Time-dependent simulations of force-free magnetospheres with *Phaedra*

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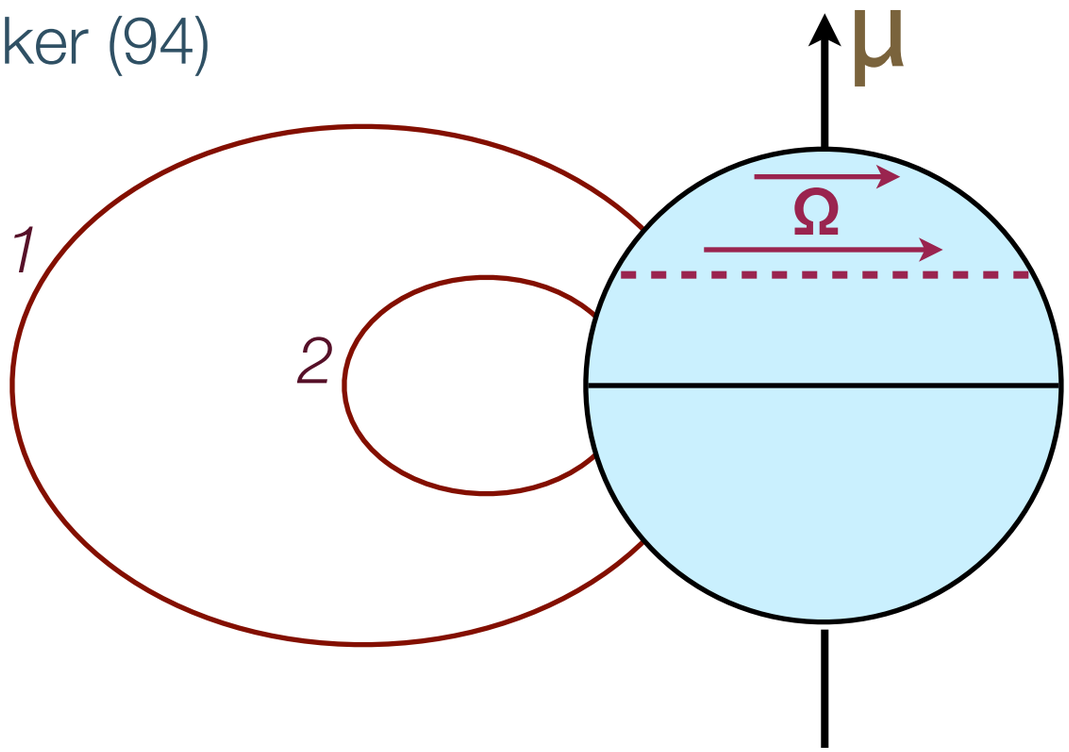
Dynamic magnetospheres

- Rotation-powered pulsars (Contopoulos et al 99, Spitkovsky 06)
- Magnetars — SGRs/AXPs: driven by crustal motions (Thompson & Duncan 95)?
  - Self-similar solutions : Low (86), Wolfson (95)
    — problematic since they involve compressive motion of crust
  - Solar corona simulations : Mikic & Linker (94)



Force-free electrodynamics

- Strong-field limit of magnetised fluids (Gruzinov 99)

$$\partial_t \vec{B} = -\nabla \times \vec{E}\,, \qquad \partial_t \vec{E} = \nabla \times \vec{B} - \vec{J}$$

$$\vec{J} = \frac{\vec{B}\cdot(\nabla\times\vec{B}) - \vec{E}\cdot(\nabla\times\vec{E})}{B^2}\vec{B} + (\nabla\cdot\vec{E})\frac{\vec{E}\times\vec{B}}{B^2}$$

$$\vec{E}\cdot\vec{B} = 0$$

$$\vec{E}\cdot\vec{J} = 0 \ \text{: no dissipation}$$

- Current sheets (discontinuities in magnetic field) are common

A spectral code for force-free electrodynamics

- *Phaedra*: *P*seudospectral *H*igh-*A*ccuracy *E*lectro*D*ynamics for *R*elativistic *A*strophysics — 2D/axisymmetric

- Existing codes: Komissarov (2004), Spitkovsky (2006), McKinney (2006), Kalapotharakos & Contopoulos (2009), Yu (2010)

- Spatial derivatives via expansion into orthogonal basis functions

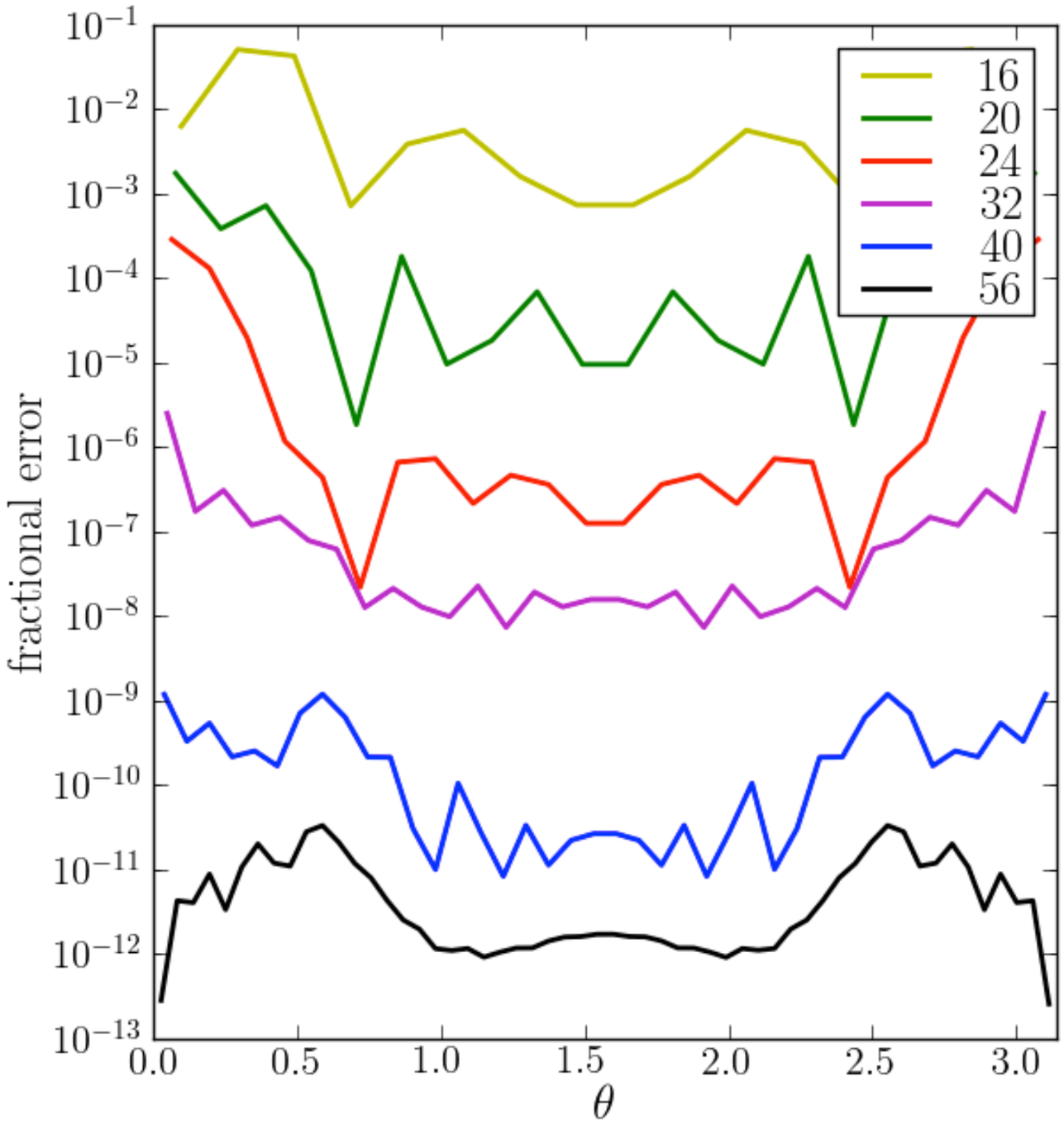
$$B^i = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} a_{nl}\, T_n(r) \begin{cases} \cos(l\theta) \\ \sin(l\theta) \end{cases}$$

- High-order spectral filtering for “shock” capturing

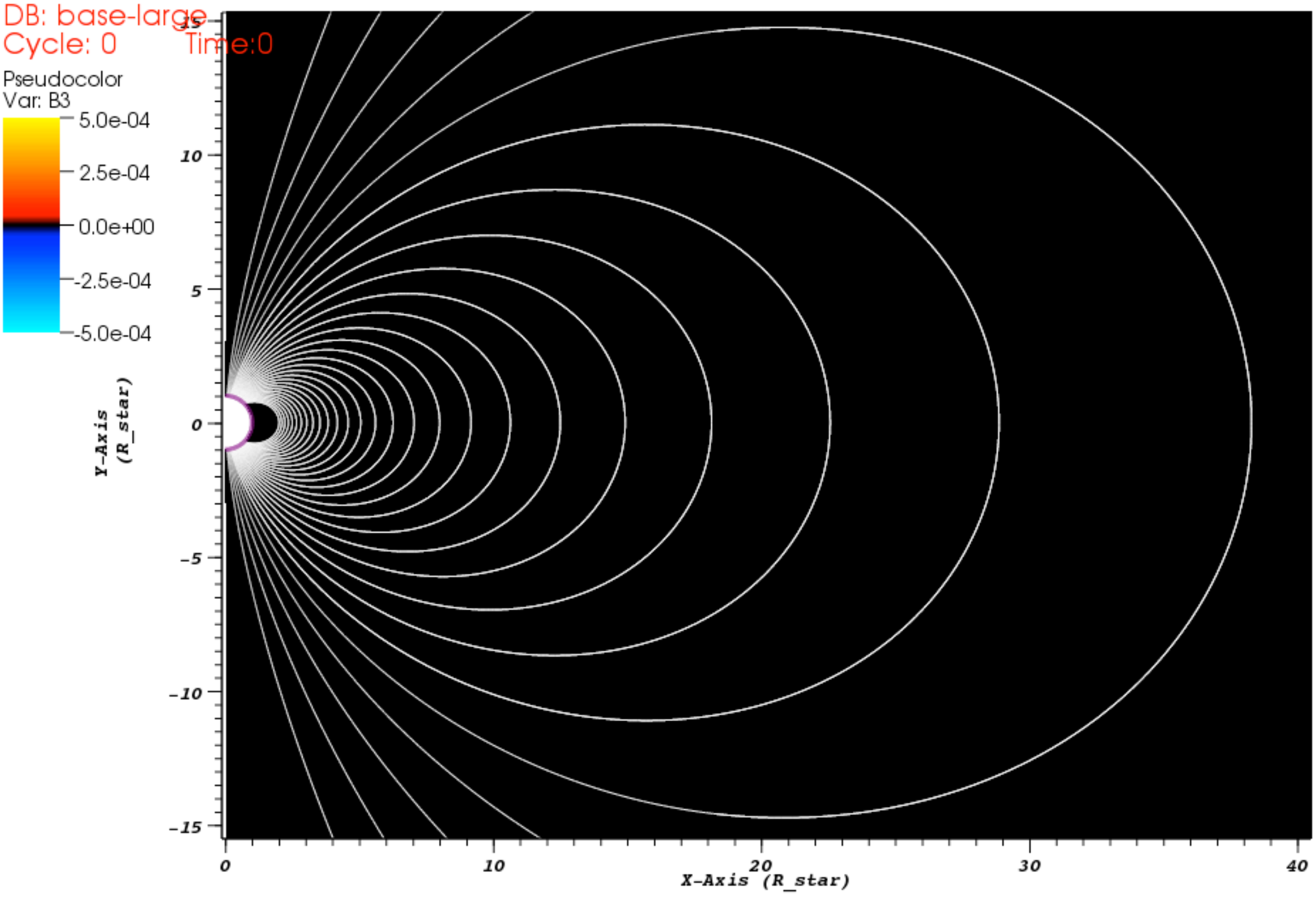
- Very low dispersion error or intrinsic dissipation

Simple 2D test: rotating monopole

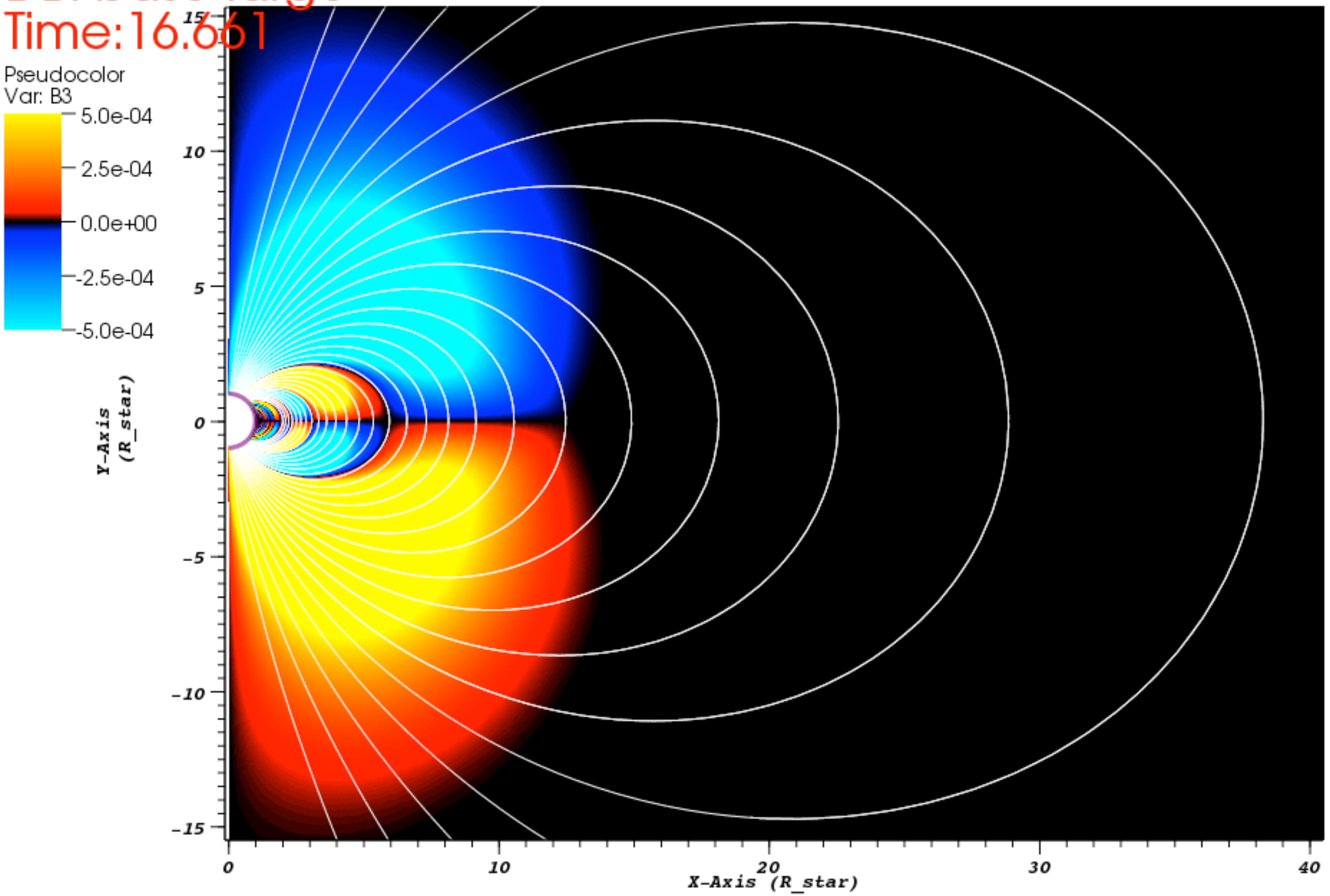
Resolution
N_θ given in legend
$N_r \sim 1\frac{1}{2} N_\theta$



Fractional errors in B^φ at $5R_\star$ versus analytic steady-state solution (Michel 72)

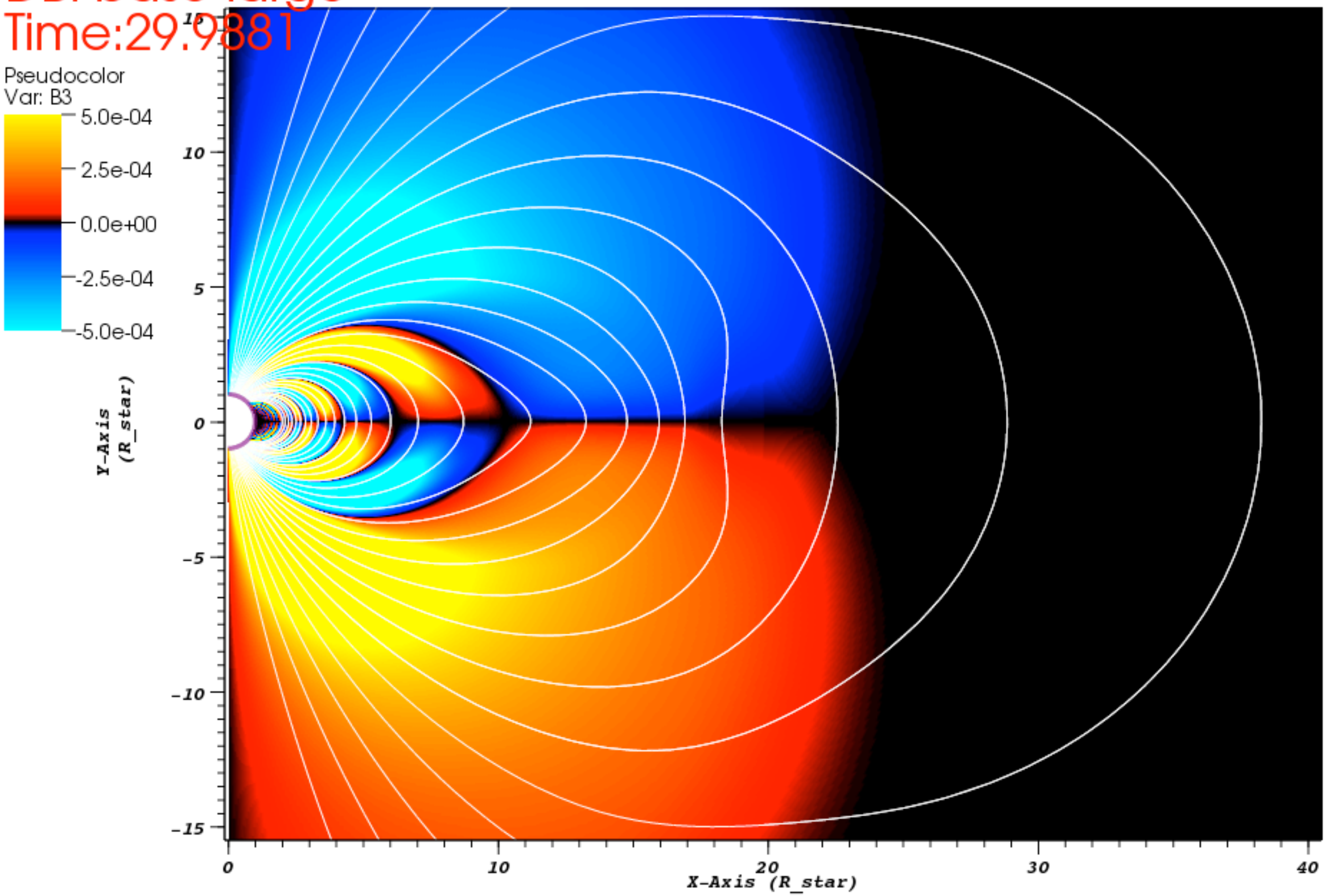


Rotating dipole — axisymmetric pulsar — light cylinder at 20 stellar radii
Shown: B^{ϕ}, poloidal field line projections

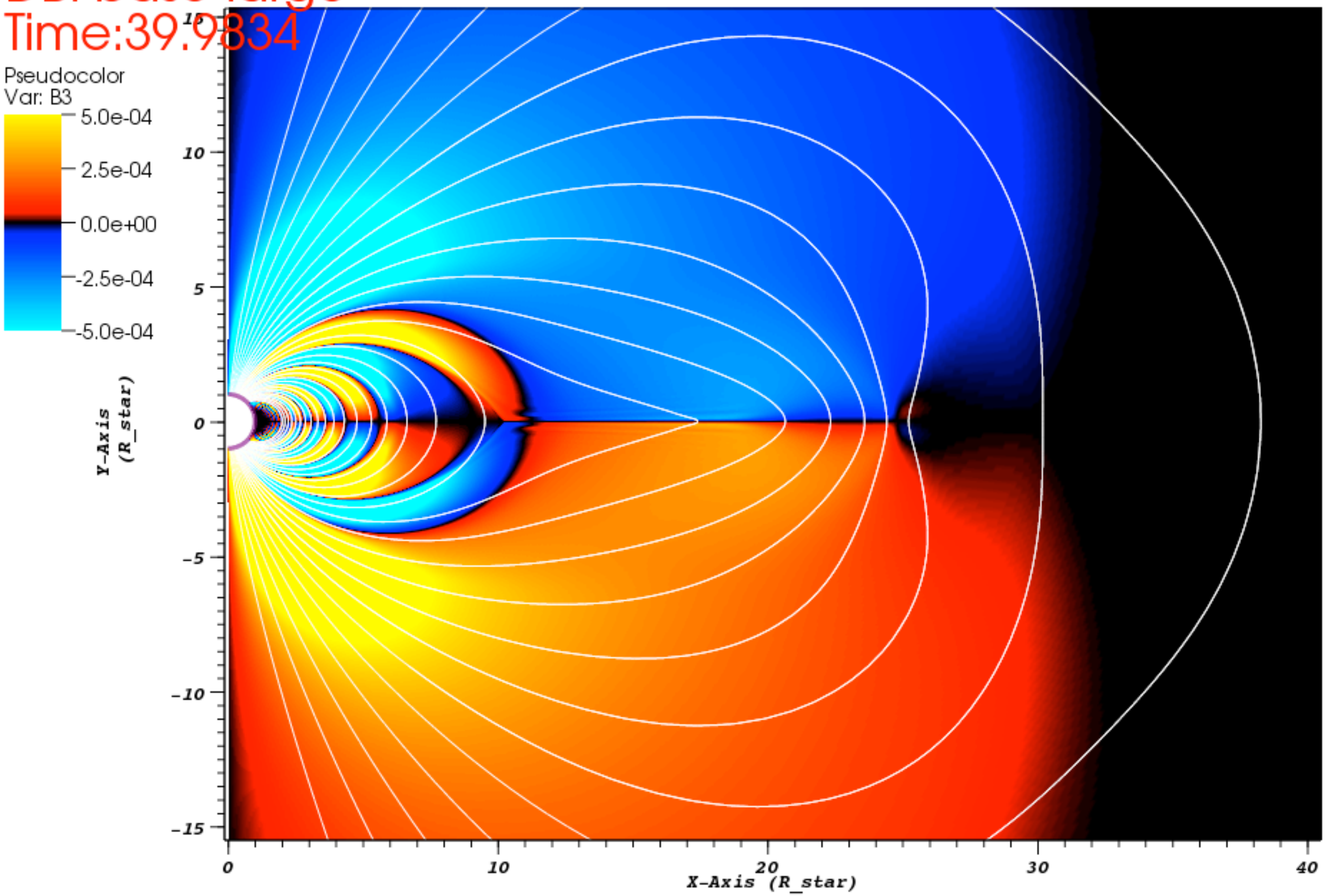


Rotating dipole — axisymmetric pulsar
Shown: B^{ϕ}, poloidal field line projections

DB: base-large
Time:29.9881

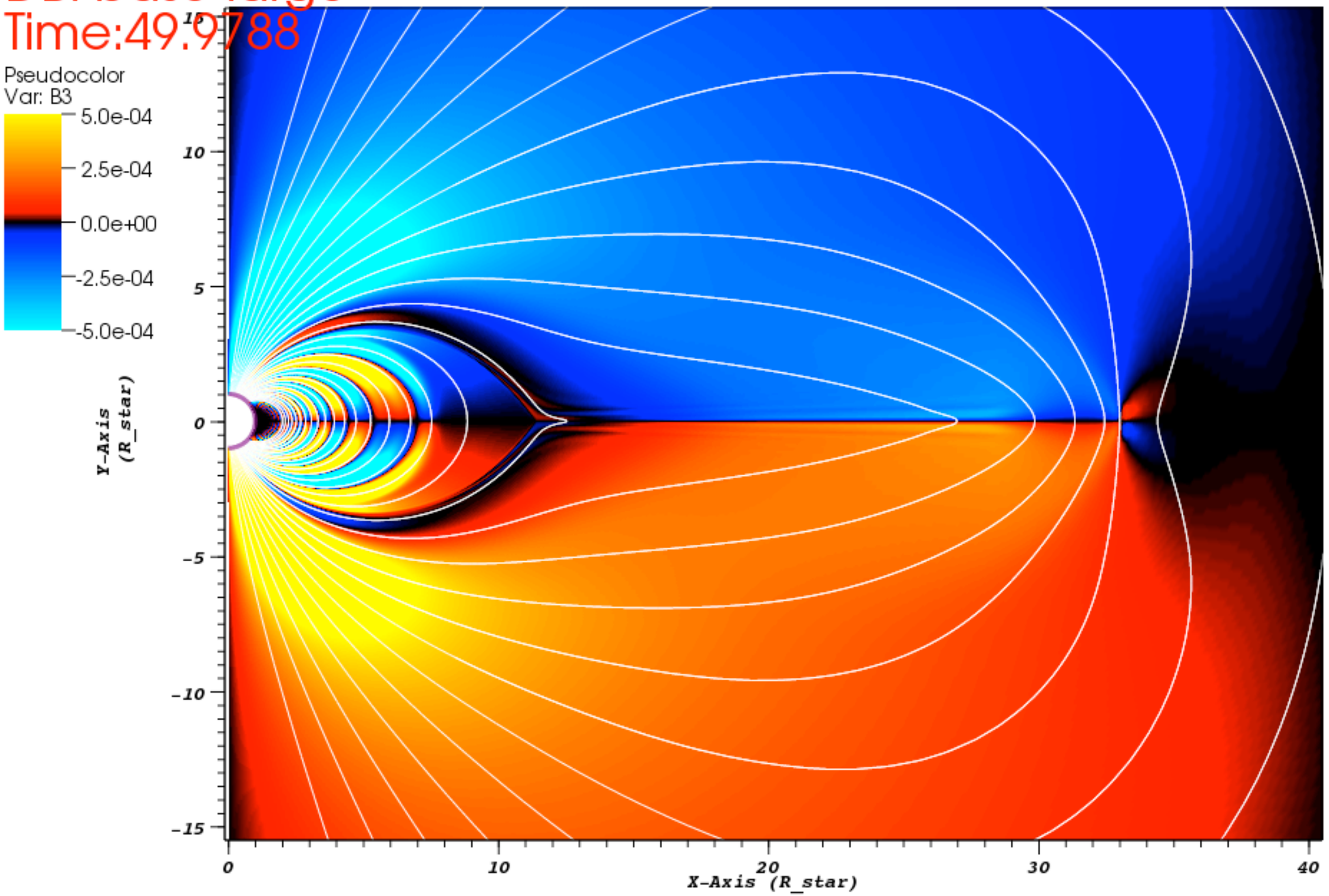


Rotating dipole — axisymmetric pulsar
Shown: B^{ϕ}, poloidal field line projections

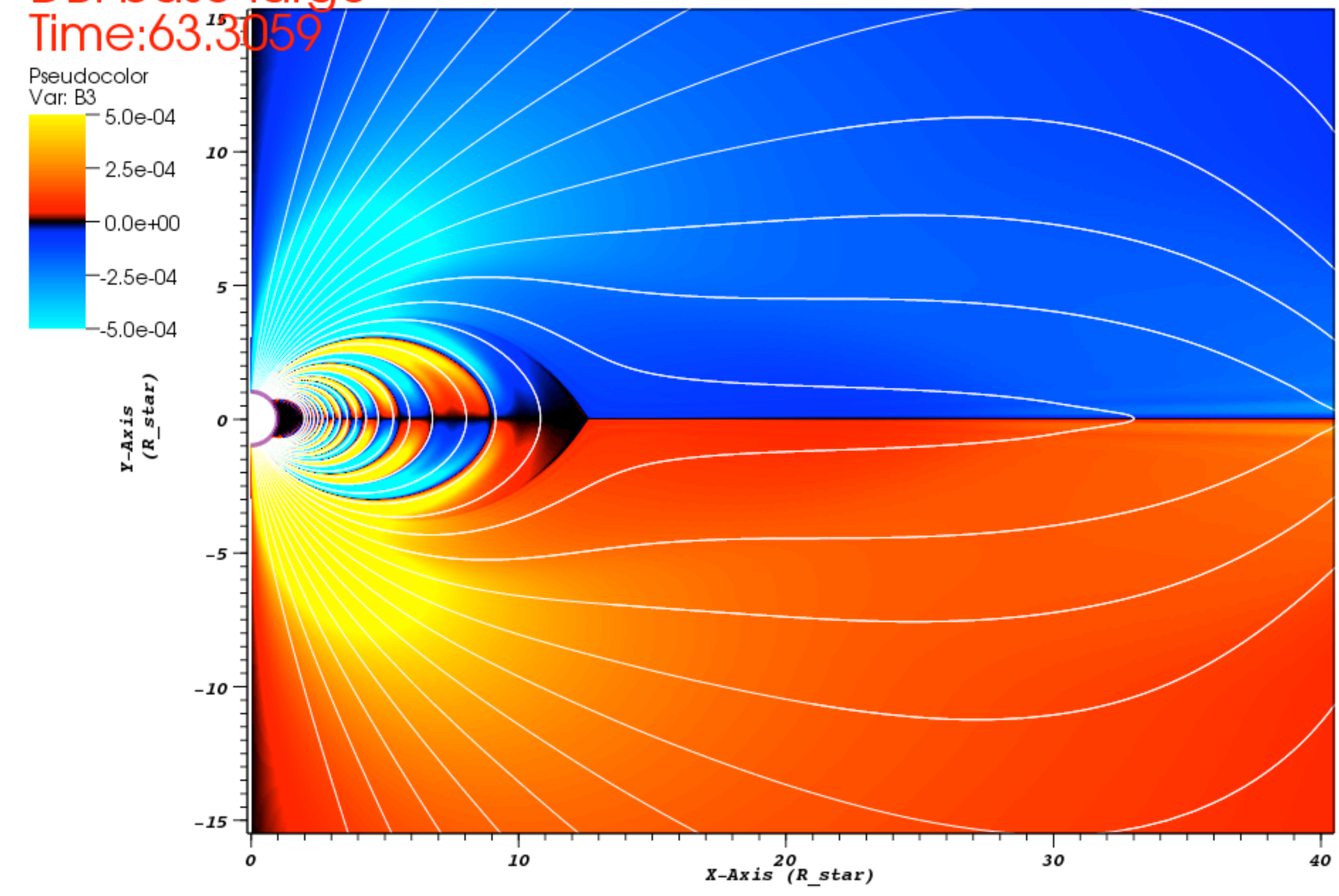


Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^{ϕ}, poloidal field line projections

DB: base-large
Time:49.9788

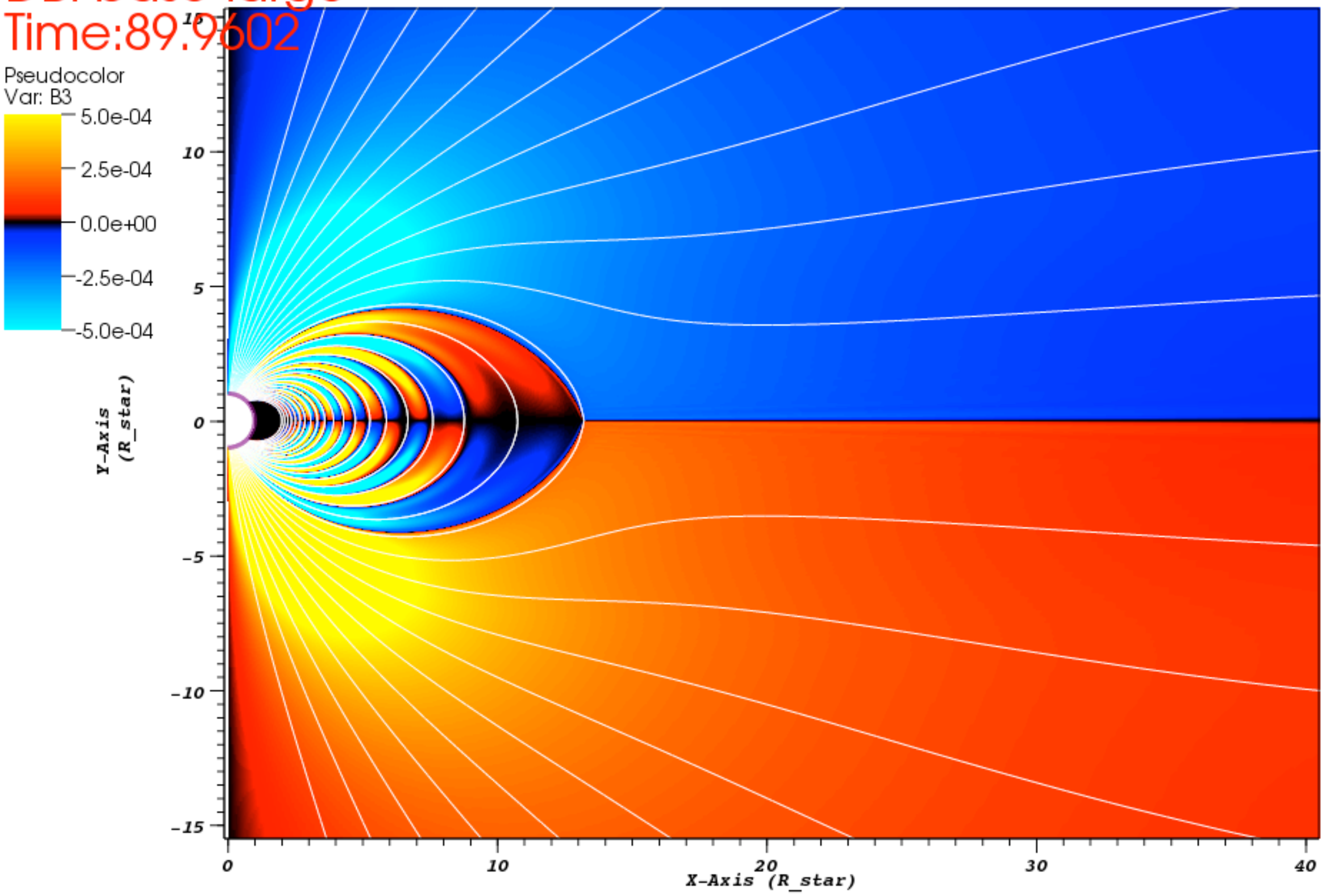


Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^{ϕ}, poloidal field line projections

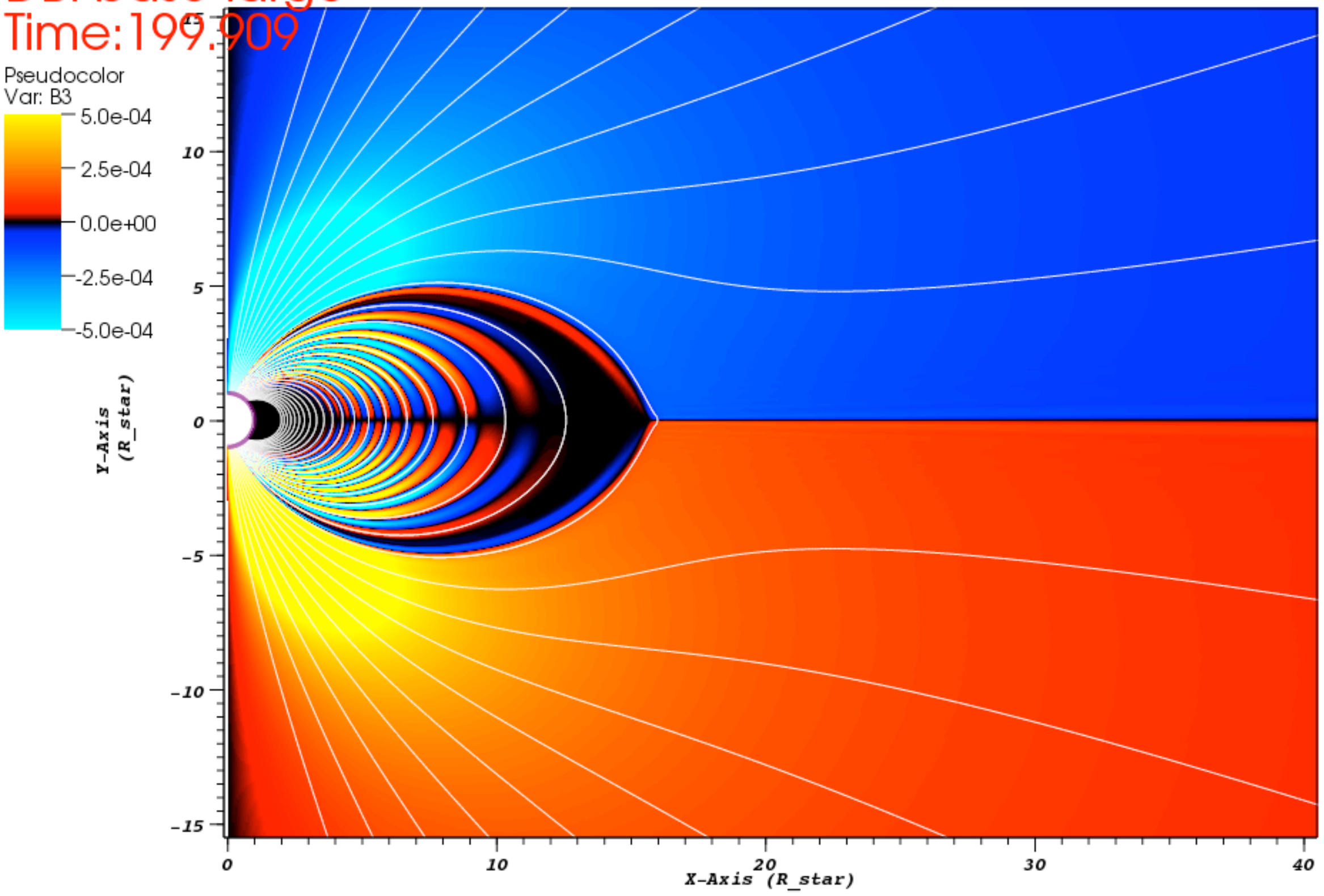


Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^{ϕ}, poloidal field line projections

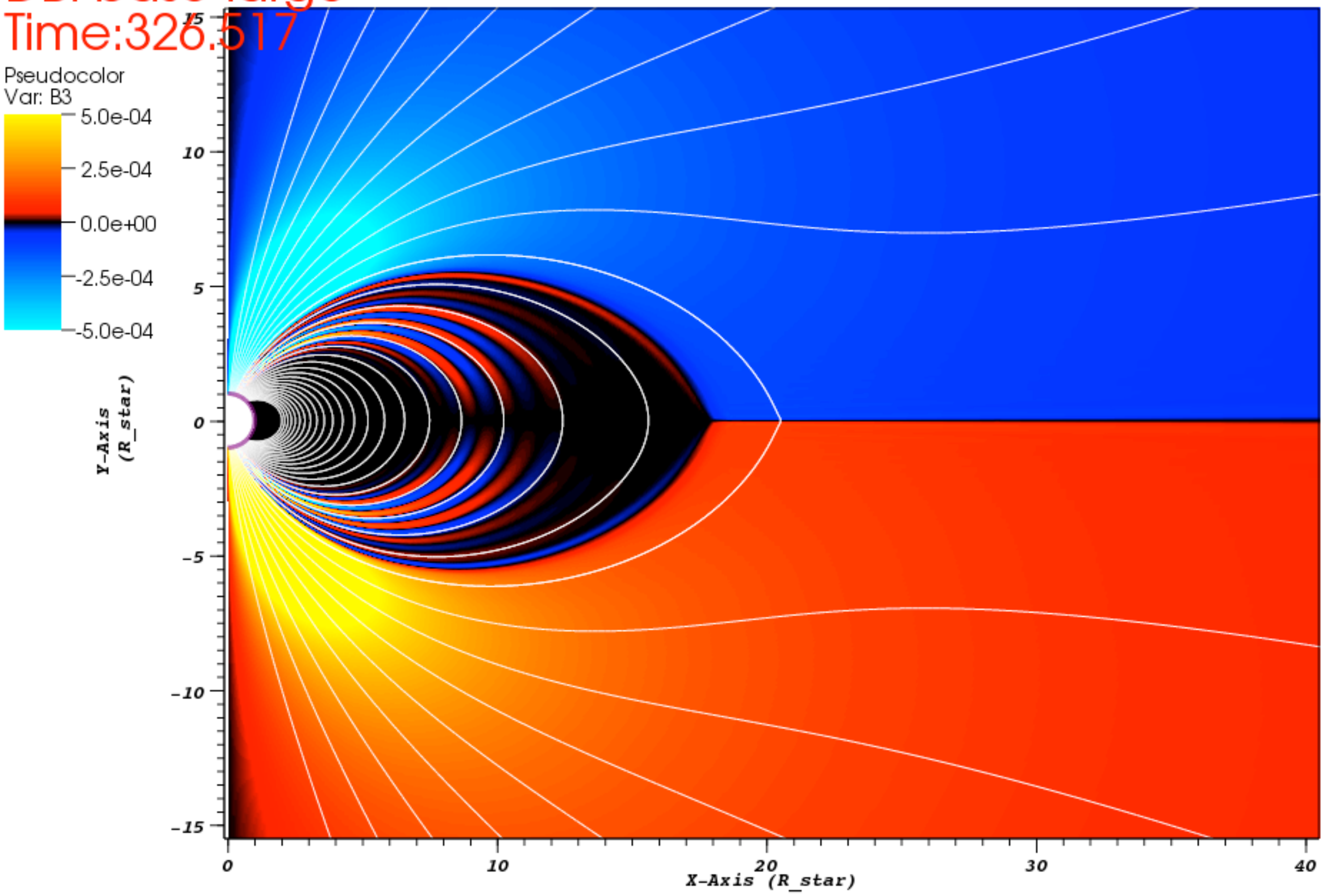
DB: base-large
Time:89.9602



Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^{ϕ}, poloidal field line projections

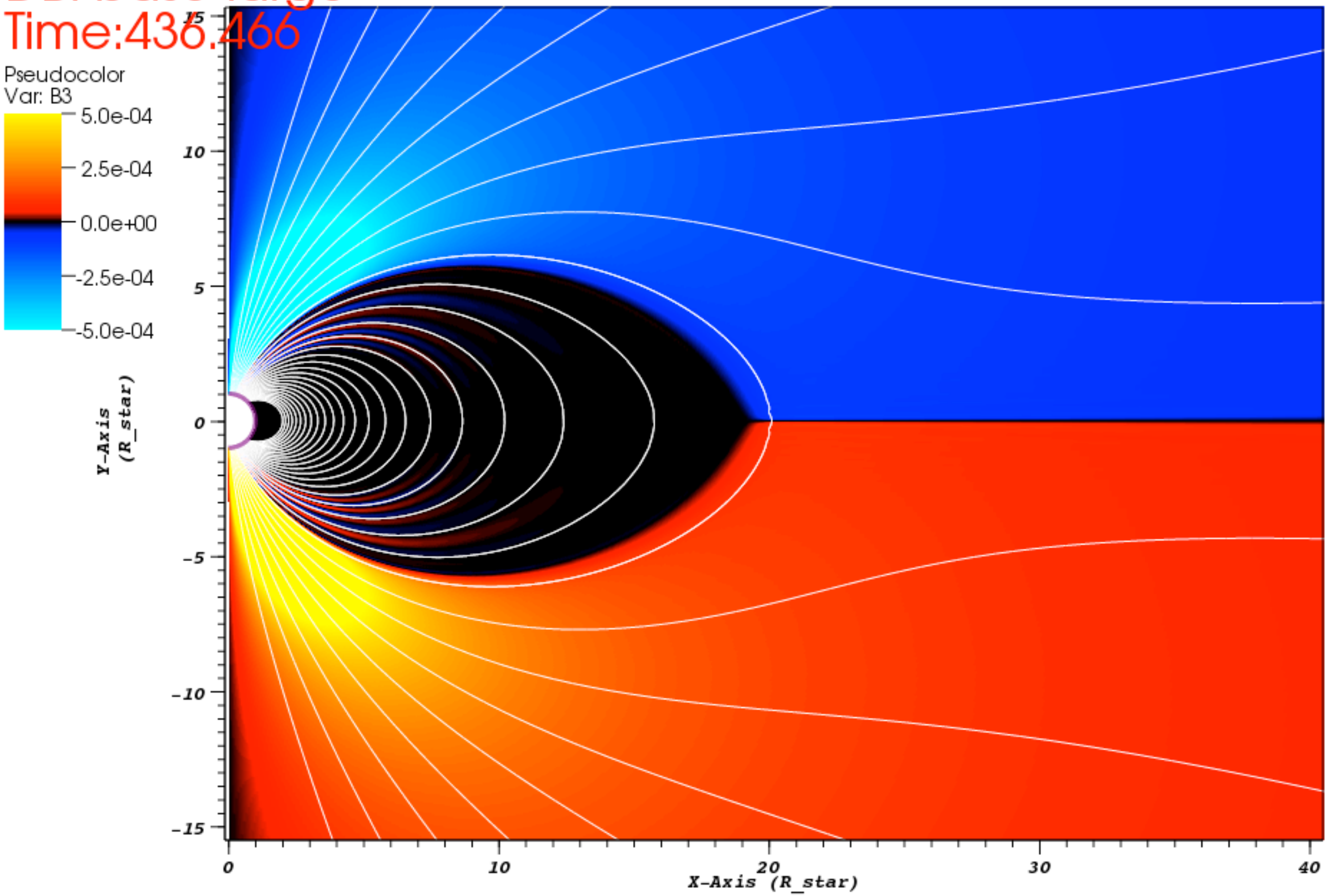


Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^{ϕ}, poloidal field line projections

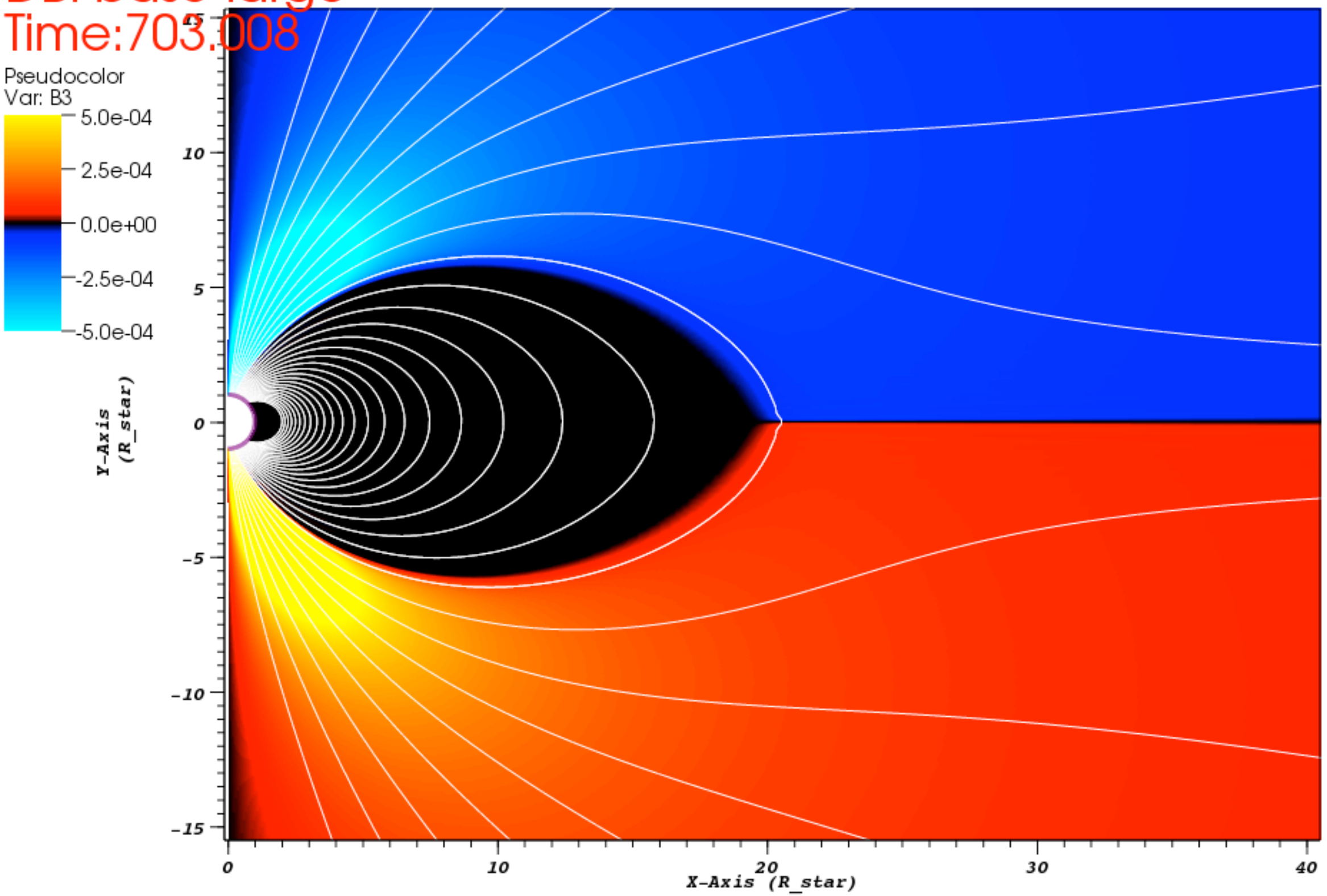


Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^{ϕ}, poloidal field line projections

DB: base-large
Time:436.466

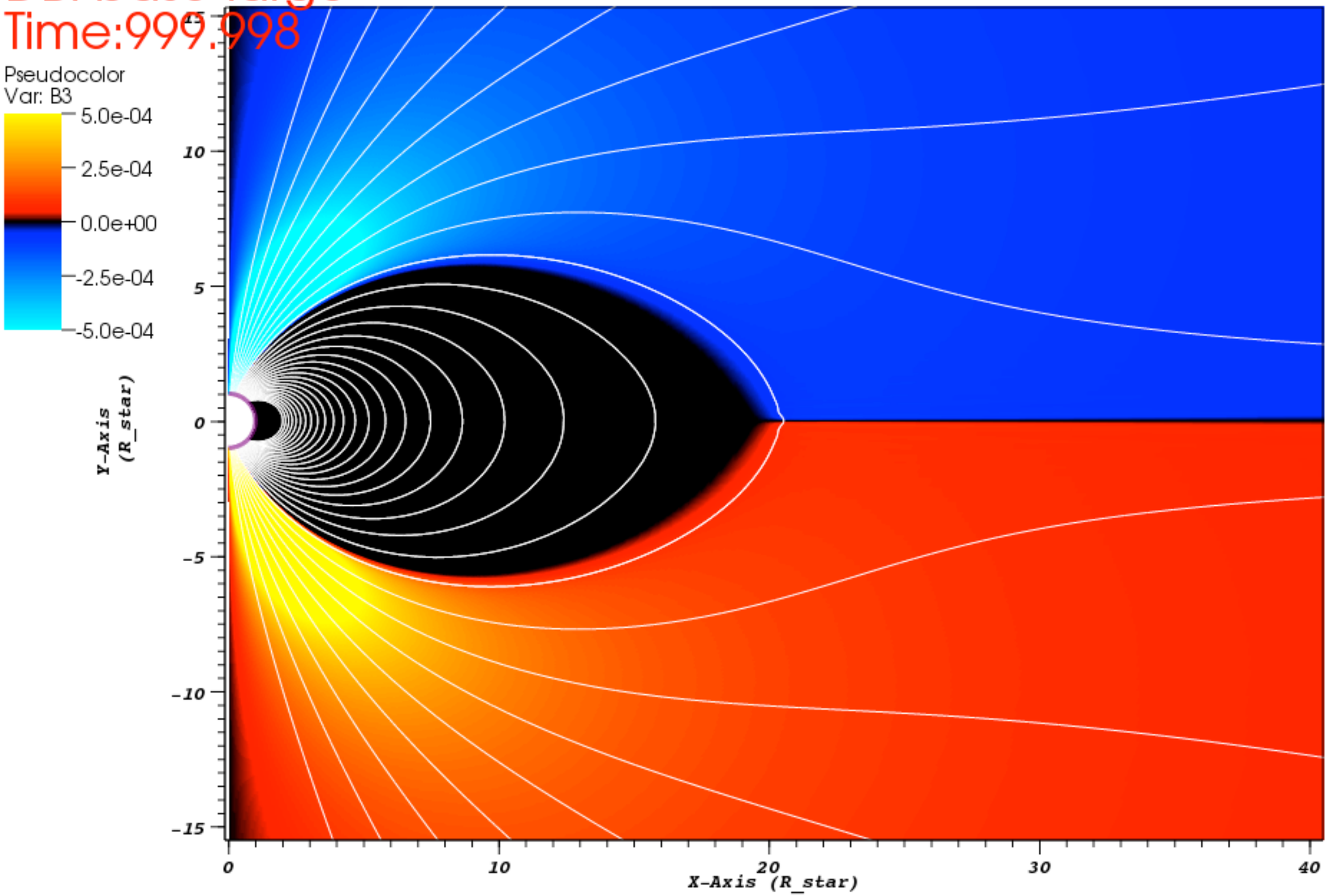


Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^{Φ}, poloidal field line projections

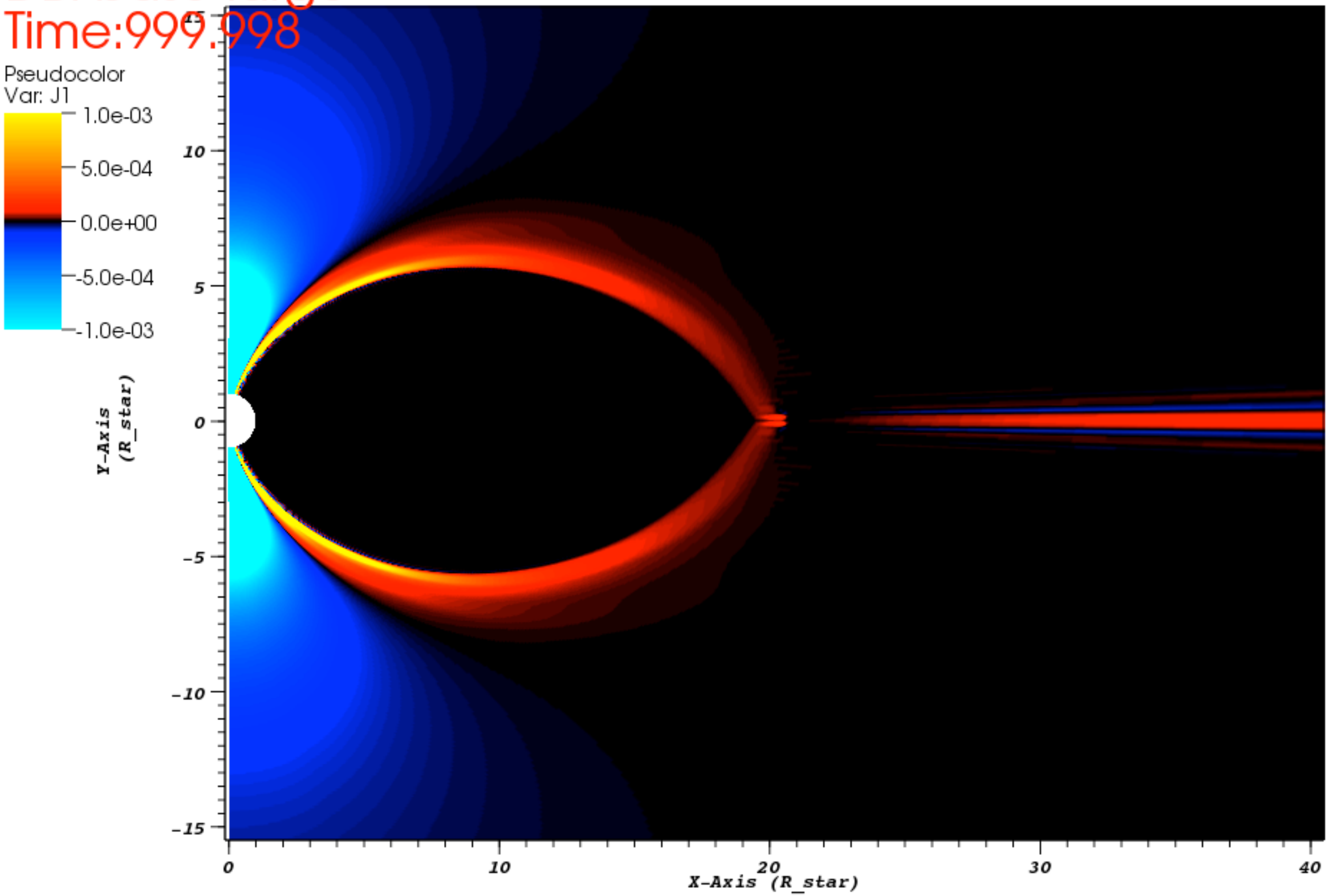


Rotating dipole — axisymmetric pulsar — steady-state
Shown: B^{ϕ}, poloidal field line projections

DB: base-large
Time:999.998



Rotating dipole — axisymmetric pulsar — steady-state
Shown: B^{ϕ}, poloidal field line projections



Rotating dipole — axisymmetric pulsar — steady-state current
Shown: J^r

Twisted magnetospheres

- Motion of a neutron star's crust will twist or deform its magnetosphere.

- Differential rotation of stellar surface: Ω(θ) in

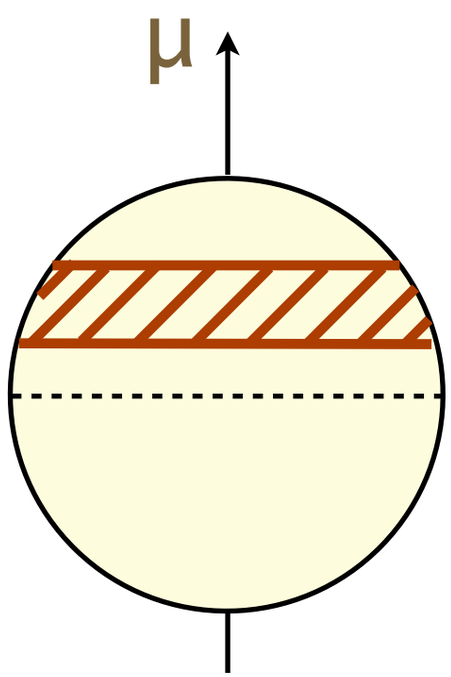
$$\vec{E} = -\left(\Omega \times \vec{r}\right) \times \vec{B}$$

  - Poloidal currents ejected from star → toroidal magnetic field

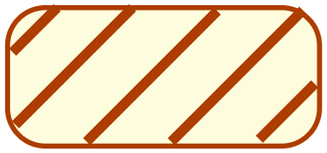
  - X-ray emission via resonant cyclotron scattering

- Reconnection if field is over-twisted : explosive energy release

Twisting profiles

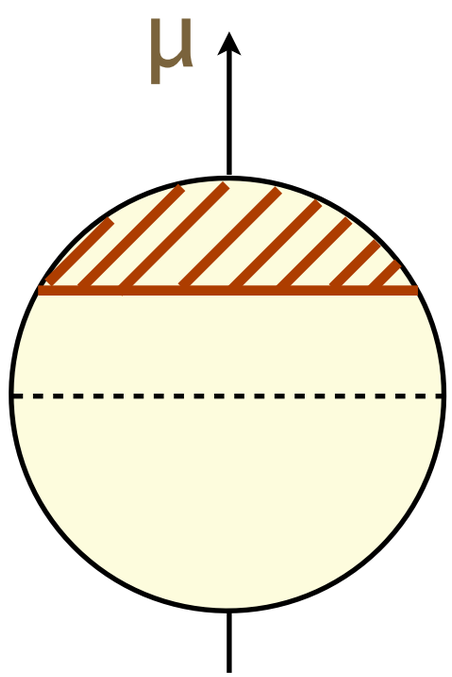


annulus

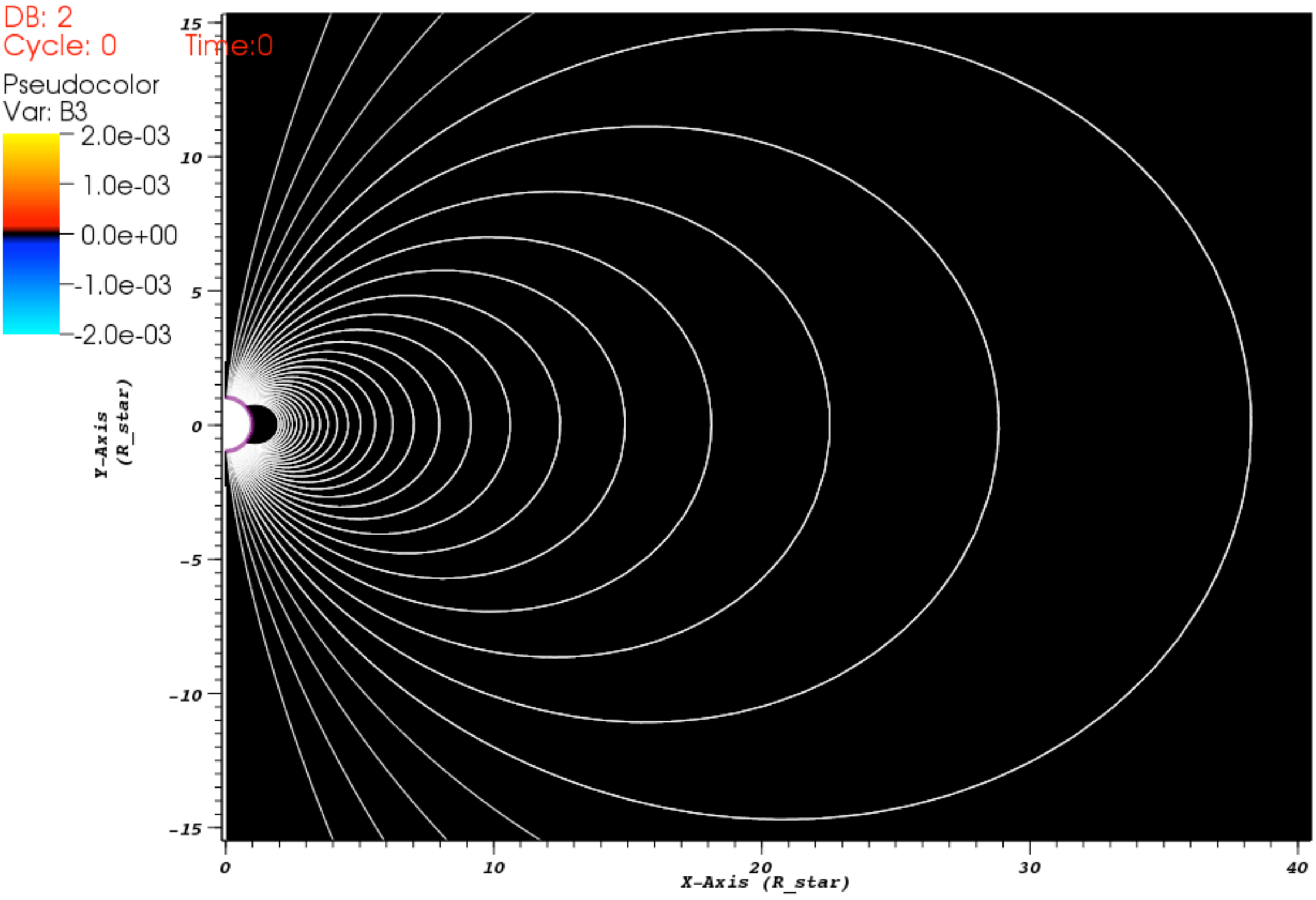


twisted region

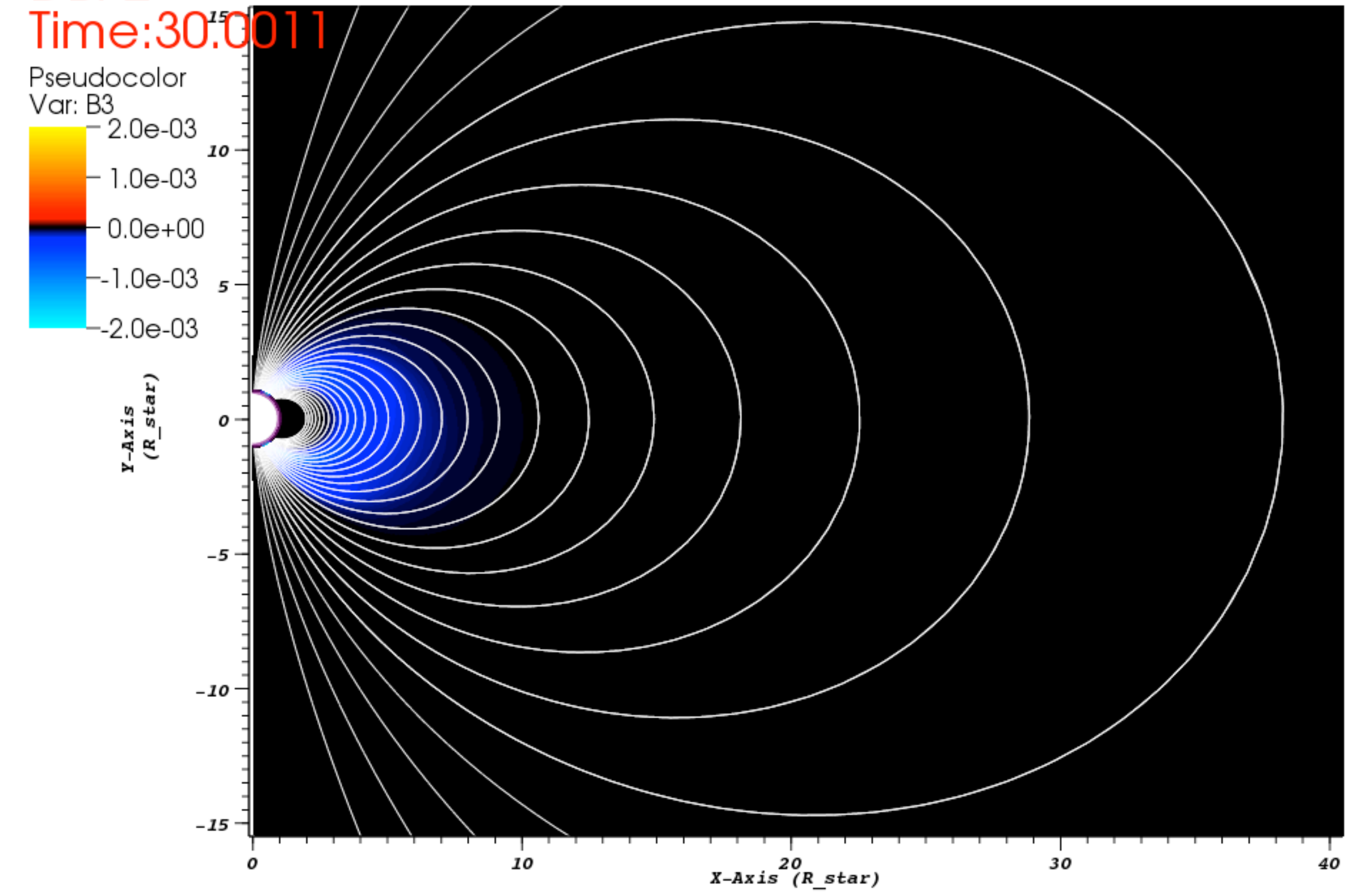
Rotates at Ω_{twist} with respect to rest of star.



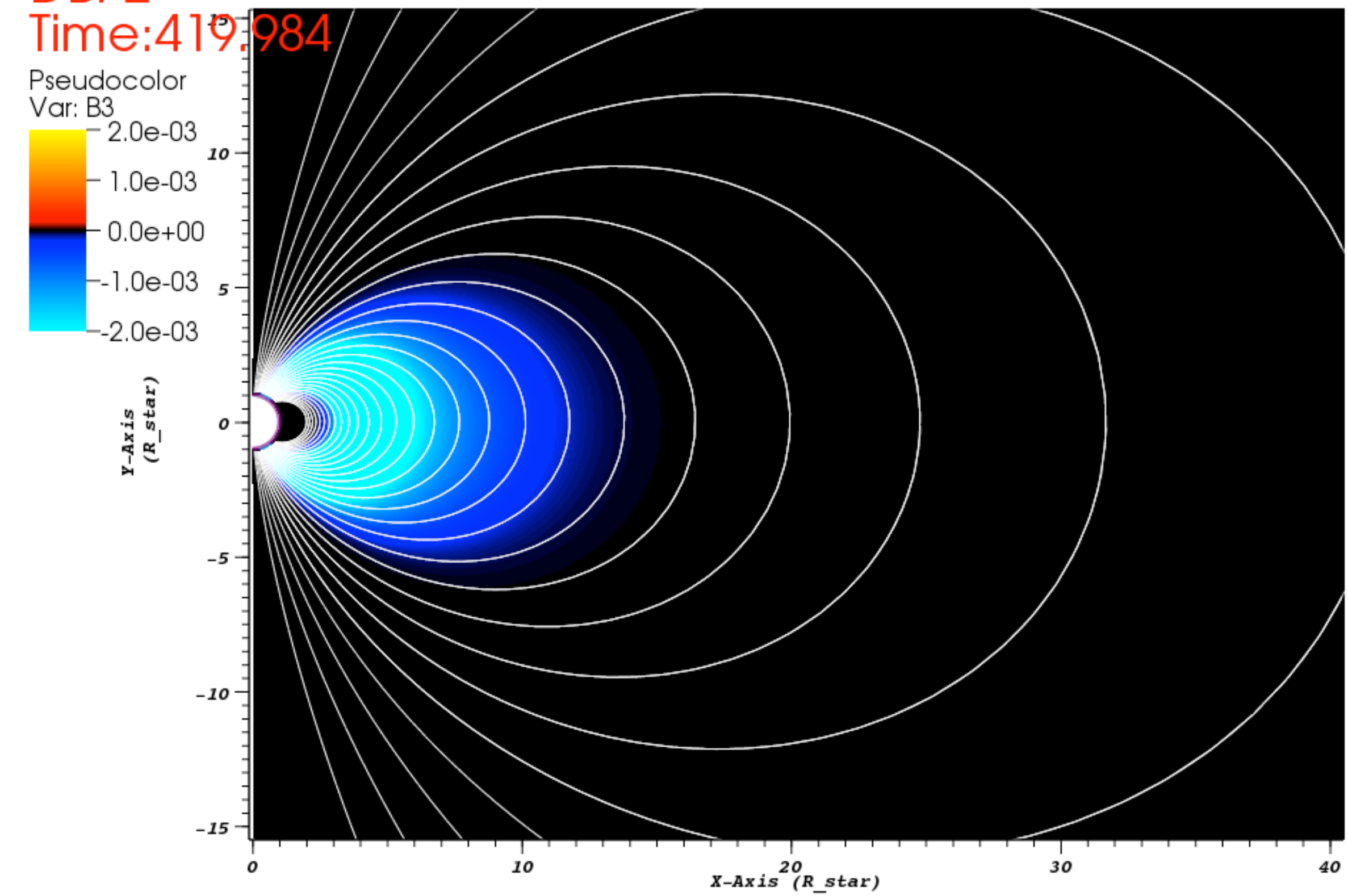
polar cap



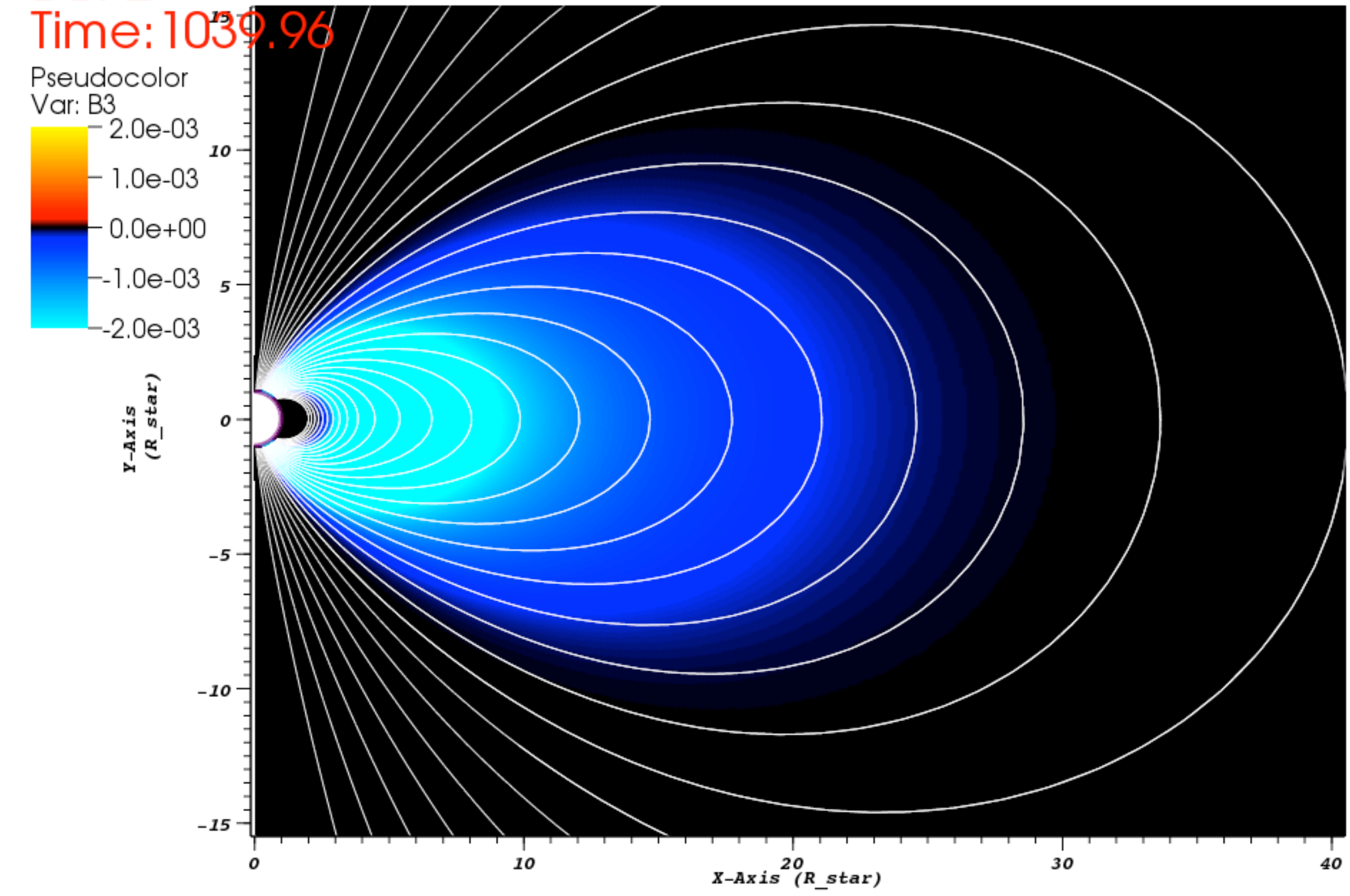
Twisted magnetosphere — annulus profile
Shown: B^{Φ}, poloidal field line projections



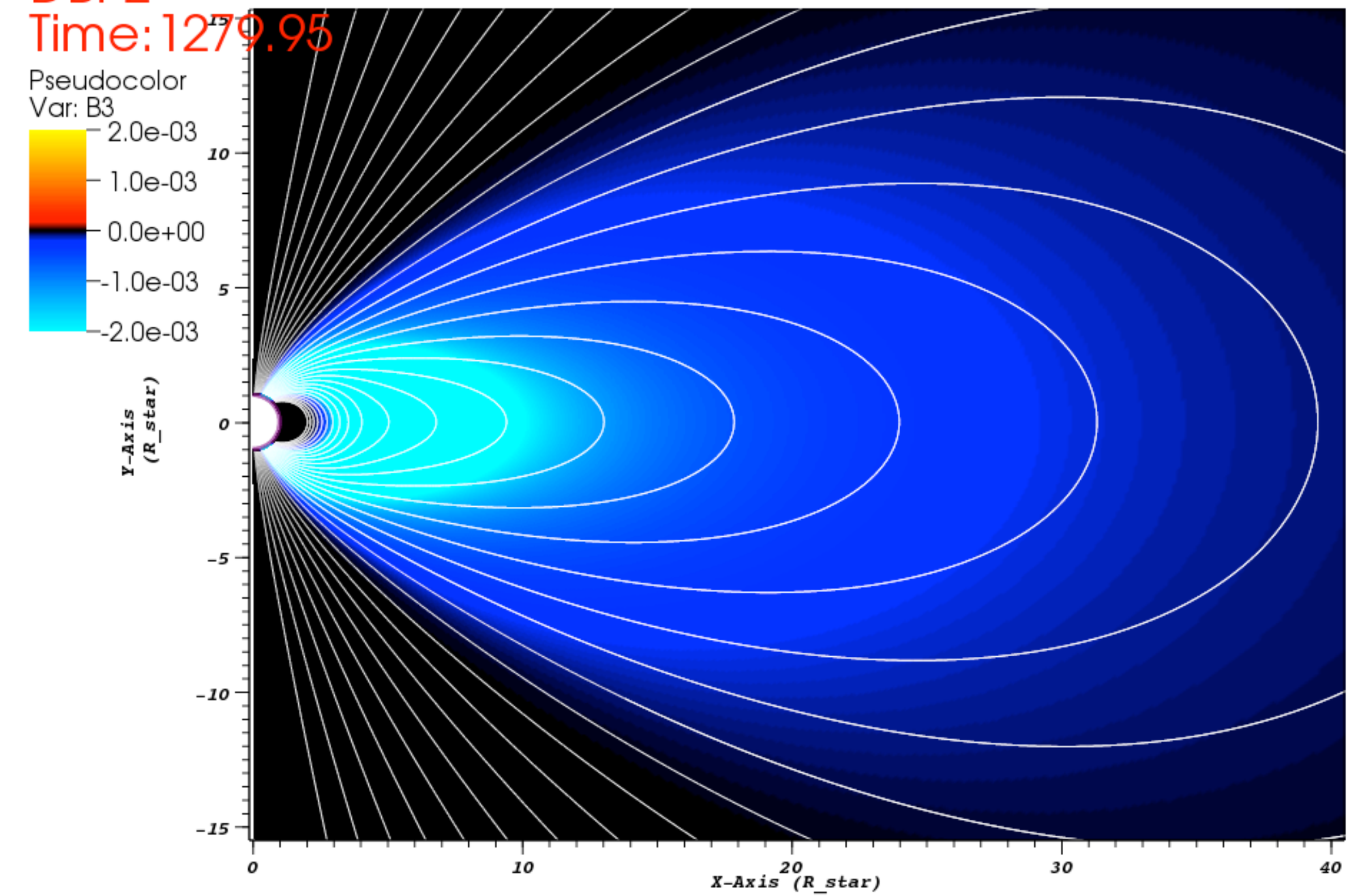
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



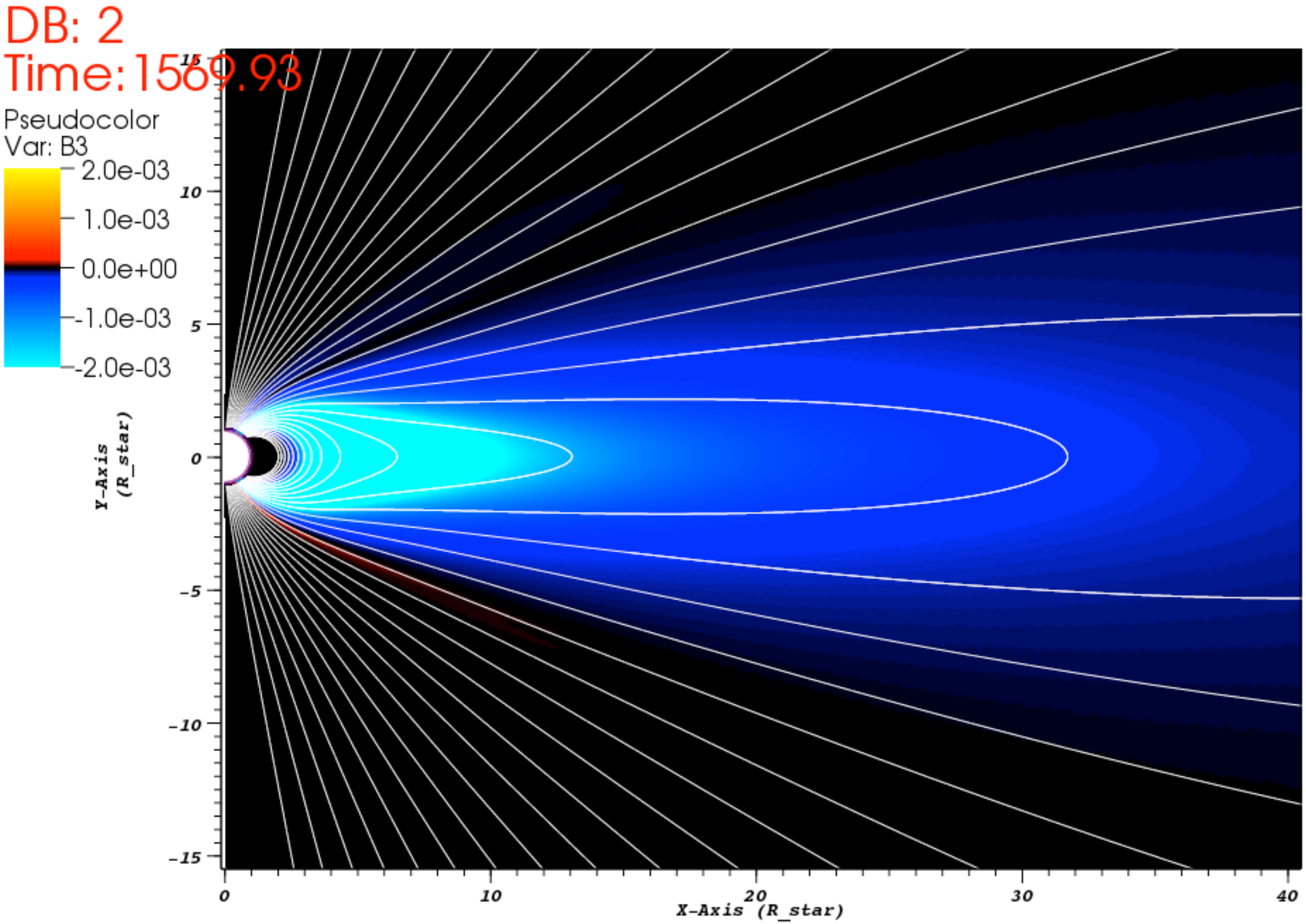
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



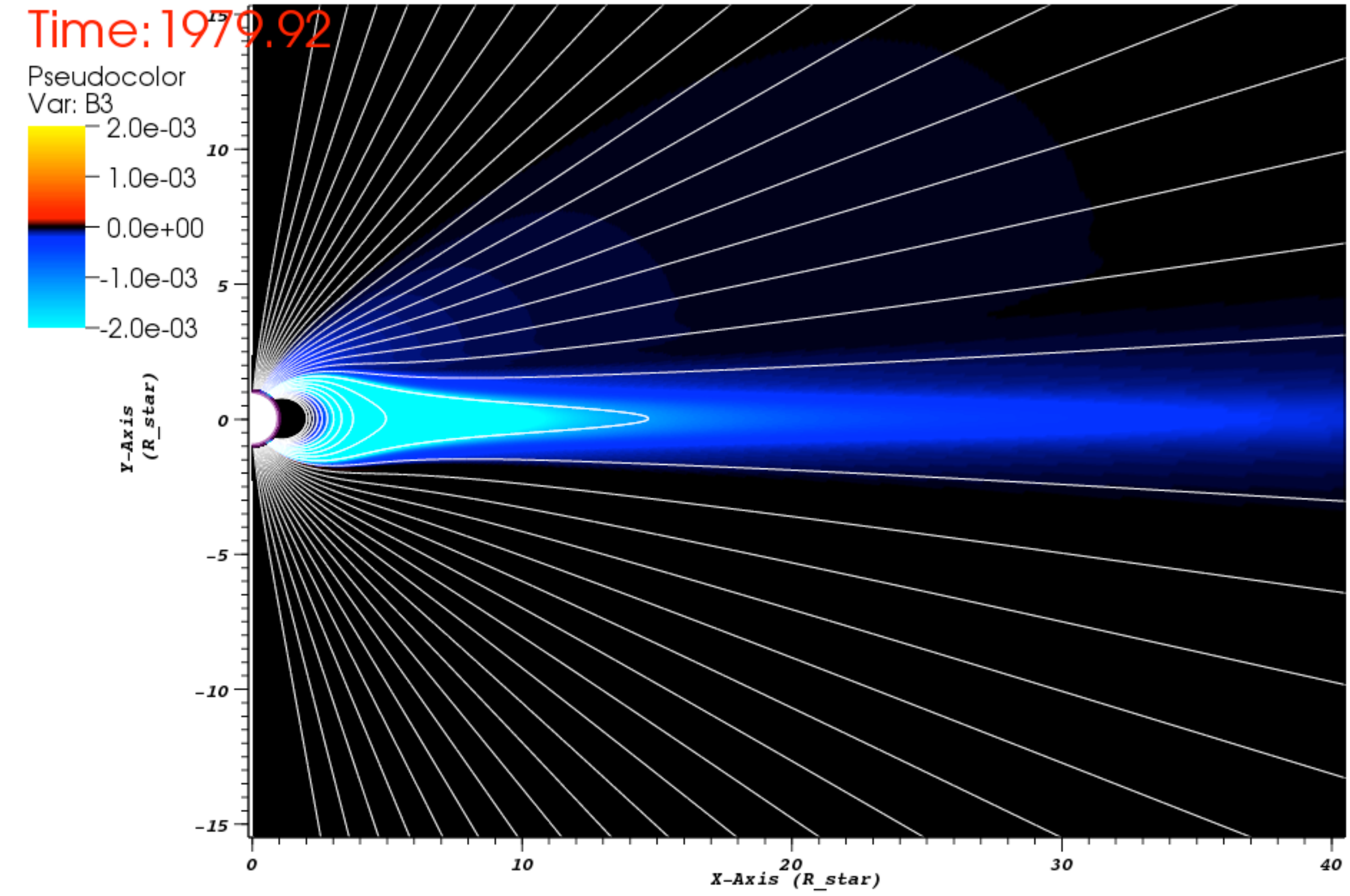
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



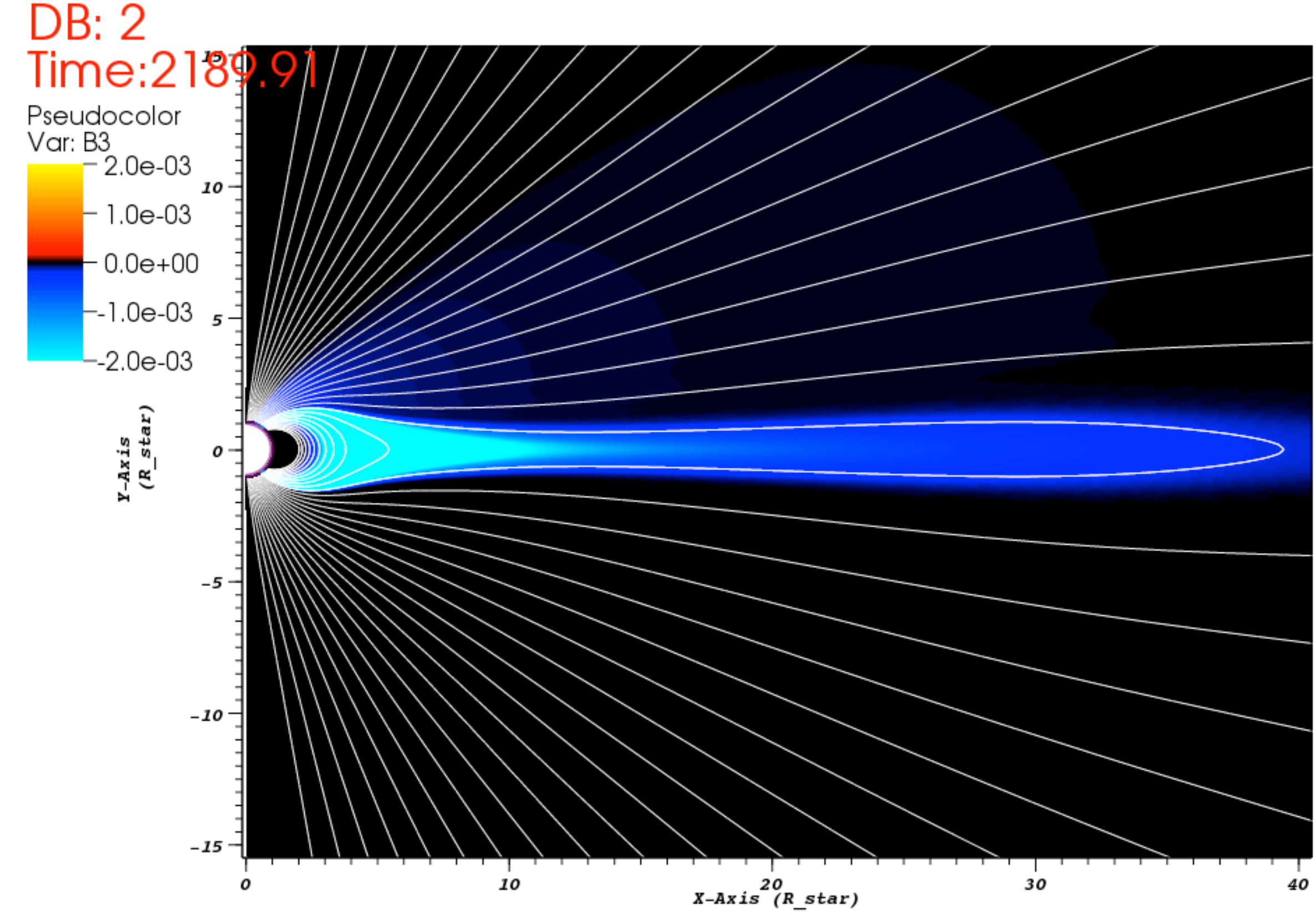
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



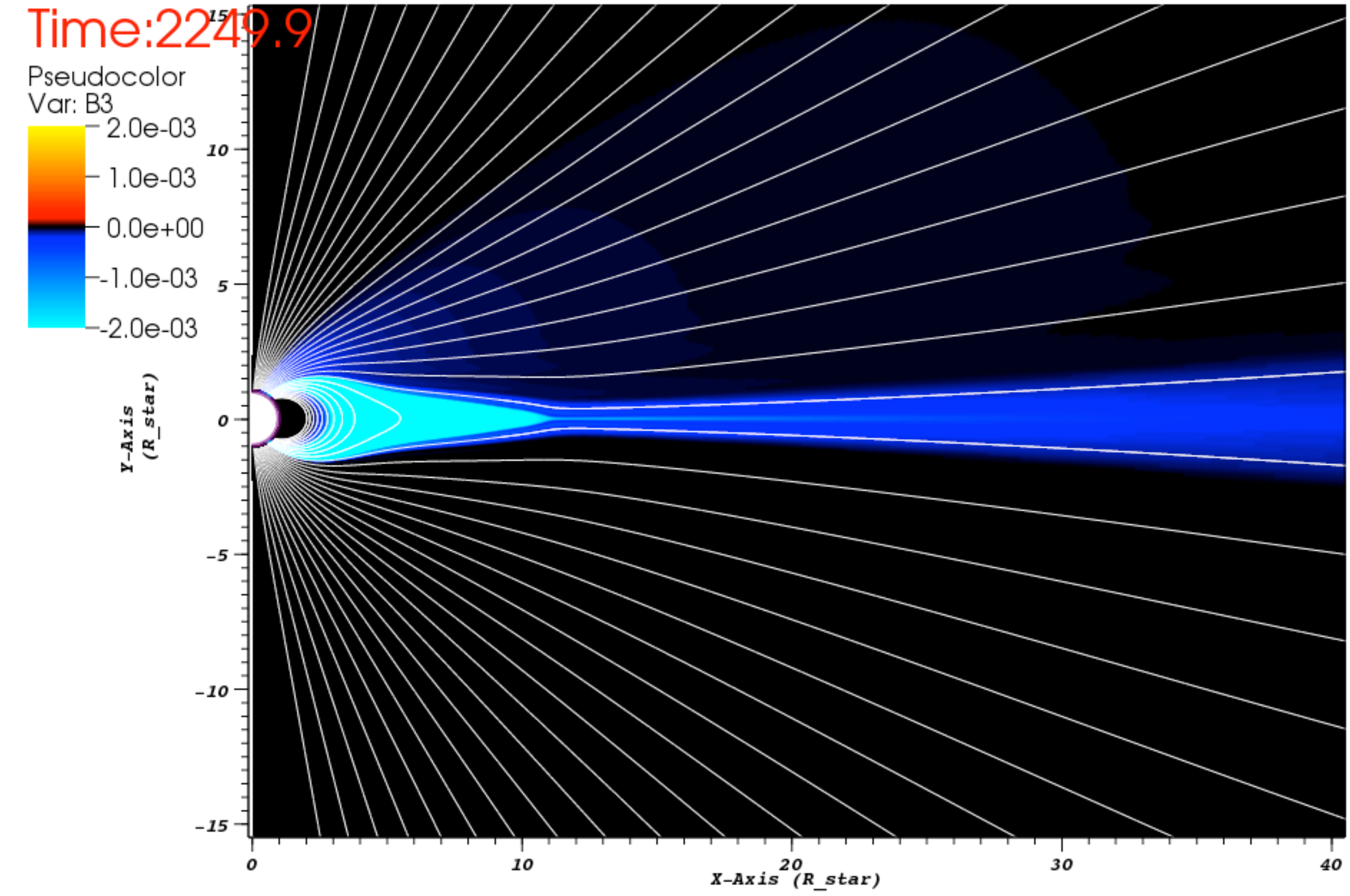
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



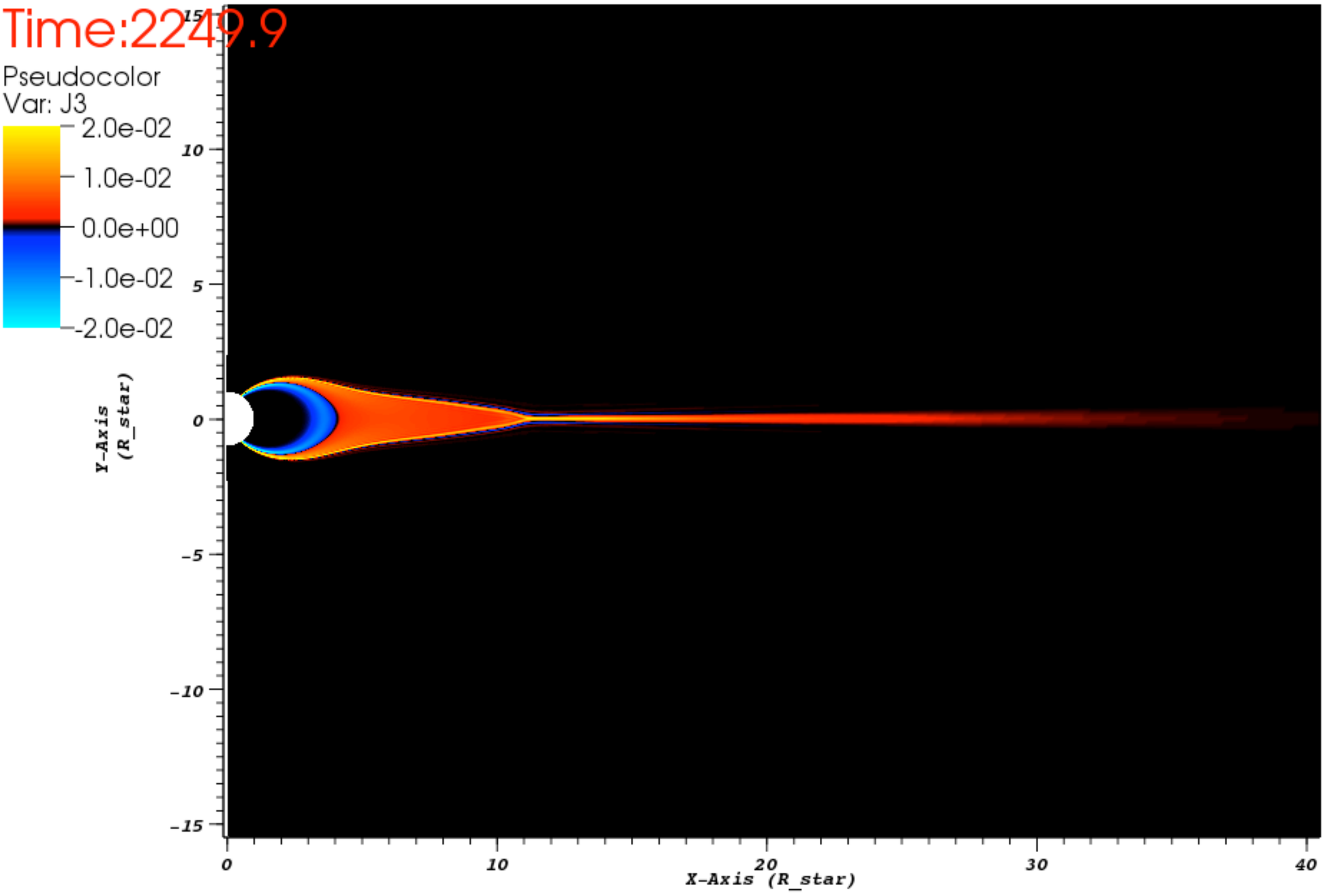
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



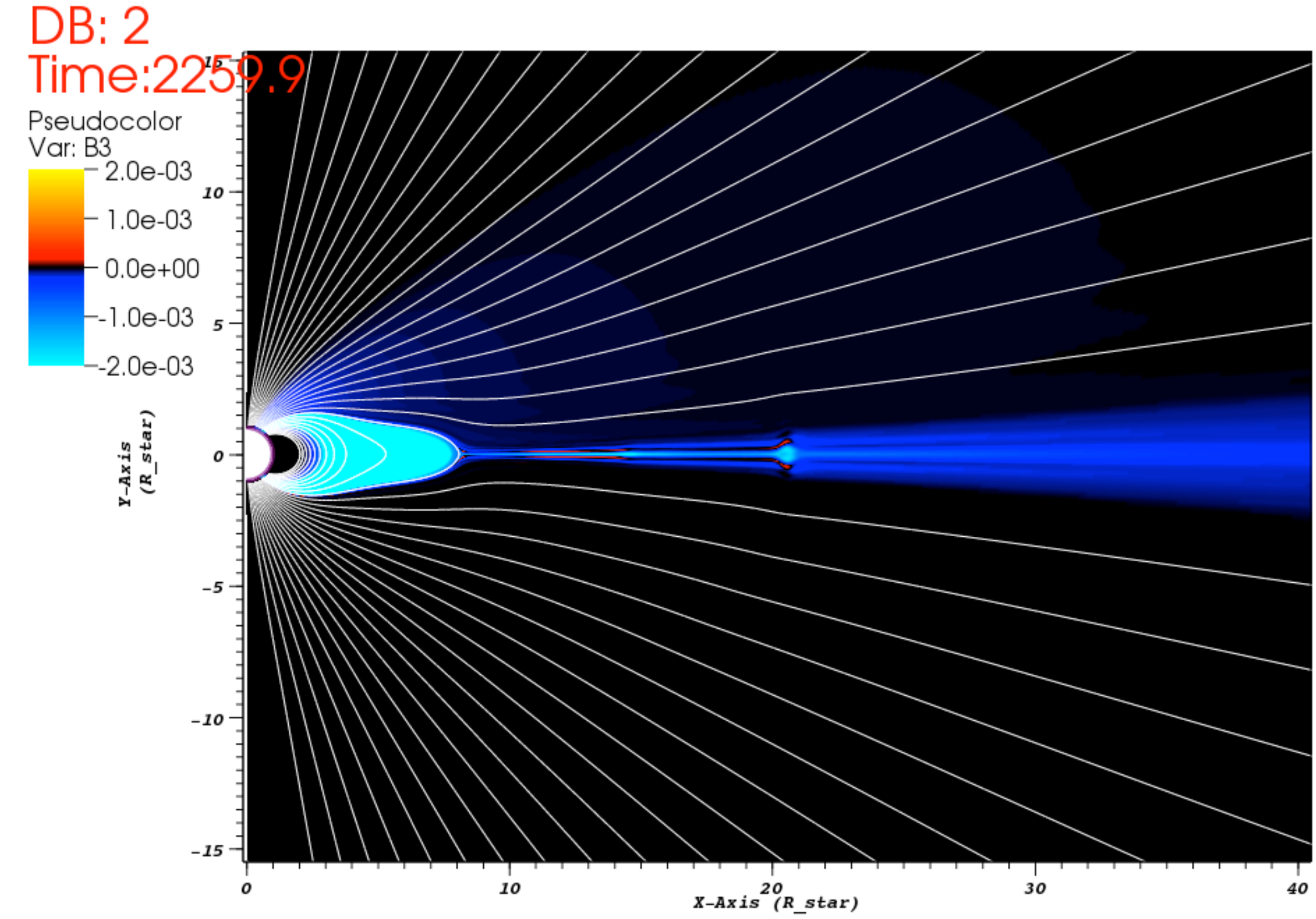
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



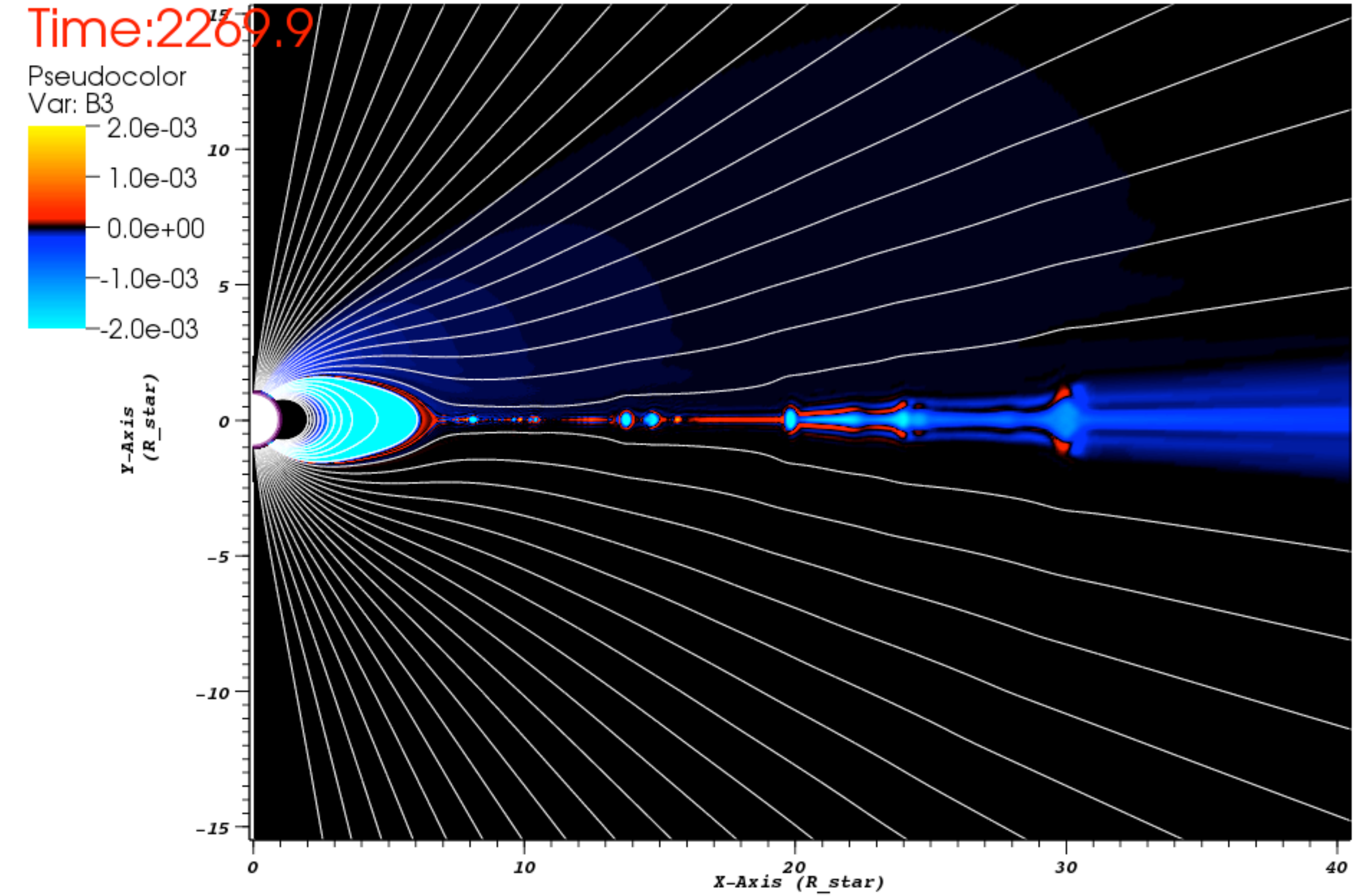
Twisted magnetosphere — open field at finite time & twist
Shown: B^{Φ}, poloidal field line projections



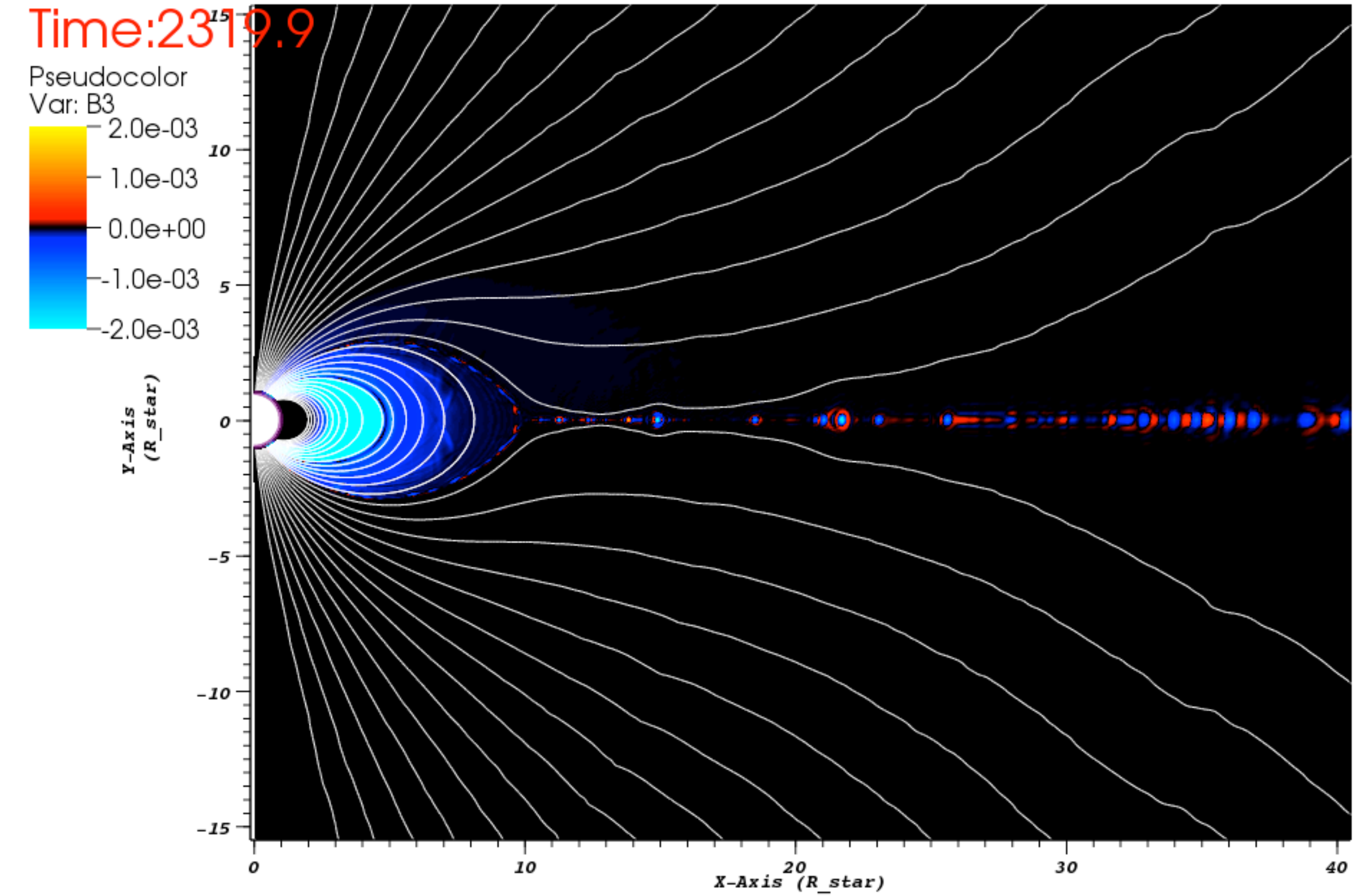
Twisted magnetosphere — current sheet formation
Shown: J^{Φ}



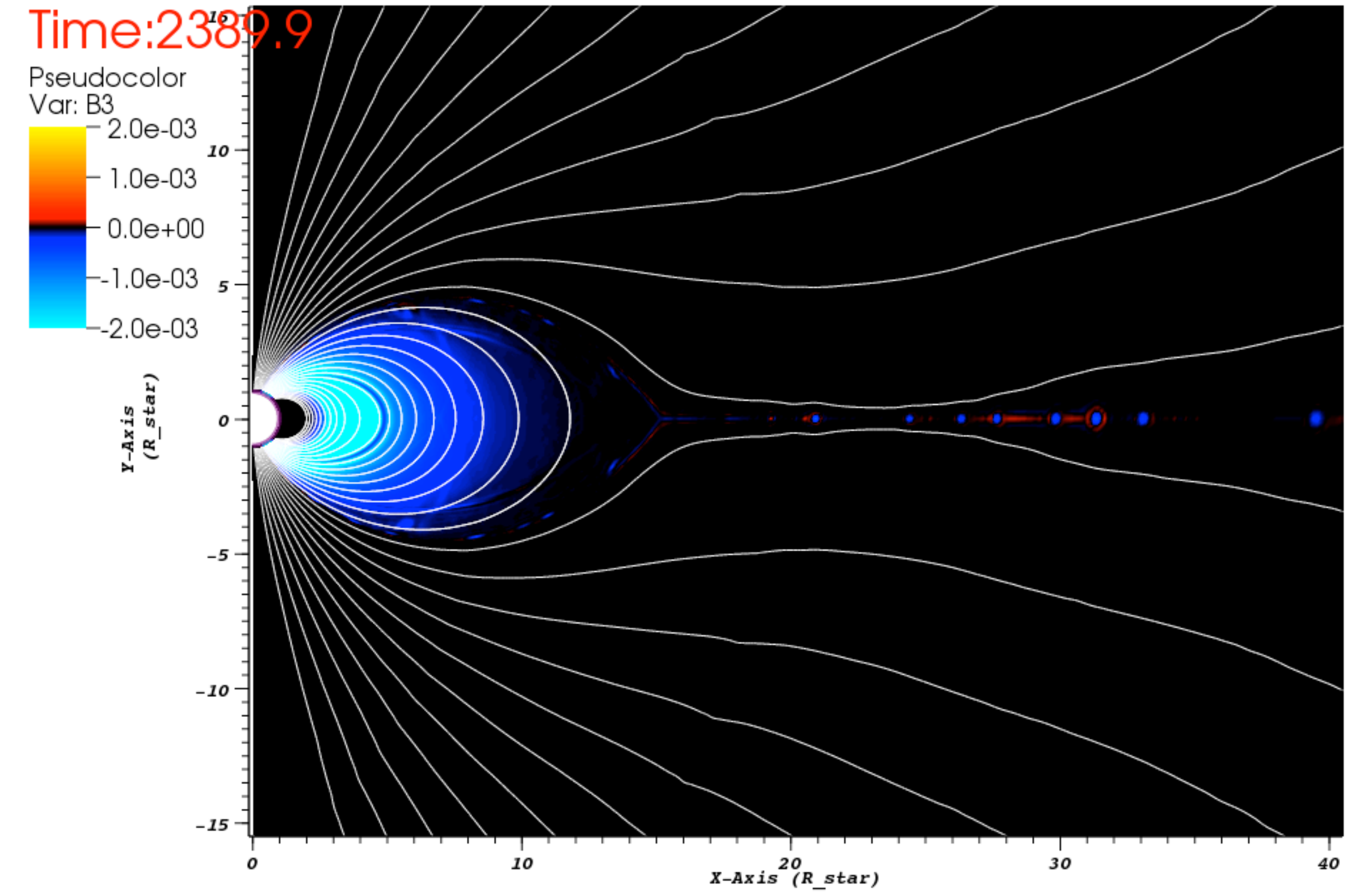
Twisted magnetosphere — current sheet formation
Shown: B^{Φ}, poloidal field line projections



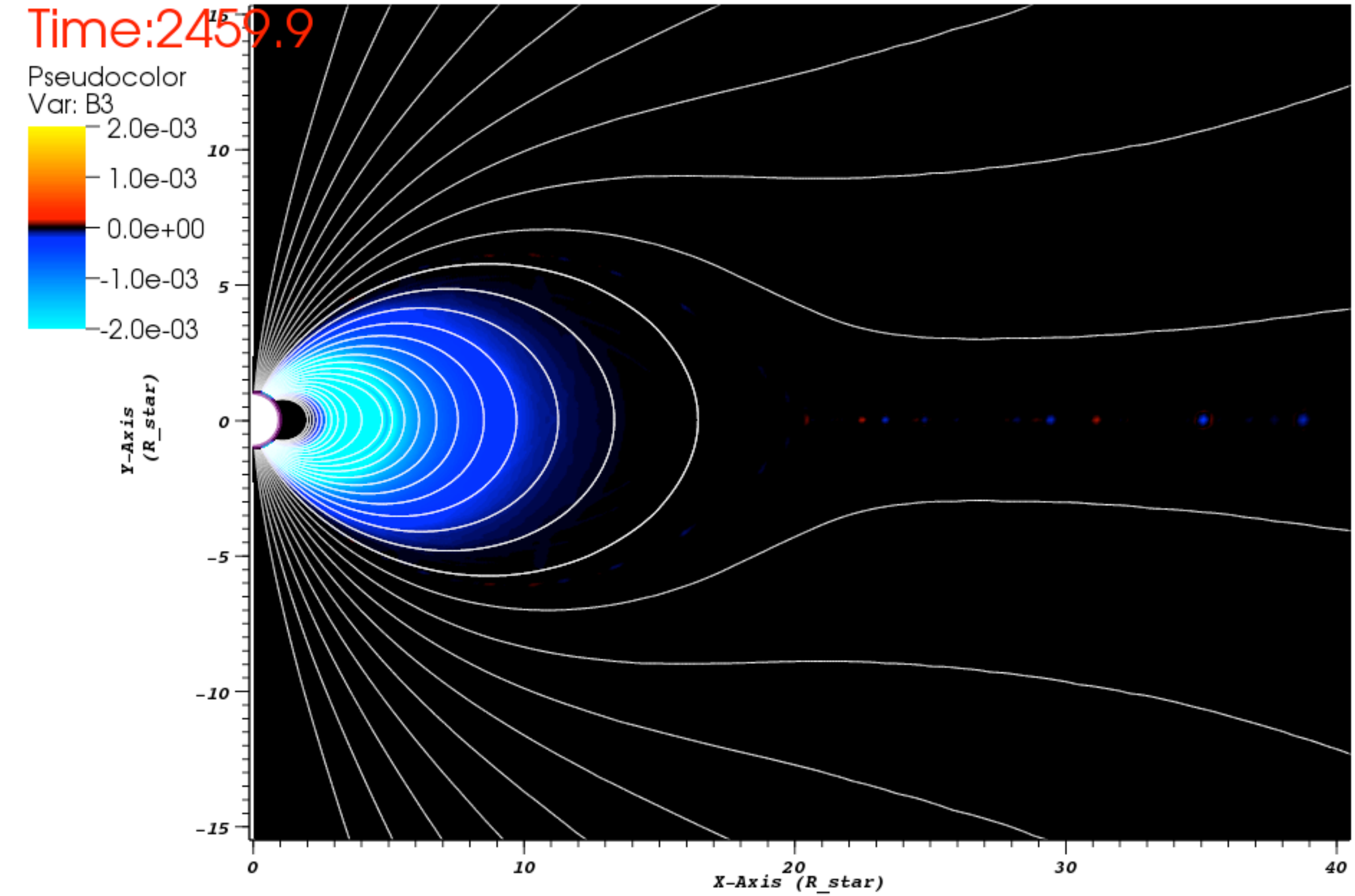
Twisted magnetosphere — first reconnection event
Shown: B^{Φ}, poloidal field line projections



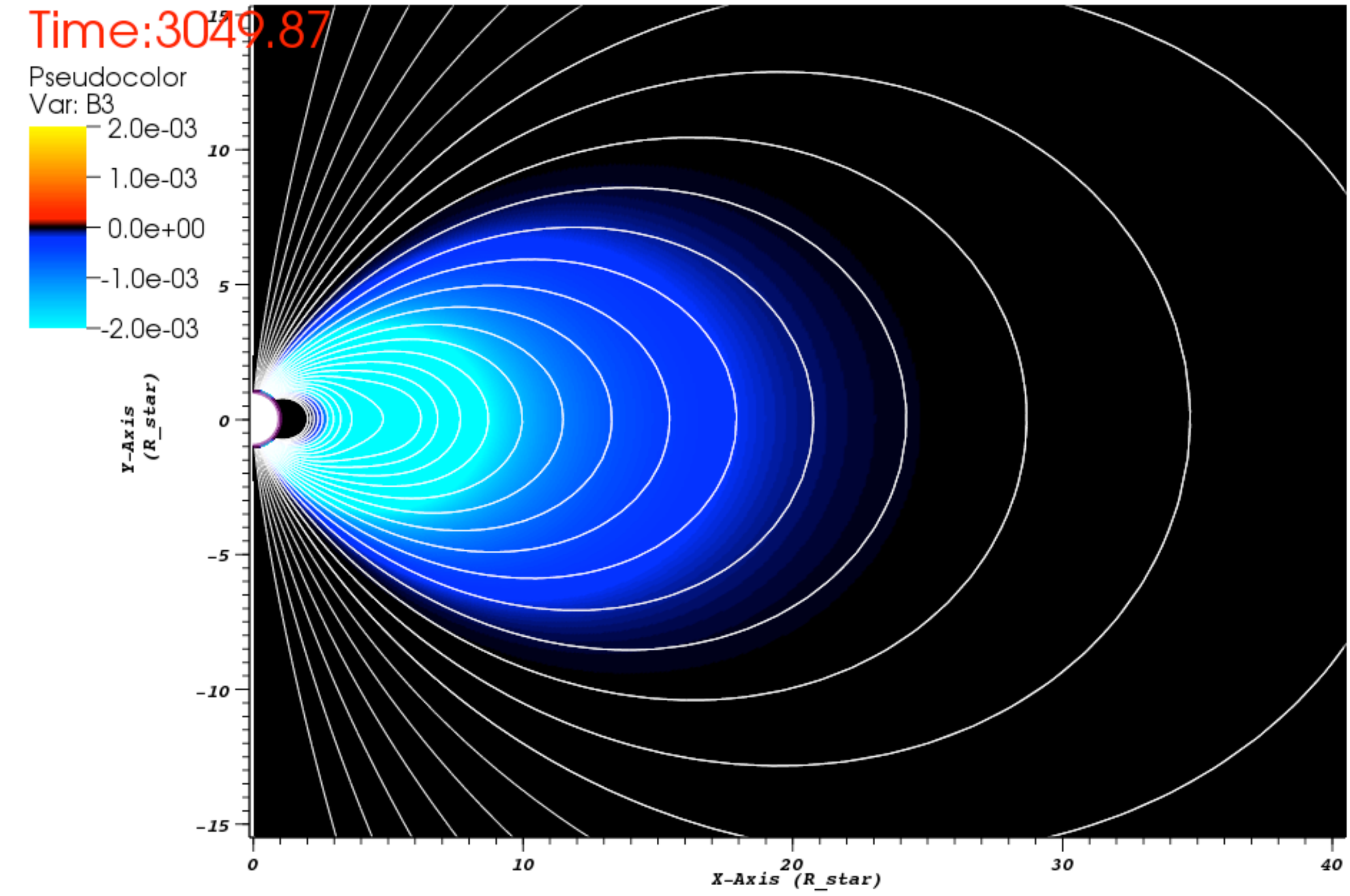
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



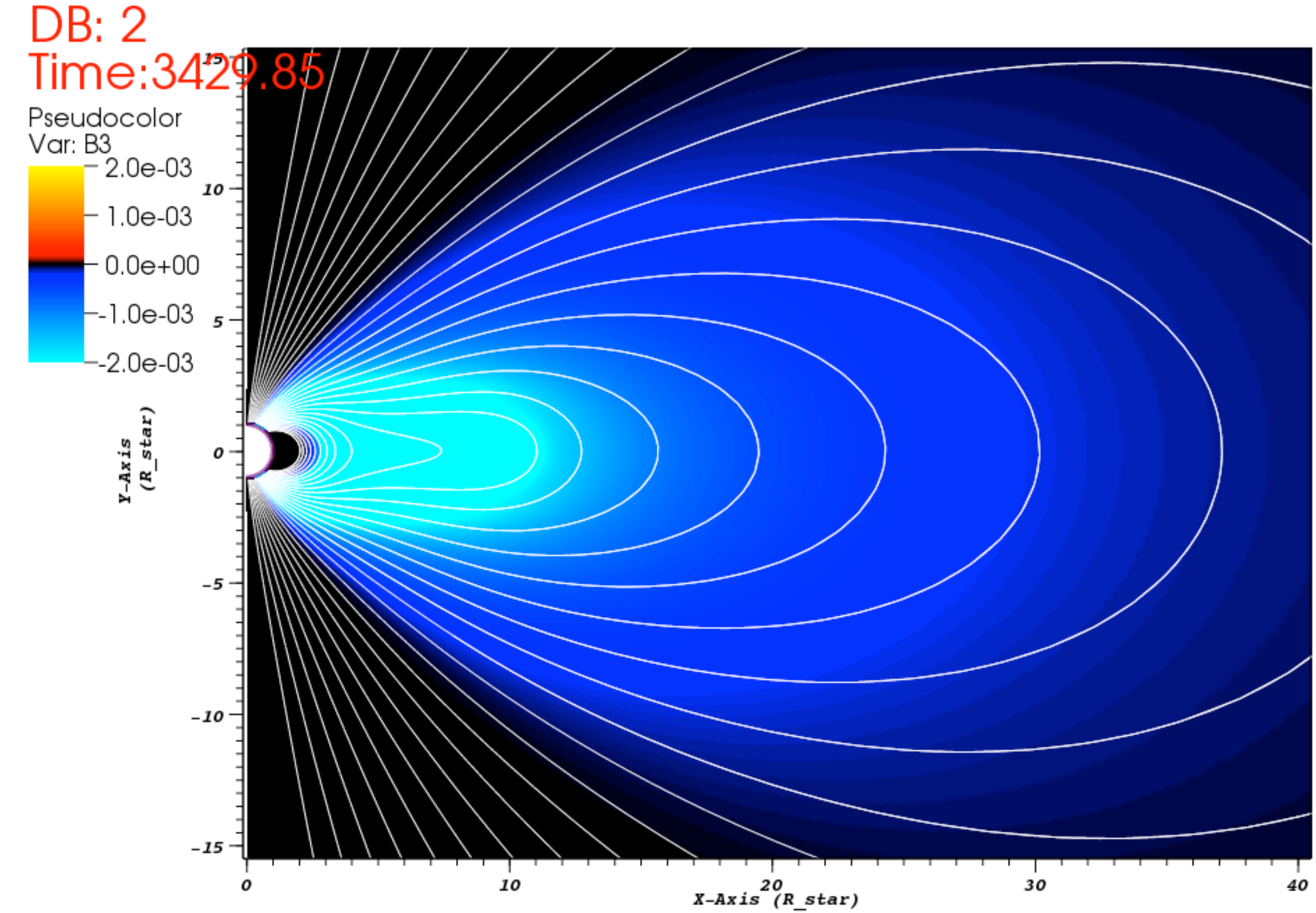
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



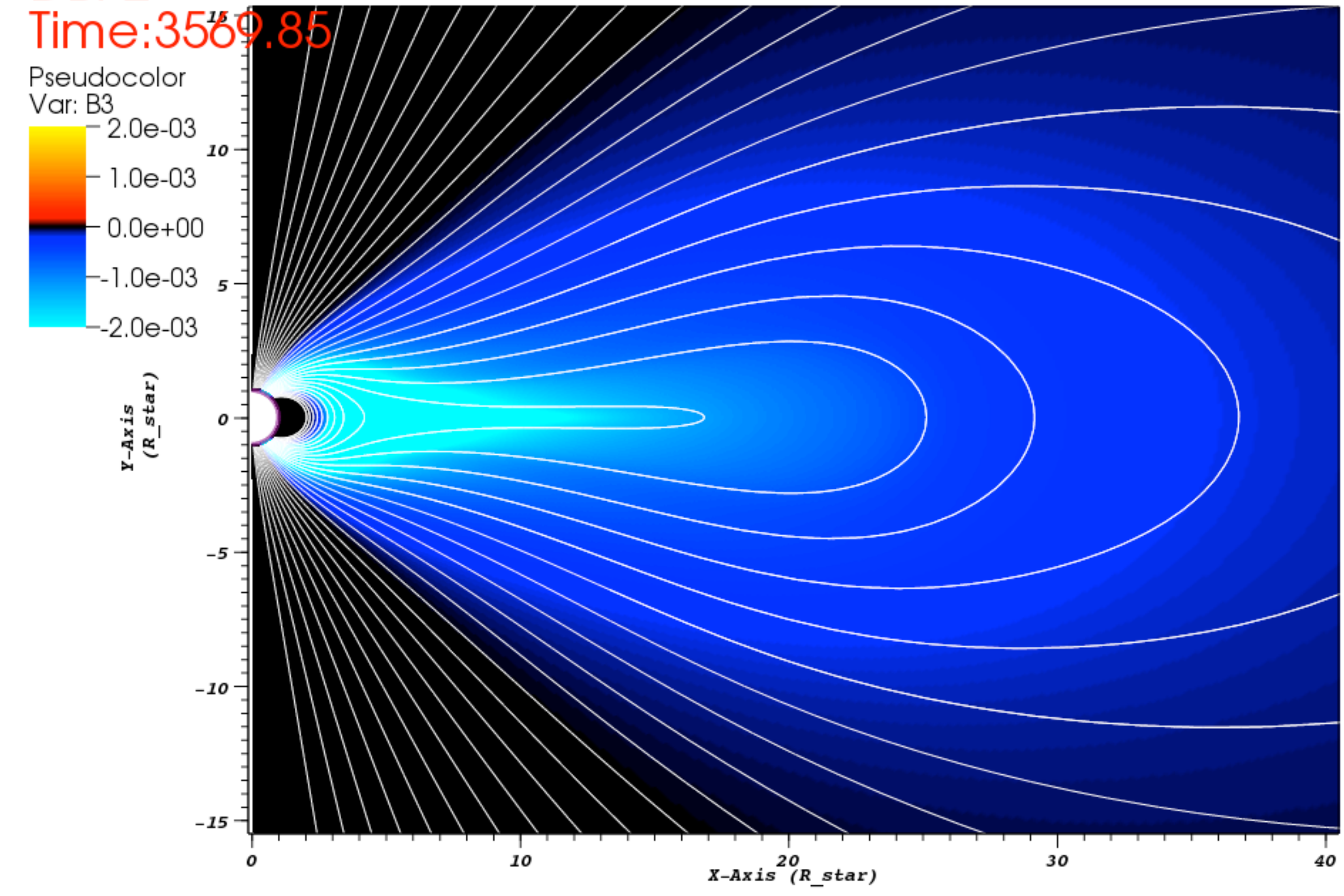
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



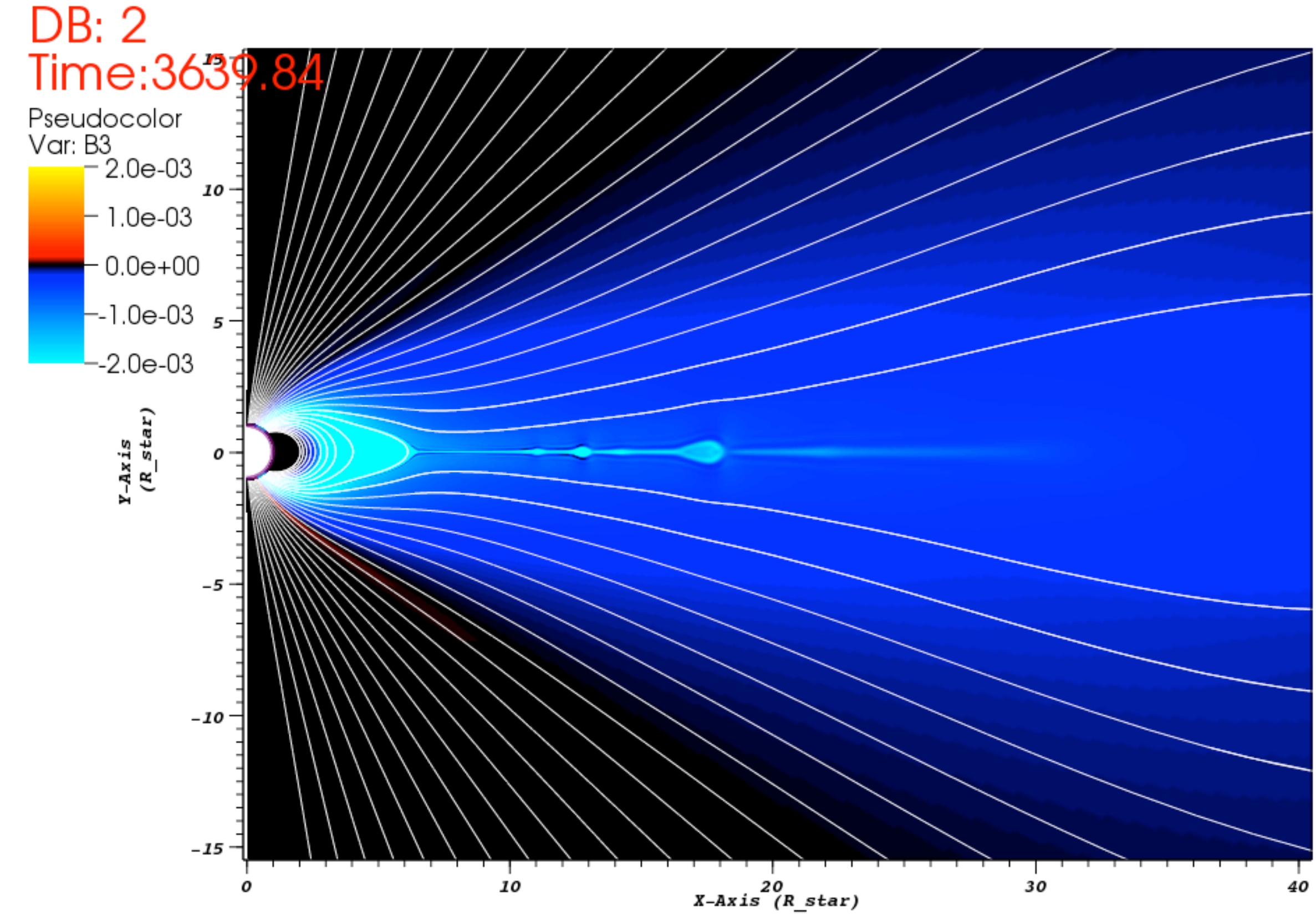
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



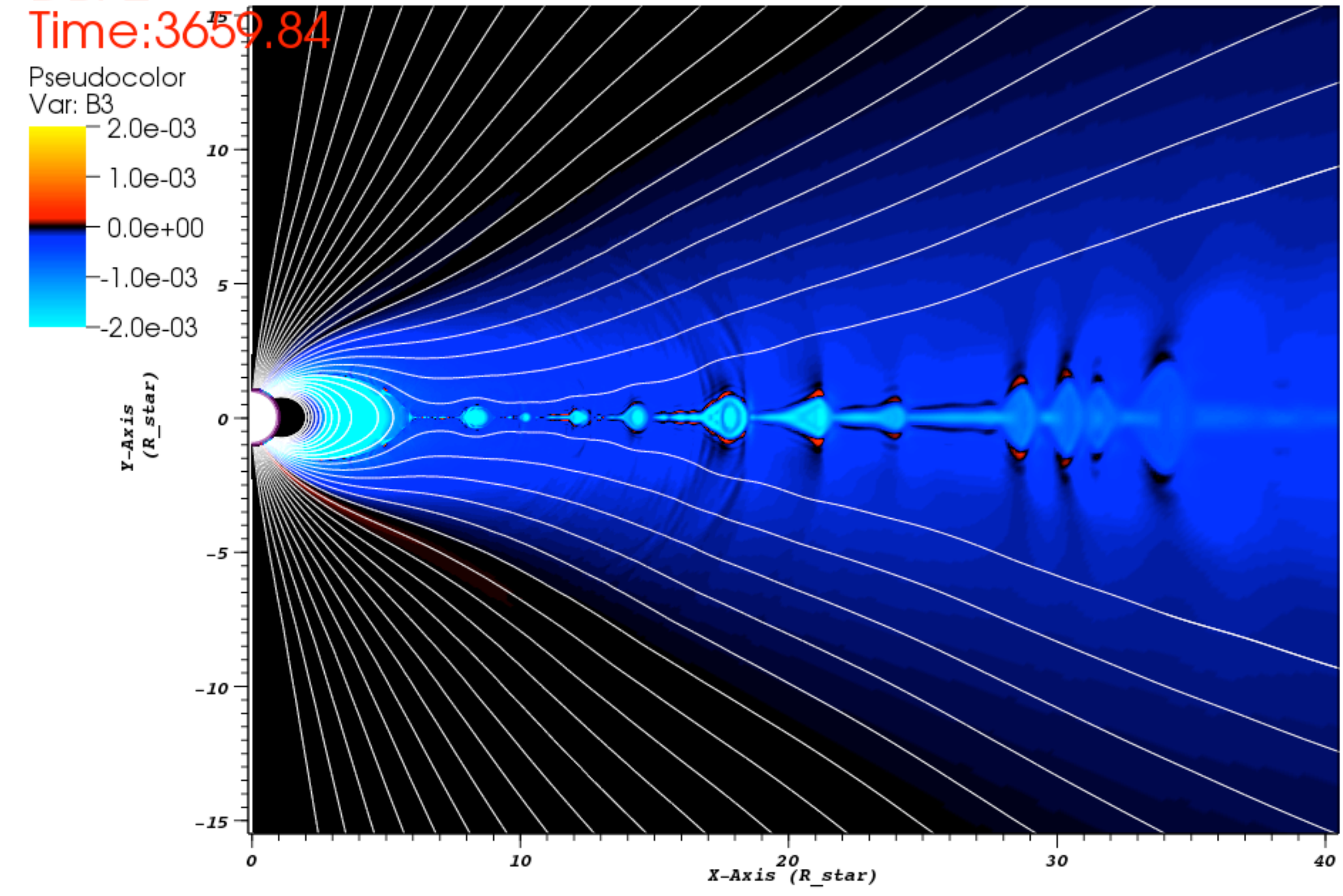
Twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



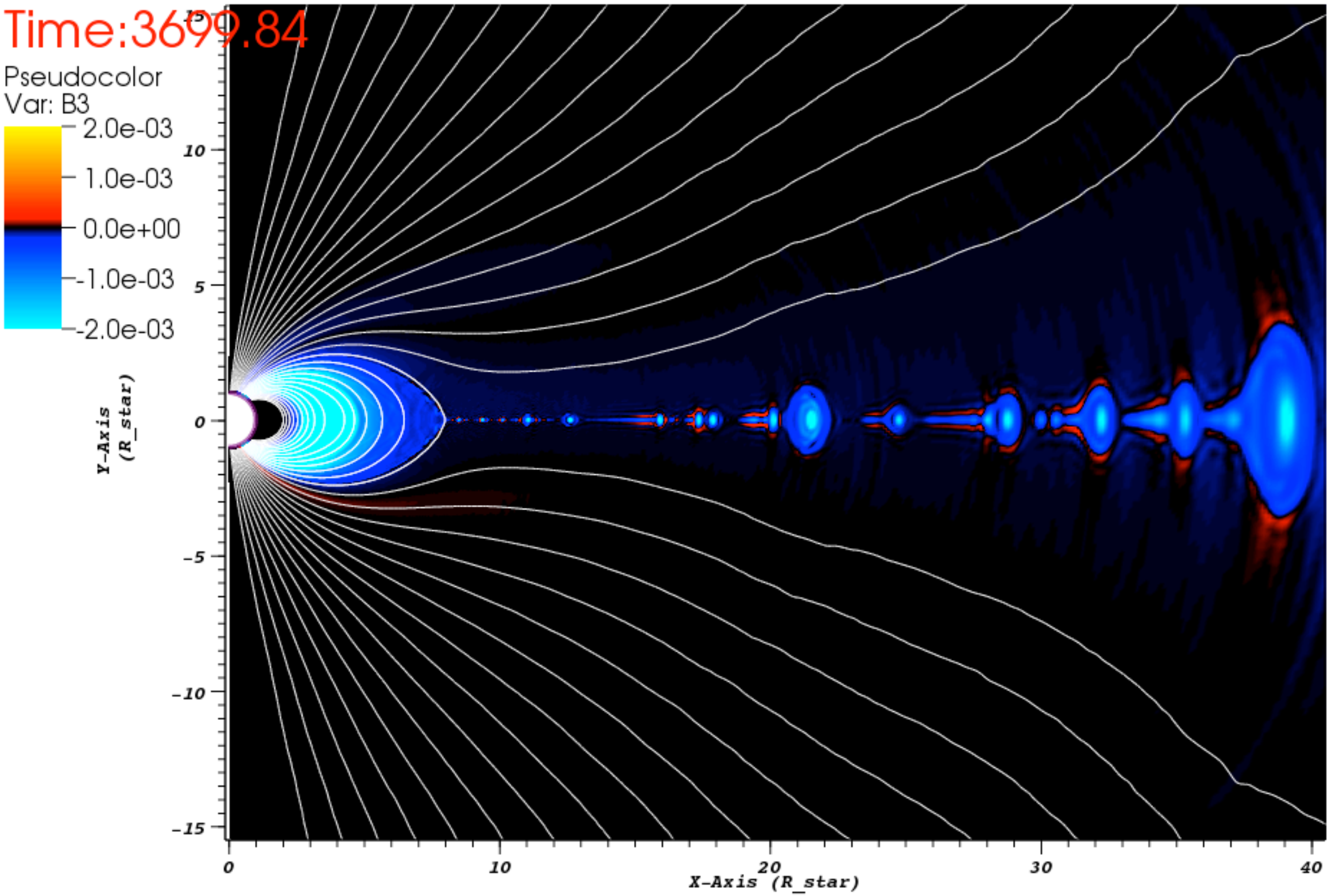
Twisted magnetosphere — impulsive opening
Shown: B^{Φ}, poloidal field line projections



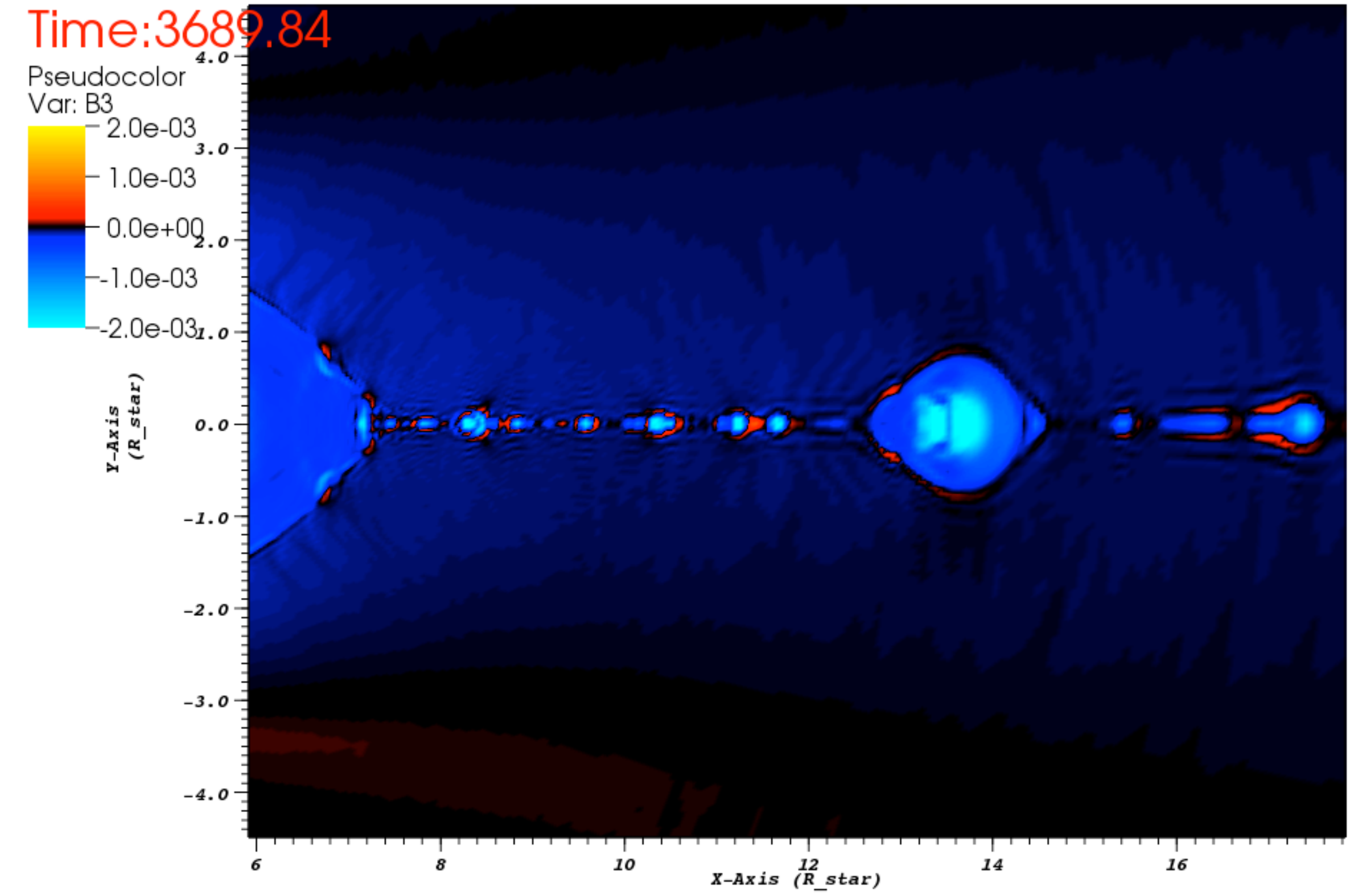
Twisted magnetosphere — second current sheet
Shown: B^{Φ}, poloidal field line projections



Twisted magnetosphere — second reconnection event
Shown: B^{Φ}, poloidal field line projections

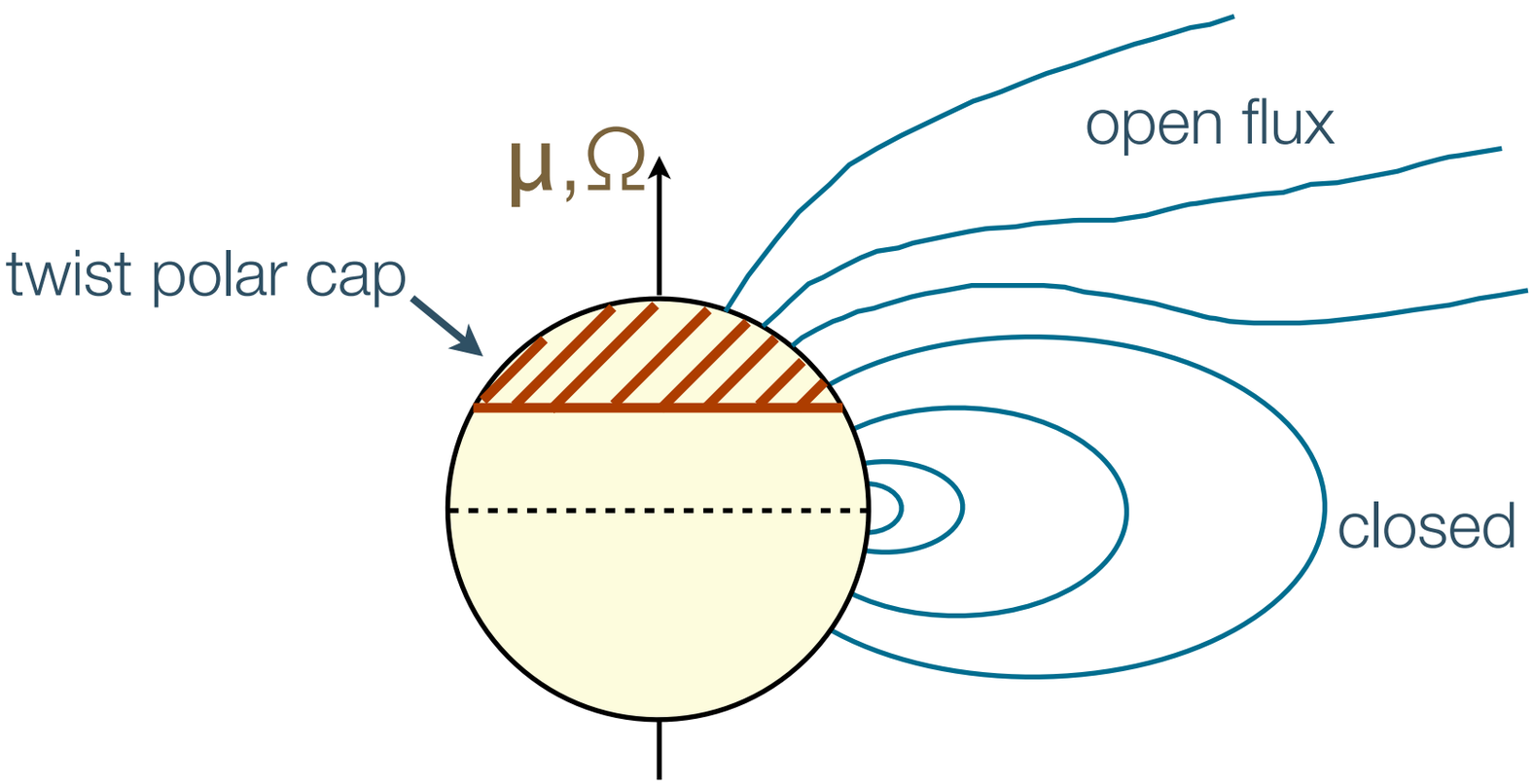


Twisted magnetosphere — giant plasmoids
Shown: B^{ϕ}, poloidal field line projections



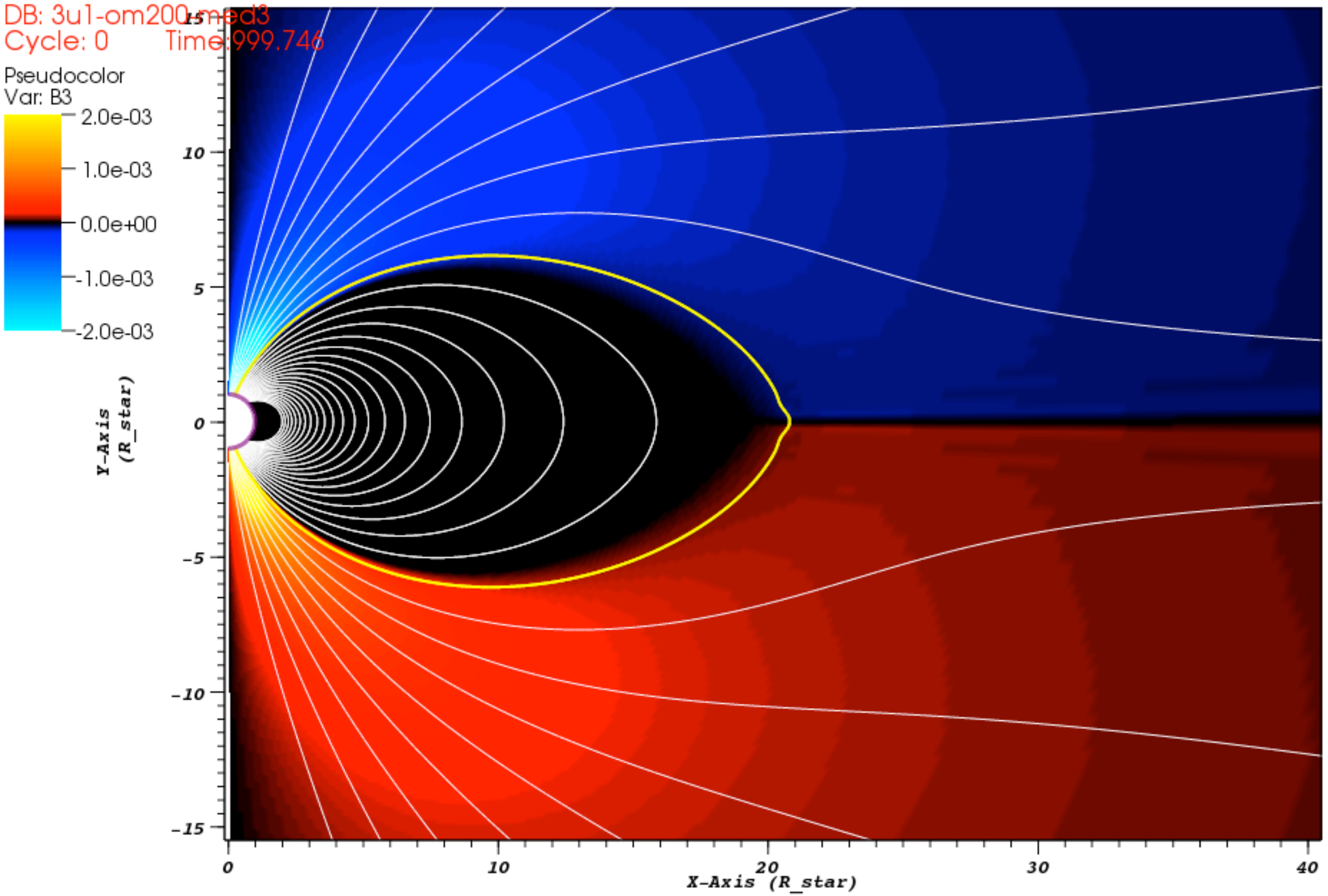
Hierarchical plasmoid formation? (Uzdensky et al 2010)

Rotation + surface differential motion

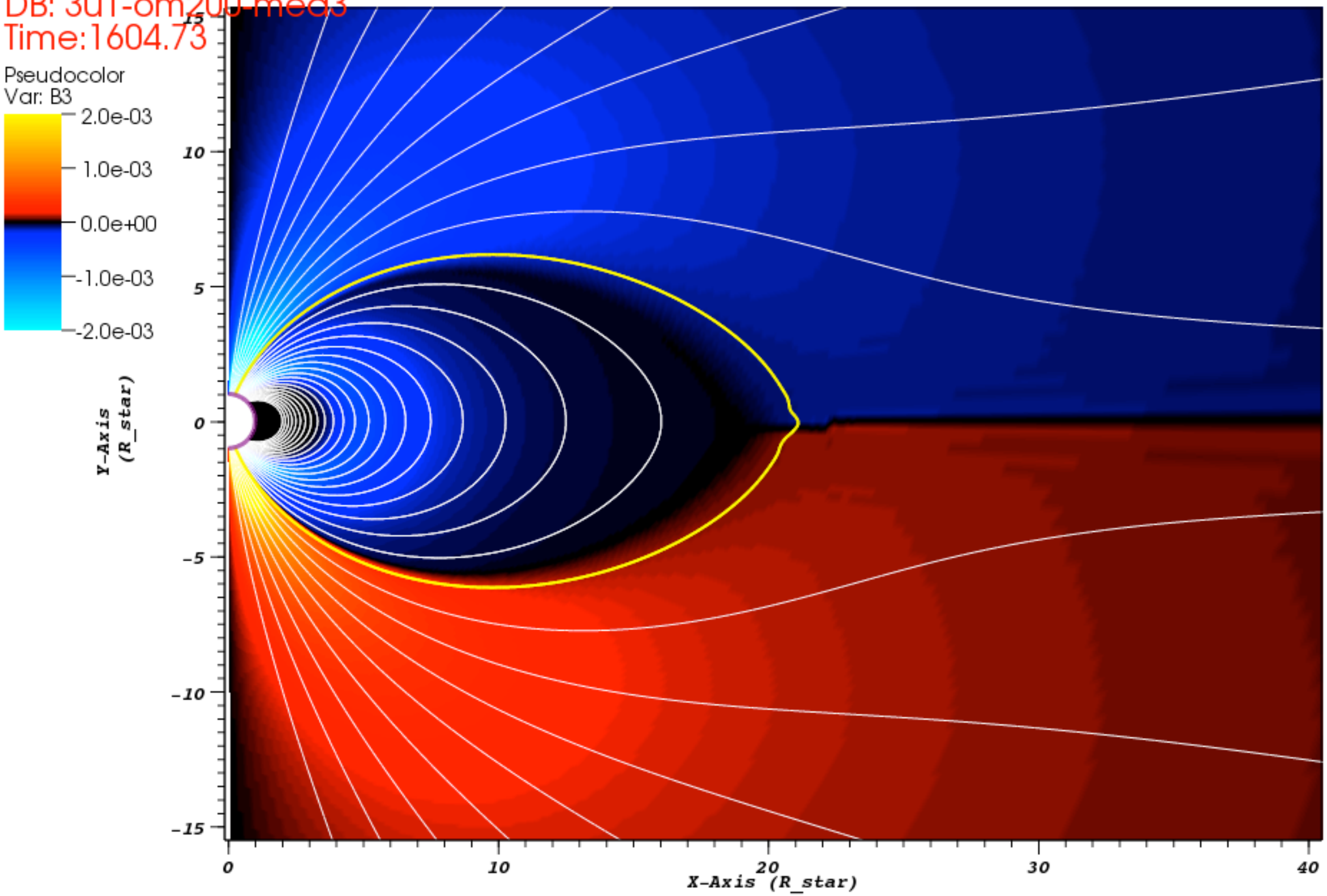


- Apply twist to open & some closed field lines
- Need to twist much slower than rotation rate

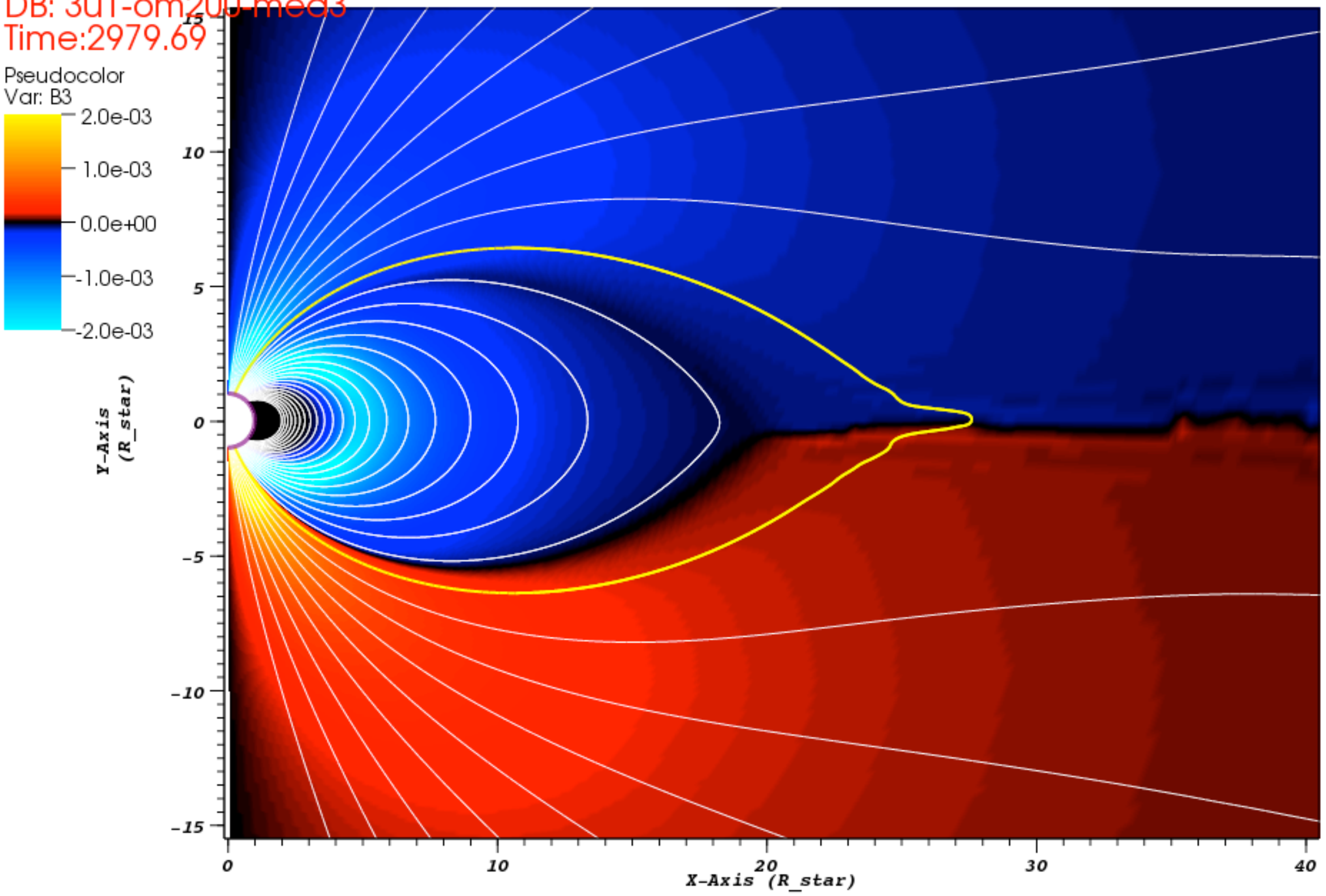
- Simulation: twisted flux = 3 x open flux, $\Omega_{twist} = \Omega_{rotate}/200$
- Example object: AXP XTE J1810-197 (Beloborodov 2009)



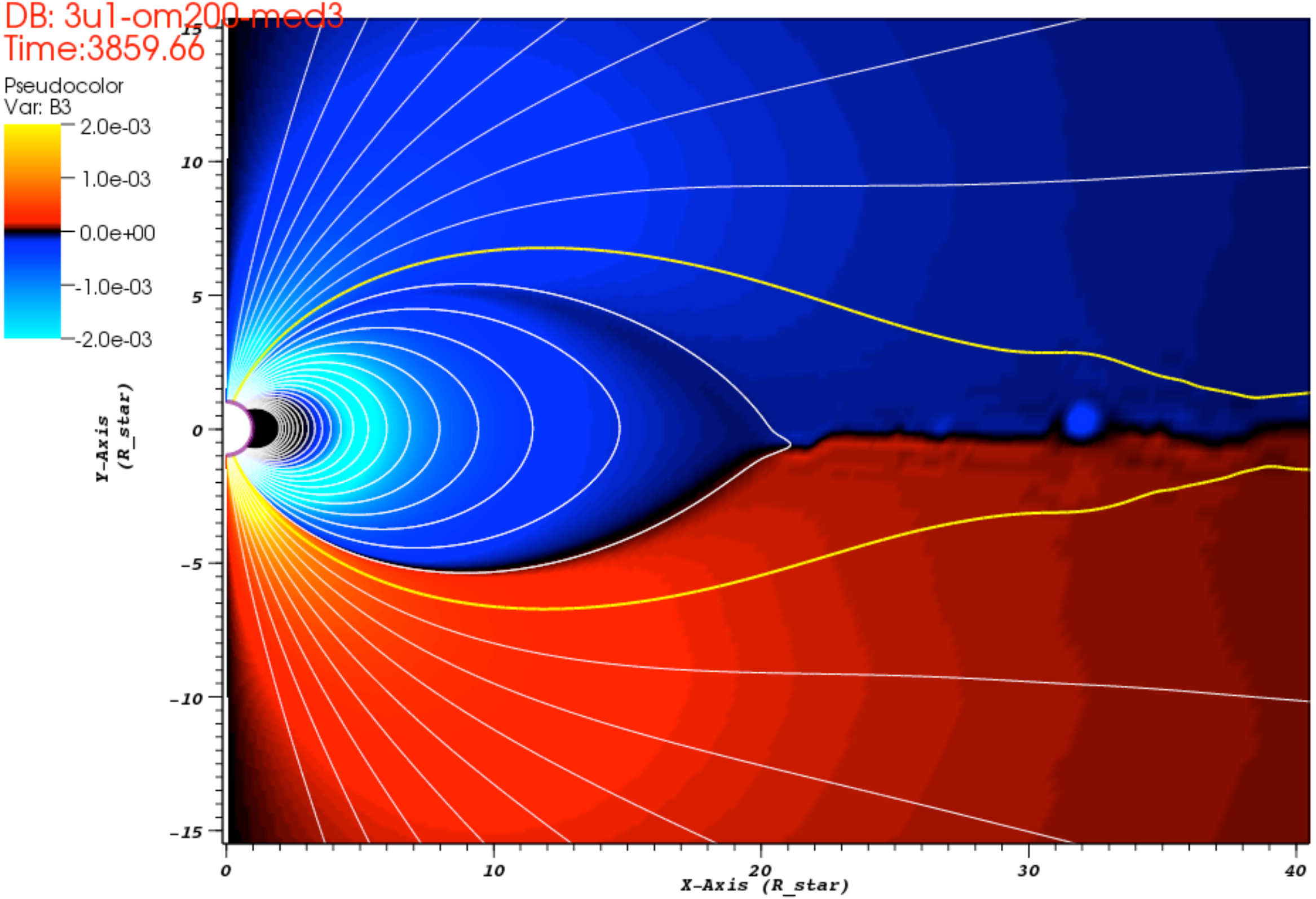
Rotating, twisted magnetosphere
Shown: B^{ϕ}, poloidal field line projections



Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



Rotating, twisted magnetosphere
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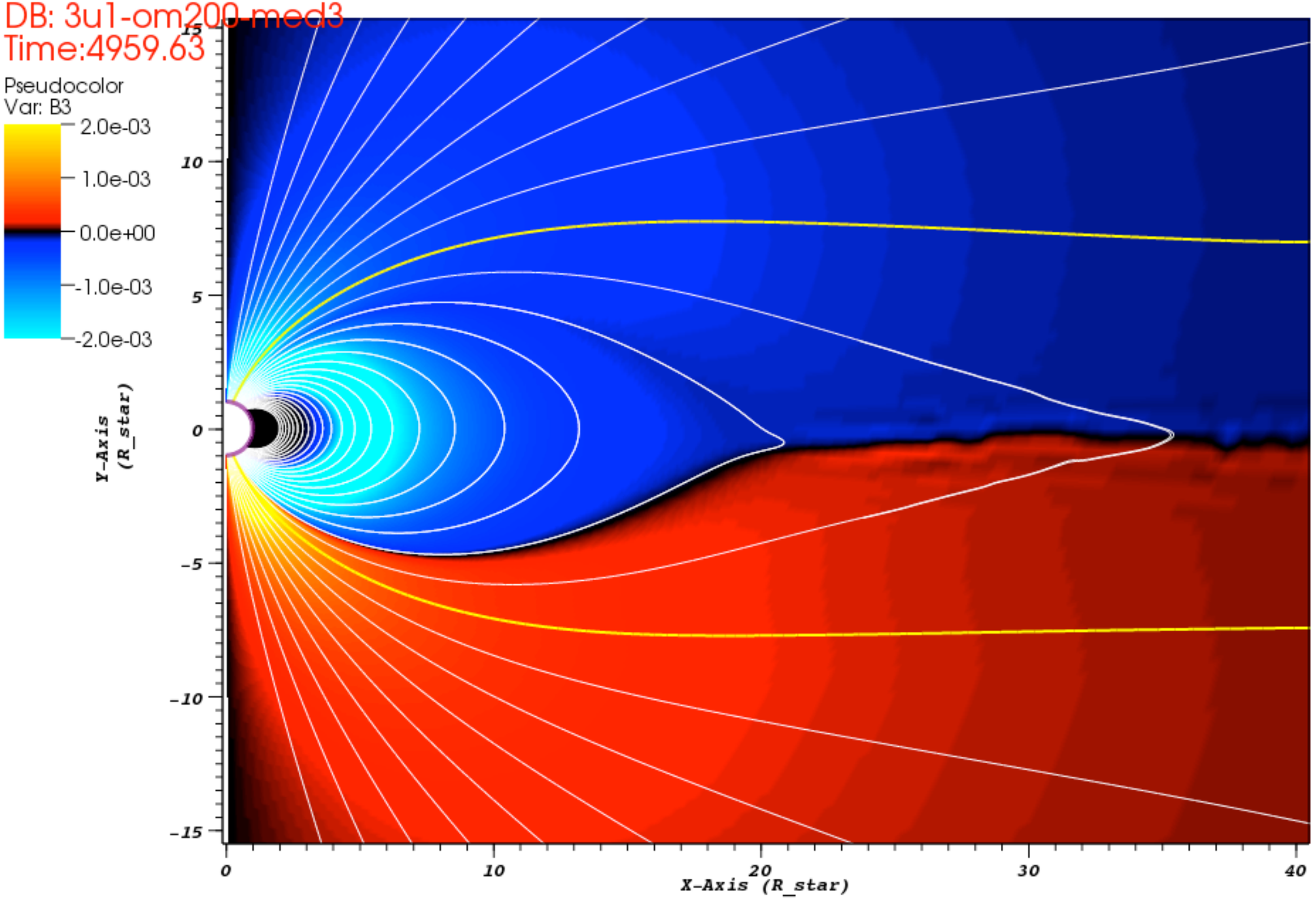

Rotating, twisted magnetosphere
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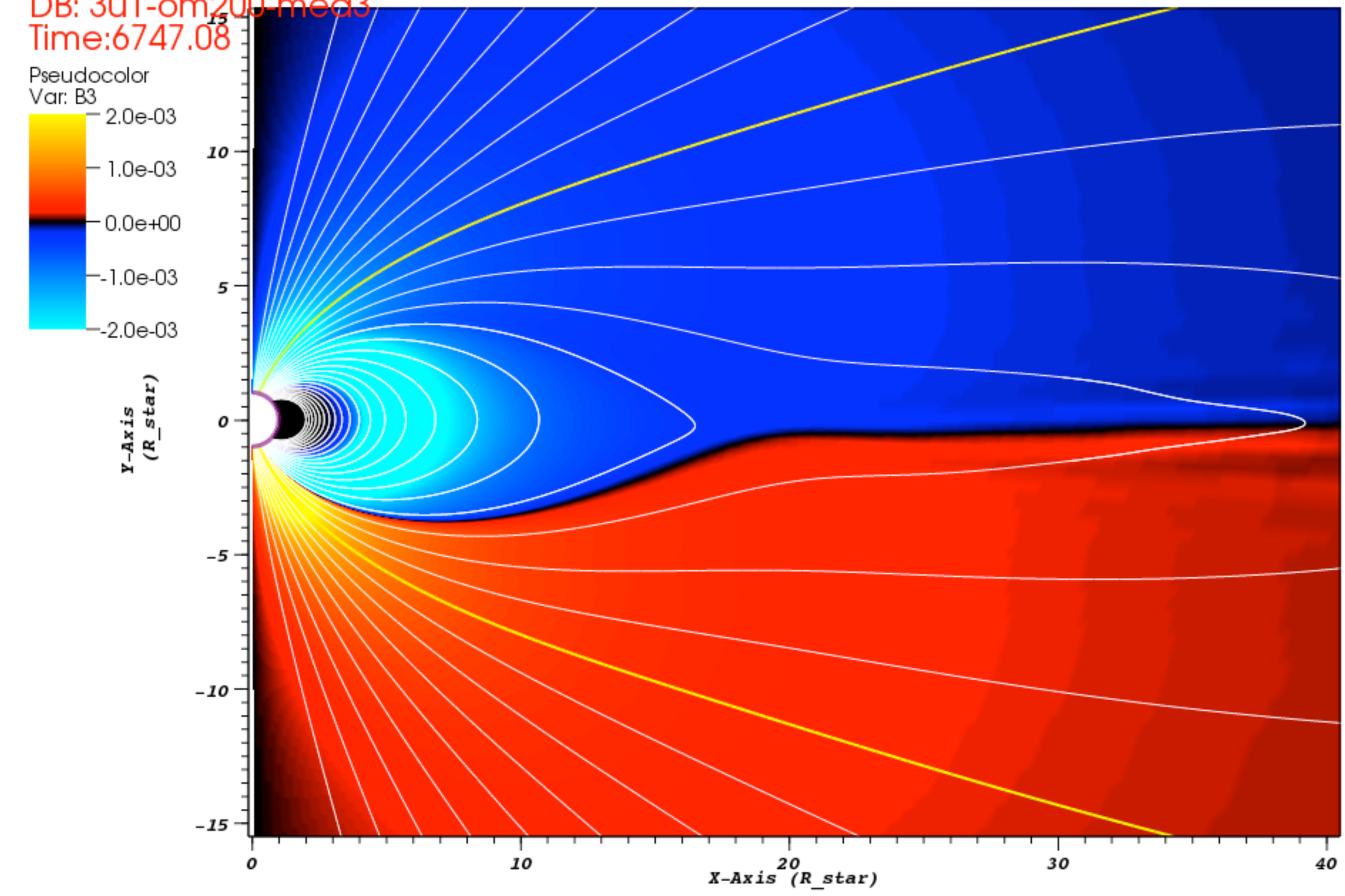
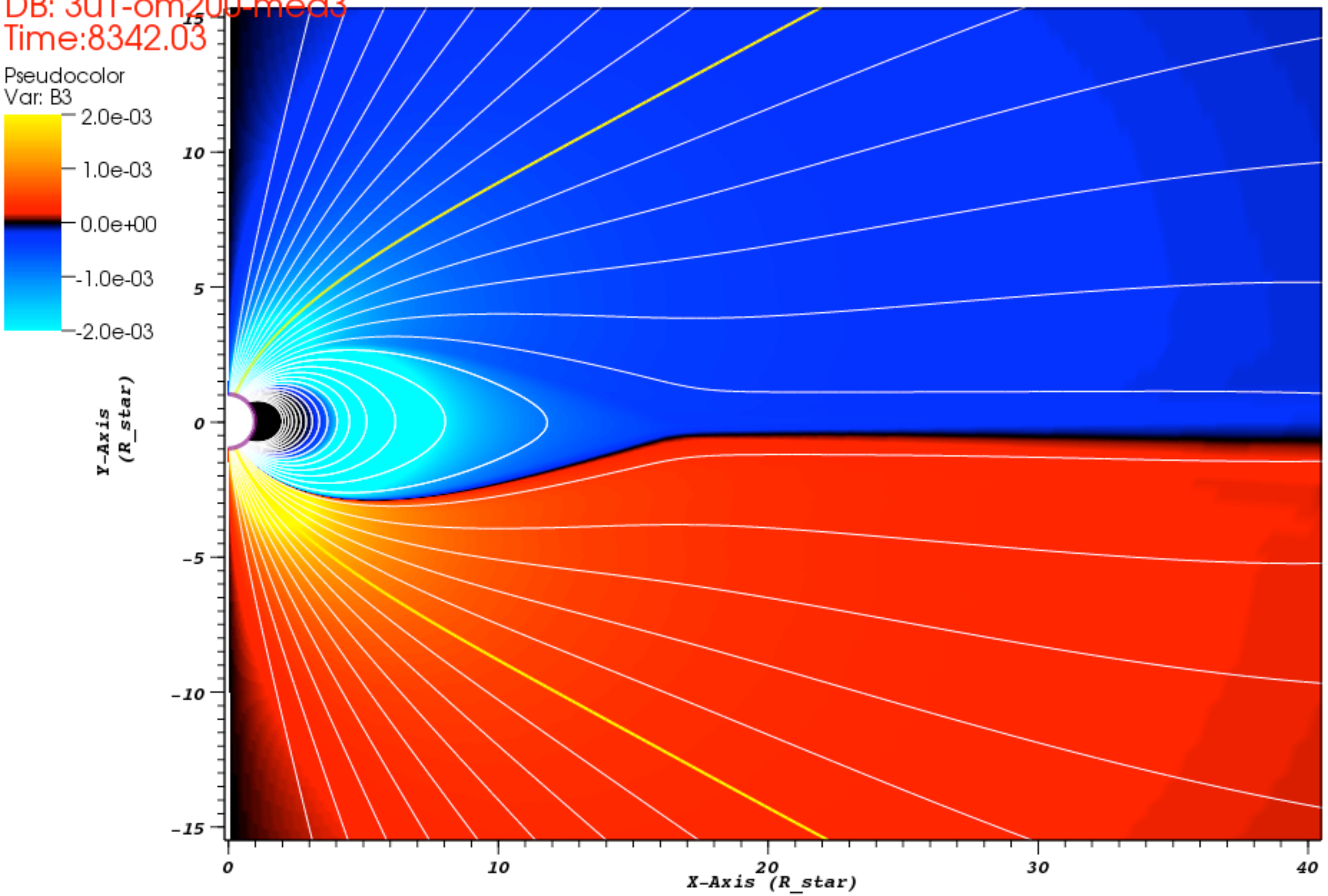
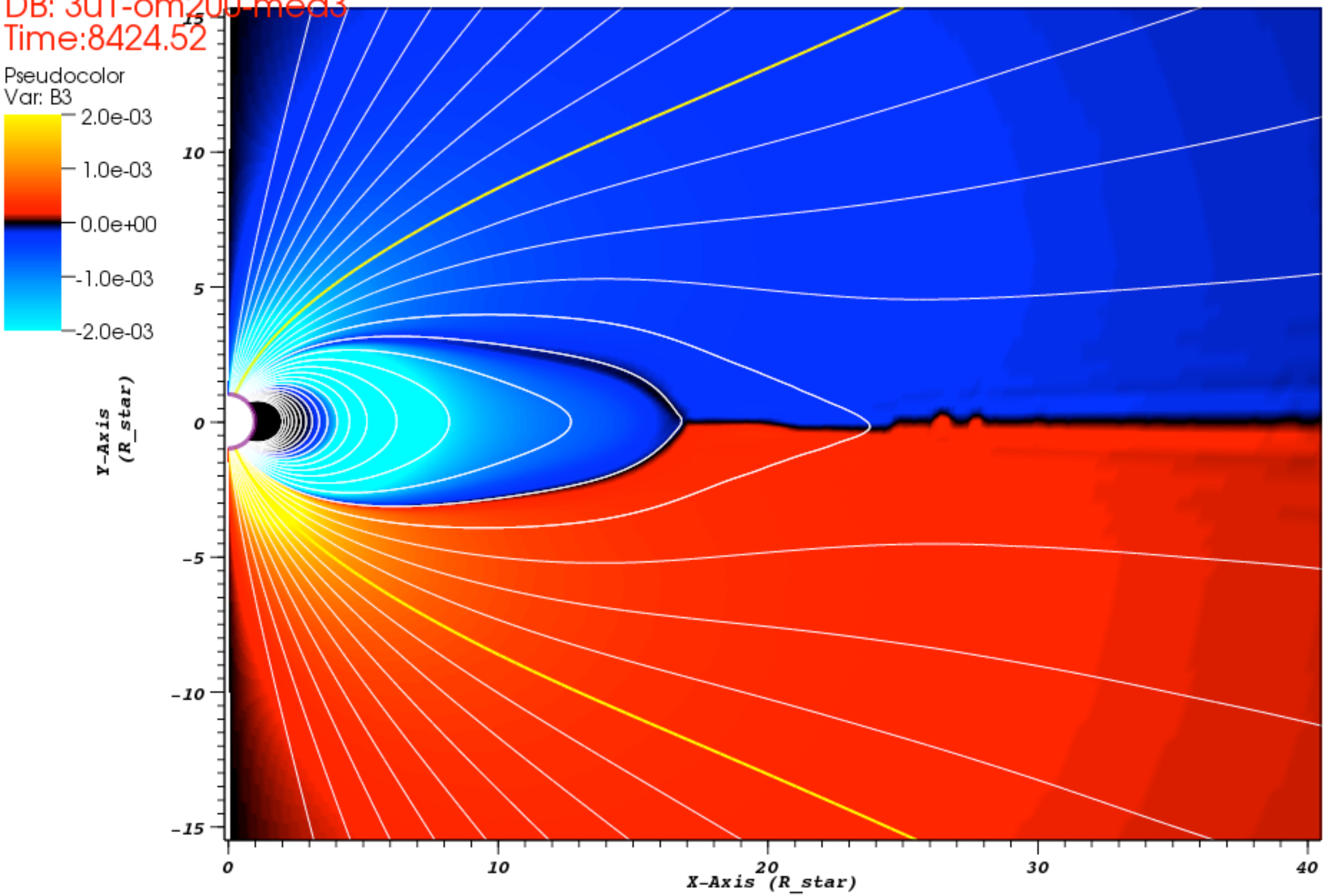


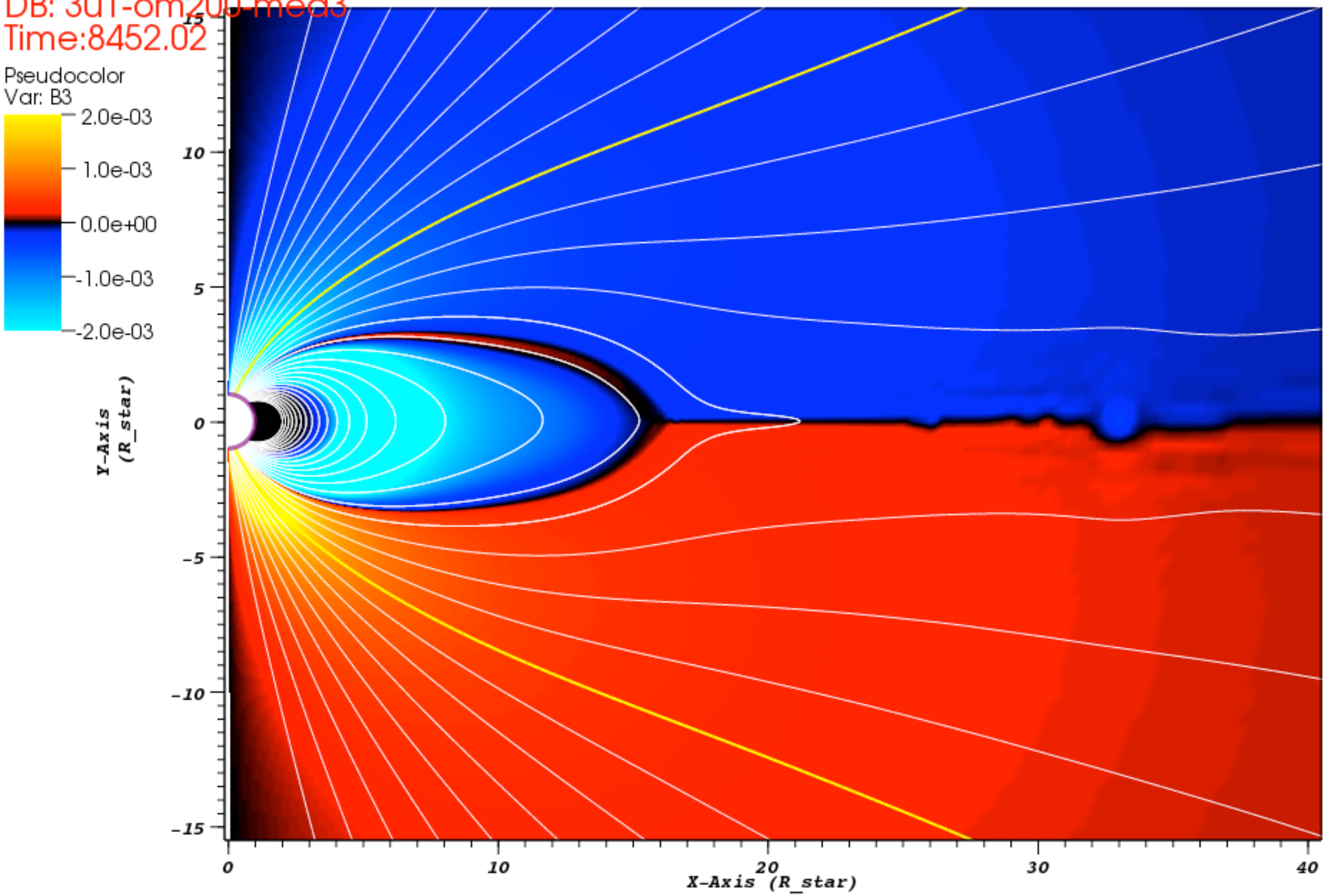
Rotating, twisted magnetosphere — unstable equilibrium
Shown: B^{Φ}, poloidal field line projections



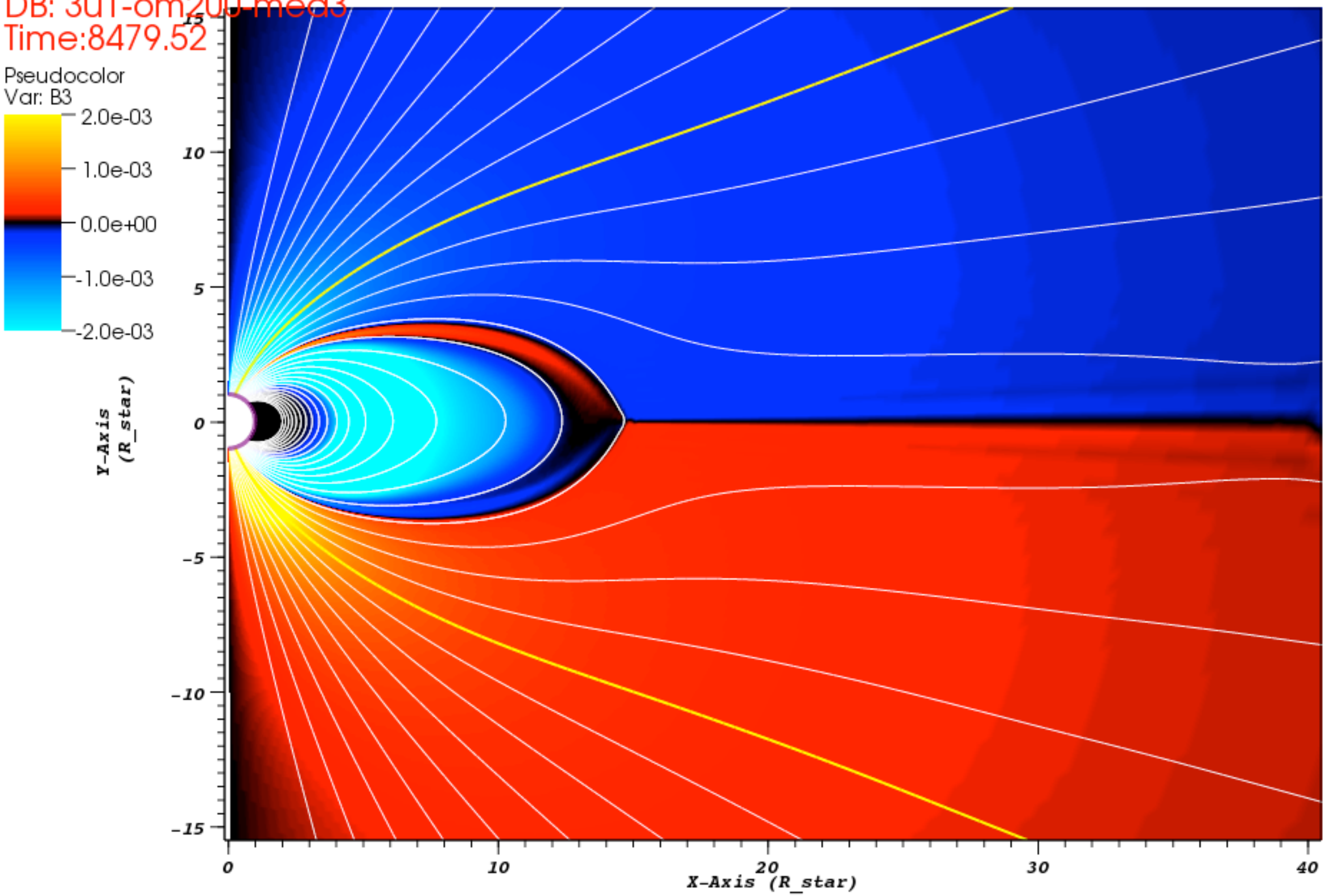
Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



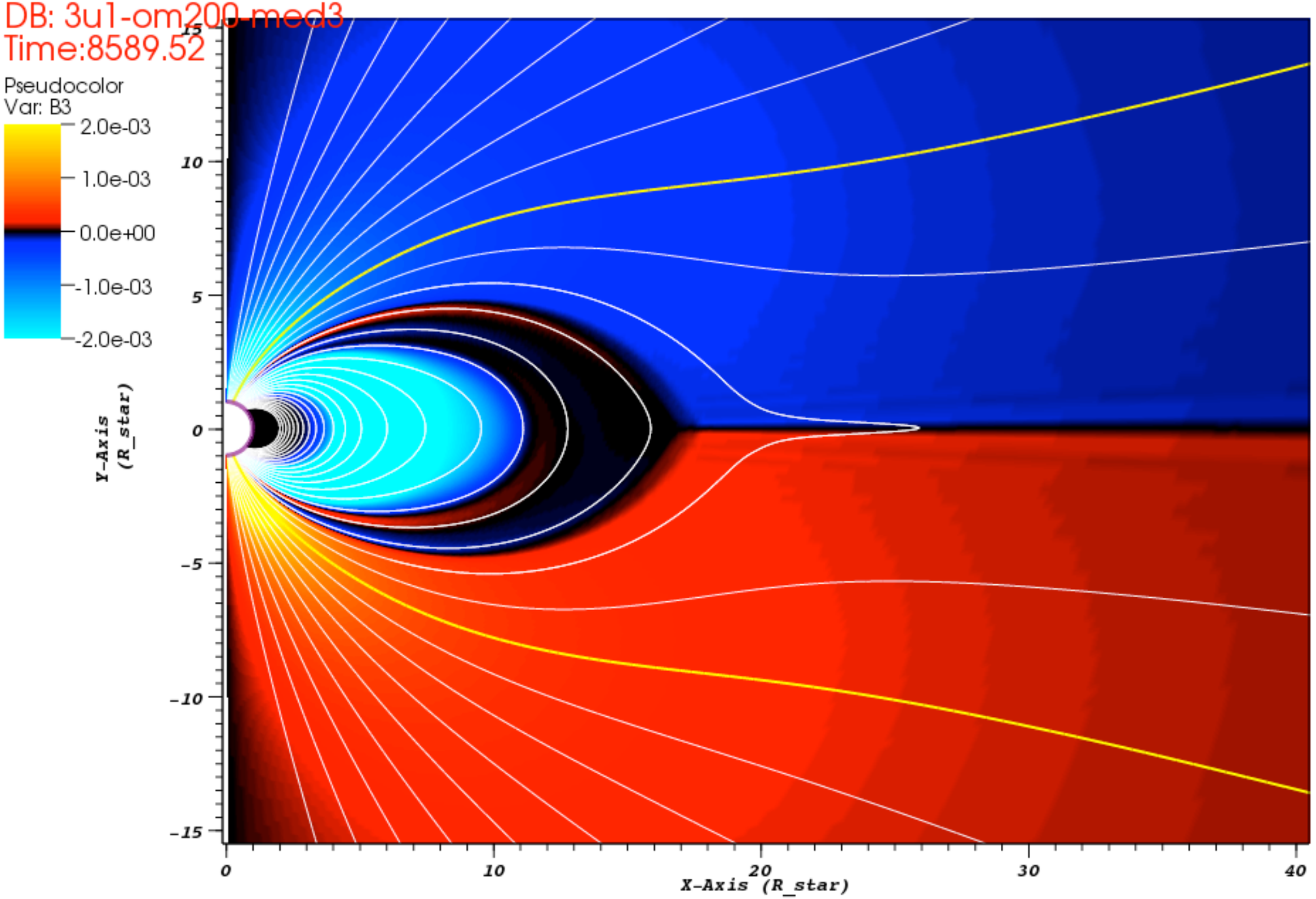
Rotating, twisted magnetosphere — first reconnection
Shown: B^{Φ}, poloidal field line projections



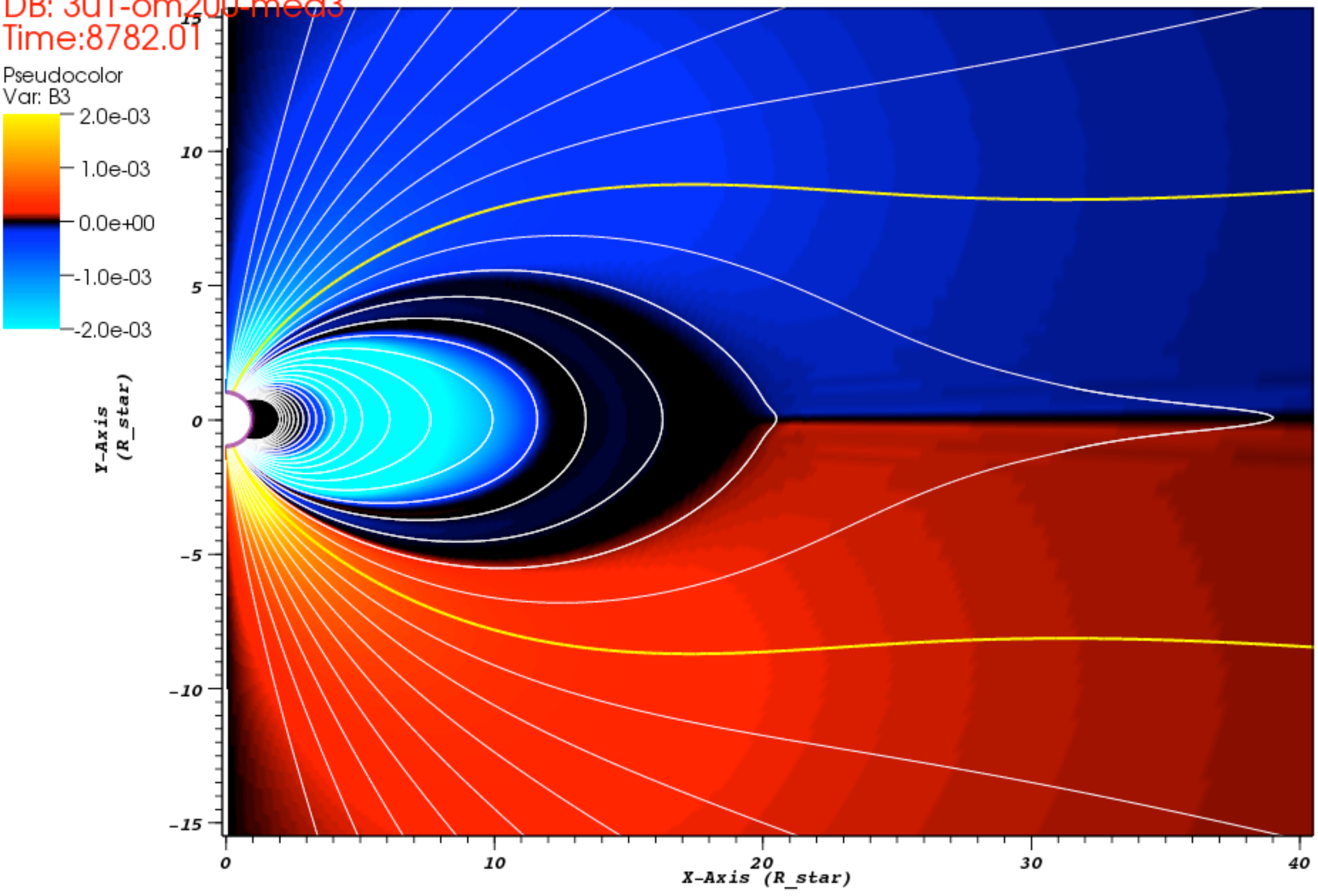
Rotating, twisted magnetosphere — first reconnection
Shown: B^{Φ}, poloidal field line projections



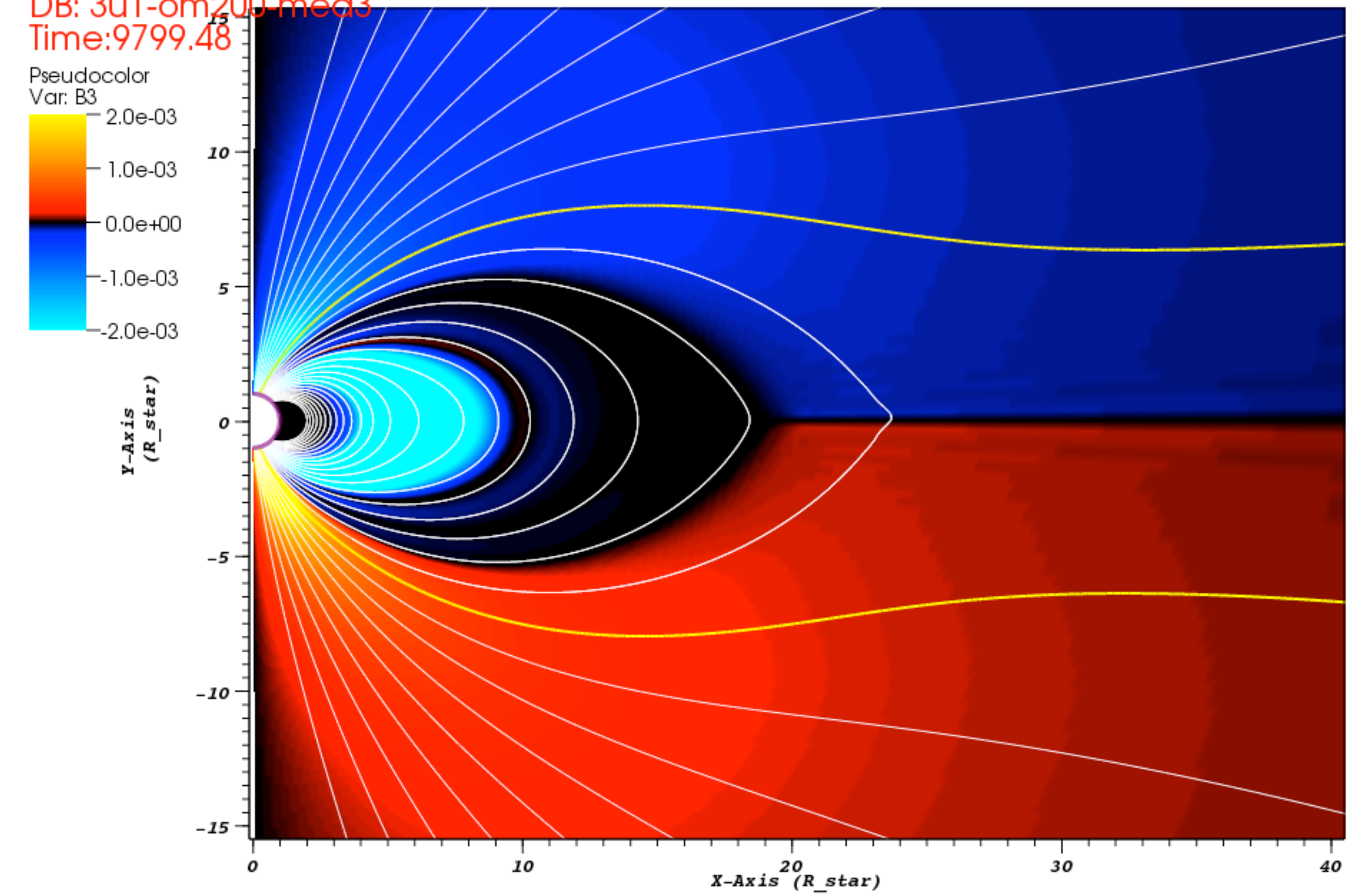
Rotating, twisted magnetosphere
Shown: B^{ϕ}, poloidal field line projections



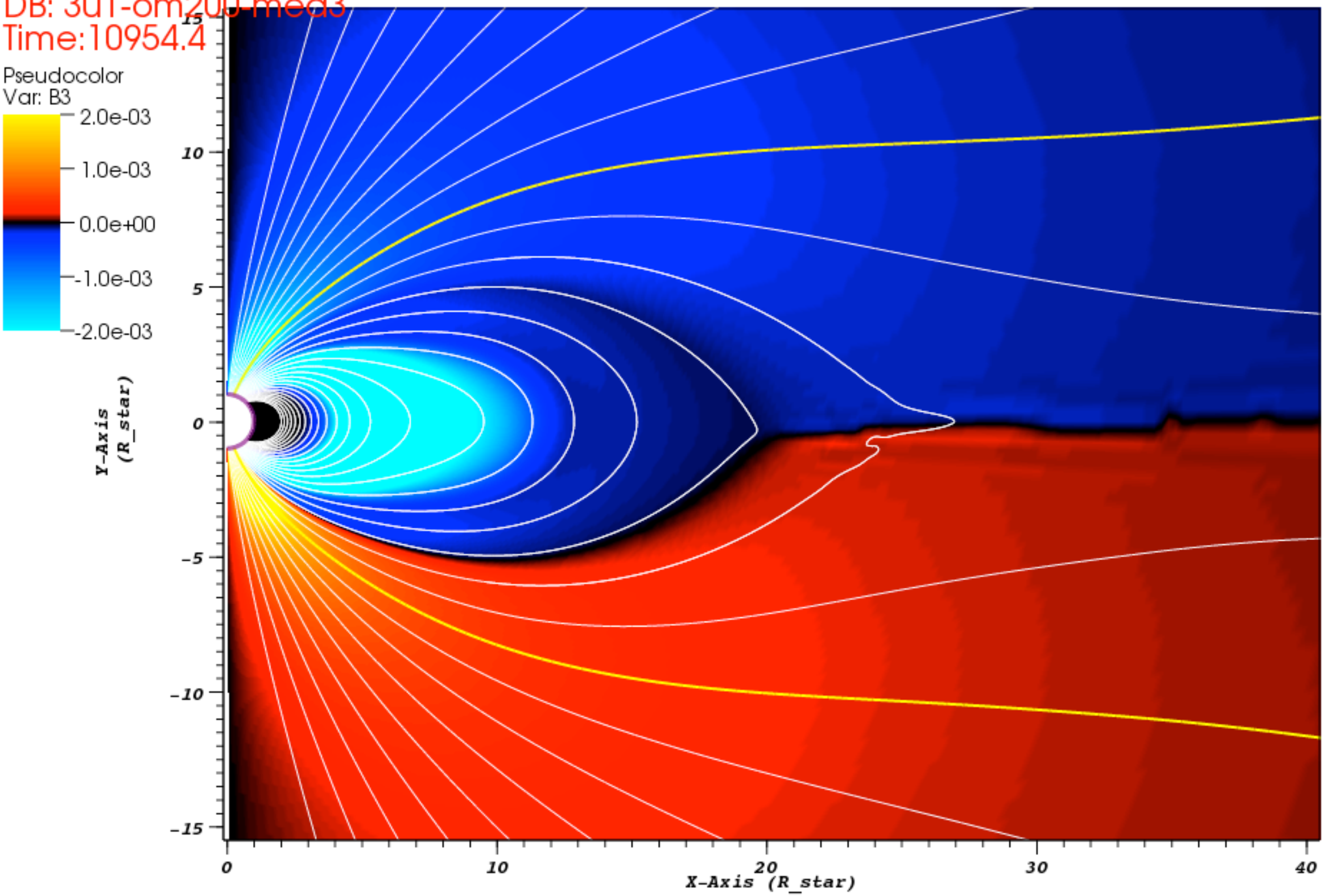
Rotating, twisted magnetosphere — cavity formation
Shown: B^{Φ}, poloidal field line projections



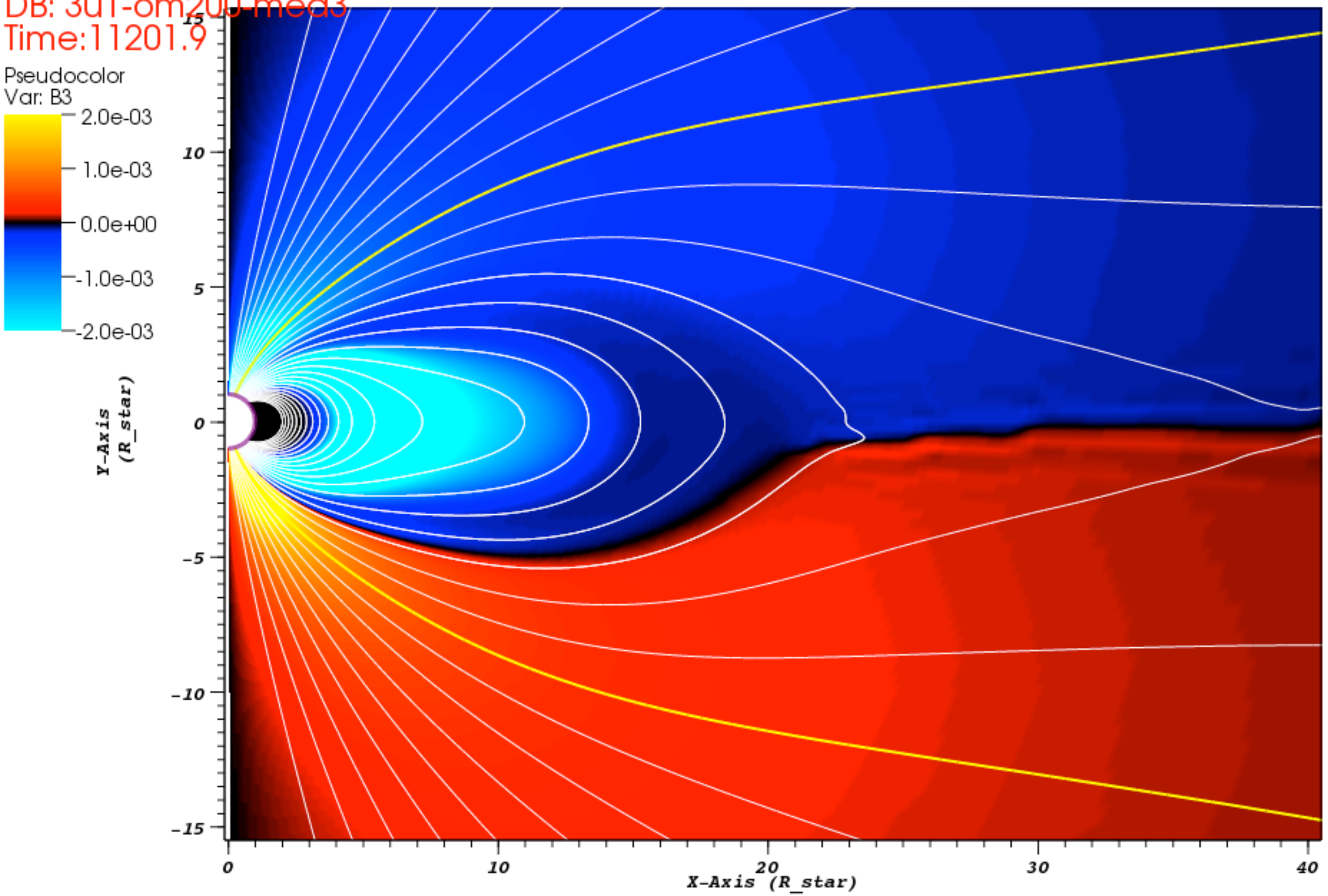
Rotating, twisted magnetosphere — cavity formation
Shown: B^{ϕ}, poloidal field line projections



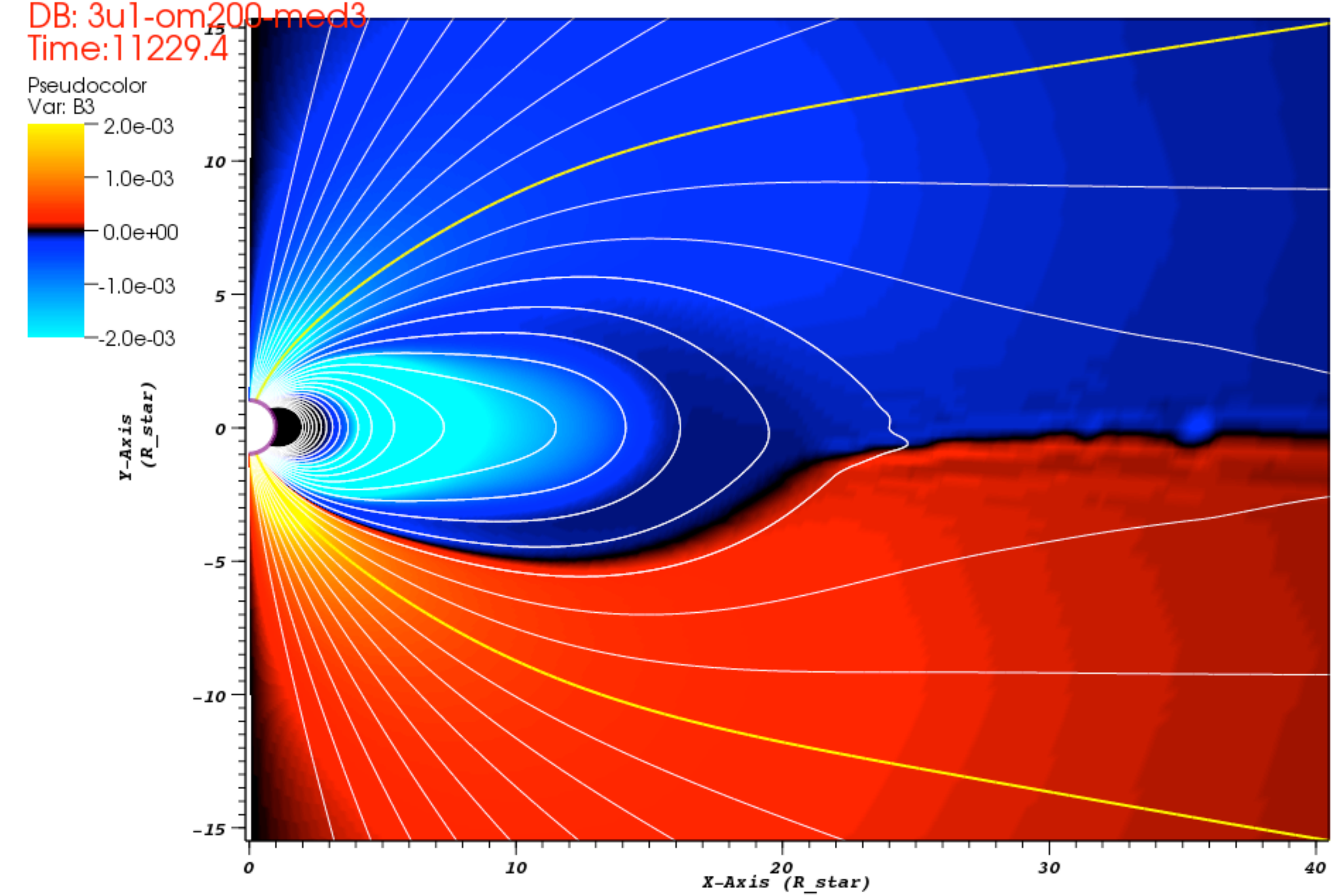
Rotating, twisted magnetosphere — cavity formation
Shown: B^{ϕ}, poloidal field line projections



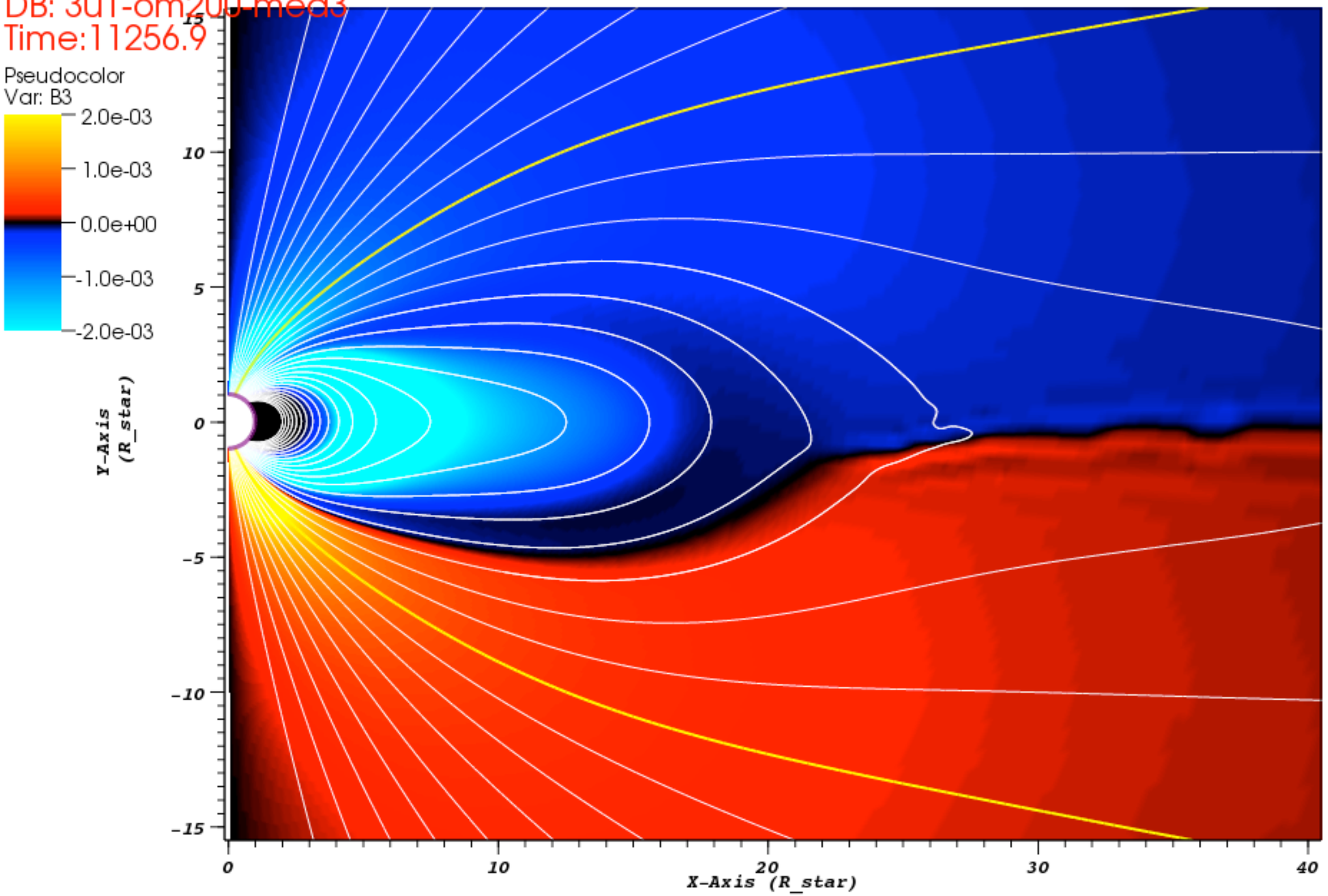
Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



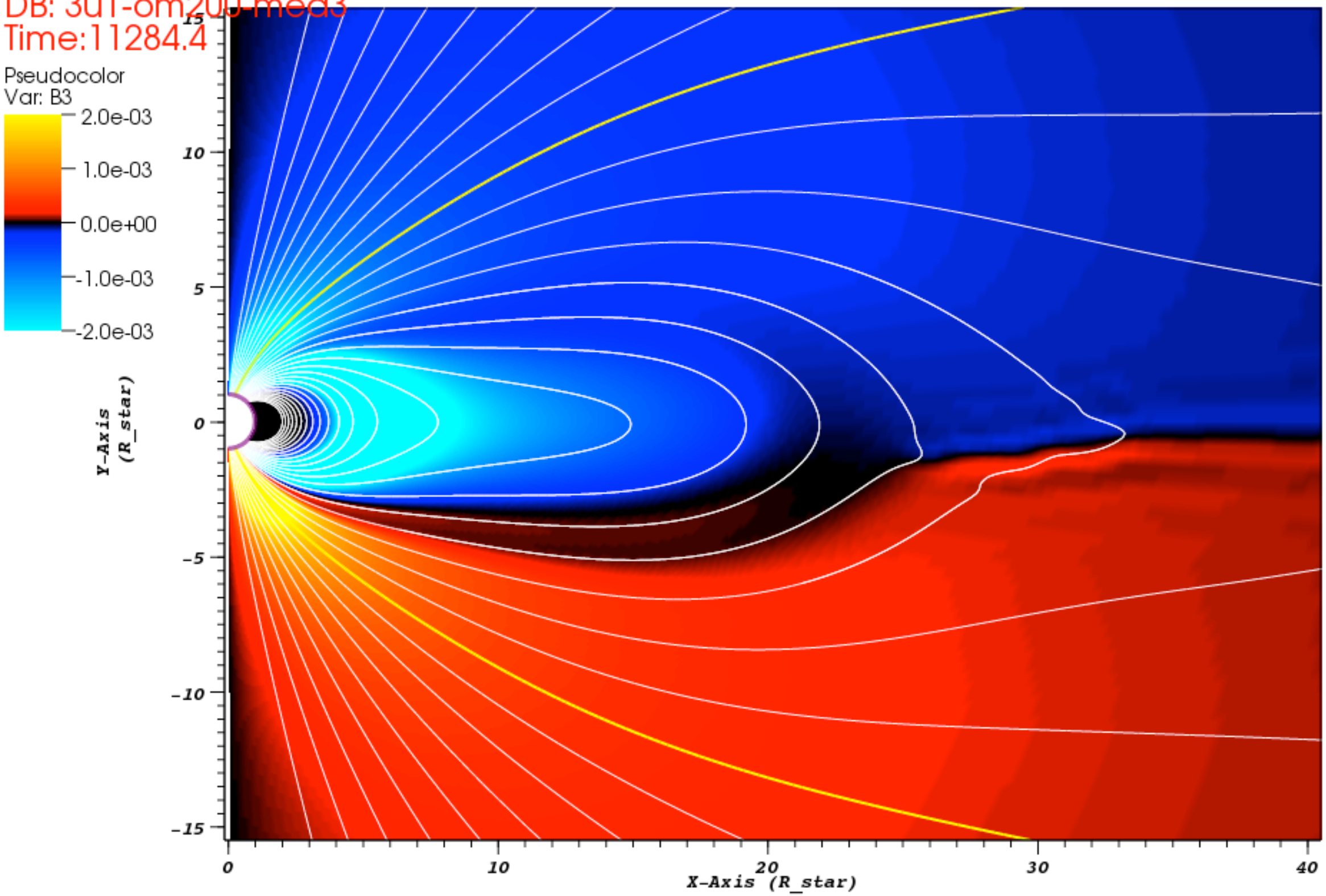
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{ϕ}, poloidal field line projections



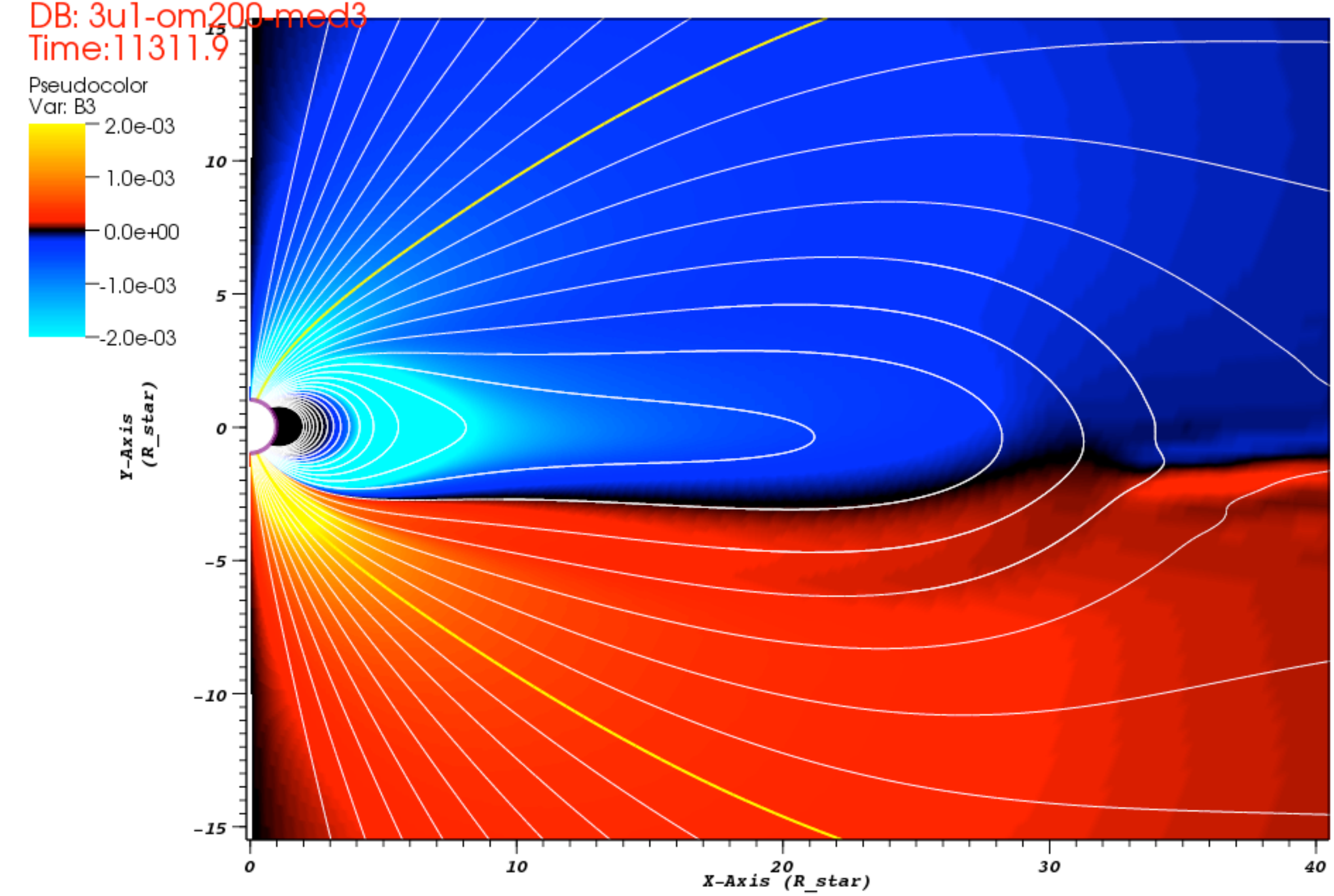
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{ϕ}, poloidal field line projections



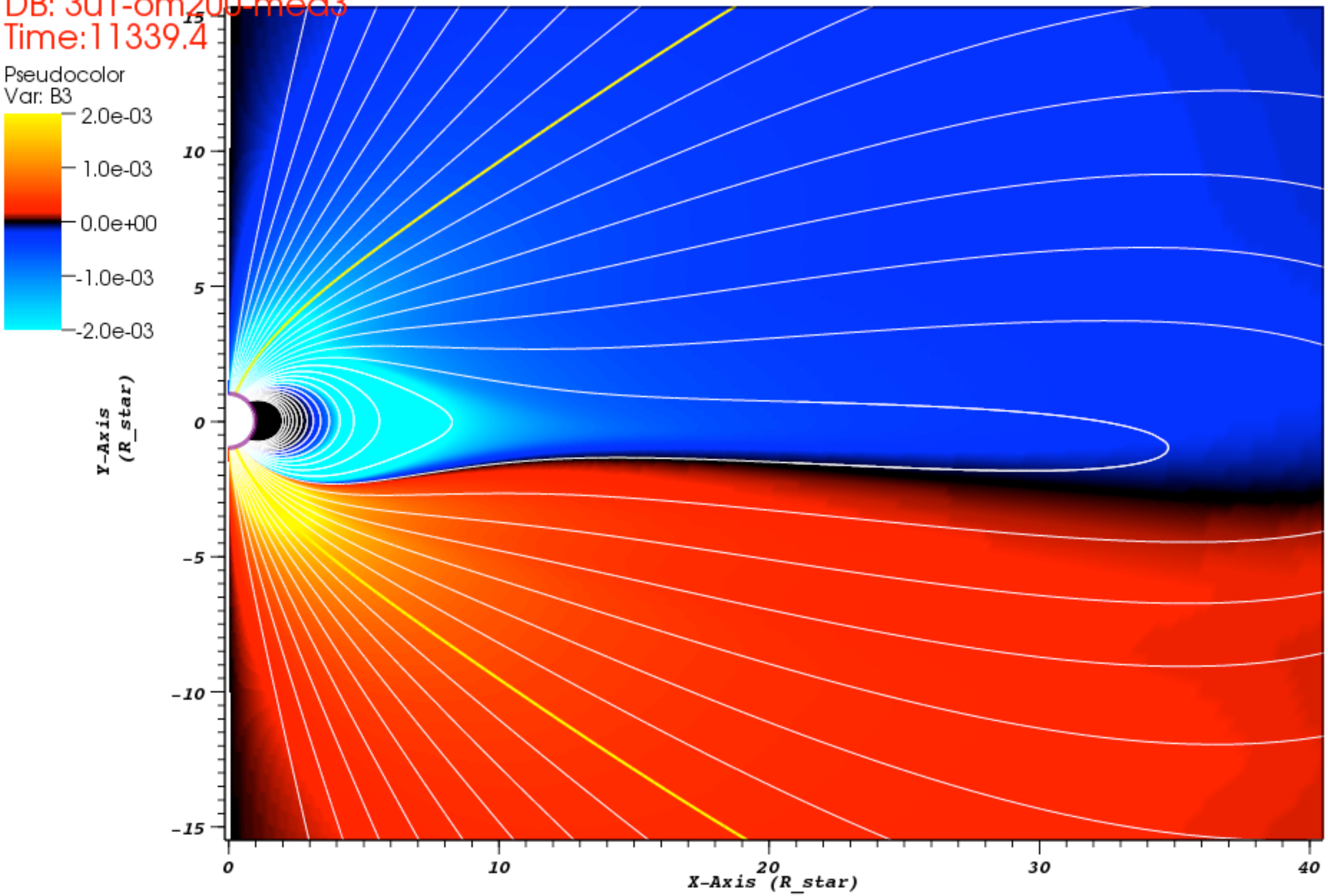
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{ϕ}, poloidal field line projections



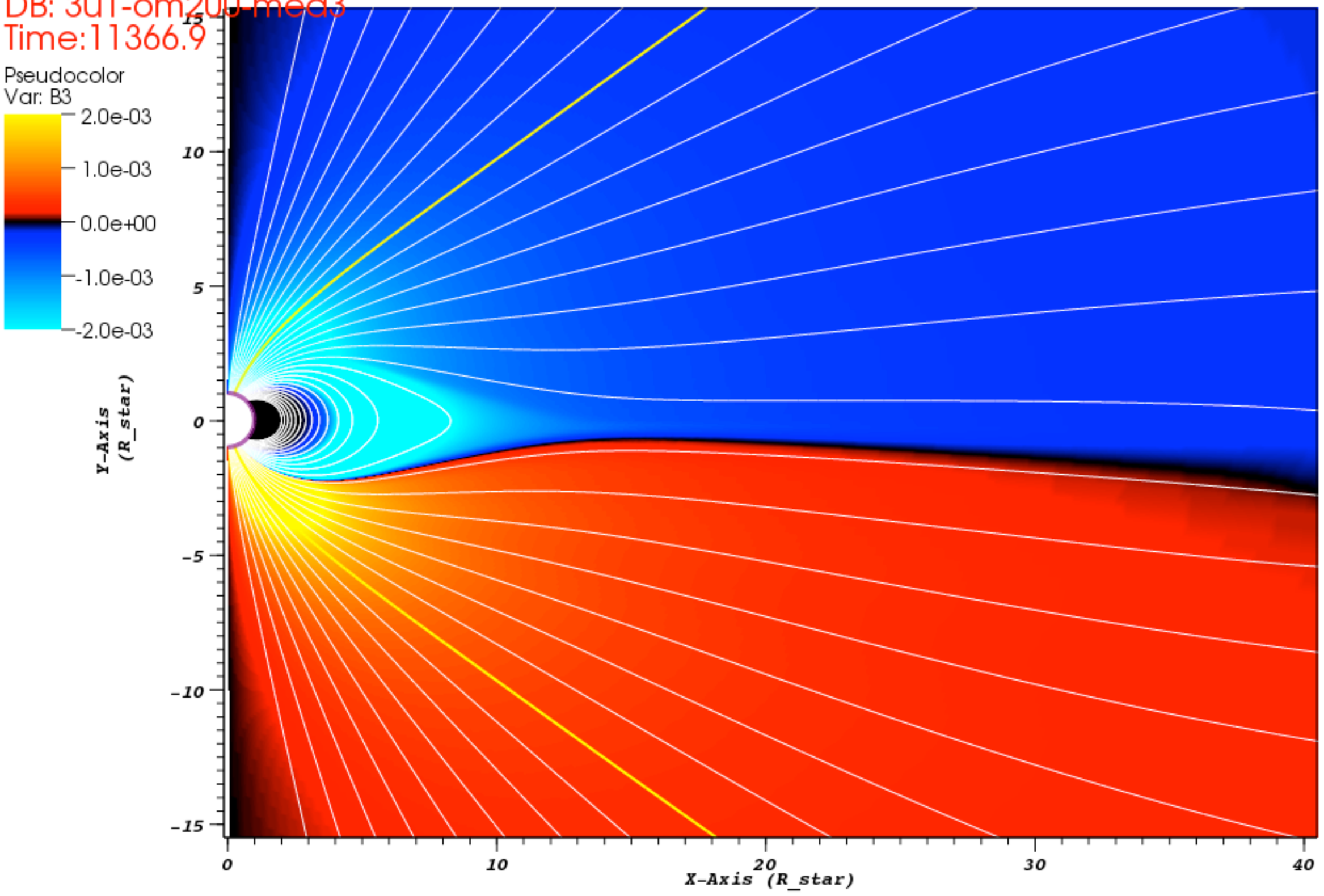
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{Φ}, poloidal field line projections



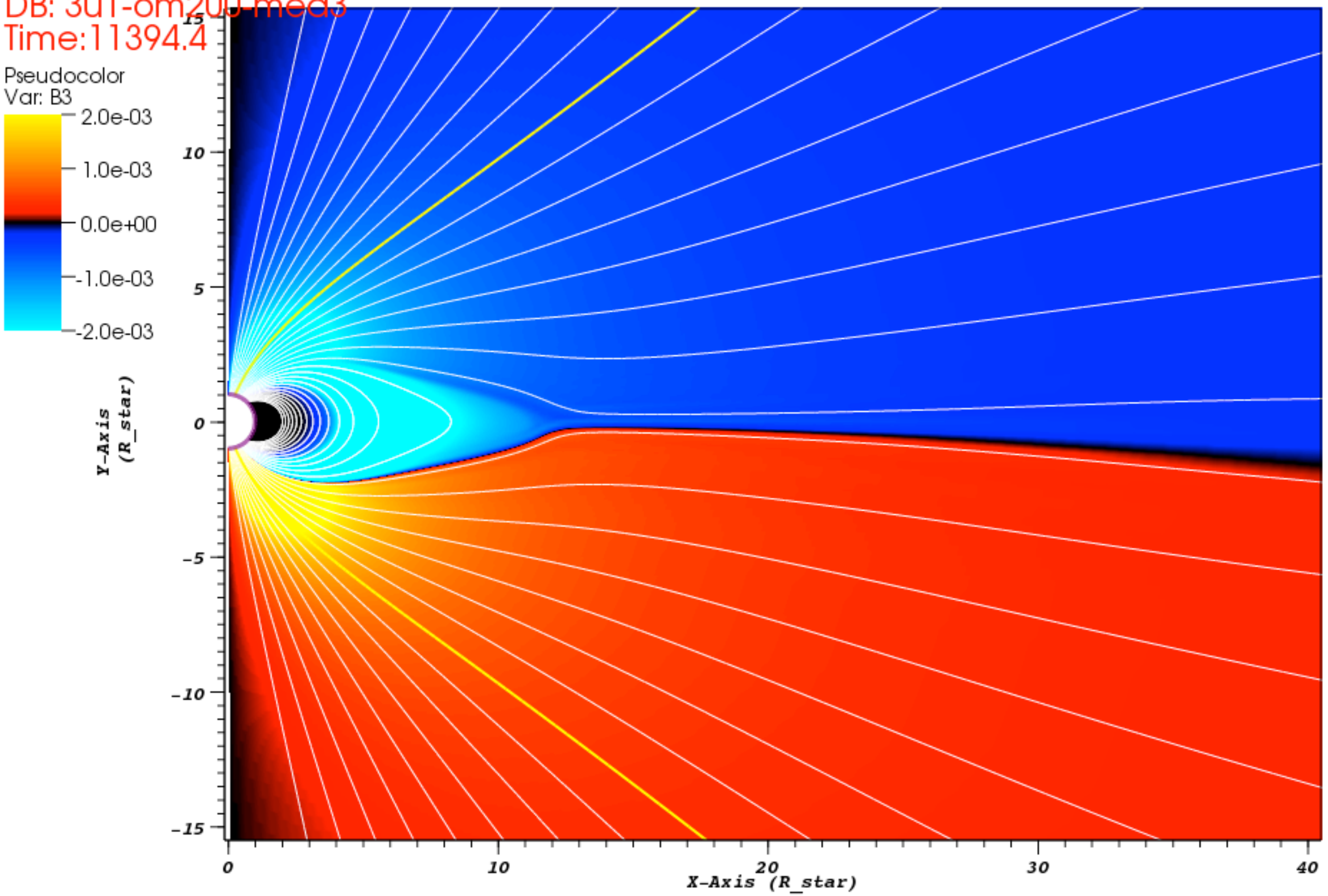
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{ϕ}, poloidal field line projections



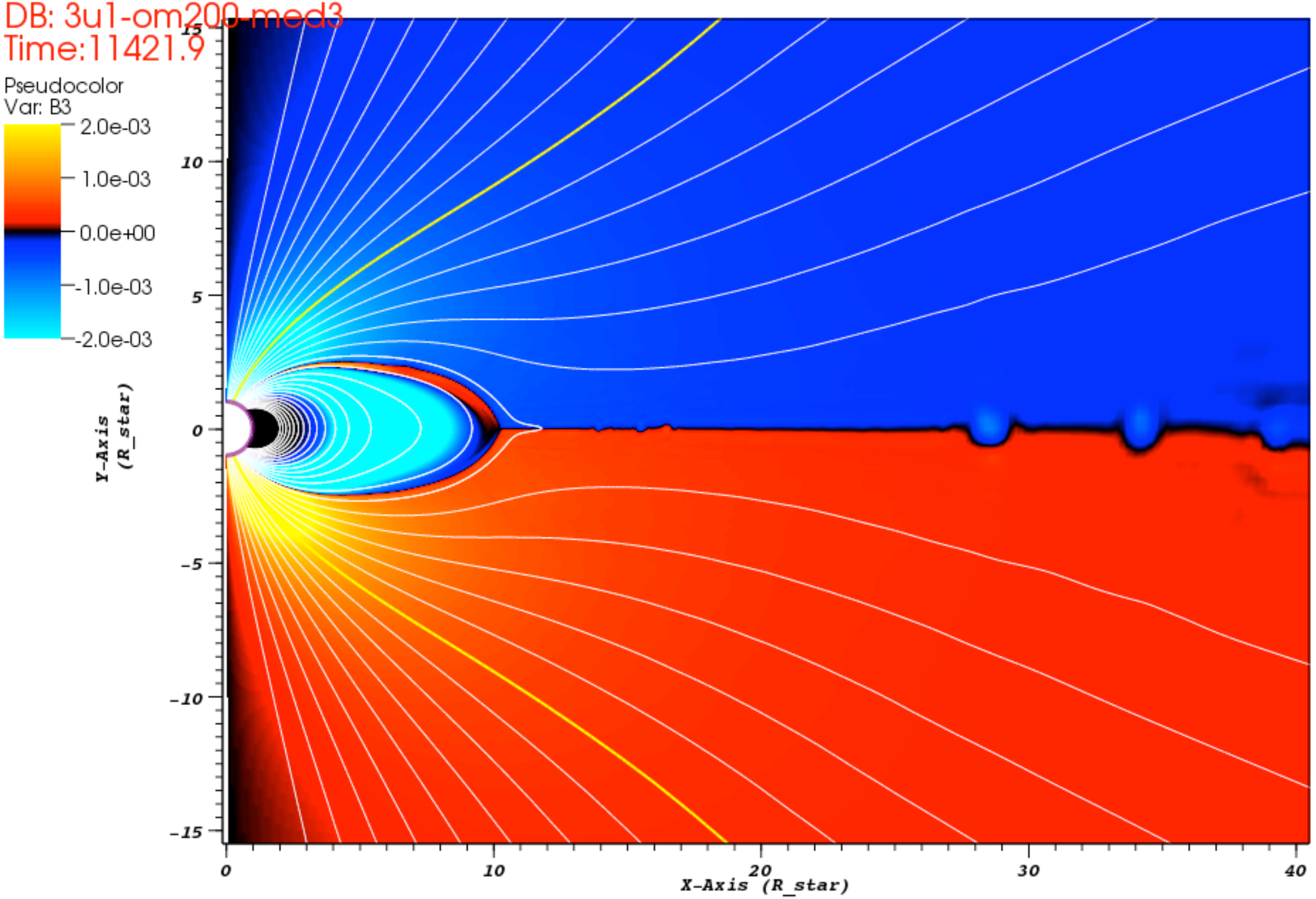
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{ϕ}, poloidal field line projections



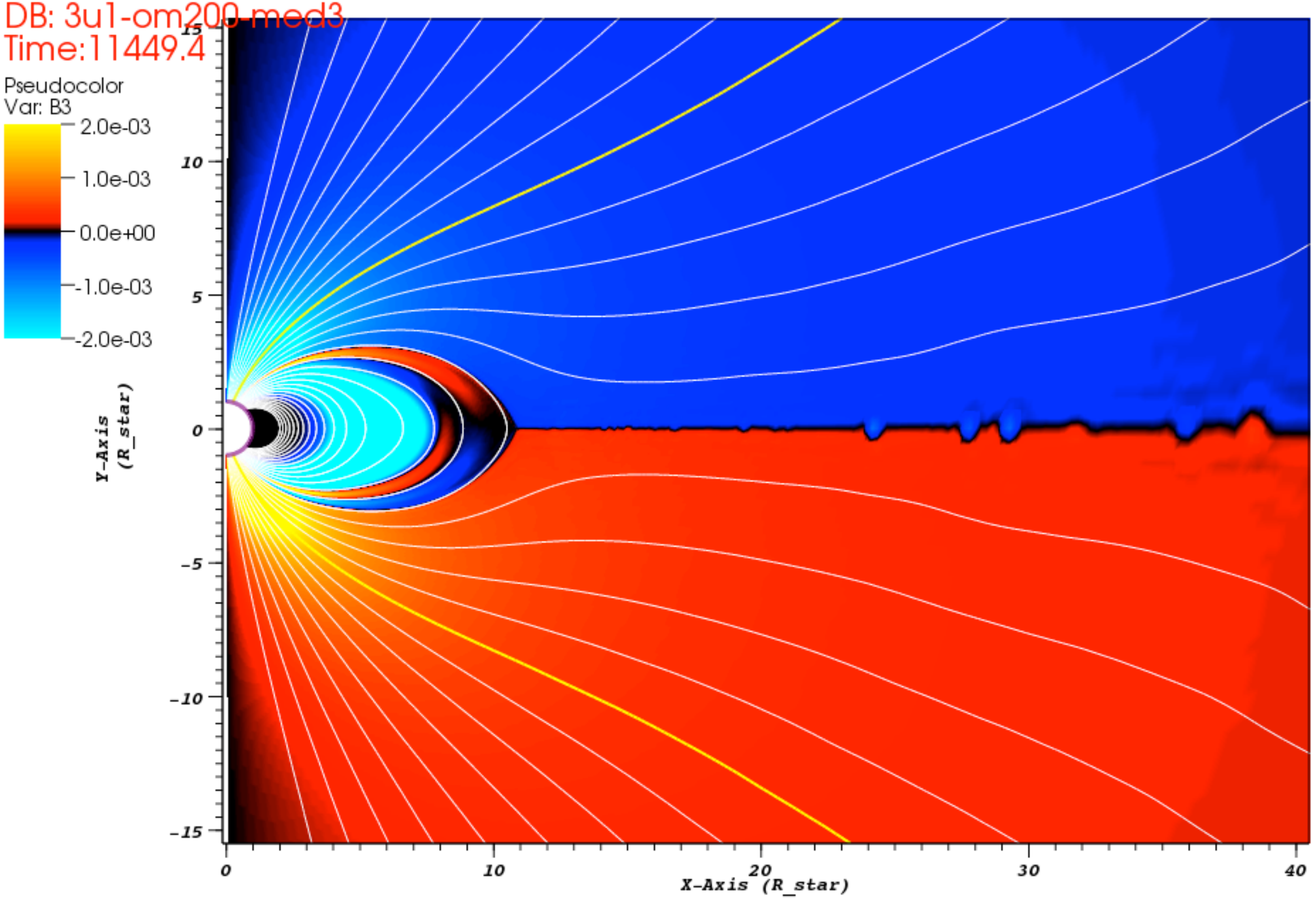
Rotating, twisted magnetosphere — impulsive opening
Shown: B^{Φ}, poloidal field line projections



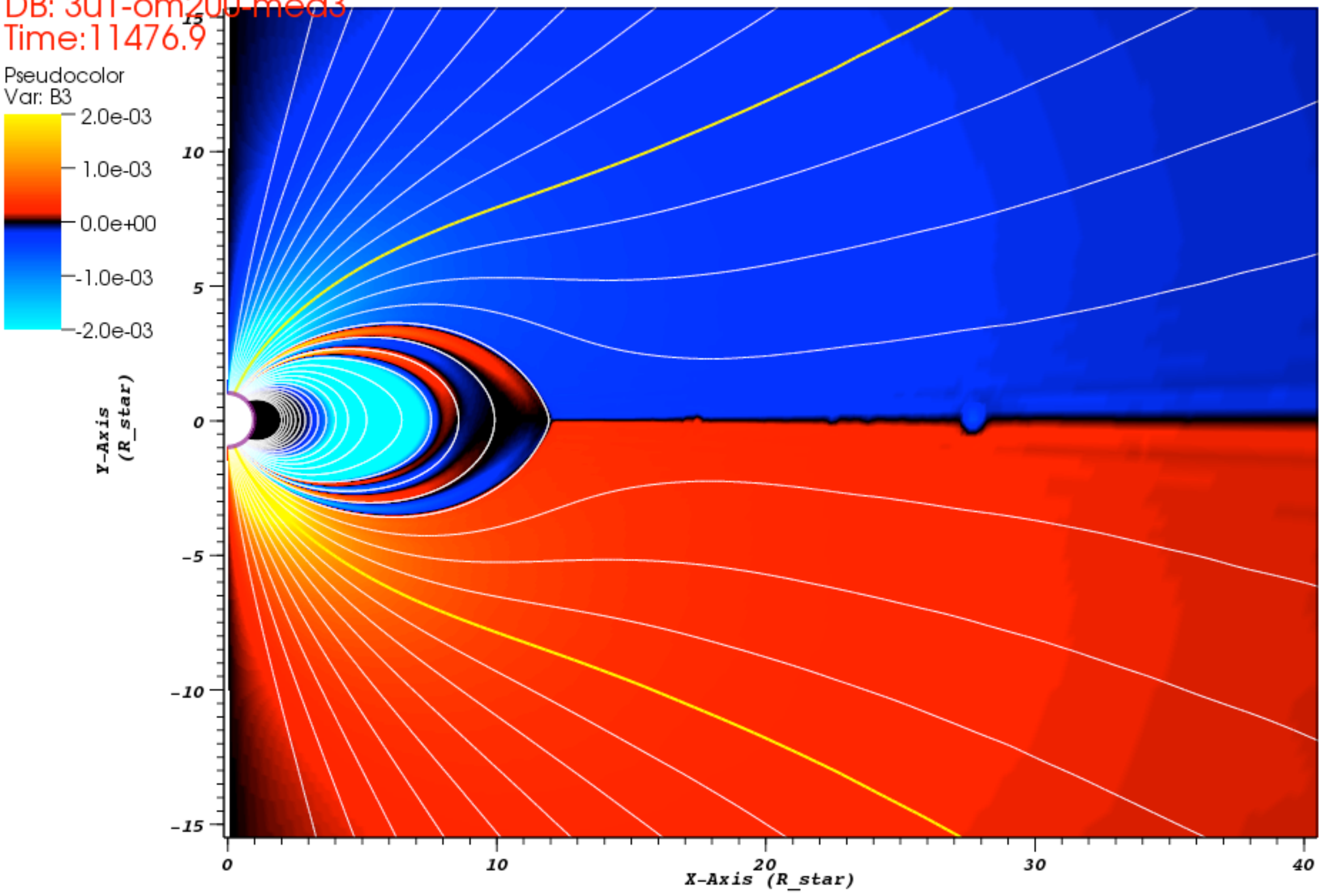
Rotating, twisted magnetosphere — second current sheet
Shown: B^{Φ}, poloidal field line projections



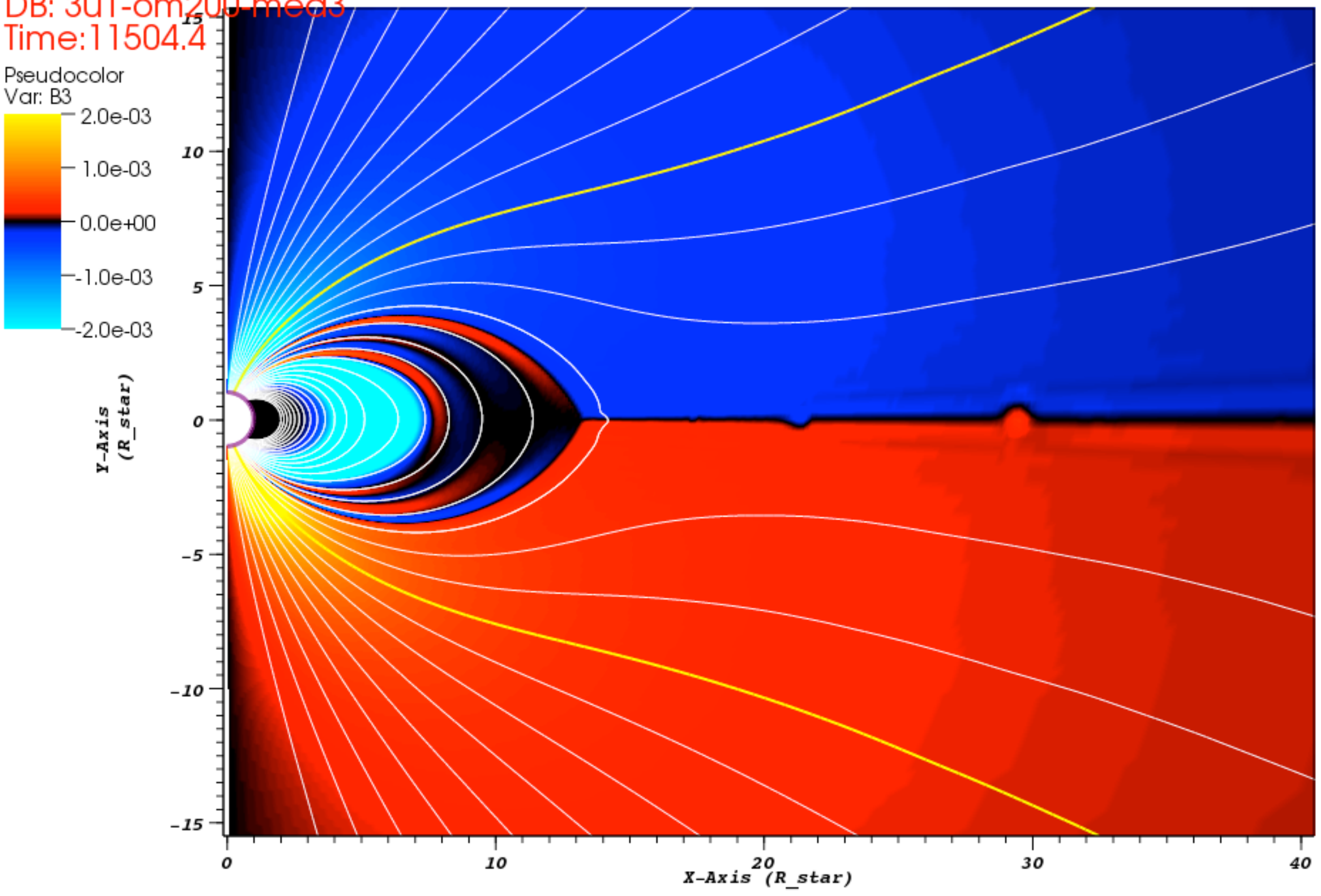
Rotating, twisted magnetosphere — second reconnection
Shown: B^{Φ}, poloidal field line projections



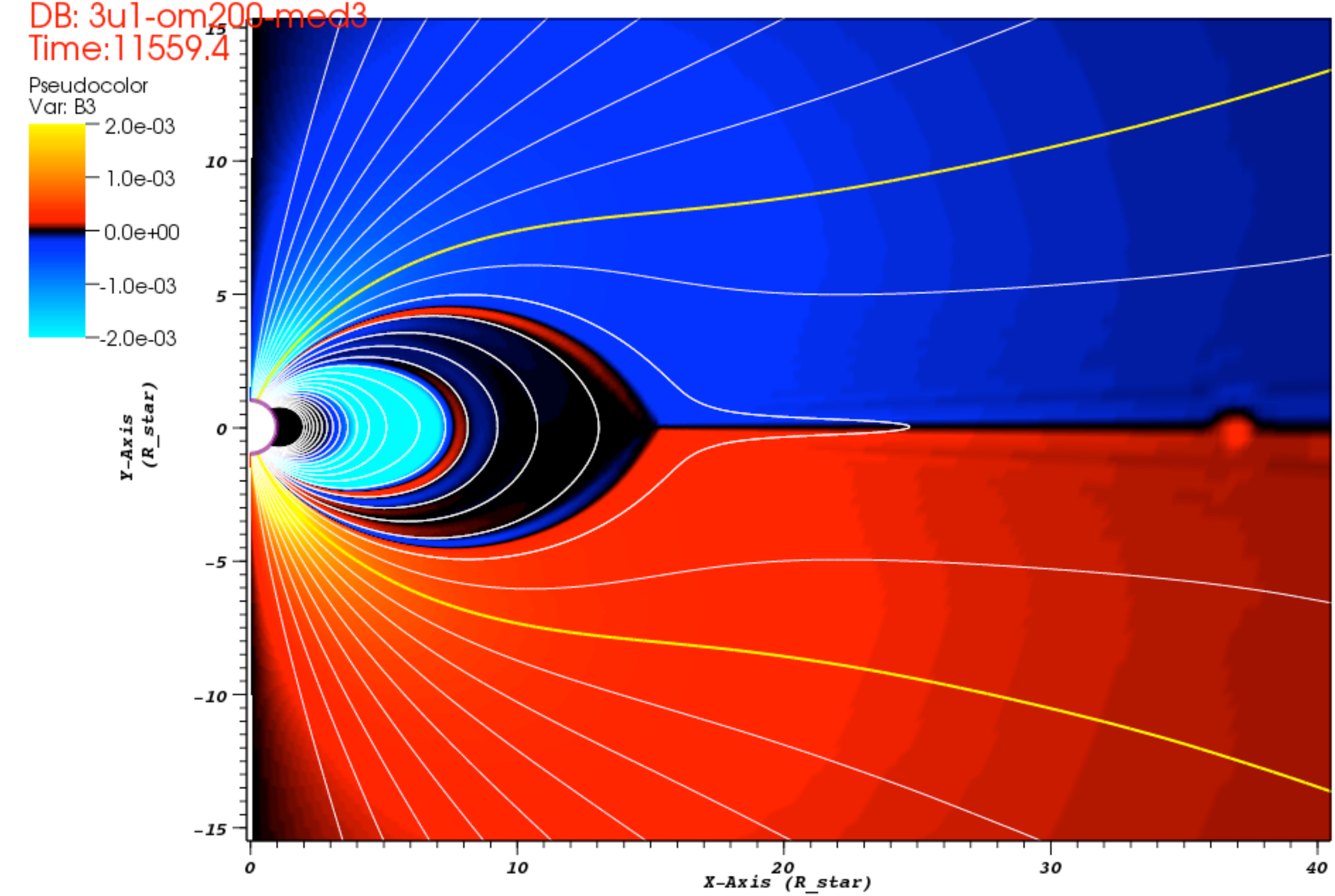
Rotating, twisted magnetosphere — second reconnection
Shown: B^{Φ}, poloidal field line projections



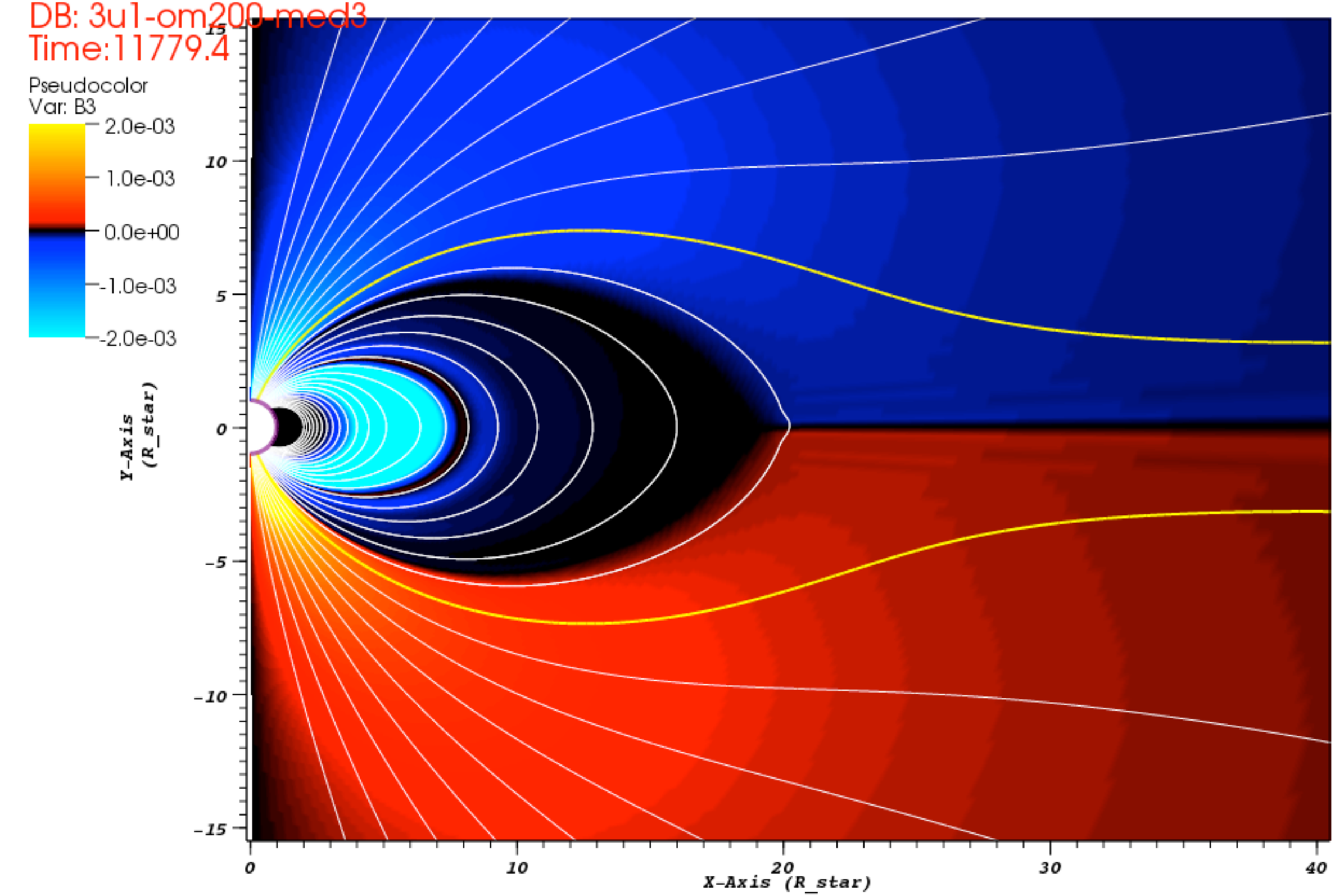
Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



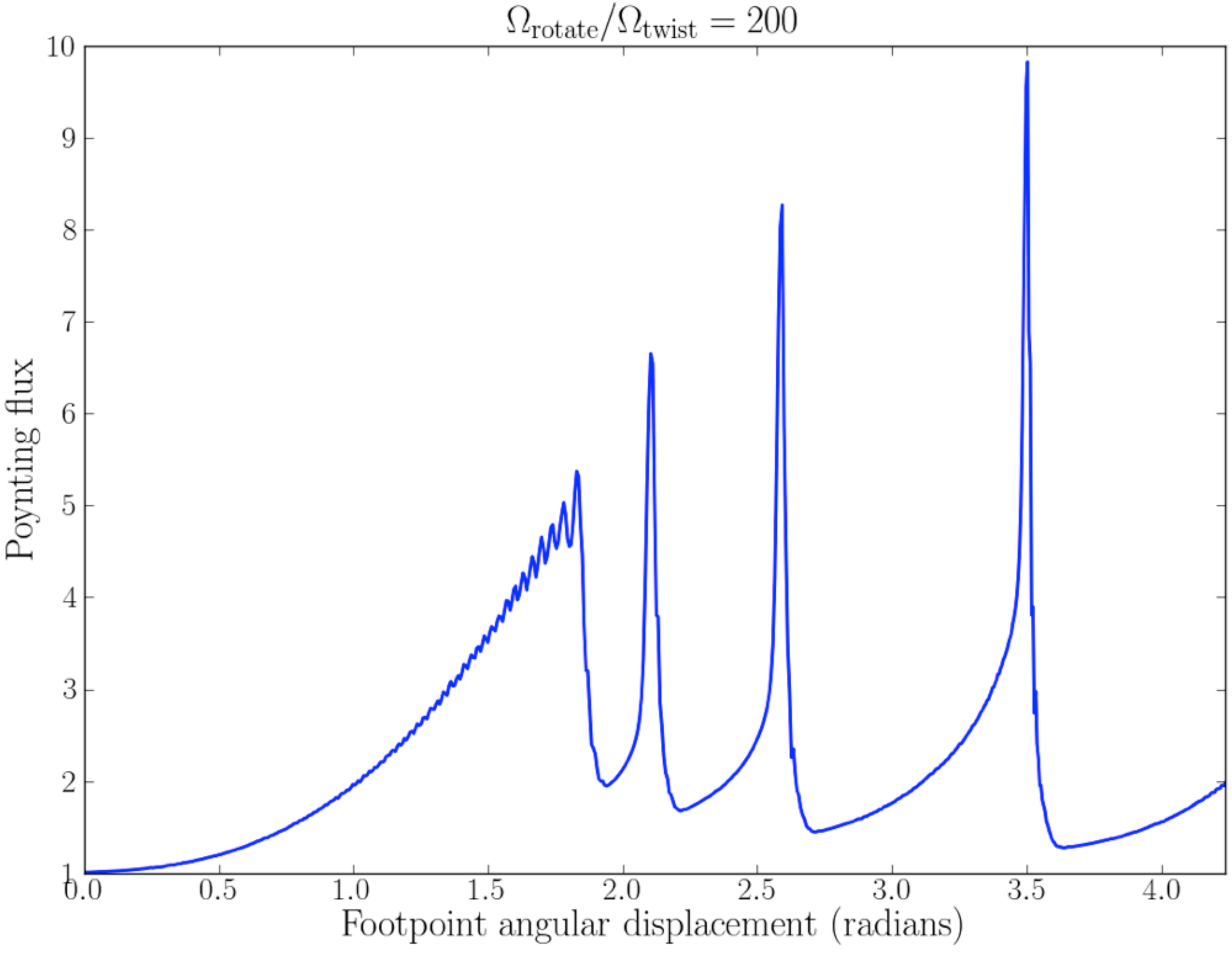
Rotating, twisted magnetosphere
Shown: B^{Φ}, poloidal field line projections



Rotating, twisted magnetosphere — second cavity
Shown: B^{ϕ}, poloidal field line projections



Rotating, twisted magnetosphere — second cavity
Shown: B^{Φ}, poloidal field line projections


$\Omega_{\text{rotate}}/\Omega_{\text{twist}} = 200$
Poynting flux
Footpoint angular displacement (radians)
10
9
8
7
6
5
4
3
2
1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Conclusions

- Pulsar: no plasmoid generation, steady Y-point & current sheet
- Plasmoid-dominated reconnection of twisted configurations
- Twisting of rotating stars can lead to large, sudden ‘anti-glitches’
- Future work
  - Full 3D simulations (spherical harmonics) — kink instability?
  - Resistivity
  - Curved spacetime (Schwarzschild, Kerr) with formalism of Komissarov (2004)

Спасибо!