Decoupling of superfluid and normal pulsation modes in neutron and hyperon stars

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Outline

- Introduction and formulation of the problem
- Superfluid hydrodynamics
- Pulsation equations in the linear approximation
- Superfluid and normal modes
- Example: radial pulsations
- Decoupling in superfluid hyperon stars
- Conclusions

Introduction and formulation of the problem

• Studying the pulsations of General Relativistic neutron stars (NSs) is very important and actively developing area of research, since comparison of pulsation theory with observations can give valuable information about the properties of superdense matter.

• Yet, it is a very difficult theoretical problem even for normal (nonsuperfluid) stars.

• Additional complication: At $T \leq 10^8 - 10^{10}\,\mathrm{K}$ baryons in the internal layers of NSs become superfluid

Lindblom and Mendell (1994) were the first who studied global pulsations of superfluid NSs.

Considering a simple model of a **Newtonian** star with **nucleon** core they numerically found two distinct classes of pulsation modes:

- normal-like modes, which practically coincide with the corresponding modes of a normal star

- superfluid modes in which the matter pulsates in such a way that the mass current density approximately vanishes

Lindblom and Mendell (1994):

Perhaps the most striking result of our numerical analysis is that one class of solutions to the full superfluid hydrodynamics equations is essentially indistinguishable from the solutions to the ordinary-fluid equations. Thus, all the work that has been done in the past on the pulsations of neutron stars using ordinary-fluid hydrodynamics is relevant even in very cold neutron stars. We do not yet understand any simple physical mechanism responsible for the similarity of these modes. We are confident that this feature is real, however, since it was found in the elementary analytical solutions to the pulsation equations described in § 6 as well as in each of our separate numerical solutions to the realistic equations described in § 7.

Subsequent numerical studies verified the result of Lindblom and Mendell though no general explanation of this result has been proposed.

In the first part of the talk we explain these numerical results for npe-matter by demonstrating that:

- Normal and superfluid modes are described by two sets of weakly coupled equations

- The coupling between them is parameterized by just ONE coupling parameter s which is small for realistic equations of state

These results allow us to formulate a simple perturbative (in parameter s) scheme which considerably simplifies the problem of calculation of the pulsation spectrum for superfluid NSs. They also lead to a number of important physical conclusions about the properties of superfluid oscillations

In the second part of the talk, we briefly discuss how these results can be extended to a more general case of superfluid ***hyperon*** stars.

The main assumptions

- We first consider ***npe-matter***
- Unperturbed star is in beta-equilibrium: $\delta\mu \equiv \mu_n - \mu_p - \mu_e = 0$

  μ_i is the chemical potential for particles *i=n, p, e*

- Quasineutrality (also for perturbed star): $n_e = n_p$

  n_i is the number density of particles *i=n, p, e*

In this talk it is additionally assumed that:

- a star is nonrotating (no Feynman-Onsager vortices)
- in the first part of the talk we also assume that protons are normal while neutrons are superfluid in some region of a NS core

These assumptions are made just to simplify the presentation and do not affect our principal results.

Superfluid hydrodynamics

The main feature of superfluid hydrodynamics is the presence of two (or more) independent velocity fields

For normal matter:

$$\begin{cases} j^{\mu}_{(e)} = n_e u^{\mu} \\ j^{\mu}_{(p)} = n_p u^{\mu} \\ j^{\mu}_{(n)} = n_n u^{\mu} \end{cases}$$

the standard expressions for particle current density

When neutrons are superfluid: ⇒ 2 velocity fields instead of one

u^{μ} is the velocity of a normal component (electrons, protons, and normal neutrons)

$V^{\mu}_{s(n)}$ is the velocity of superfluid neutrons

$$\Rightarrow \boxed{j^{\mu}_{(n)} = n_n u^{\mu} + \underline{Y_{nn} w^{\mu}_{(n)}}}$$

where the “superfluid” four-vector $w^{\mu}_{(n)}$ is given by:

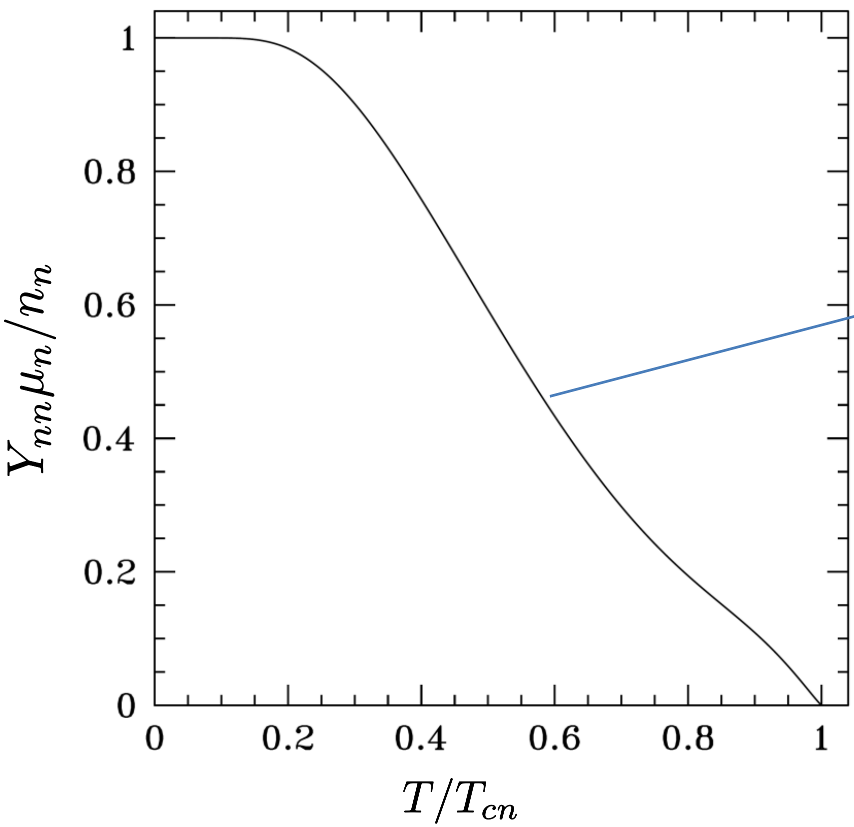
$$w^{\mu}_{(n)} \equiv \mu_n \left(V^{\mu}_{s(n)} - u^{\mu} \right)$$

proportional to a difference between the superfluid and normal velocity

$$j^{\mu}_{(n)} = n_n u^{\mu} + Y_{nn} w^{\mu}_{(n)}$$

The physical meaning of the coefficient Y_{nn} :

$$\mu_n Y_{nn} = \text{effective number density of superfluid neutrons}$$



strong temperature dependence!

$$Y_{nn} \in [0, n_n/\mu_n]$$

(*Gusakov, Kantor & Haensel , PRC,* **80**, 015803, 2009)

The basic equations

- Conservation of baryons and electrons

$$(n_b\, U^\mu)_{;\mu} = 0$$

$$(n_e\, u^\mu)_{;\mu} = 0$$

$$U^\mu \equiv u^\mu + Y_{nn} w^\mu_{(n)} / n_b$$

velocity of baryons

- Einstein equation

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G\, T^{\mu\nu}$$

$$T^{\mu\nu} = (P+\varepsilon)\, u^\mu u^\nu + P g^{\mu\nu} + \underline{Y_{nn} (w^\mu_{(n)} w^\nu_{(n)} + \mu_n\, w^\mu_{(n)} u^\nu + \mu_n\, w^\nu_{(n)} u^\mu)}$$

- Potentiality condition for motion of superfluid neutrons

$$\partial_\nu \left[w_{(n)\mu} + \mu_n u_\mu \right] - \partial_\mu \left[w_{(n)\nu} + \mu_n u_\nu \right] = 0$$

- The second law of thermodynamics

$$d\varepsilon = \mu_n\, dn_b - \delta\mu\, dn_e + T\, dS$$

Pulsation equations in the linear approximation

One can derive the following linearized "superfluid" equation:

$$\boxed{i\,\omega\,(\mu_n\,Y_{nn} - n_b)\,\delta w_{(n)j} = \sqrt{-g_{00}}\;n_e\;\partial_j(\delta\mu)} \qquad j = 1, 2, 3$$

$$\delta\mu = \delta\mu(\delta g^{\mu\nu}, \delta U^{\mu}, \delta w^{\mu}_{(n)})$$

depends on the perturbations of $\delta g^{\mu\nu}$, δU^{μ}, and $\delta w^{\mu}_{(n)}$

The linearized Einstein equations can be presented (<u>very schematically</u>) in the following symbolic form:

$$\underbrace{\delta(R^{\mu\nu} - 1/2\;g^{\mu\nu}\;R)}_{\text{contains only the metric}} = 8\pi G\;\delta T^{\mu\nu}_{\rm norm} + \underline{\underline{s}} \times \left(\text{terms depending on } \delta w^{\mu}_{(n)}\right)$$

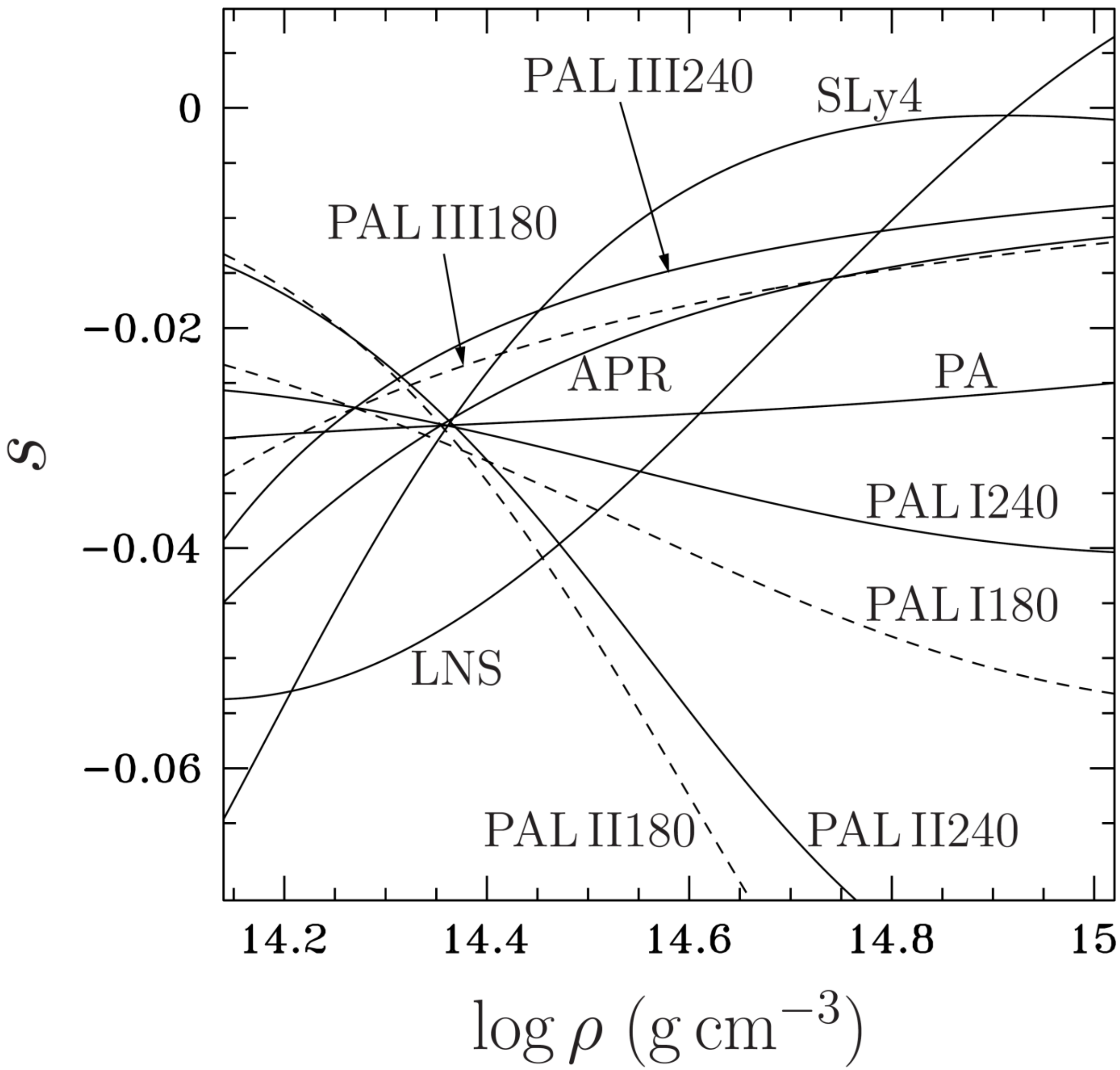
However, it can be shown that if *s* is small, then one can present the solution to the system of pulsation equations in the form of a ***<u>series</u>*** in parameter *s*.
So, let us inspect the coupling parameter *s*.

$$s \equiv \frac{n_e}{n_b}\,\frac{\partial P/\partial n_e}{\partial P/\partial n_b}$$

coupling parameter

Generally, both these equations are coupled!

The coupling parameter versus density



The stiffer EOS, the smaller s.

Recent measurements of the mass of the pulsar PSR J1614-2230 by P. Demorest et al. (2010) indicate that EOS is stiff

$$M_{\max} > 1.97 M_{\odot}!$$

$$|s| \sim 0.01 - 0.05$$

approximation of vanishing *s* should be very reasonable

Superfluid and normal modes

What will happen if *s* vanishes?

If ***s=0*** then the normal equation does not depend on $\delta w^{\mu}_{(n)}$ and has ***exactly*** the same form as in the absence of superfluidity

$$\delta(\underbrace{R^{\mu\nu} - 1/2\ g^{\mu\nu}\ R}_{\text{contains only the metric perturbations } \delta g^{\mu\nu}}) = 8\pi G\ \delta T^{\mu\nu}_{\rm norm}$$

$$\delta T^{\mu\nu}_{\rm norm} = \delta T^{\mu\nu}_{\rm norm}(\delta g^{\mu\nu}, \delta U^{\mu})$$

this quantity is formally given by exactly the same expression as the perturbation of $\delta T^{\mu\nu}$ for <u>normal</u> matter

The solution (the spectrum of eigenfunctions δU^{μ} and eigeinfrequencies ω) is indistinguishable from that for a nonsuperfluid star.

We obtained the *normal* modes

Assume that *s* still vanishes. Is it possible for a NS to oscillate on a frequency which is *not* an eigenfrequency of a normal star?

If **yes** then the Einstein equations will be satisfied only if:

$$\boxed{\delta U^{\mu} = 0 \text{ and } \delta g^{\mu\nu} = 0}$$

$$\Rightarrow \quad \delta\mu = \frac{n_e\,(\partial\delta\mu/\partial n_e)}{i\,\omega\,U^0}\left[\frac{\partial_j n_e}{n_e}\,\frac{Y_{nn}\,\delta w^j_{(n)}}{n_b} + \left(\frac{Y_{nn}\,\delta w^j_{(n)}}{n_b}\right)_{;j}\right]$$

depends only on $\delta w^{\mu}_{(n)}$

$$\boxed{i\,\omega\,(\mu_n\,Y_{nn} - n_b)\,\delta w_{(n)j} = \sqrt{-g_{00}}\;n_e\;\partial_j(\delta\mu)} \qquad j = 1, 2, 3$$

In this approximation the superfluid equation is self-contained and allows to determine the eigenfrequencies ω and the eigenfunctions $\delta w^{\mu}_{(n)}$

We obtained the *superfluid* modes

Properties of the superfluid modes in the zero approximation (*s=0*)

- They do not emit gravitational waves, $\delta g^{\mu\nu} = 0$

- The oscillating quantity is $\delta\mu = \mu_n - \mu_p - \mu_e$; the pressure and baryon current density are not perturbed, $\delta P = 0; \ \delta j^{\mu}_{(b)} = 0$

- The modes are entirely localized in the superfluid region of a star (in particular, they do not appear at the NS surface)

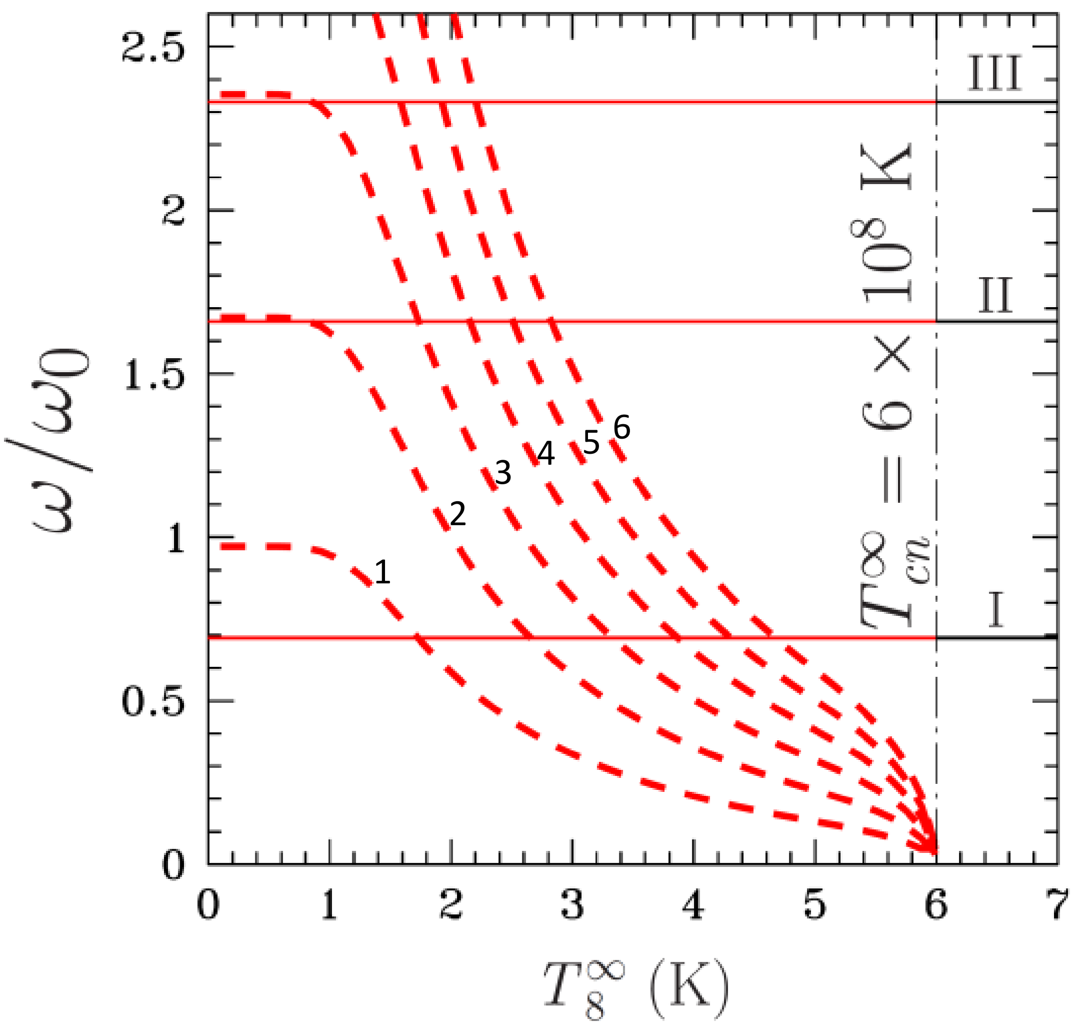
In the consideration above we assumed that *s=0.* Clearly, superfluid and normal modes should remain approximately decoupled also at small but finite coupling parameter *s.*

Moreover, since $|s| \sim 0.01 - 0.05$, the approximation of vanishing s is already sufficient to calculate the spectrum within accuracy $\sim s$ (i.e., a few per cent).

Example: Radial pulsations

Superfluid (dashes) and normal (solid lines) modes for a radially pulsating NS. The spectrum is calculated assuming *s=0*.

Six superfluid modes (1,…,6) and three normal modes (I, II, III)



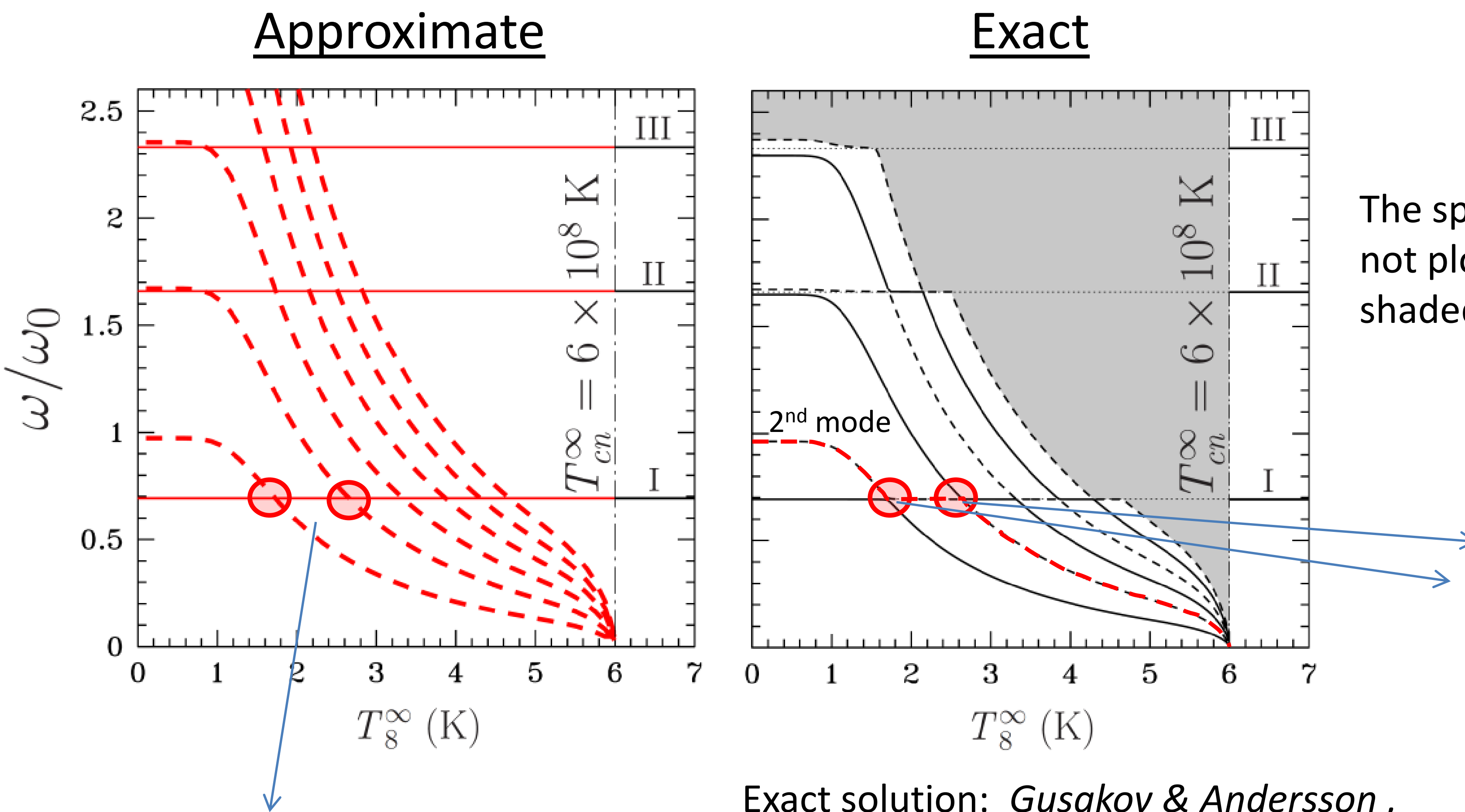
$M = 1.4M_\odot$

EOS APR
(Akmal et al. 1998)

$\omega_0 = \frac{c}{R}$ || $R = 12.17$ km

Neutrons in the core are superfluid at $T \leq T_{cn}^\infty = 6 \times 10^8$ K .

Comparison with the exact solution to the system of coupled pulsation equations



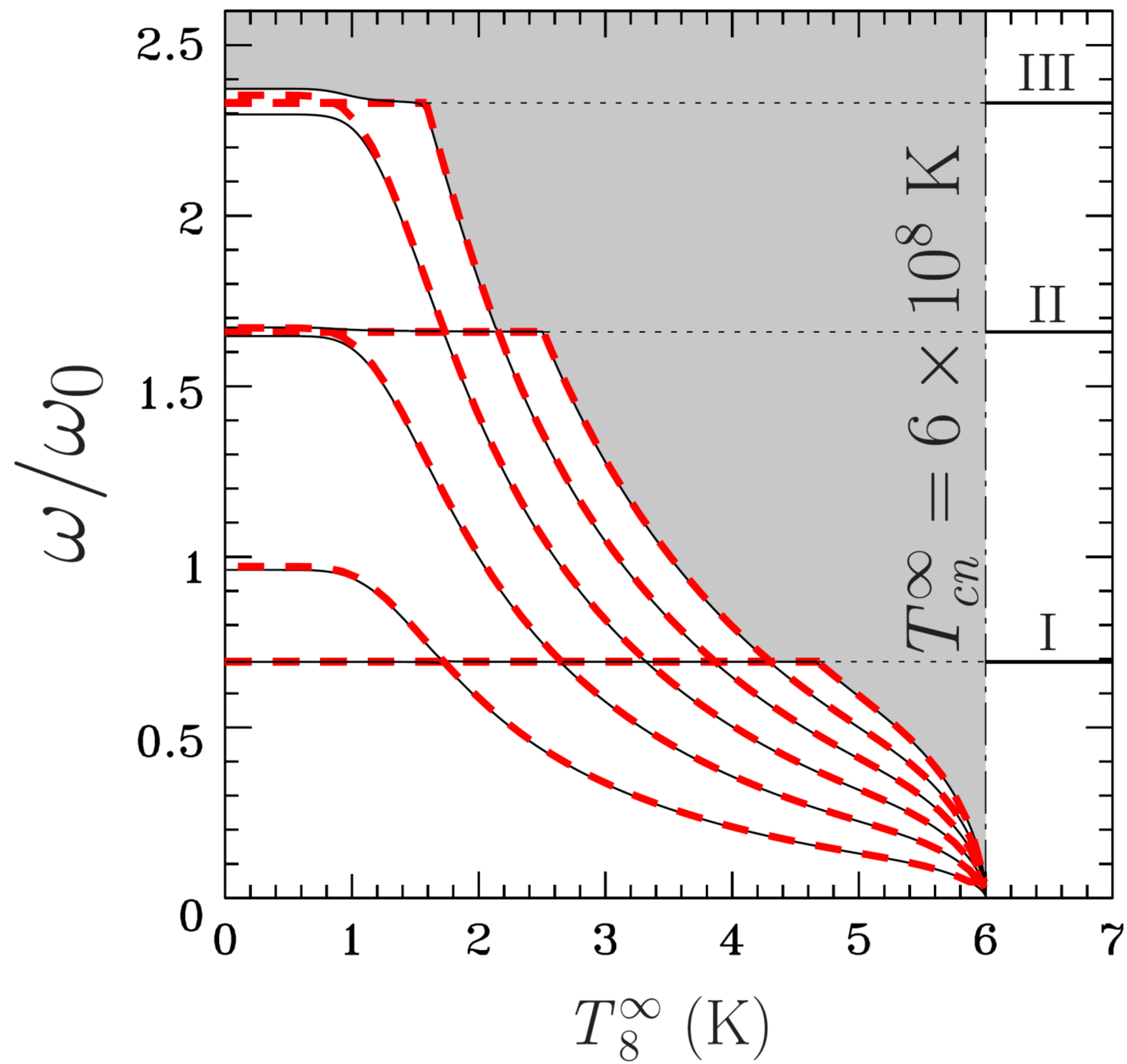
The spectrum is not plotted in the shaded region

avoided crossings

ordinary crossings

Exact solution: *Gusakov & Andersson , MNRAS, **372**, 1776 (2006)*

Approximate (***dashes***) and exact (***solid lines***) spectra



Approximate solution fits very well the exact solution and differs from it by 1—3 %

Decoupling in superfluid hyperon stars

Could this approach be generalized to hyperon matter?

The answer is YES!

However, in this case one has ***TWO*** coupling parameters instead of one:

$$s_l \equiv \frac{n_e}{n_b}\frac{\partial P/\partial n_e}{\partial P/\partial n_b} + \frac{n_\mu}{n_b}\frac{\partial P/\partial n_\mu}{\partial P/\partial n_b}$$

"lepton" coupling parameter

$$s_{\rm str} \equiv \frac{n_{\rm str}}{n_b}\frac{\partial P/\partial n_{\rm str}}{\partial P/\partial n_b}$$

"strange" coupling parameter

$$n_{\rm str} = \sum_{i \in \text{baryons}} s_i\, n_i$$

$$\boxed{i = n,\ p,\ \Lambda,\ \Xi,\ \Sigma}$$

s_i - strangeness

Pressure: $P = P(n_b, n_{\rm str}, n_e, n_\mu)$

Are these coupling parameters small?

To answer this question we used 2 stiff hyperon EOSs:

The "old" $\sigma\omega\rho$ -model of Glendenning (his model III)

hereafter **EOS GIII**

$$\boxed{M_{\max} = 1.975 M_{\odot}}$$

Recent highly nonlinear $\sigma\omega\rho\sigma^{*}\phi$ -model of Bednarek & Manka (2009)

(hereafter **EOS MB16**)

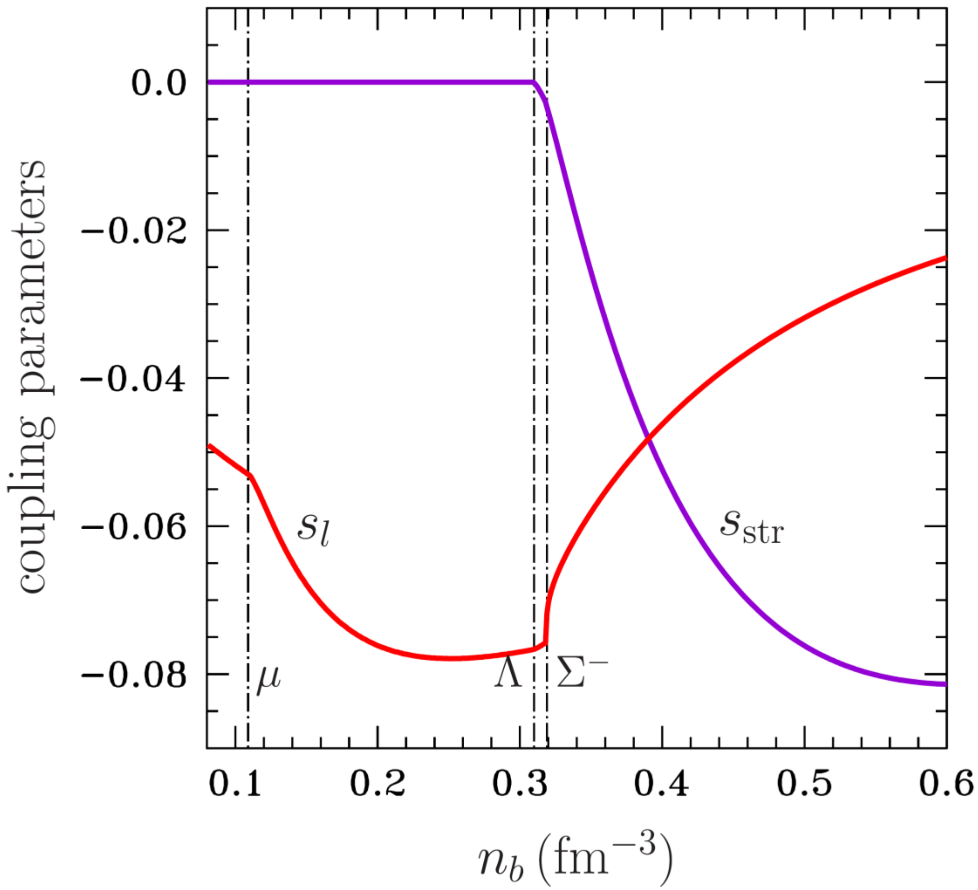
$$\boxed{M_{\max} = 1.988 M_{\odot}}$$

Three types of hyperons for stable neutron-star configurations: Λ, Ξ^{-}, and Σ^{-}

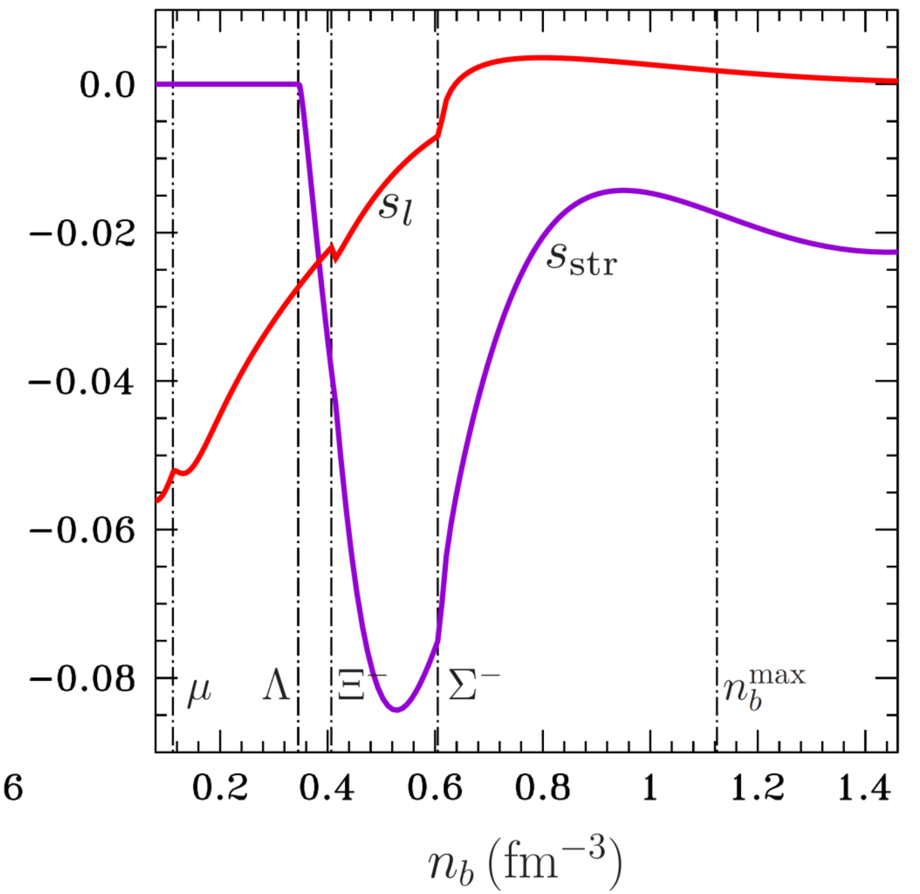
Based on TM1 parameter set

Coupling parameters

$\sigma\omega\rho$ -model (***EOS GIII***)



$\sigma\omega\rho\sigma^*\phi$ -model (***EOS MB16***)



$$|s_l|,\ |s_{\mathrm{str}}| < 0.1$$

Thus, the approximation of $s_l = 0$ and $s_{\mathrm{str}} = 0$ should be reasonable.

Let us illustrate this conclusion by considering sound waves in superfluid nucleon-hyperon matter

It can be shown that generally there are ***3*** sound modes in such matter (*Kantor & Gusakov, PRD, **79**, 043004, 2009*)

2 superfluid sound modes ***1*** normal mode

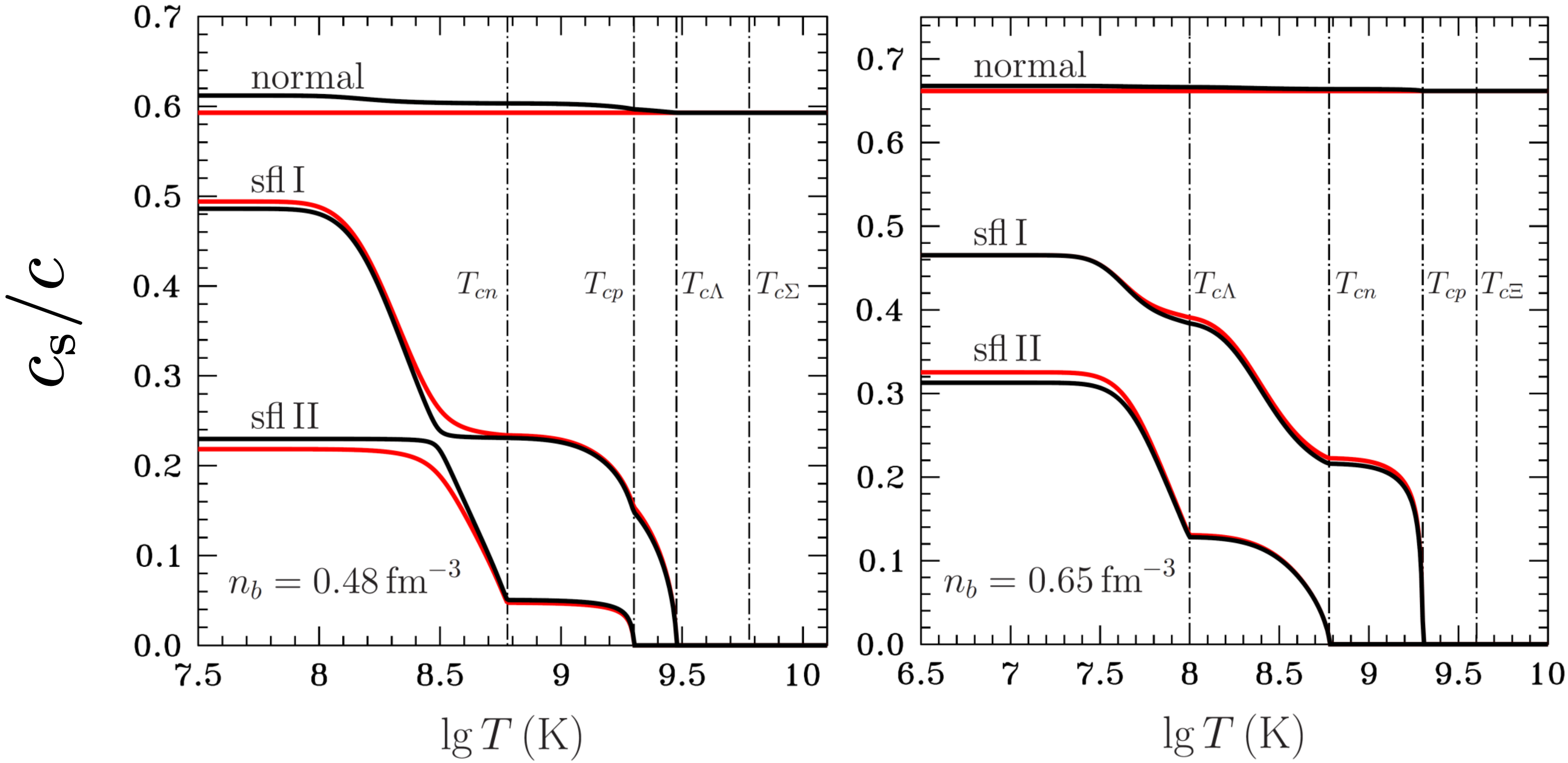
Speed of sound modes versus temperature

Exact solution to the system of coupled pulsation equations

Approximate solution of completely decoupled superfluid and normal equations

$\sigma\omega\rho$ -model (***EOS GIII***)

$\sigma\omega\rho\sigma^*\phi$ -model (***EOS MB16***)



The approximation of decoupled pulsation equations works well also for hyperon matter

Conclusions

• We demonstrate that equations governing oscillations of superfluid neutron and hyperon stars can be split into two systems of weakly coupled equations. One system of equations describes normal modes, another one – superfluid modes.

• The coupling of these systems is small for realistic EOSs. We have 1 coupling parameter for neutron stars ($|s| \sim 0.01 - 0.05$) and 2 coupling parameters for hyperon stars ($|s_l|,\ |s_{\rm str}| < 0.1$).

• Already an approximation of ***vanishing*** coupling parameters is sufficient to calculate the pulsation spectrum within accuracy of a few per cent.

• Pulsation spectra for normal modes can be calculated using the ordinary nonsuperfluid hydrodynamics. (However, to study ***dissipation*** of normal modes one has to take into account the specific superfluid dissipative terms! We know these terms since we can easily calculate the "superfluid" eigenfunctions using the method proposed in this talk.)

• The obtained results suggest a simple perturbative (in coupling parameters) scheme which considerably simplifies the problem of calculation of the pulsation spectrum for superfluid neutron and hyperon stars.

• The proposed approach allows to take into account realistic EOSs, temperature effects, dissipation, baryon superfluidity, density-dependent profiles of critical temperatures, and stellar rotation.

Properties of superfluid modes

- In the zero approximation ($s = 0$ or $s_l,\ s_{\rm str} = 0$) superfluid modes:

(a) do not perturb pressure and baryon current density;

(b) do not appear at the stellar surface
(localized in a superfluid region of a star);

(c) do not perturb metric (no gravitational radiation).

- These results indicate that superfluid modes should be very difficult to observe at small but finite coupling parameters. This means that observational properties of a pulsating superfluid star and a normal star of the same mass should be very similar, so that it will be very hard to discriminate one from the other.

- Gravitational radiation from the global superfluid modes is possible only in the next (first) order of perturbation theory in coupling parameters.

Thus, it should be suppressed in comparison to that from normal modes by a factor of $s^2 \sim 10^{-3}$ for neutron stars and $s^2_{l,\,\rm str} \sim 10^{-2}$ for hyperon stars.

For more details see: *M.E. Gusakov & E.M. Kantor*, ***PRD 83, 081304(R) (2011)***

and a poster by A.I. Chugunov & M.E. Gusakov ***at this conference!***

$$u^{\nu}\{\partial_{\mu}[w_{(n)\nu}+\mu_n u_{\nu}]-\partial_{\nu}[w_{(n)\mu}+\mu_n u_{\mu}]\}=O_{\mu\nu}w^{\nu}$$

Using this hydrodynamics one obtains the following superfluid equations

2 main equations: $i,\, k = n,\, p,\, \Lambda,\, \Xi,\, \Sigma$

$$\begin{cases} i\,\omega\,(\mu_i\, Y_{ik}\delta w_{(k)j} - n_b\,\delta w_{(n)j}) = \sqrt{-g_{00}}\;\;[n_e\;\partial_j(\delta\mu_e) + n_\mu\;\partial_j(\delta\mu_\mu) + n_{\rm str}\;\partial_j(\delta\mu_b)] \\ i\,\omega(\delta w_{(\Lambda)j} - \delta w_{(n)j}) = \partial_j(\sqrt{-g_{00}}\;\delta\mu_b) \end{cases}$$

and **3** supplementary equations independent of pulsation frequency ω :

$$\delta w_{(n)j} + \delta w_{(\Lambda)j} = \delta w_{(p)j} + \delta w_{(\Sigma)j}$$

consequence of the requirement of equilibrium with respect to the fast reaction $n + \Lambda \leftrightarrow p + \Sigma$

$$2\,\delta w_{(\Lambda)j} = \delta w_{(p)j} + \delta w_{(\Xi)j}$$

consequence of the requirement of equilibrium with respect to the fast reaction $\Lambda + \Lambda \leftrightarrow p + \Xi$

$$(Y_{pi} - Y_{\Sigma i} - Y_{\Xi i})\,\delta w^{\mu}_{(i)} = 0$$

consequence of charge neutrality

5 equations for **5** superfluid four-vectors: $\delta w^{\mu}_{(n)},\, \delta w^{\mu}_{(p)},\, \delta w^{\mu}_{(\Lambda)},\, \delta w^{\mu}_{(\Xi)},\, \delta w^{\mu}_{(\Sigma)}.$

Y_{ik} is the relativistic entrainment matrix, generalization of Y_{nn} to the case of superfluid mixtures [see the poster of ***M. Timofeeva et al.*** at this conference].

Following the same strategy as in the case of npe-matter, one can present the linearized Einstein equation in the form

$$\delta(R^{\mu\nu} - 1/2\ g^{\mu\nu}\ R) = 8\pi G\ \delta T^{\mu\nu}_{\rm norm} + \underline{\underline{s_l}} \times \left(\text{terms depending on } \sum_{i\in\text{baryons}} Y_{ik}\,\delta w^{\mu}_{(k)}\right)$$

$$+\underline{\underline{s_{\rm str}}} \times \left(\text{terms depending on } s_i\, Y_{ik}\,\delta w^{\mu}_{(k)}\right)$$

contains only the metric perturbations $\delta g^{\mu\nu}$

strangeness

$$\delta T^{\mu\nu}_{\rm norm} = \delta T^{\mu\nu}_{\rm norm}(\delta g^{\mu\nu}, \delta U^{\mu})$$

coincides with $\delta T^{\mu\nu}$ for normal matter

Two coupling parameters instead of one!

$$s_l \equiv \frac{n_e}{n_b}\frac{\partial P/\partial n_e}{\partial P/\partial n_b} + \frac{n_\mu}{n_b}\frac{\partial P/\partial n_\mu}{\partial P/\partial n_b}$$

"lepton" coupling parameter

$$s_{\rm str} \equiv \frac{n_{\rm str}}{n_b}\frac{\partial P/\partial n_{\rm str}}{\partial P/\partial n_b}$$

"strange" coupling parameter

Again, as for npe-matter, if $s_l = 0$ and $s_{\rm str} = 0$ then superfluid degrees of freedom are completely decoupled from metric perturbations $\delta g^{\mu\nu}$ and the baryon four-velocity δU^{μ}

Let 's look, whether these coupling parameters are small or not