3\right)^{\frac{1}{3}}}-1\right)+\frac{2}{3}\pi\rho_b\left(R_s^2-R_c^2\right)$$

The dynamical evolution of the quark core is illustrated in Fig. 3. Currently we generalize these equations by including first and second viscosity.

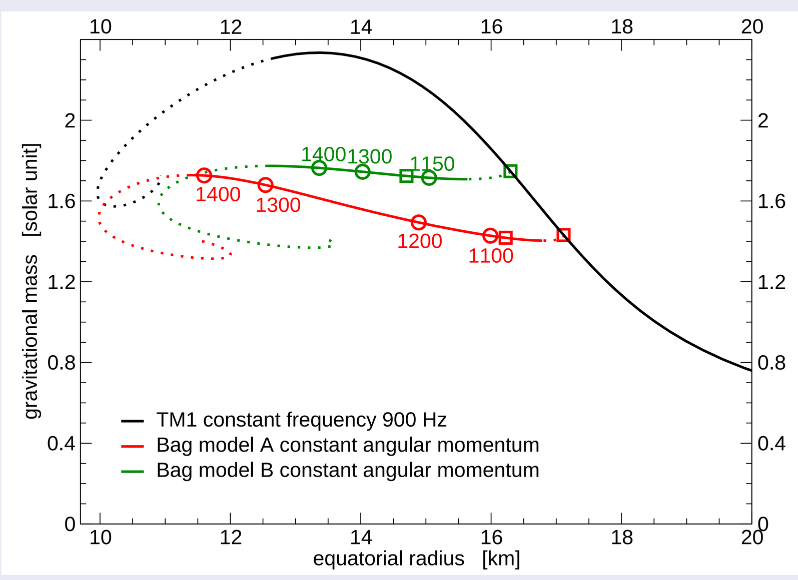

FIG. 2: Mass-radius diagramm for rotating hybrid stars. Twin star configurations of equal baryon number and angular momentum are denoted by squares. The circles mark stars, for which the value of the angular frequency is given in units of Hz.

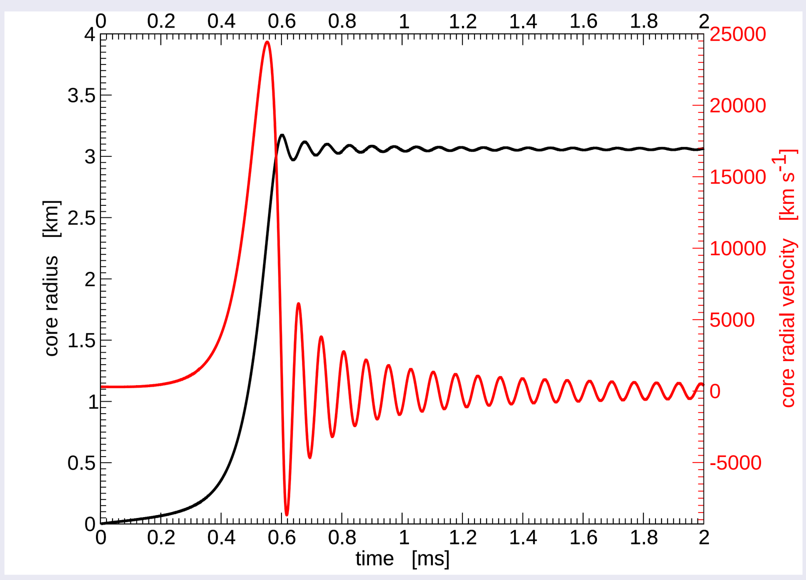

FIG. 3: Core radius and radial velocity as function of time for $M=2M_{\odot}$, $\rho=\rho_0$, $\lambda=2$.

Hartle-Thorne approach for rotating stars in GR

We have considered a rotating star with oscillating core radius. Rotating equilibria are computed using an extended version [7] of Hartle's formalism [8], suitable for frequencies below mass-shedding. Rotation breaks the spherical symmetry and introduces latitudinal dependence for all fluid variables. The central values for Lense-Thirring frequency and pressure provide a parameterisation for stellar configurations. Corresponding twin stars are selected by demanding equal baryon number and angular momentum. In most cases total mass-energy of the equilibrium belonging to the third stable branch is reduced. The gained energy is released during the transistion process. The gravitational wave signal due to time-varying quadrupole moment is computed in linearized gravity $h_+=\frac{\ddot{Q}_{yy}-\ddot{Q}_{zz}}{r}$ $\quad Q_{ij}=\int\rho\left(x^ix^j-\frac{1}{3}\delta^{ij}r^2\right)d^3x$. Due to axisymmetry only plus-polarised waves contribute.

GR hydrodynamics model

The spherically symmetric collapse of an ideal liquid in GR was studied by Misner and Sharp [4] [5] in the 60's using Lagrangian coordinates and subsequently generalized by Bekenstein for effects of electromagnetic type [6]. Using Eulerian coordinates for the isentropic case, it is possible to obtain evolution equations generalizing the Oppenheimer-Volkoff equations for the case of a moving charged fluid. The time-dependent metric $g_{\mu\nu}=diag[-e^{2\Phi},e^{2\Lambda},r^2,r^2sin^2\theta]$ yields a non-diagonal Ricci tensor.

$$R_{00}=\partial_\rho\Gamma^\rho_{00}-\partial_0\Gamma^\rho_{0\rho}+\Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{00}-\Gamma^\rho_{0\lambda}\Gamma^\lambda_{0\rho}=e^{2\Phi-2\Lambda}\left[-\Phi'\Lambda'+\Phi''+\Phi'^2+2r^{-1}\Phi'\right]+\dot{\Phi}\dot{\Lambda}-\dot{\Lambda}^2-\ddot{\Lambda}$$

$$R_{01}=\partial_\rho\Gamma^\rho_{01}-\partial_1\Gamma^\rho_{0\rho}+\Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{01}-\Gamma^\rho_{1\lambda}\Gamma^\lambda_{0\rho}=2r^{-1}\dot{\Lambda}$$

$$R_{11}=\partial_\rho\Gamma^\rho_{11}-\partial_1\Gamma^\rho_{1\rho}+\Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{11}-\Gamma^\rho_{1\lambda}\Gamma^\lambda_{1\rho}=e^{2\Lambda-2\Phi}\left[-\dot{\Phi}\dot{\Lambda}+\ddot{\Lambda}+\dot{\Lambda}^2\right]-\Phi''+\Phi'\Lambda'+2r^{-1}\Lambda'-\Phi'^2$$

$$R_{22}=\partial_\rho\Gamma^\rho_{22}-\partial_2\Gamma^\rho_{2\rho}+\Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{22}-\Gamma^\rho_{2\lambda}\Gamma^\lambda_{2\rho}=-e^{-2\Lambda}+r\Lambda'e^{-2\Lambda}-r\Phi'e^{-2\Lambda}+1$$

$$R_{33}=\partial_\rho\Gamma^\rho_{33}-\partial_3\Gamma^\rho_{3\rho}+\Gamma^\rho_{\lambda\rho}\Gamma^\lambda_{33}-\Gamma^\rho_{3\lambda}\Gamma^\lambda_{3\rho}=sin^2\theta\left[-e^{-2\Lambda}+r\Lambda'e^{-2\Lambda}-r\Phi'e^{-2\Lambda}+1\right]$$

The energy-momentum tensor reads $T_{\mu\nu}=(\rho+P)u_\mu u_\nu+Pg_{\mu\nu}+\frac{1}{4\pi}\left[F_\mu{}^\alpha F_{\nu\alpha}-\frac{1}{4}g_{\mu\nu}F^{\zeta\xi}F_{\zeta\xi}\right]$ with $u_2=u_3=0$. Then $G_{\mu\nu}=8\pi T_{\mu\nu}$ and $T^{\mu\nu}_{\ ;\mu}=0$ reduce to

$$\frac{\partial m}{\partial r}=4\pi r^2\frac{\rho e^{2\Phi}+Pe^{2\Lambda}v^2}{e^{2\Phi}-e^{2\Lambda}v^2}+\frac{QQ'}{r}$$

$$\frac{\partial m}{\partial t}=-\frac{4\pi r^2\left(\rho+P\right)e^{2\Phi}v}{e^{2\Phi}-e^{2\Lambda}v^2}+\frac{Q\dot{Q}}{r}$$

$$\frac{\partial \Phi}{\partial r}=\frac{mr-Q^2+4\pi r^4\frac{Pe^{2\Phi}+\rho e^{2\Lambda}v^2}{e^{2\Phi}-e^{2\Lambda}v^2}}{r^3-2mr^2+Q^2r}$$

$$\frac{\partial P}{\partial r}=\frac{\sqrt{e^{2\Phi-2\Lambda}-v^2}\,Q\rho_{ch}e^{\Phi}-e^{2\Phi}(\rho+P)\left[6m-2r+8\pi Pr^3+2mrv^{-1}v'-r^2v^{-1}v'-Q^2\left(4r^{-1}+v^{-1}v'\right)\right]}{e^{2\Phi}(r^2-2mr+Q^2)-r^2v^2}$$

$$\frac{\partial P}{\partial t}=\begin{cases}-\left(\rho+P\right)v' & \text{if } r=0\\ -vP' & \text{if } r\neq 0\end{cases}$$

$$\frac{\partial Q}{\partial r}=\frac{4\pi\rho_{ch}r^2e^{\Phi+\Lambda}}{\left(e^{2\Phi}-e^{2\Lambda}v^2\right)^{\frac{1}{2}}}$$

$$\frac{\partial Q}{\partial t}=-vQ'$$

with v being the radial 3-velocity. The evolution equation for v follows from conservation of baryon charge. $\dot{\Phi}$ is determined implicitly. The analysis of these equations is in progess.

Summary

- Stable third family equilibria for recent equations of state are constructed
- Energy release up to 10^{52} erg during transition is found
- The GW emission due to the star rearrangement is evaluated.
- The obtained GW signal is to low for events at 10 Mpc and present detectors

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