Relativistic numerical models for stationary superfluid neutron stars

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MOTIVATIONS

- Interiors of neutron stars : very complex physical system, far beyond experimental capacities of Earth-based laboratories.
- Observations can be done in many parts of the electro-magnetic spectrum, from neutrinos and, possibly with gravitational wave emission.

⇒ Need for realistic (GR + microphysics) stationary models, used to determine some of the observable data (maximal rotation frequency, mass, ...).

⇒ Need for initial data for dynamical models: collapse to a black hole, oscillations and glitches (superfluidity, two-stream instability).

Inversely, having detailed models permits some "inversion" of observational data to infer composition of neutron stars and very dense matter properties (*e.g.* if gravitational waves from oscillations are observed).

Model

Two-fluids model

NEUTRONS

Superfluid neutrons in the crust and the outer core. No viscosity, so they can flow freely through the other component.

"PROTONS"

Nuclei, electrons, muons and protons are locked together on short timescales by viscosity and magnetic field.

$\Rightarrow$ these two components are coupled together by strong nuclear force.

- General Relativity for the gravitational field
- stationarity and axisymmetry
- uniform rotation of both components / common axis, but different rotation rates.

Relativistic gravity

Bonazzola *et al.* 1993

Need for General Relativity?

- deviation from Newton's law given by the compactness

$$\frac{2GM}{Rc^2} \sim 0.3 - 0.4 \text{ for cold neutron stars,}$$

- notion of maximal mass does not appear in Newtonian gravity,
- Tolmann-Oppenheimer-Volkoff (spherical symmetry) system easy to solve numerically,
- other tools publicly available for rotating stars (`rotstar`/LORENE, `rns`)

* stationary, axially symmetric system $\Rightarrow$4 coupled, Poisson-like, non-linear PDEs for the gravitational field

* no local notion of mass or angular momentum

Two-fluid hydrodynamics

from Carter, Langlois, *et al.*

- For each fluid define the conserved 4-current n_{n}^{μ} and n_{p}^{μ},
- The Lagrangian density $\Lambda = -\mathcal{E}$ depends only on the three possible scalar products between these 4-vectors.
- Define momenta as conjugates of currents:

$$\mathrm{d}\Lambda = p_{\mu}^{\mathrm{n}}\mathrm{d}n_{\mathrm{n}}^{\mu} + p_{\mu}^{\mathrm{p}}\mathrm{d}n_{\mathrm{p}}^{\mu}.$$

- The equations of motions (in the absence of direct dissipative forces) are:

$$n_{\mathrm{n}}^{\mu}\nabla_{[\mu}p_{\mu]}^{\mathrm{n}} = 0 \text{ and } n_{\mathrm{p}}^{\mu}\nabla_{[\mu}p_{\mu]}^{\mathrm{p}} = 0$$

- The stress-energy tensor $T_{\mu}{}^{\nu} = p_{\mu}^{\mathrm{n}}n_{\mathrm{n}}^{\nu} + p_{\mu}^{\mathrm{p}}n_{\mathrm{p}}^{\nu} + \Psi\delta_{\mu}{}^{\nu}$,
- with the generalized pressure $\Psi = -\mathcal{E} - p_{\mu}^{\mathrm{n}}n_{\mathrm{n}}^{\mu} - p_{\mu}^{\mathrm{p}}n_{\mathrm{p}}^{\mu}$

Equation of state

The EOS depends only on densities and "relative speed" Δ: $\mathcal{E}(n_\mathrm{n}, n_\mathrm{p}, \Delta^2)$, and the first law of thermodynamics reads (defining the chemical potentials μ^n and μ^p)

$$\mathrm{d}\mathcal{E} = \mu^\mathrm{n}\mathrm{d}n_\mathrm{n} + \mu^p\mathrm{d}n_\mathrm{p} + e\,\mathrm{d}\Delta^2,$$

and the equations of motion take the integral form:

$$\frac{N}{\Gamma_\mathrm{n}}\mu^\mathrm{n} = C^\mathrm{n} \text{ and } \frac{N}{\Gamma_\mathrm{p}}\mu^\mathrm{p} = C^\mathrm{p}$$

We have used a simple (2-fluid polytrope) EOS

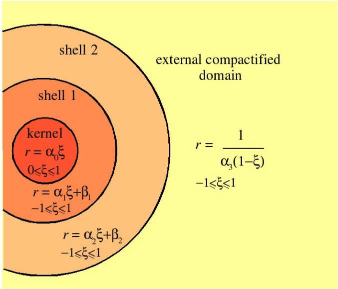
$$\mathcal{E} = \rho c^2 + \frac{1}{2}\kappa_\mathrm{n} n_\mathrm{n}^2 + \frac{1}{2}\kappa_\mathrm{p} n_\mathrm{p}^2 + \kappa_\mathrm{np} n_\mathrm{n} n_\mathrm{p} + \kappa_\Delta n_\mathrm{n} n_\mathrm{p} \Delta^2.$$

$\Rightarrow$all physical features: entrainment + symmetry energy, and the inversion $(\mu^\mathrm{n}, \mu^\mathrm{p}) \leftrightarrow (n_\mathrm{n}, n_\mathrm{p})$ is made easy (linear system).

Numerical methods

spectral methods (Grandclément & Novak 2009)

Need: solve Poisson-like PDEs with sources of non-compact support.
$\Rightarrow$use a linear Poisson solver with iteration and relaxation.



Decomposition $f(r,\theta)$:

Chebyshev polynomials for ξ,
Spherical harmonics $Y_\ell(\theta)$ for the angular part.

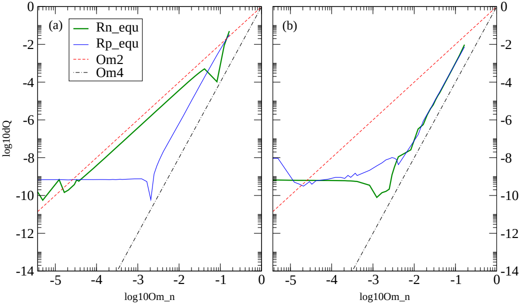
- symmetries and coordinate singularity at the origin and on the axis of spherical coordinates
- compactified variable for elliptic PDEs $\Rightarrow$boundary conditions are well imposed

Crude initial guess $\longrightarrow$ $T^{\mu\nu}$ $\xrightarrow{\text{grav. eq.}}$ metric $\xrightarrow{\text{eq. of motion}}$ $(\mu^{\mathrm{n}}, \mu^{\mathrm{p}})$ $\xrightarrow{\text{EOS}}$ $T^{\mu\nu} \dots$

Comparison to previous works

Most models have been devised in the "slow-rotation" approximation:

- Prix *et al.* 2002 in the Newtonian regime,
- Anderson & Comer 2001 in Relativistic theory.



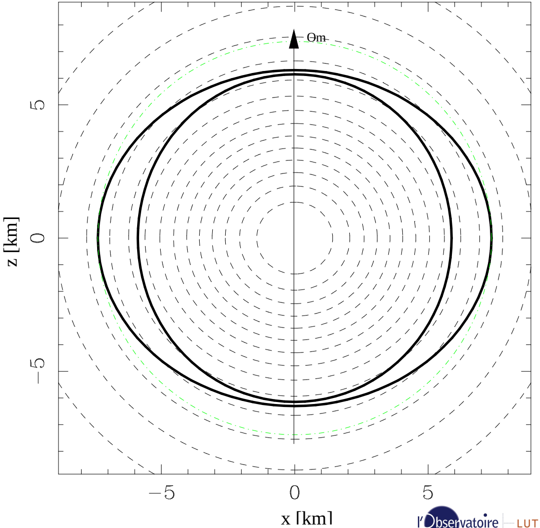
In the Newtonian case, one can obtain an analytical expression for the solution and, depending on the type of EOS inversion, the behavior of the difference as a function of Ω is recovered.

In the relativistic case, the agreement on gauge-independent quantities ranges from 10^{-4} to a few percent, depending on the rotation rate.

Results

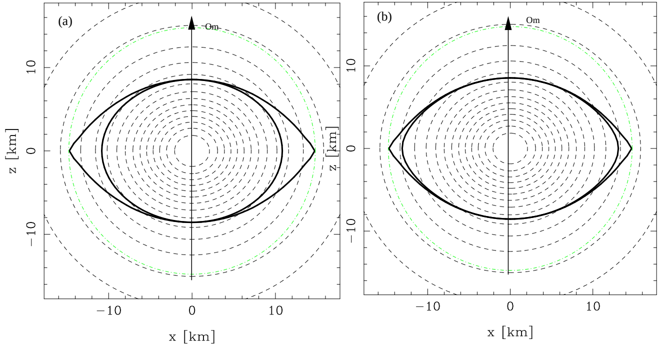
oblate / prolate configurations

- It is possible with non-realistic parameters to get a configuration with one fluid surface having oblate shape, while the other has a prolate one.
- made possible by counter-rotation and the effective interaction potential, which tends to “separate” both fluids.



Results

Kepler limit



- Slow-rotation approximations overestimate the Kepler frequencies by $\sim 15\%$.
- If no chemical equilibrium at the center, the Kepler limit is determined by the outer fluid, even if it is rotating slower than the inner one.

Outlook

- Allow for differential rotation of superfluid component.
- Need for more realistic nuclear-physics EOS, particularly for the entrainment term (Anderson *et al.* 2006, Goriely *et al.* 2010).
- What about mutual friction in such situations?
- Add a solid crust…
- Study the dynamical evolution: oscillation modes and gravitational wave emission.

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