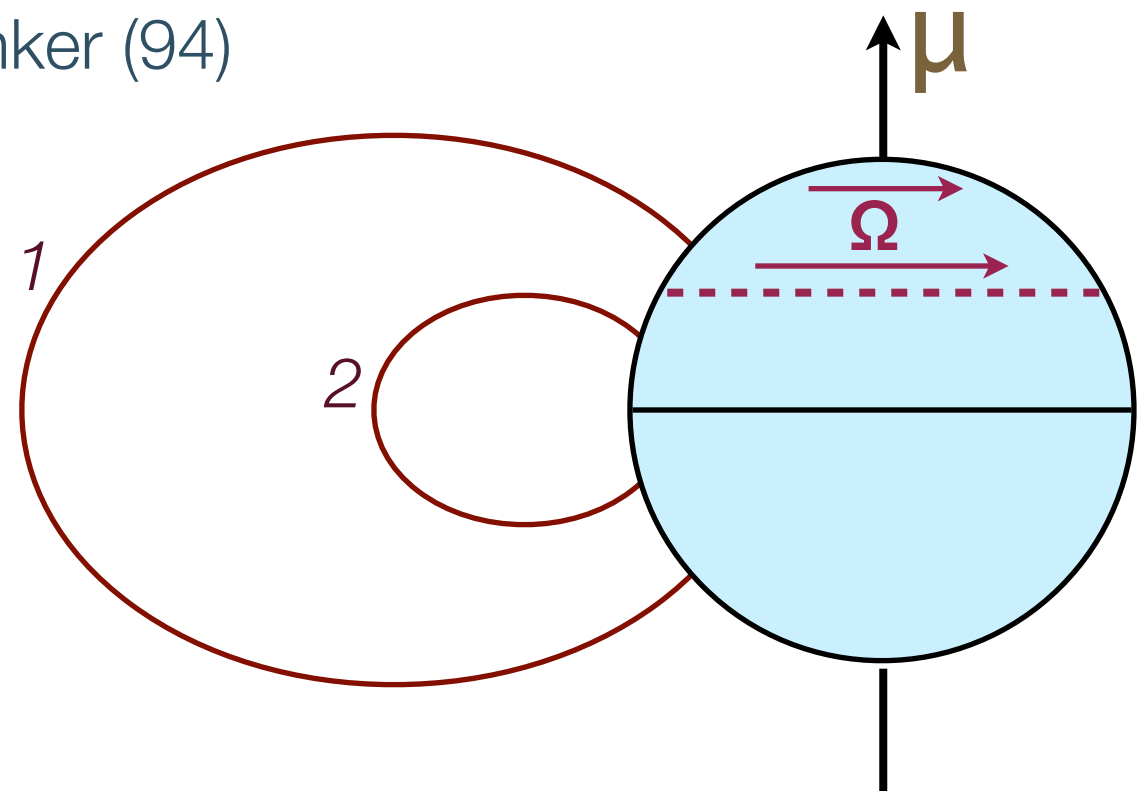


Time-dependent simulations of force-free magnetospheres with *Phaedra*

Kyle Parfrey, Andrei Beloborodov, Lam Hui

Dynamic magnetospheres

- Rotation-powered pulsars (Contopoulos et al 99, Spitkovsky 06)
- Magnetars — SGRs/AXPs: driven by crustal motions (Thompson & Duncan 95)?
 - Self-similar solutions : Low (86), Wolfson (95)
 - problematic since they involve compressive motion of crust
 - Solar corona simulations : Mikic & Linker (94)



Force-free electrodynamics

- Strong-field limit of magnetised fluids (Gruzinov 99)

$$\partial_t \vec{B} = -\nabla \times \vec{E}, \quad \partial_t \vec{E} = \nabla \times \vec{B} - \vec{J}$$

$$\vec{J} = \frac{\vec{B} \cdot (\nabla \times \vec{B}) - \vec{E} \cdot (\nabla \times \vec{E})}{B^2} \vec{B} + (\nabla \cdot \vec{E}) \frac{\vec{E} \times \vec{B}}{B^2}$$

$$\vec{E} \cdot \vec{B} = 0$$

$$\vec{E} \cdot \vec{J} = 0 : \text{no dissipation}$$

- Current sheets (discontinuities in magnetic field) are common

A spectral code for force-free electrodynamics

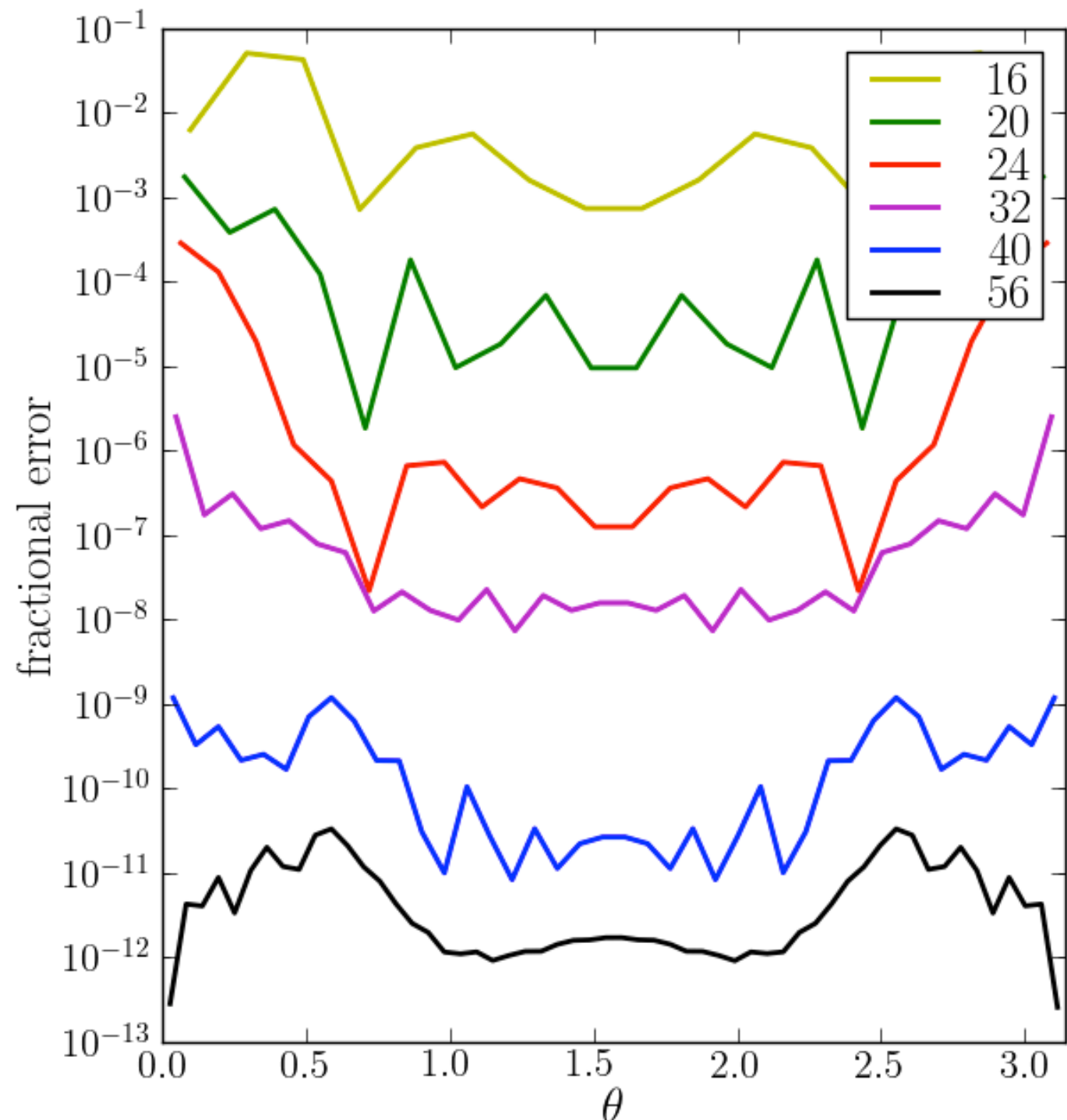
- *Phaedra: Pseudospectral High-Accuracy ElectroDynamics for Relativistic Astrophysics* — 2D/axisymmetric
- Existing codes: Komissarov (2004), Spitkovsky (2006), McKinney (2006), Kalapotharakos & Contopoulos (2009), Yu (2010)
- Spatial derivatives via expansion into orthogonal basis functions

$$B^i = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} a_{nl} T_n(r) \begin{cases} \cos(l\theta) \\ \sin(l\theta) \end{cases}$$

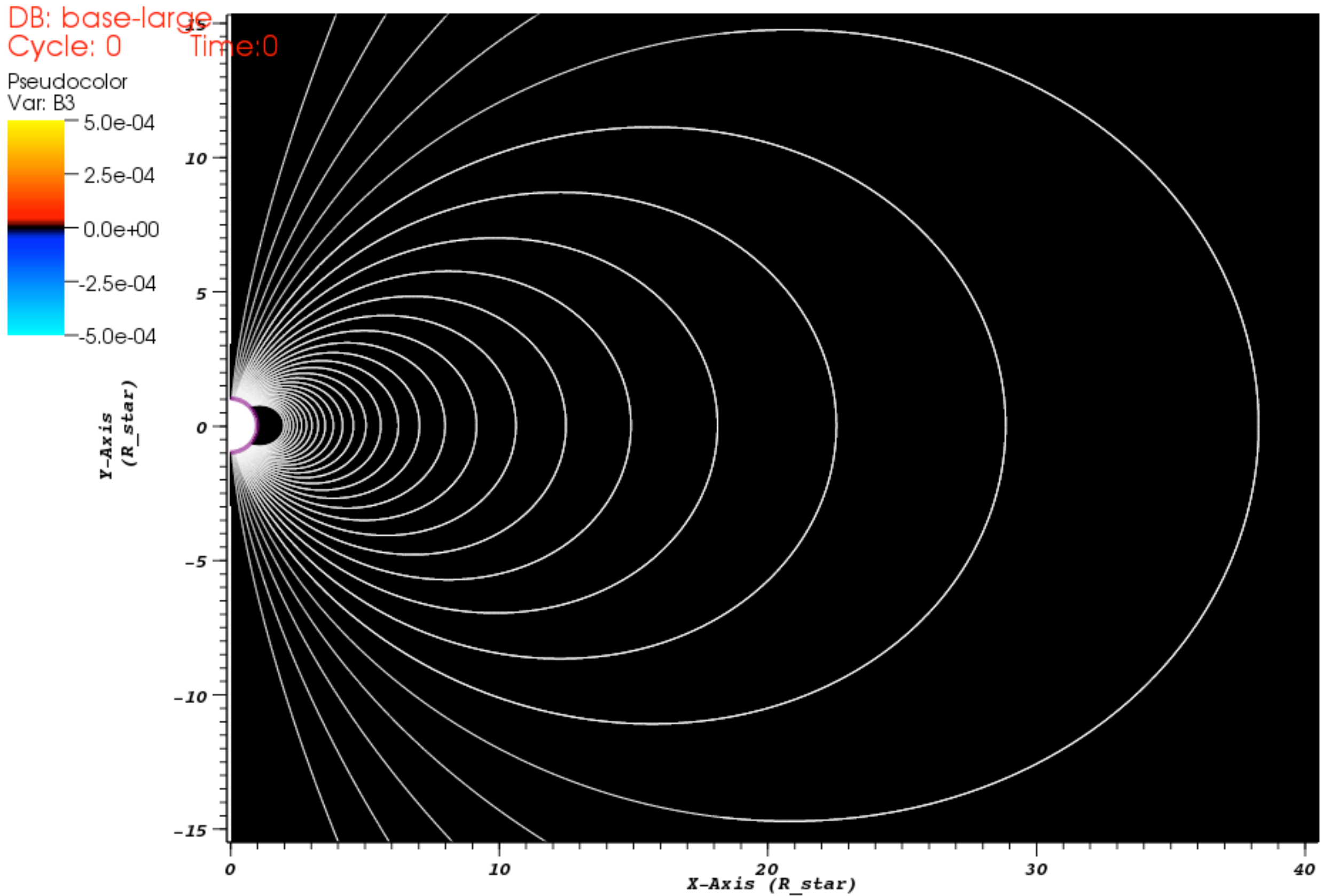
- High-order spectral filtering for “shock” capturing
- Very low dispersion error or intrinsic dissipation

Simple 2D test: rotating monopole

Resolution
 N_θ given in legend
 $N_r \sim 1\frac{1}{2} N_\theta$



Fractional errors in B^φ at $5R_\star$ versus analytic steady-state solution (Michel 72)

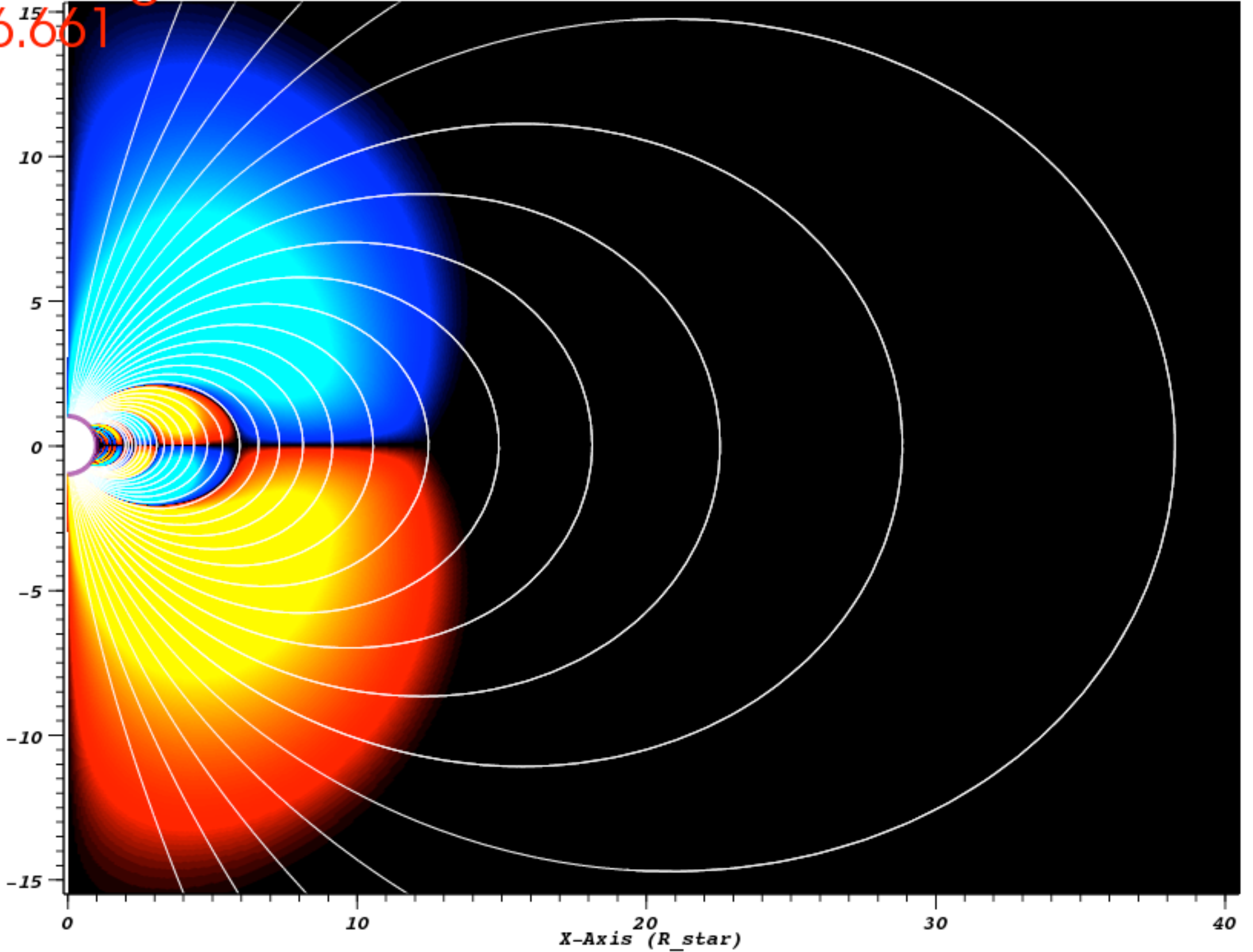


Rotating dipole — axisymmetric pulsar — light cylinder at 20 stellar radii
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 16.661

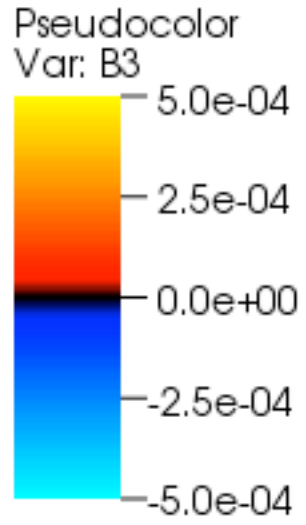
Pseudocolor
Var: B3
5.0e-04
2.5e-04
0.0e+00
-2.5e-04
-5.0e-04

Y-Axis
(R_star)

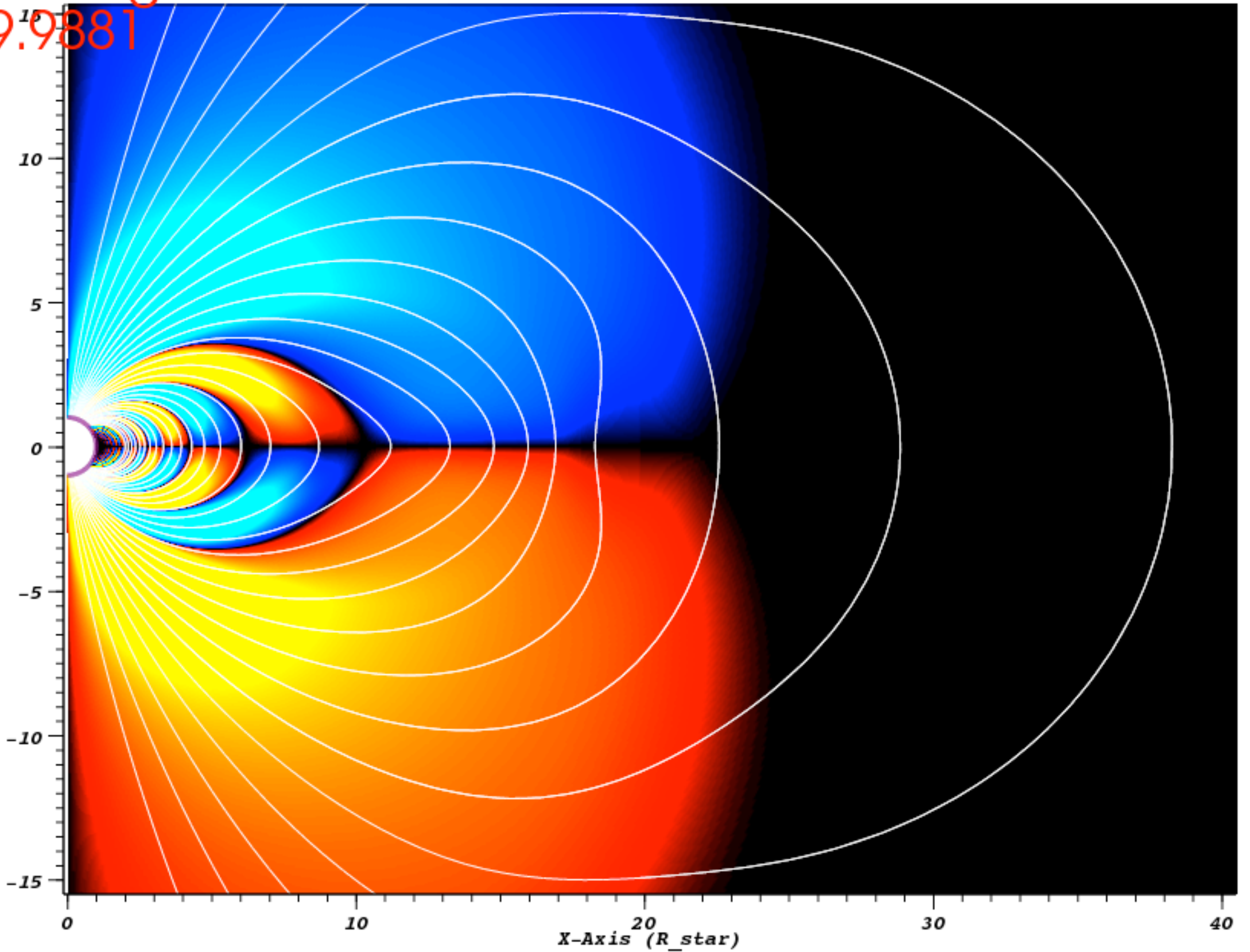


Rotating dipole — axisymmetric pulsar
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 29.9881



Y-Axis
(R_star)

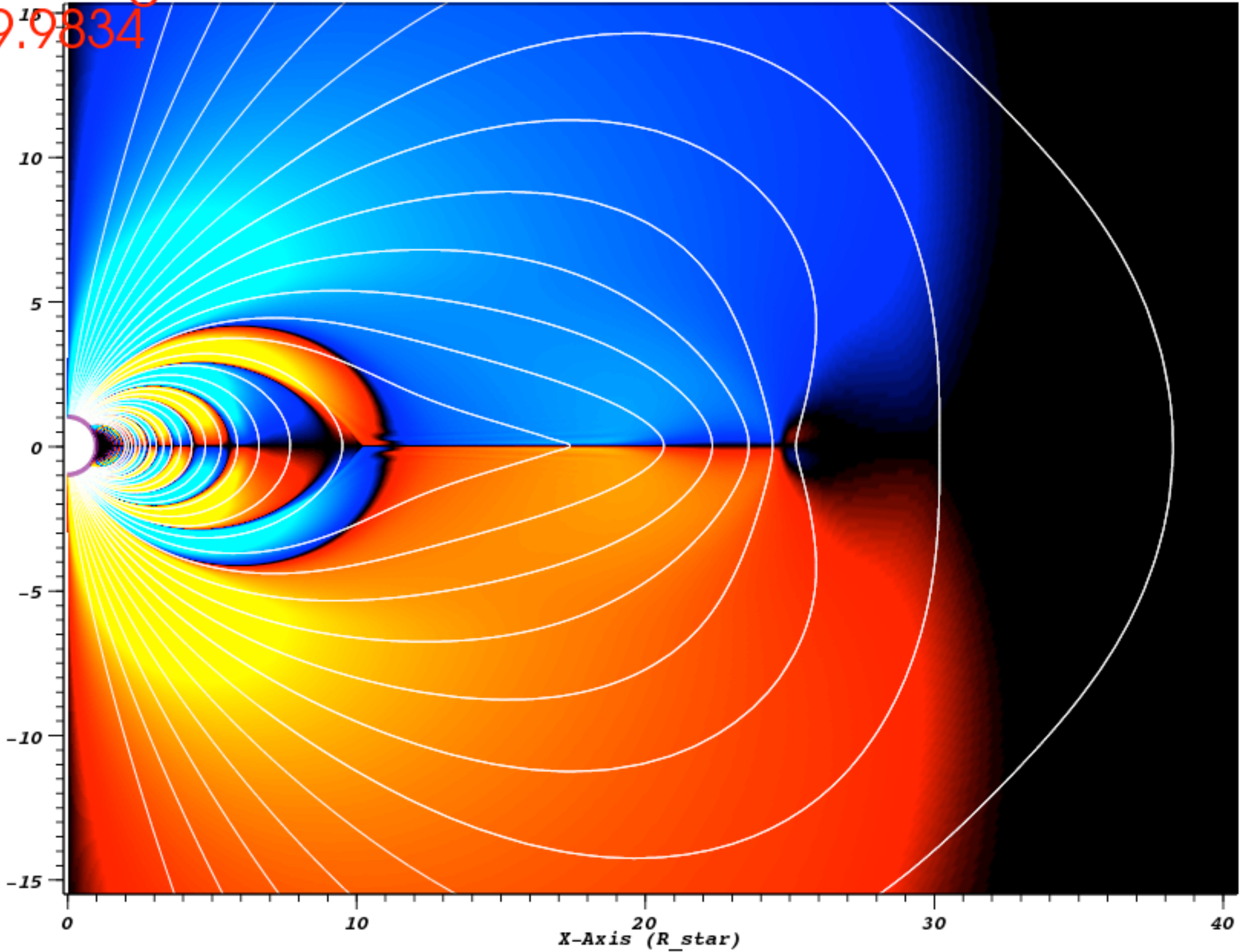


Rotating dipole — axisymmetric pulsar
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 39.9834

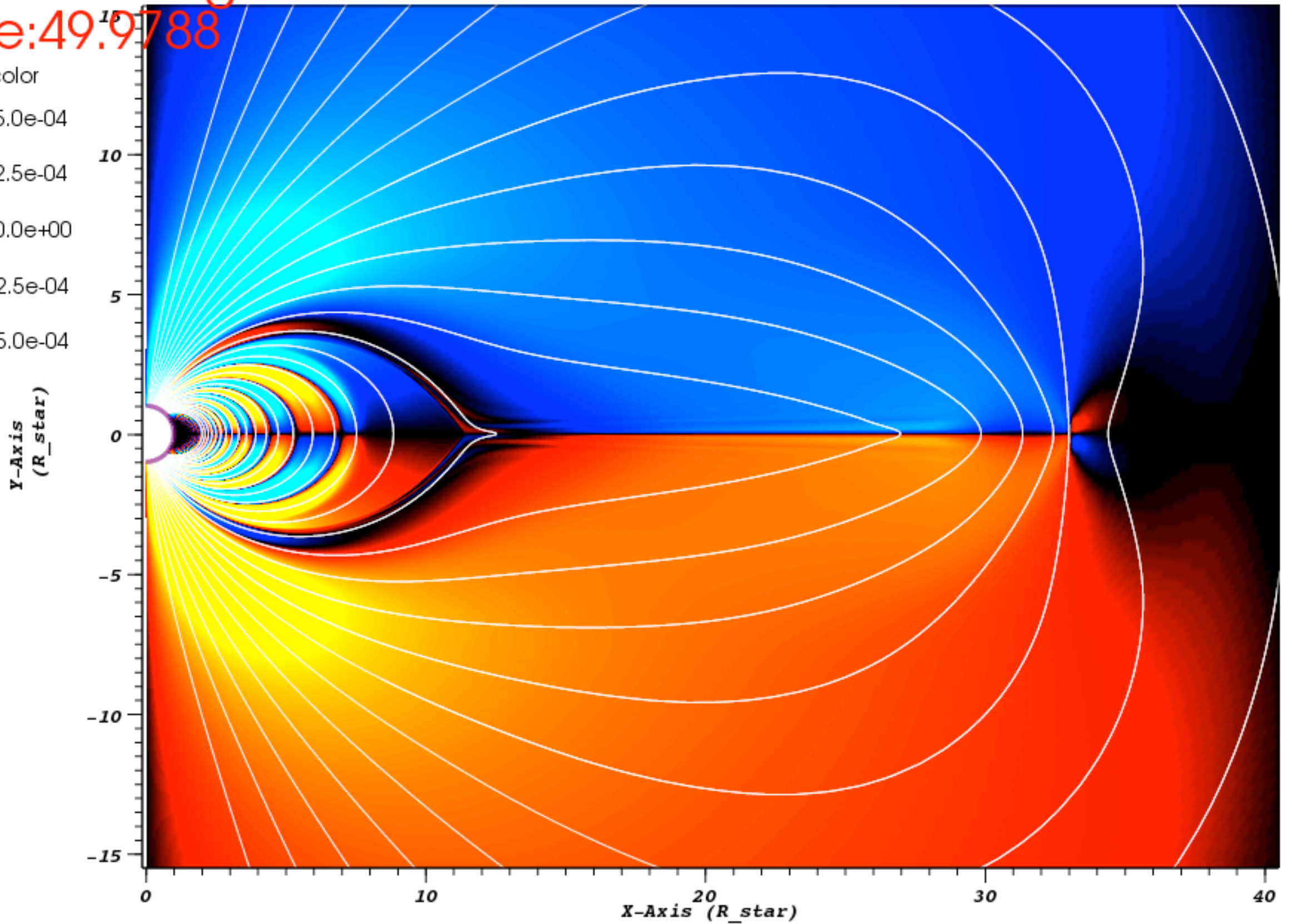
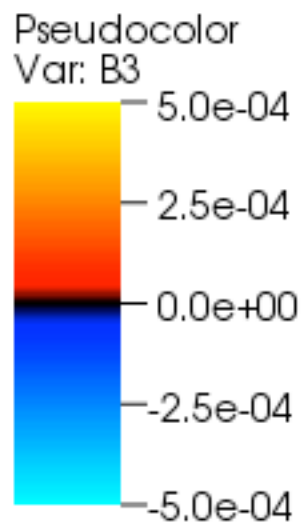
Pseudocolor
Var: B3
5.0e-04
2.5e-04
0.0e+00
-2.5e-04
-5.0e-04

Y-Axis
(R_star)



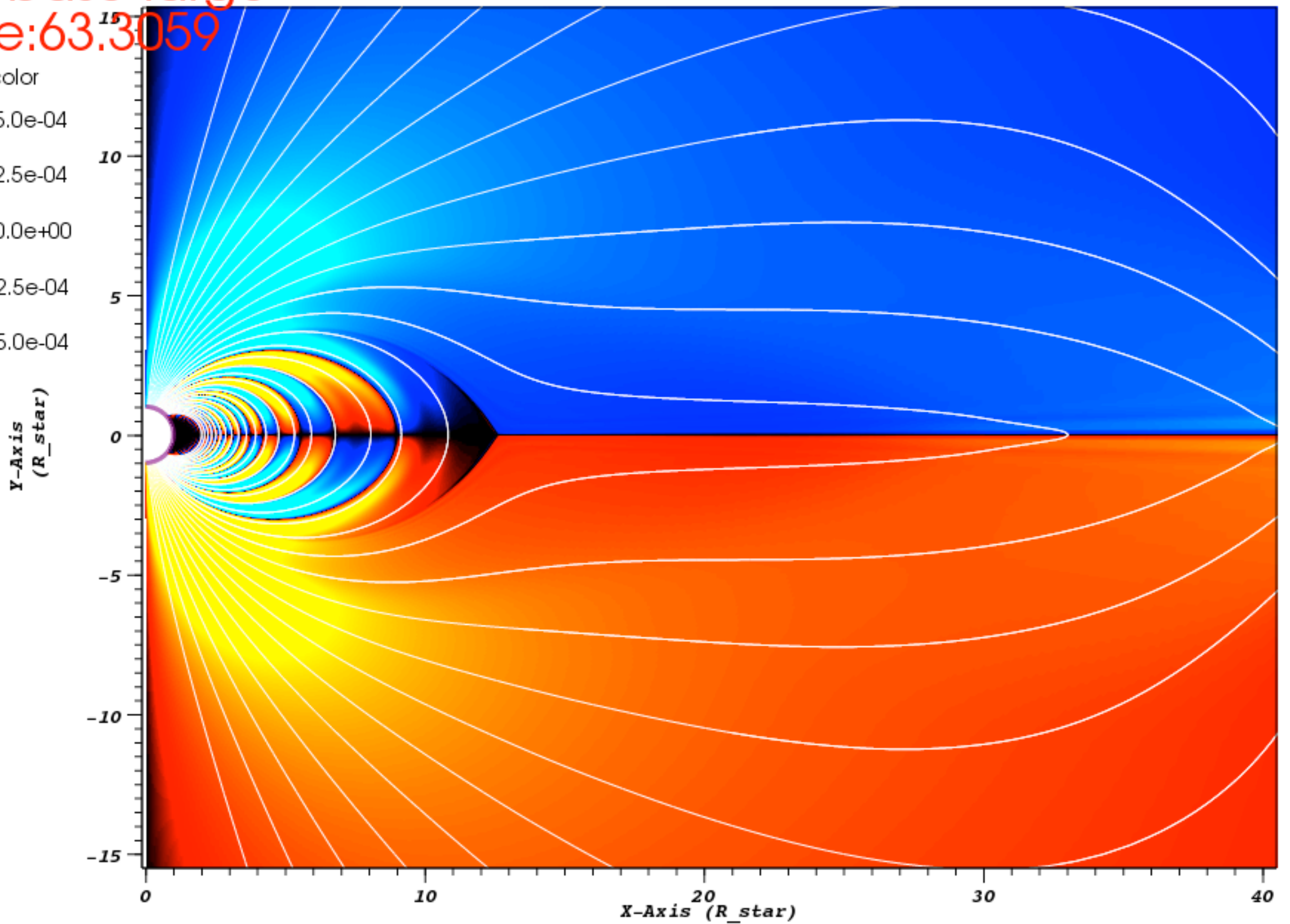
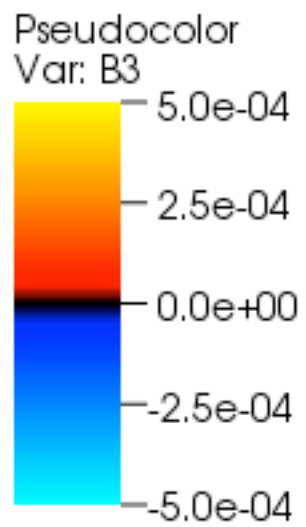
Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 49.9788



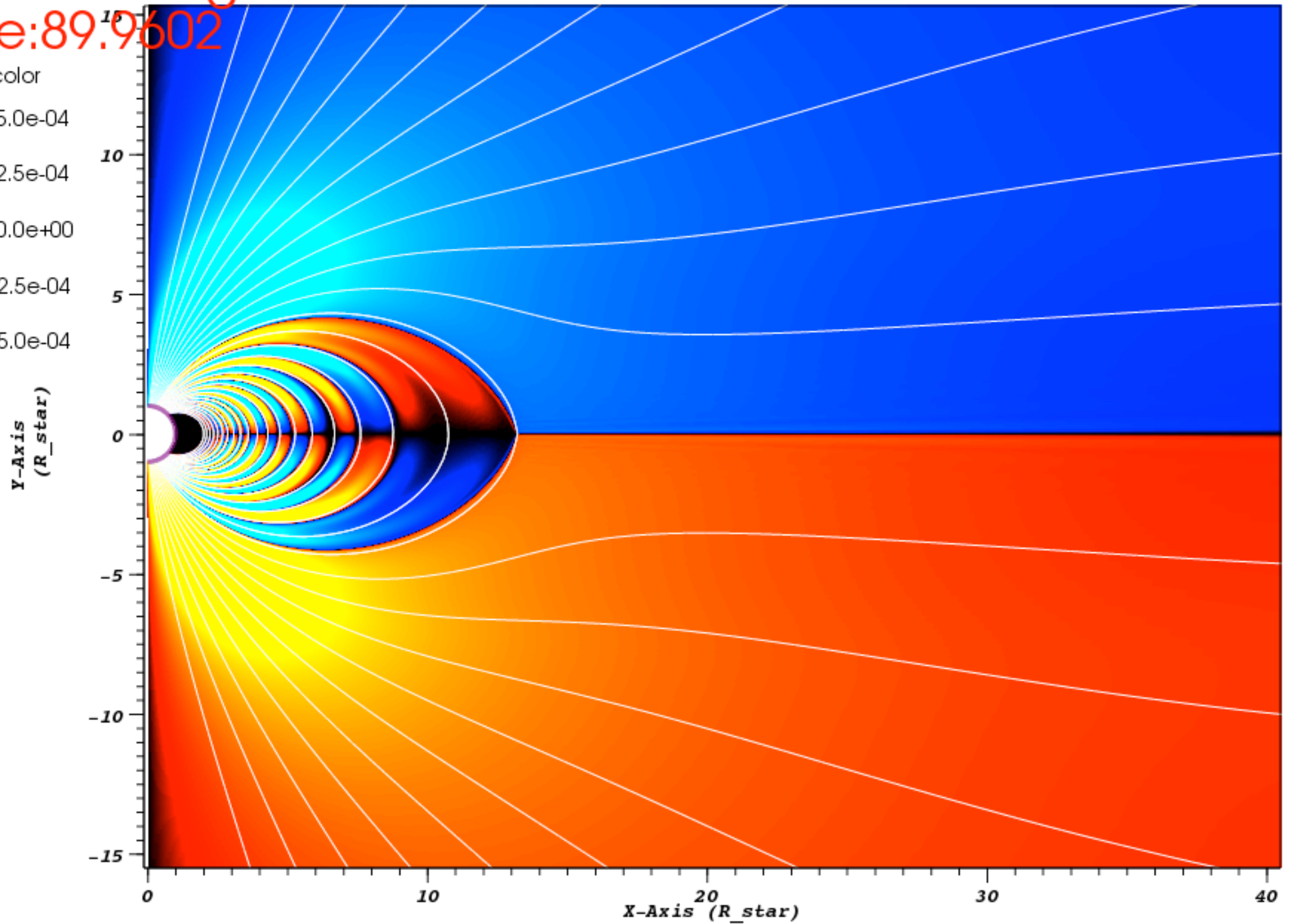
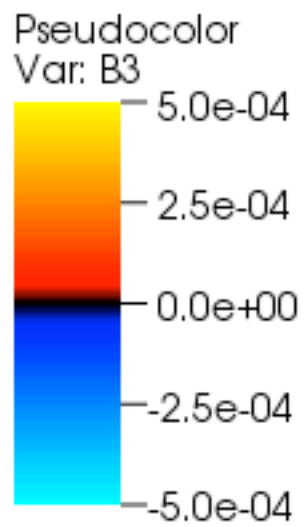
Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 63.3059



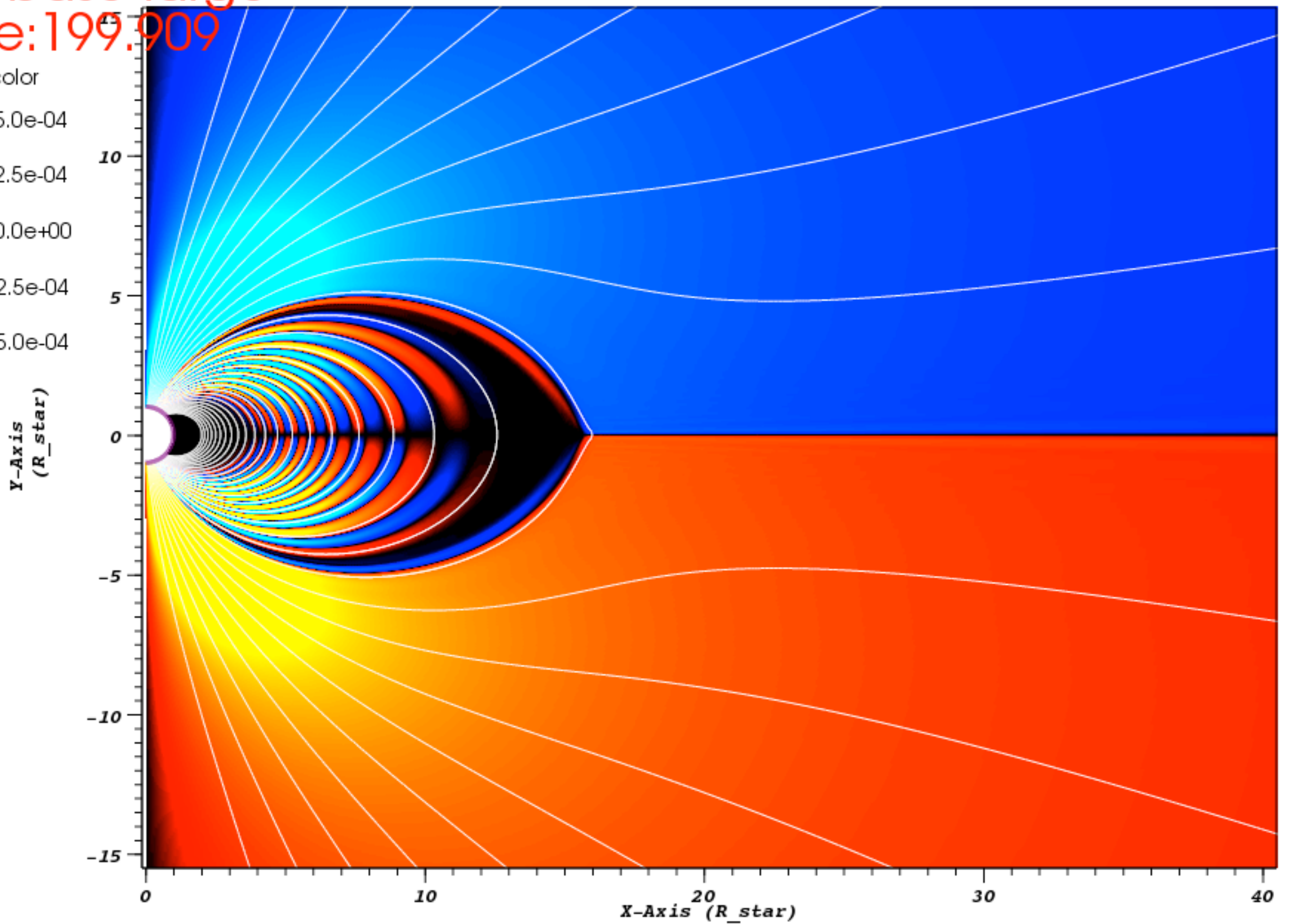
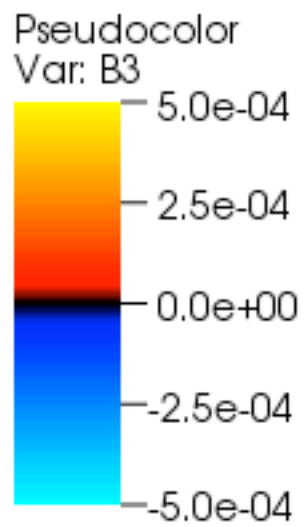
Rotating dipole — axisymmetric pulsar — Y-point formation
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 89.9602



Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^ϕ , poloidal field line projections

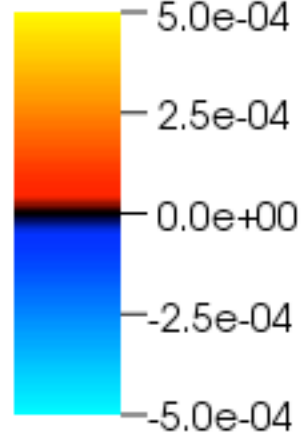
DB: base-large
Time: 199.909



Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^ϕ , poloidal field line projections

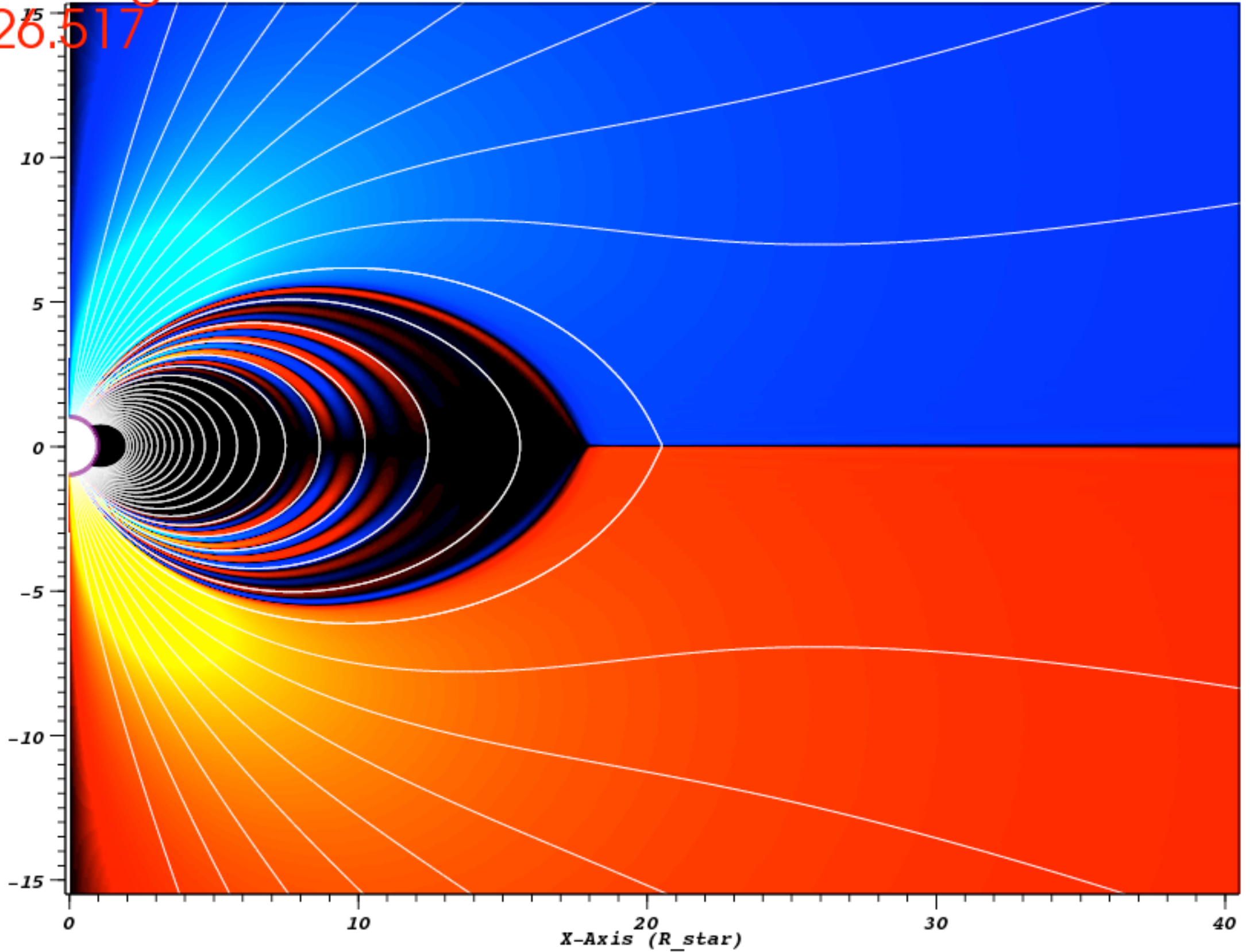
DB: base-large
Time: 326.517

Pseudocolor
Var: B3



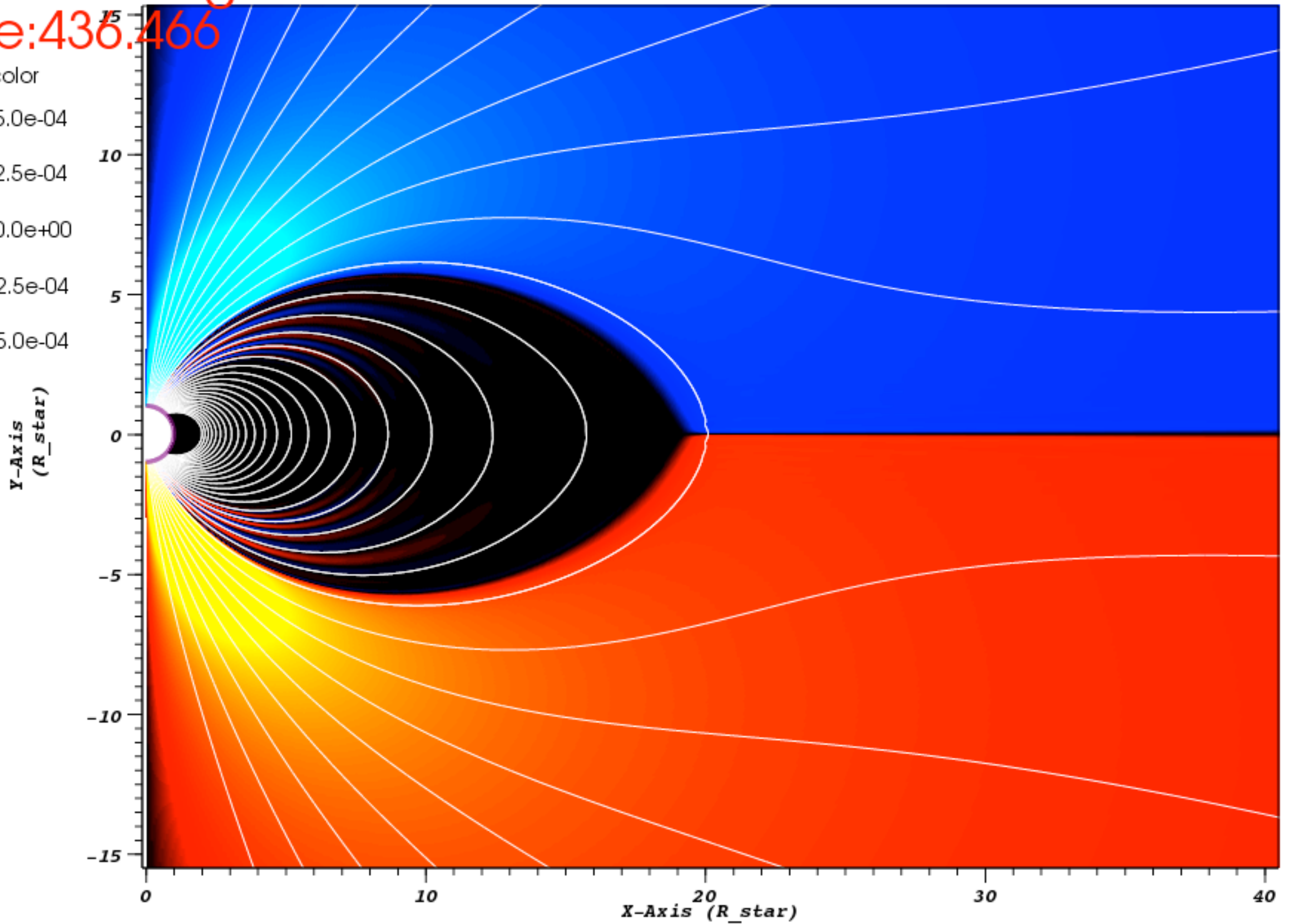
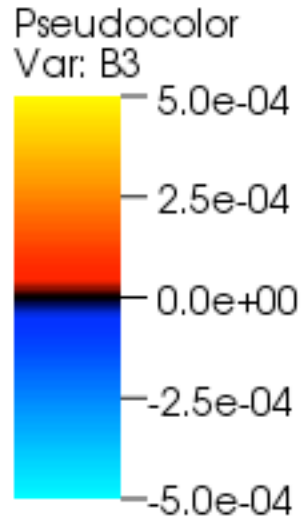
5.0e-04
2.5e-04
0.0e+00
-2.5e-04
-5.0e-04

Y-Axis
(R_star)



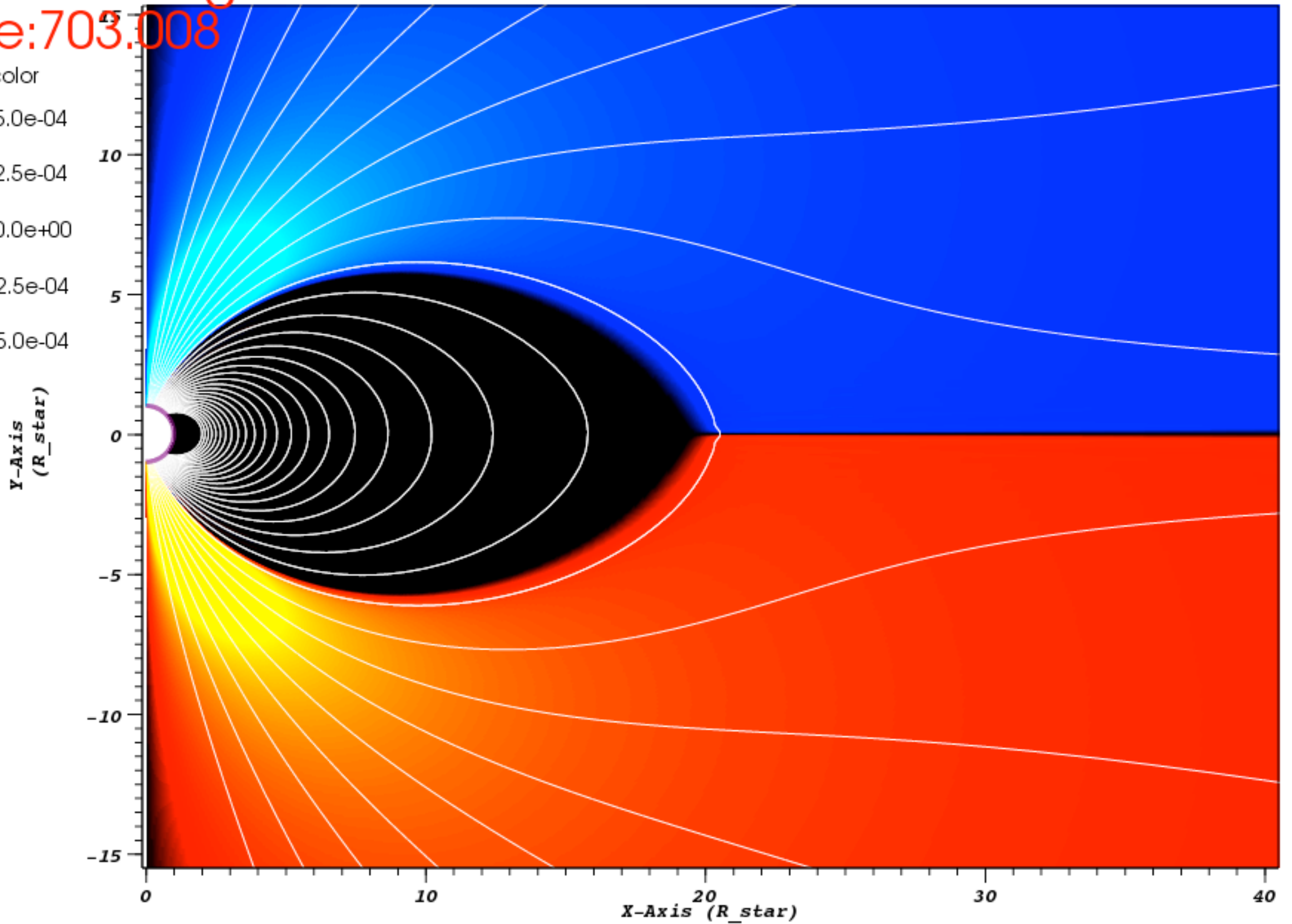
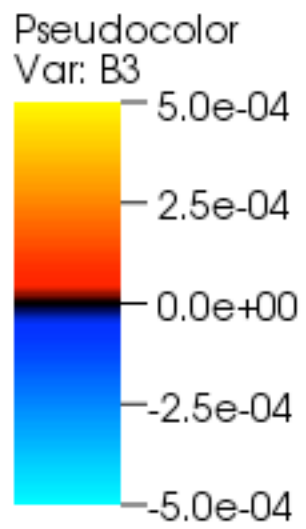
Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 436.466



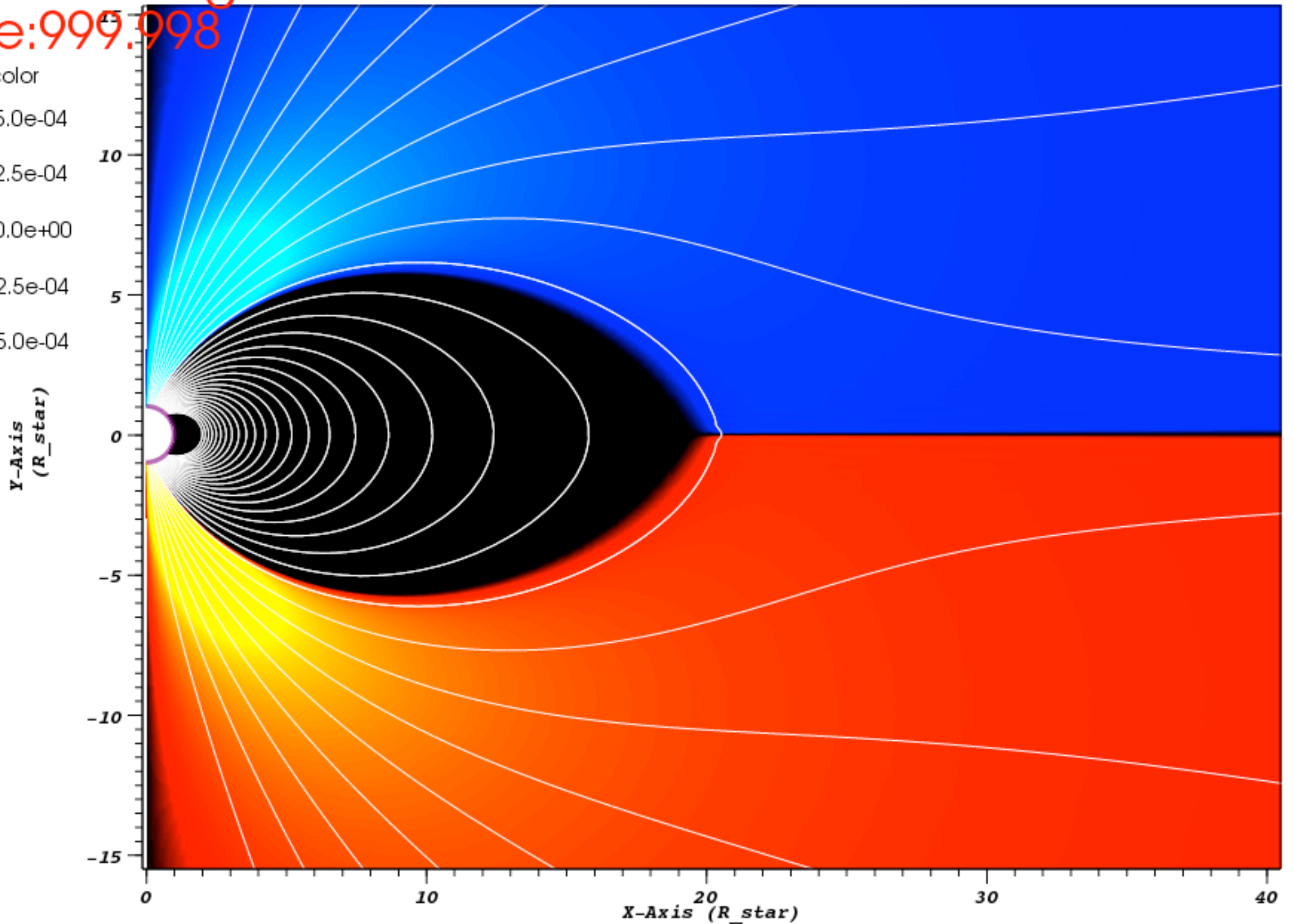
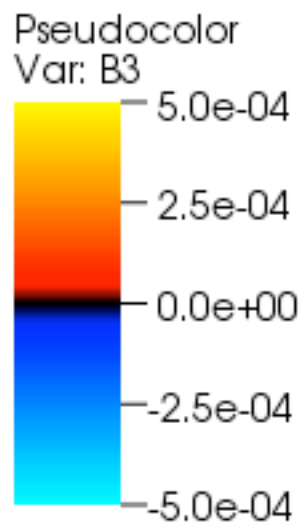
Rotating dipole — axisymmetric pulsar — quasi-static evolution
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 703.008



Rotating dipole — axisymmetric pulsar — steady-state
Shown: B^ϕ , poloidal field line projections

DB: base-large
Time: 999.998

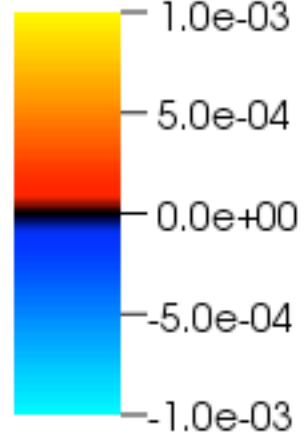


Rotating dipole — axisymmetric pulsar — steady-state
Shown: B^ϕ , poloidal field line projections



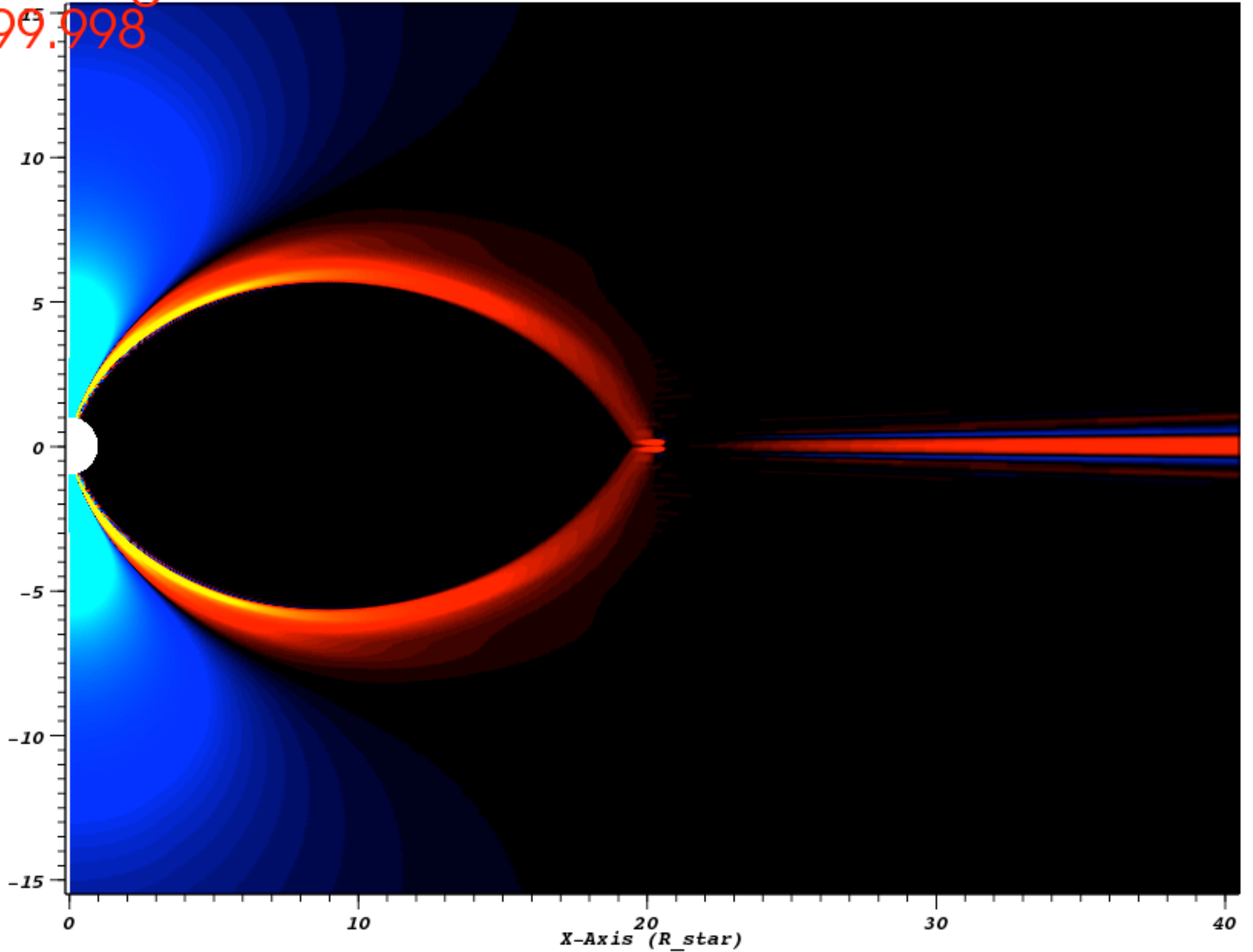
DB: base-large
Time: 999.998

Pseudocolor
Var: J1



1.0e-03
5.0e-04
0.0e+00
-5.0e-04
-1.0e-03

Y-Axis
(R_star)



Rotating dipole — axisymmetric pulsar — steady-state current
Shown: J^r

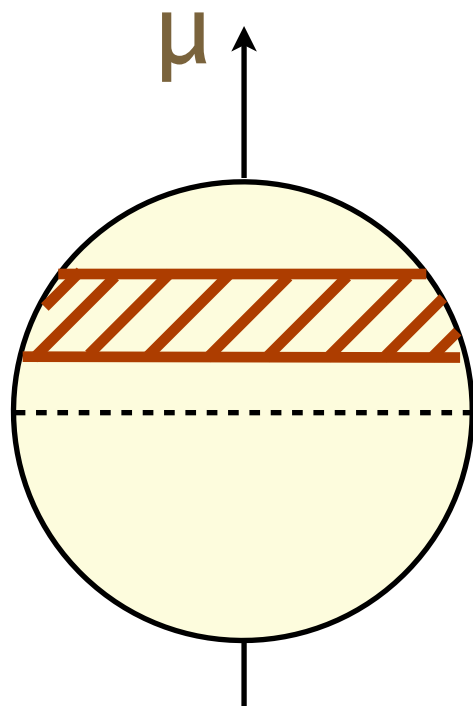
Twisted magnetospheres

- Motion of a neutron star's crust will twist or deform its magnetosphere.
- Differential rotation of stellar surface: $\Omega(\theta)$ in

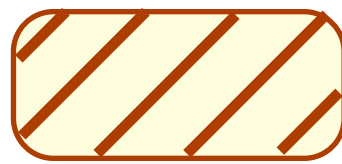
$$\vec{E} = - (\Omega \times \vec{r}) \times \vec{B}$$

- Poloidal currents ejected from star \rightarrow toroidal magnetic field
- X-ray emission via resonant cyclotron scattering
- Reconnection if field is over-twisted : explosive energy release

Twisting profiles

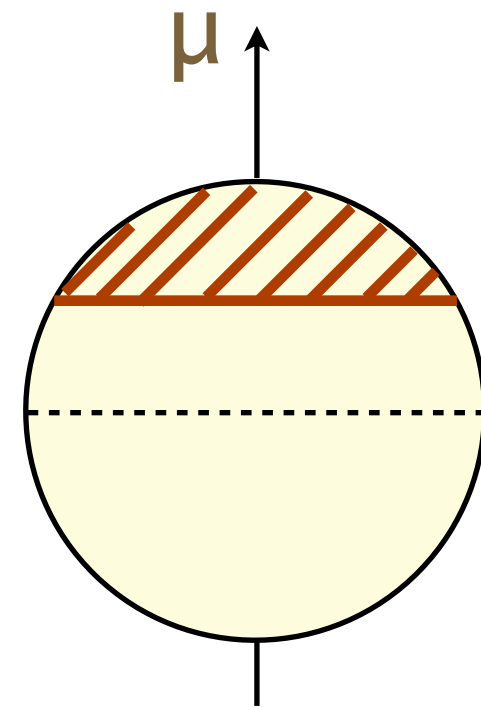


annulus

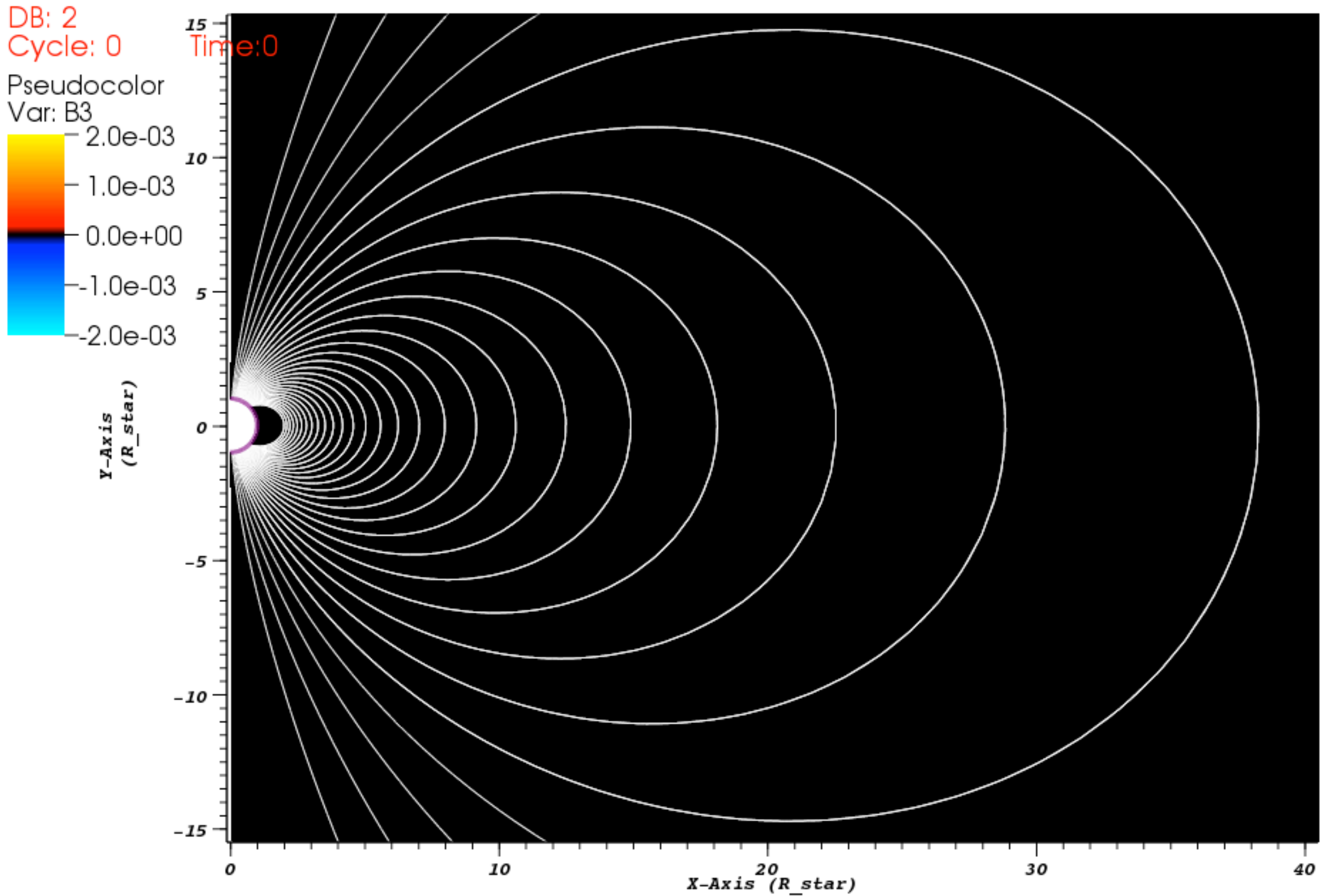


twisted region

Rotates at Ω_{twist} with respect to rest of star.



polar cap



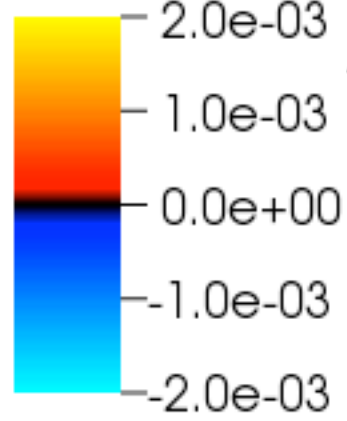
Twisted magnetosphere — annulus profile
Shown: B^ϕ , poloidal field line projections

DB: 2

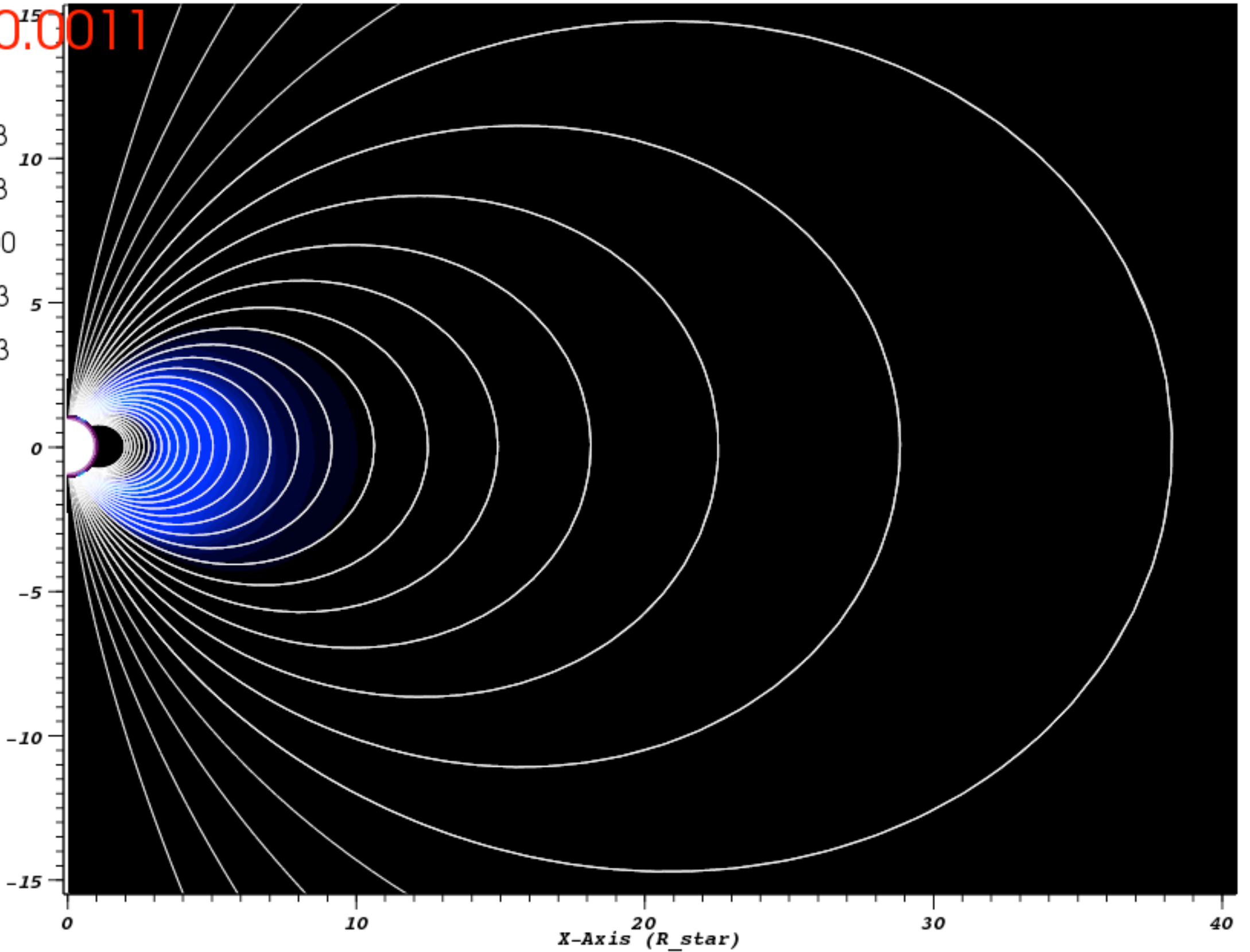
Time: 30.0011

Pseudocolor

Var: B3



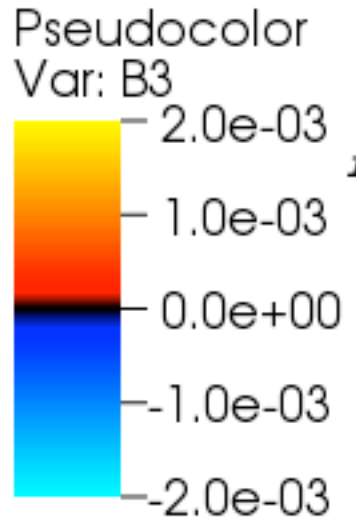
Y-Axis
(R_star)



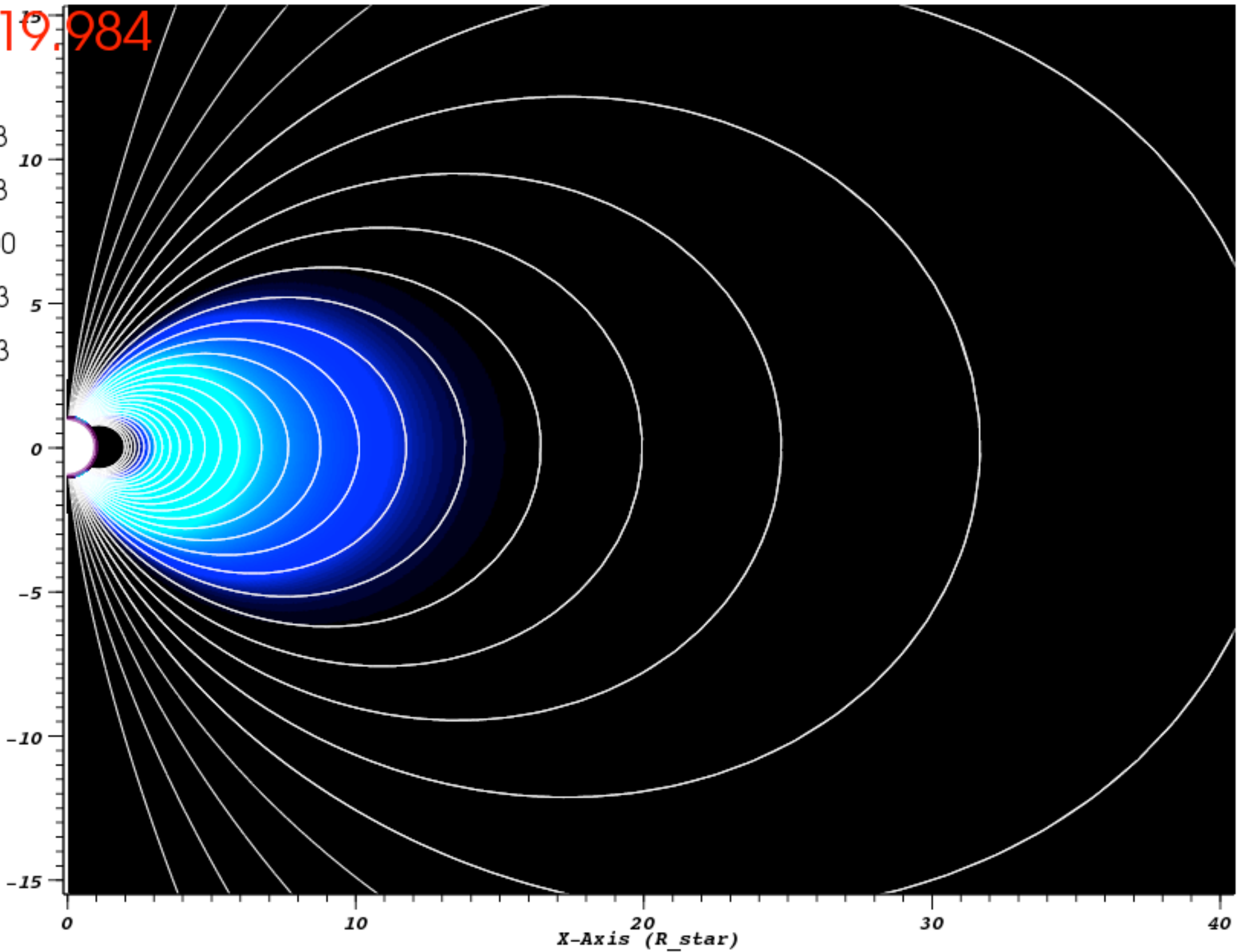
Twisted magnetosphere

Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 419.984

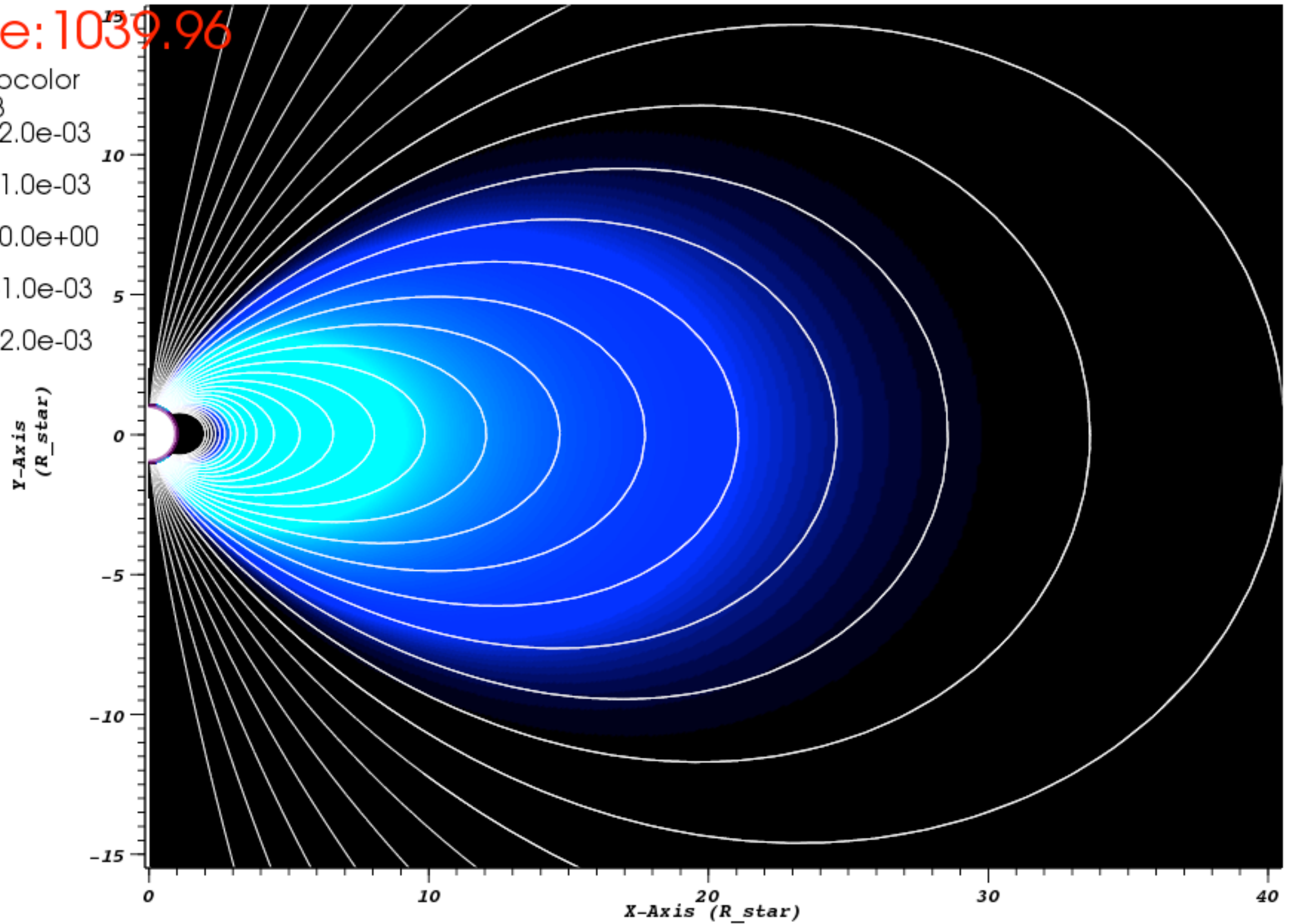
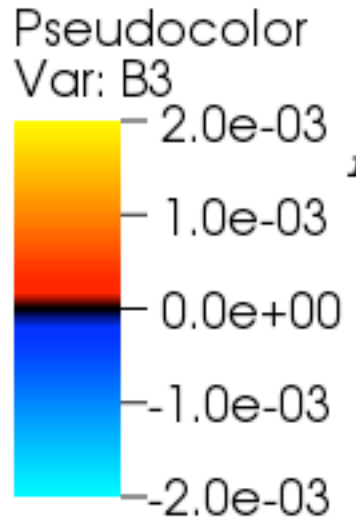


Y-Axis
(R_star)



Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 1039.96

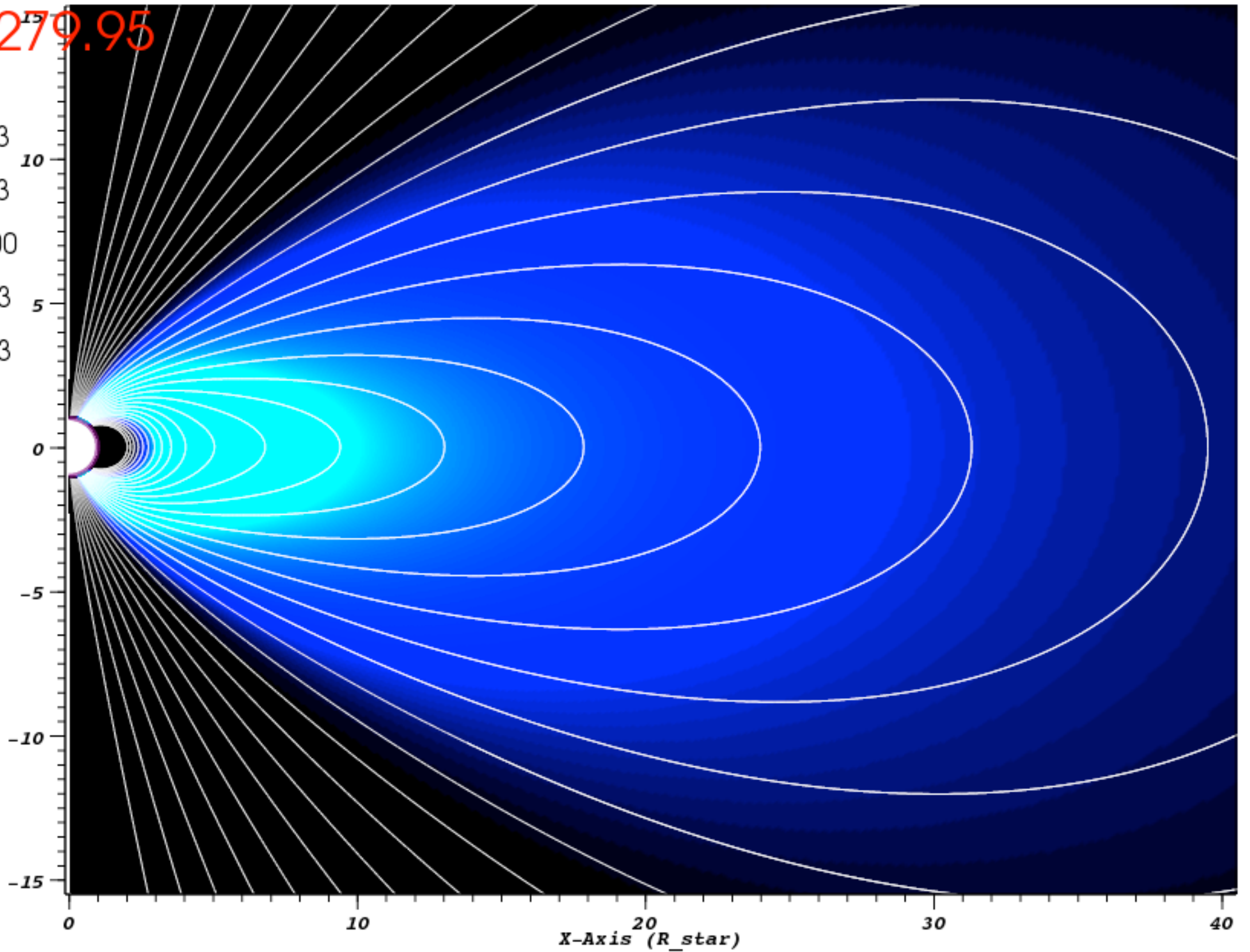


Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 1279.95

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

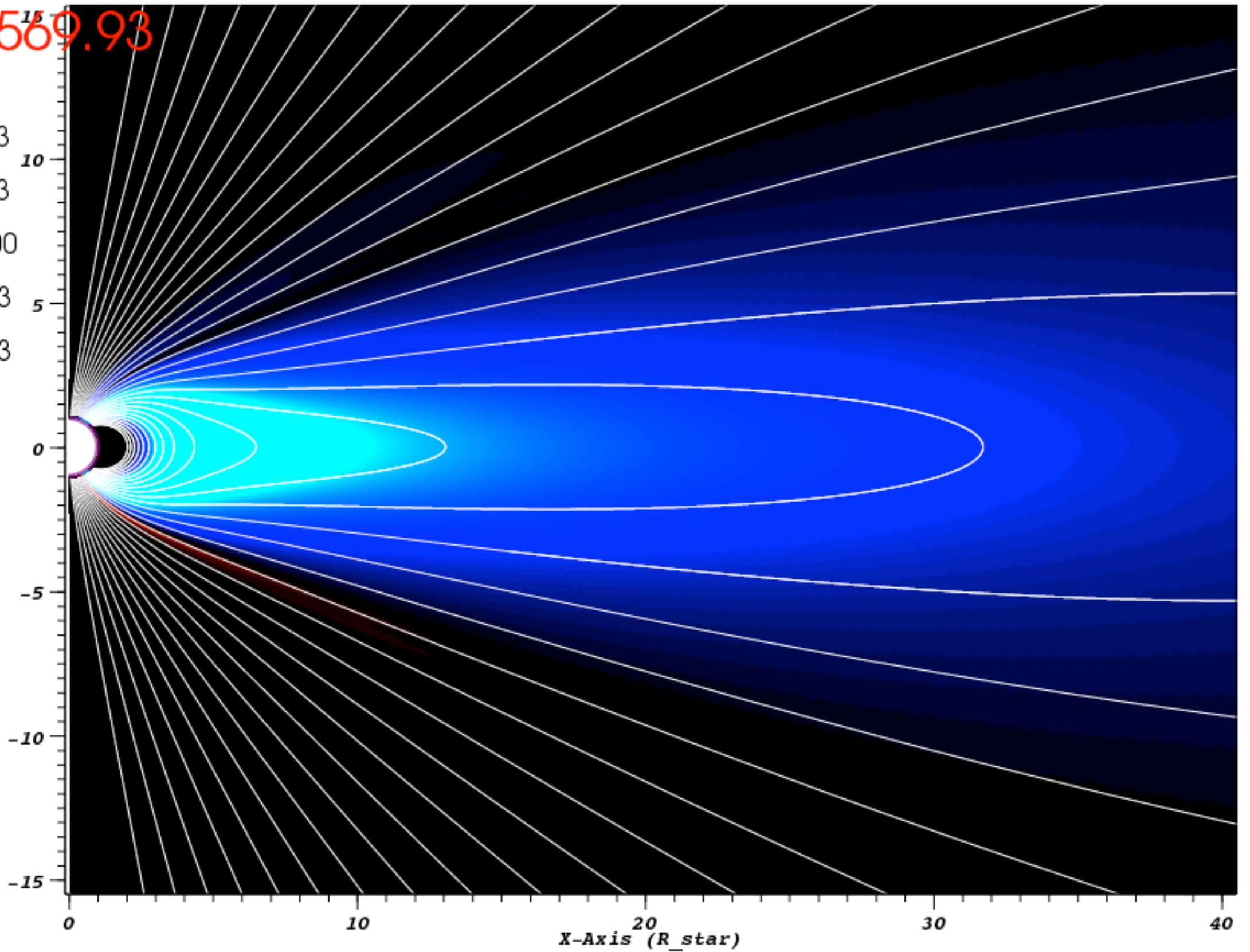


Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 1569.93

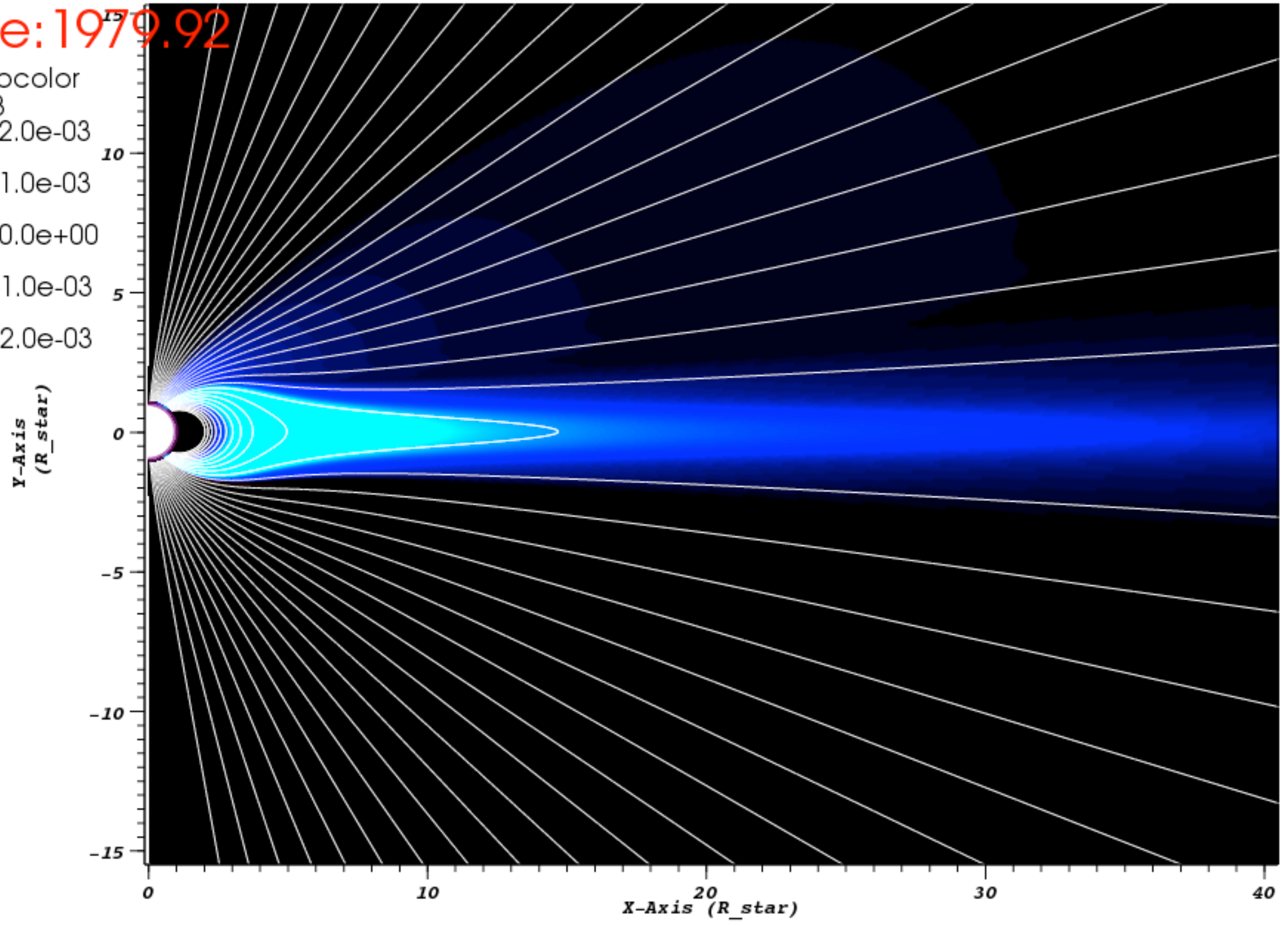
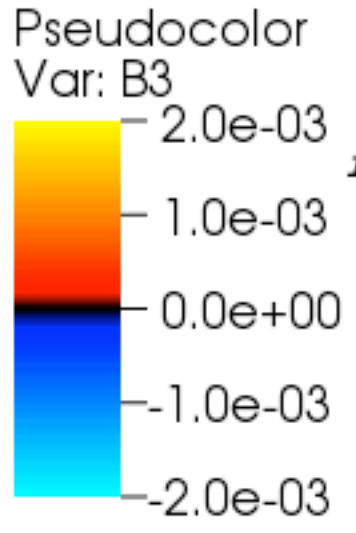
Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 1979.92

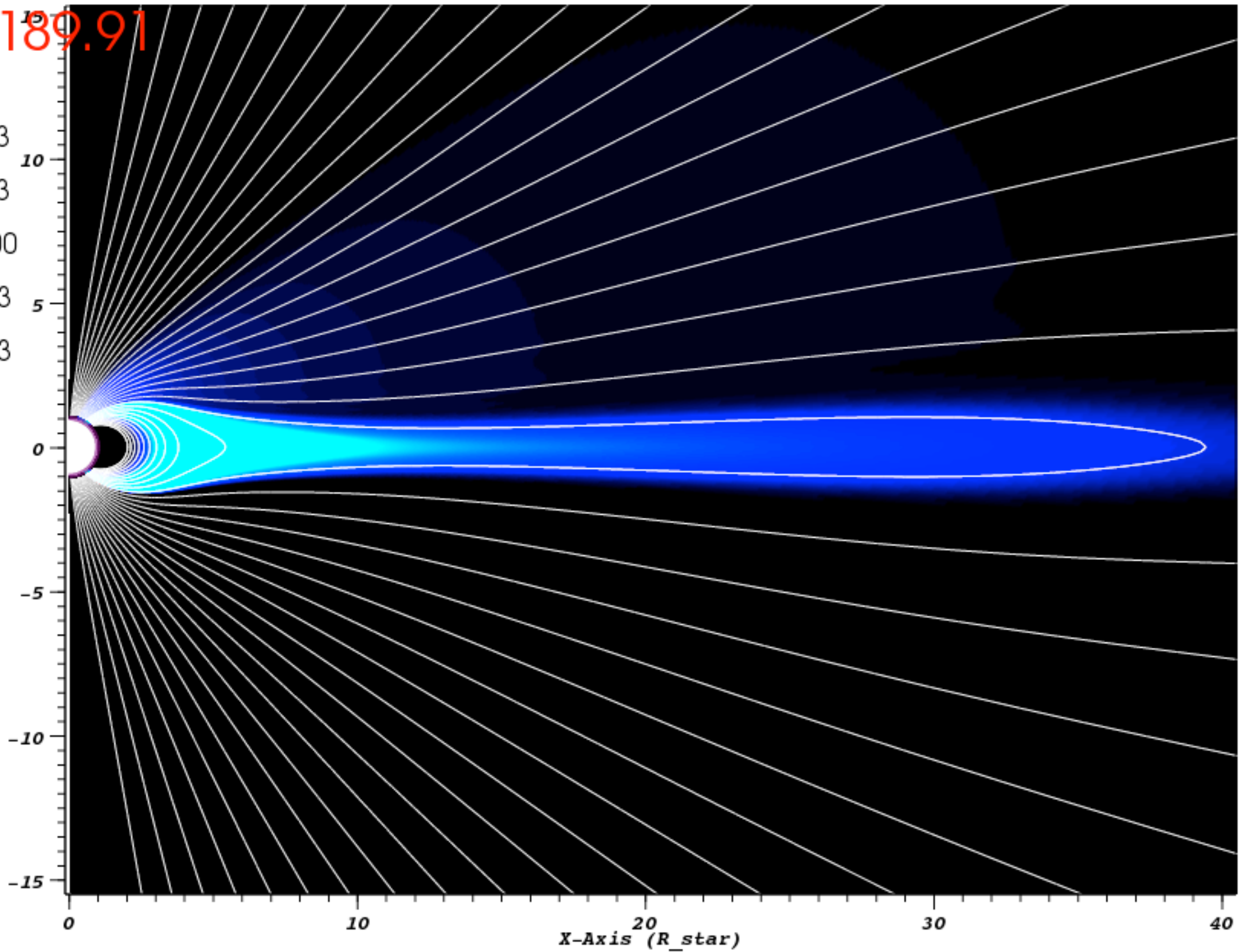


Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2189.91

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



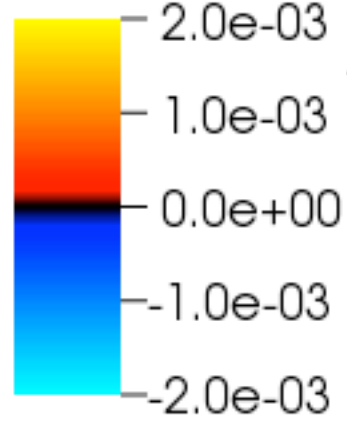
Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2

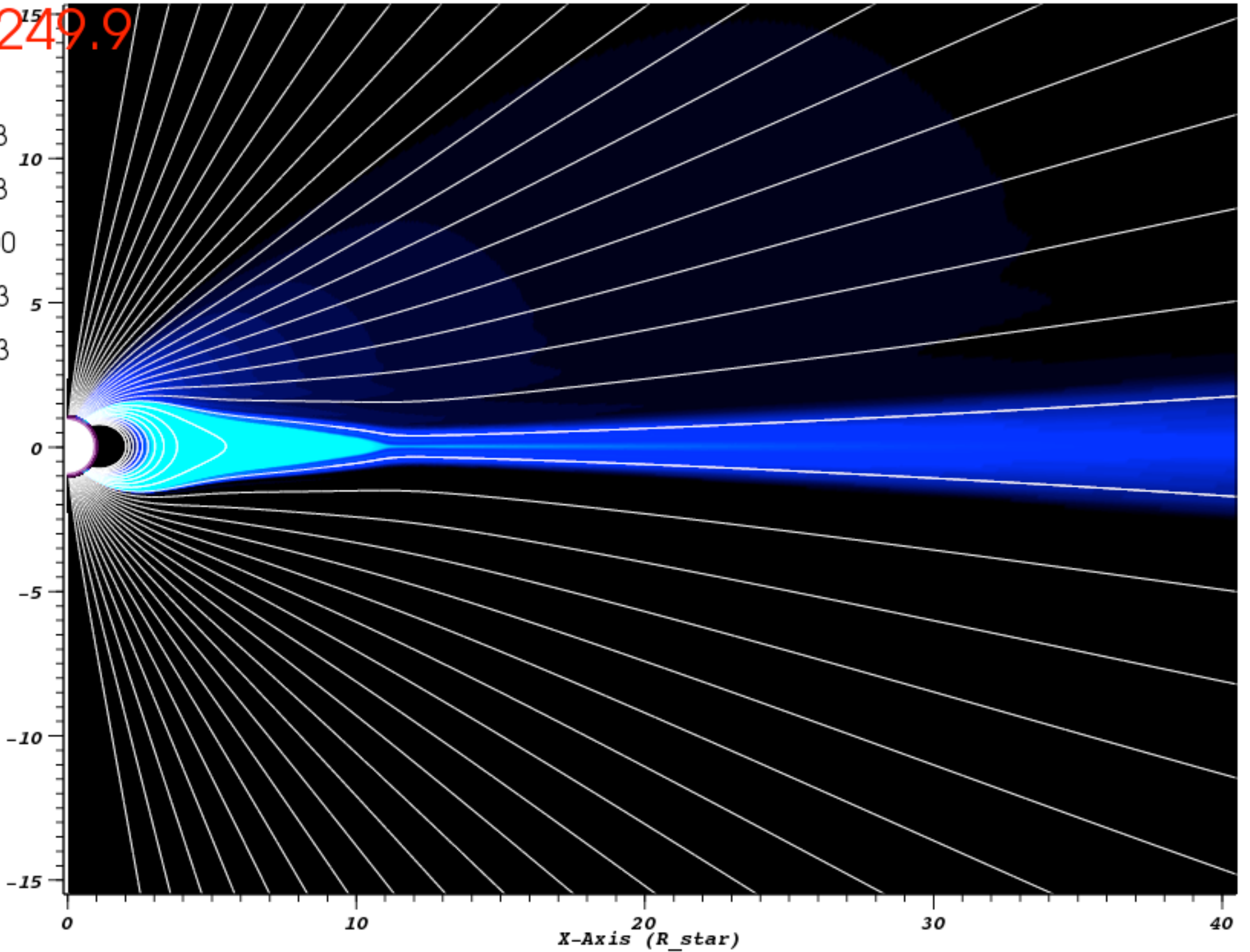
Time: 2249.9

Pseudocolor

Var: B3

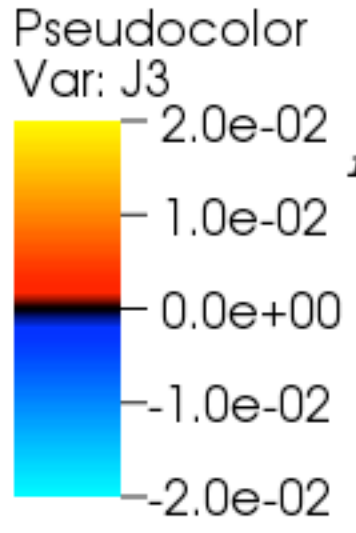


Y-Axis
(R_star)

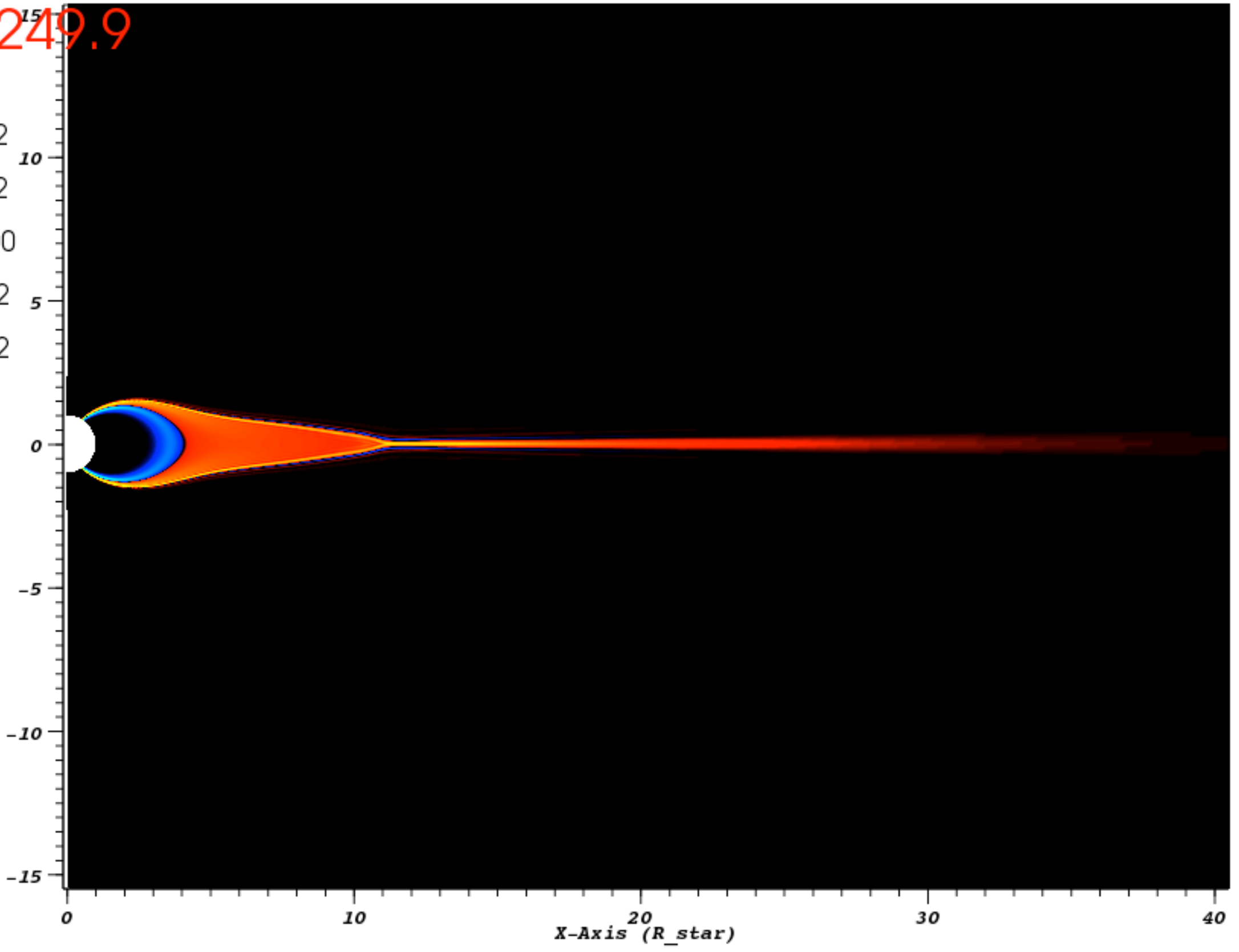


Twisted magnetosphere — open field at finite time & twist
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2249.9



Y-Axis
(R_star)

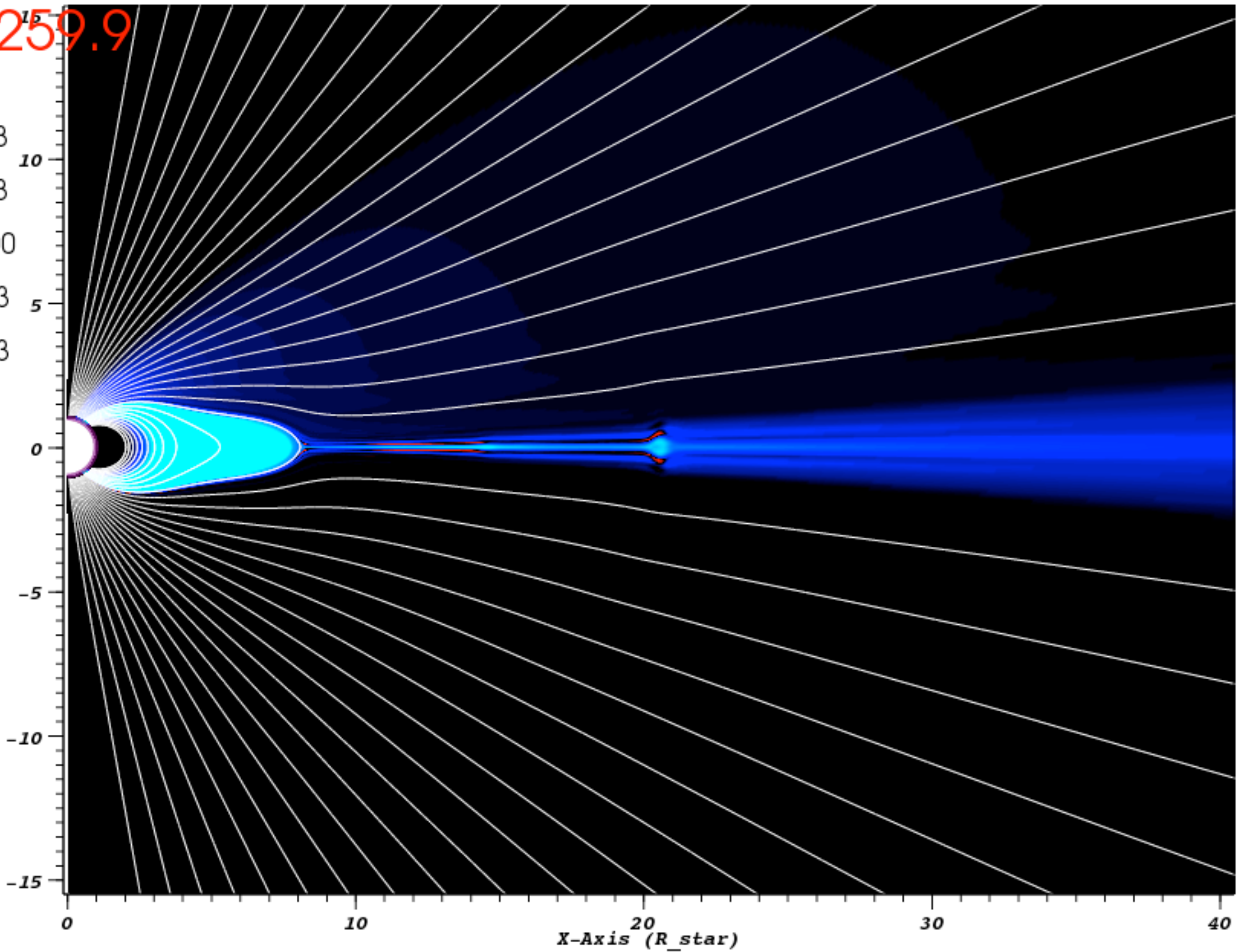


Twisted magnetosphere — current sheet formation
Shown: $J\phi$

DB: 2
Time: 2259.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

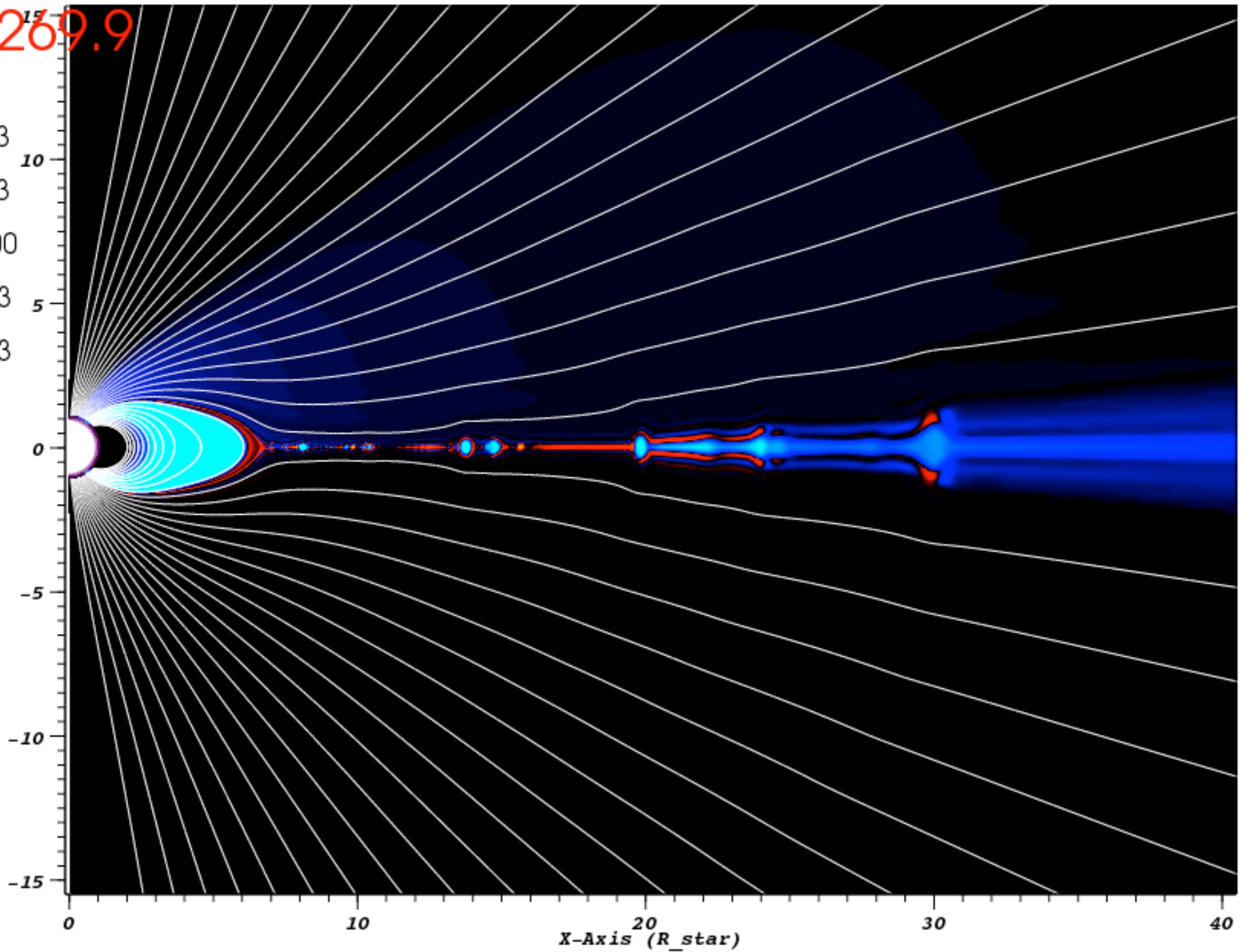


Twisted magnetosphere — current sheet formation
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2269.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

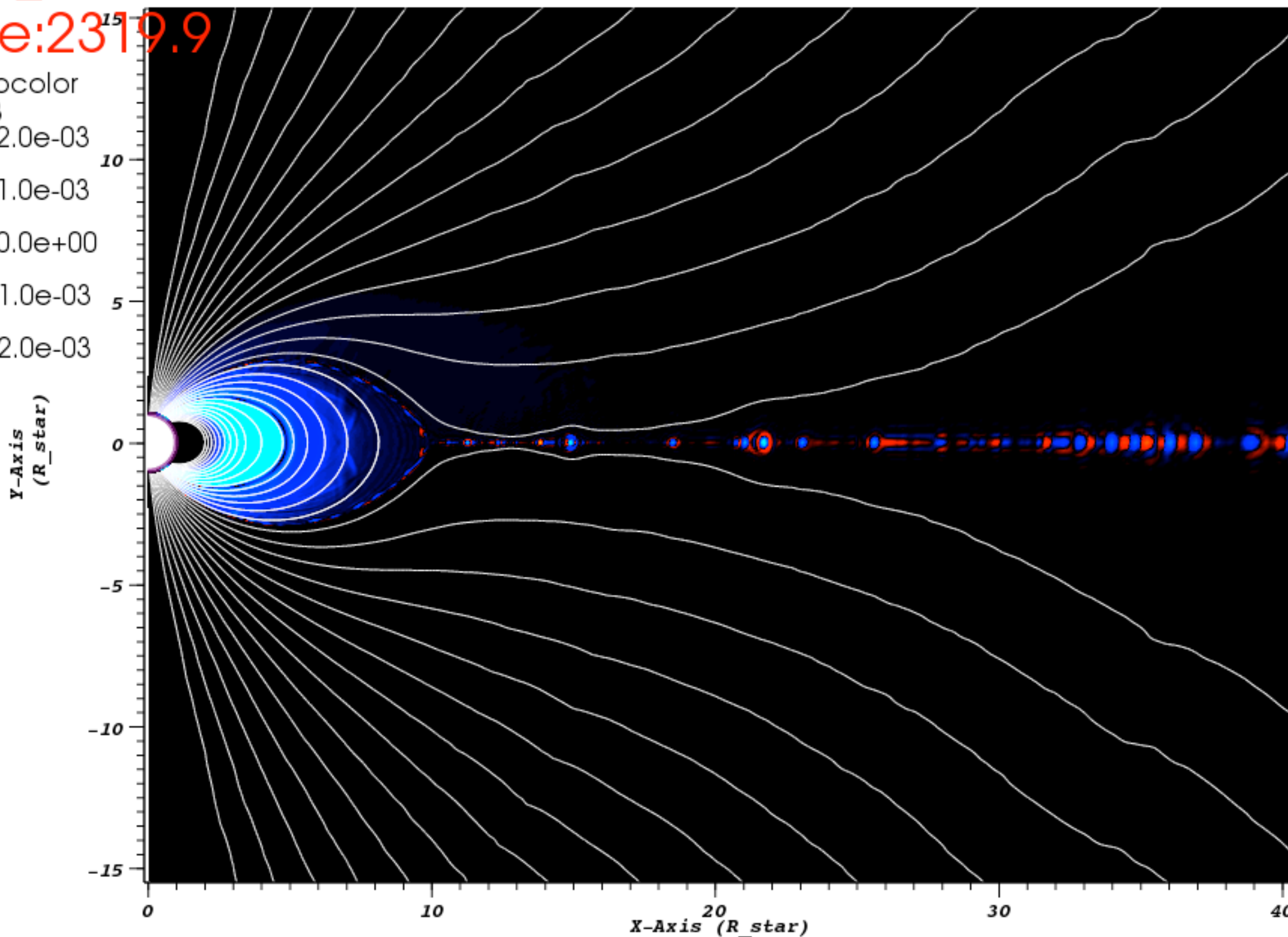
Y-Axis
(R_star)



Twisted magnetosphere — first reconnection event
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2319.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

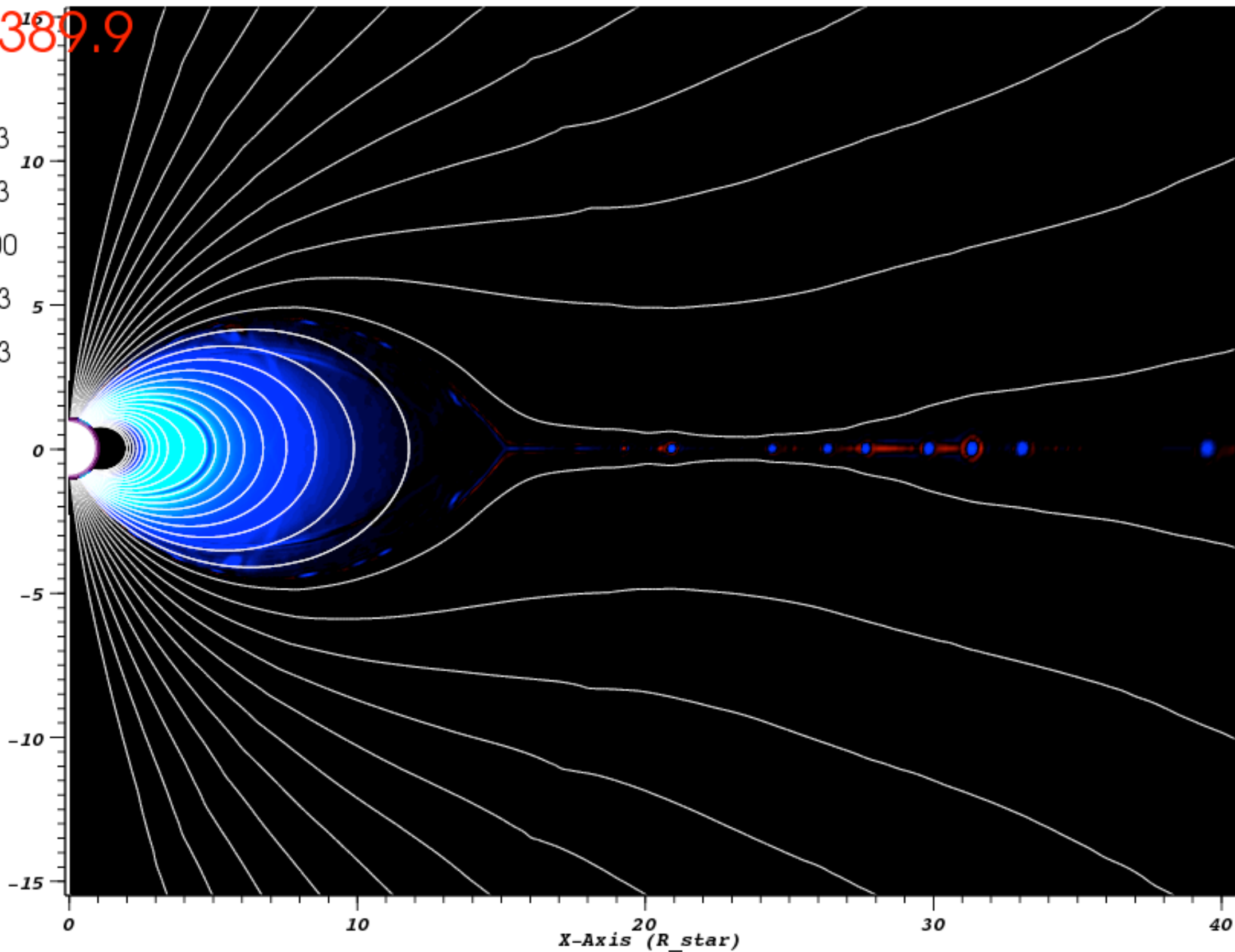


Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2389.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

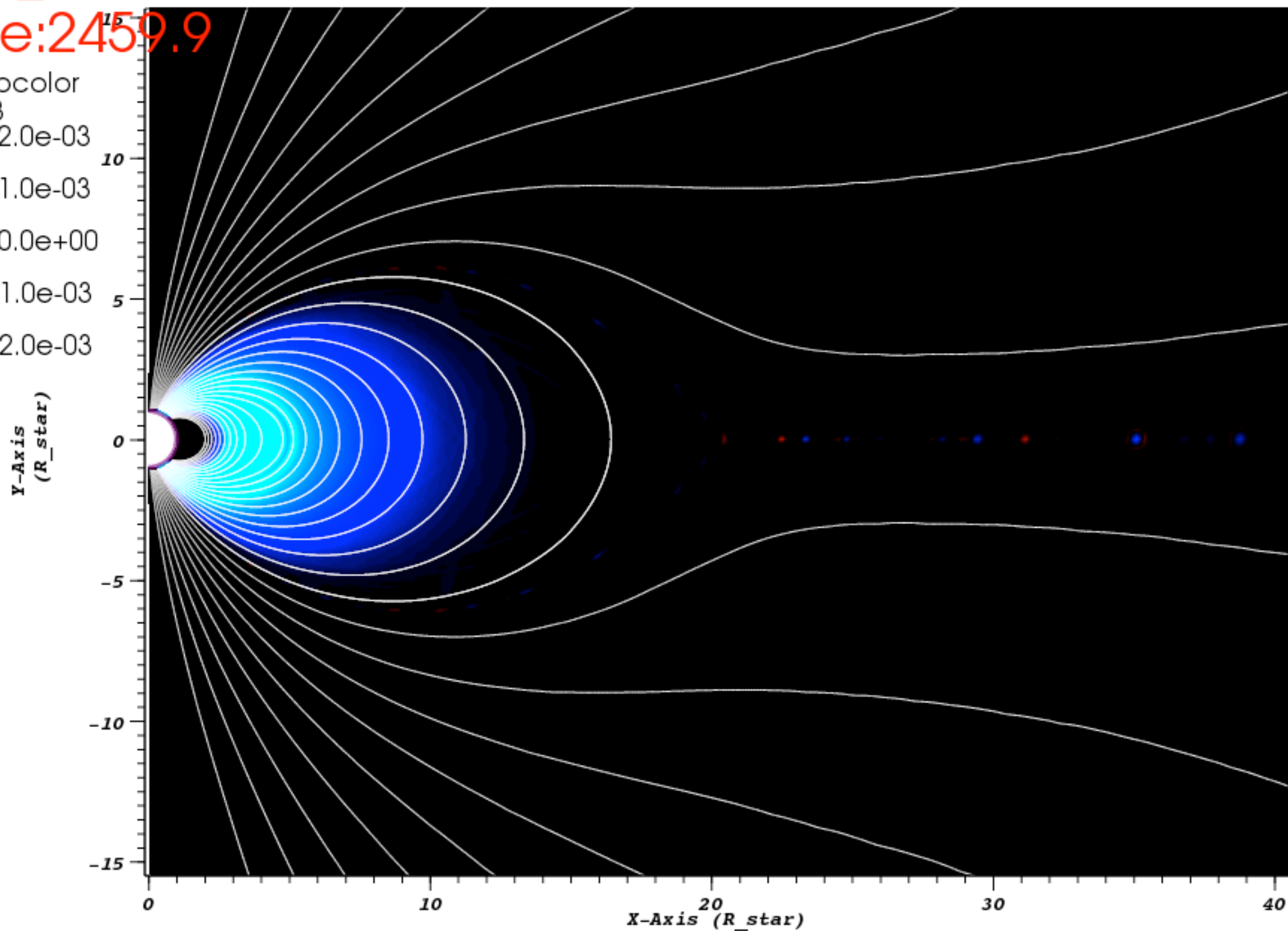
Y-Axis
(R_star)



Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 2459.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03



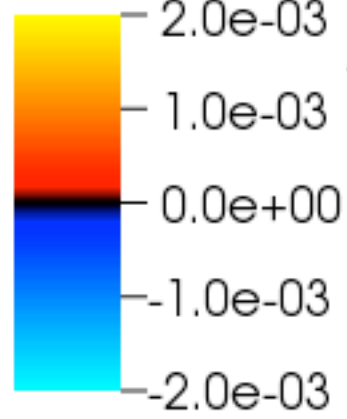
Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2

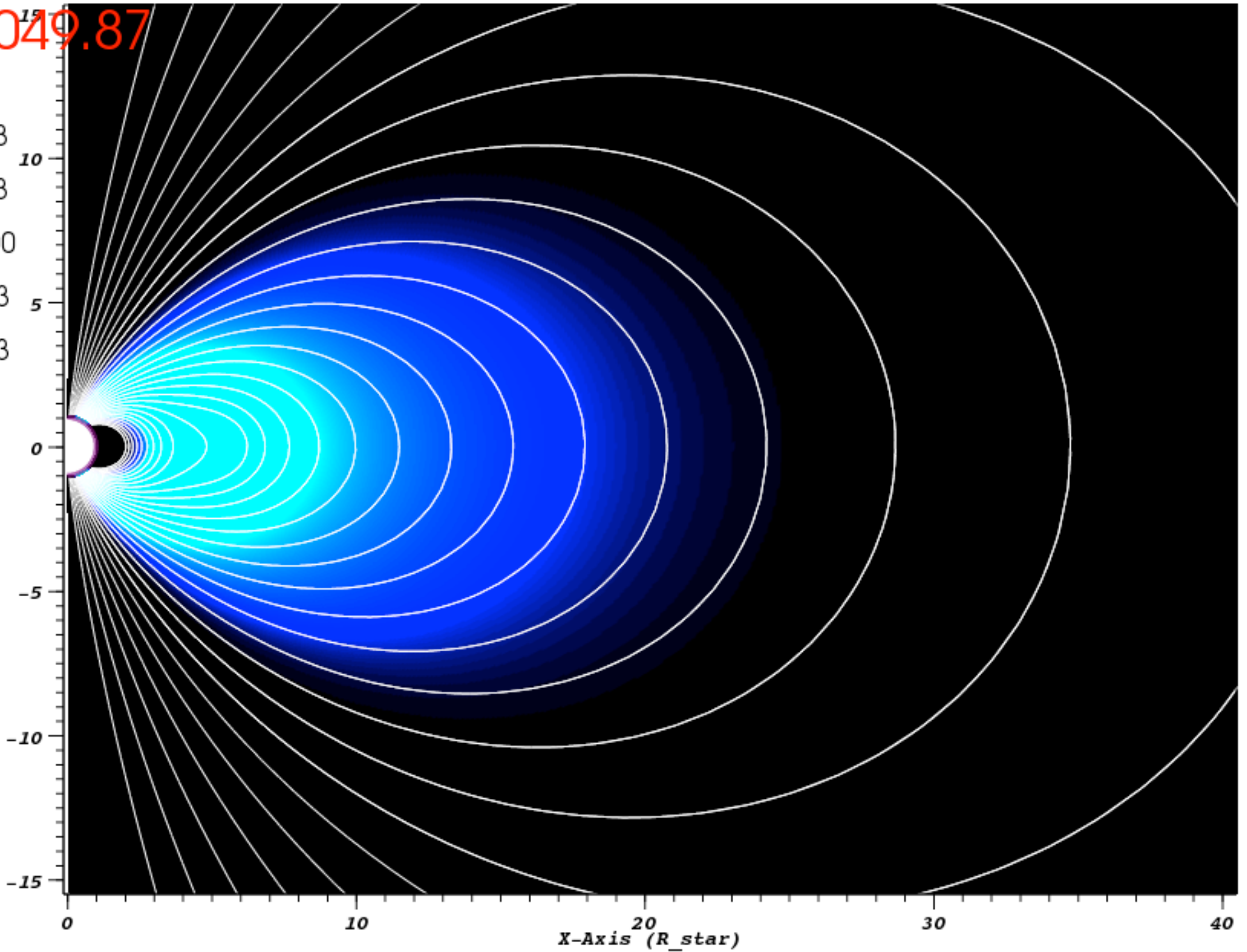
Time: 3049.87

Pseudocolor

Var: B3



Y-Axis
(R_star)



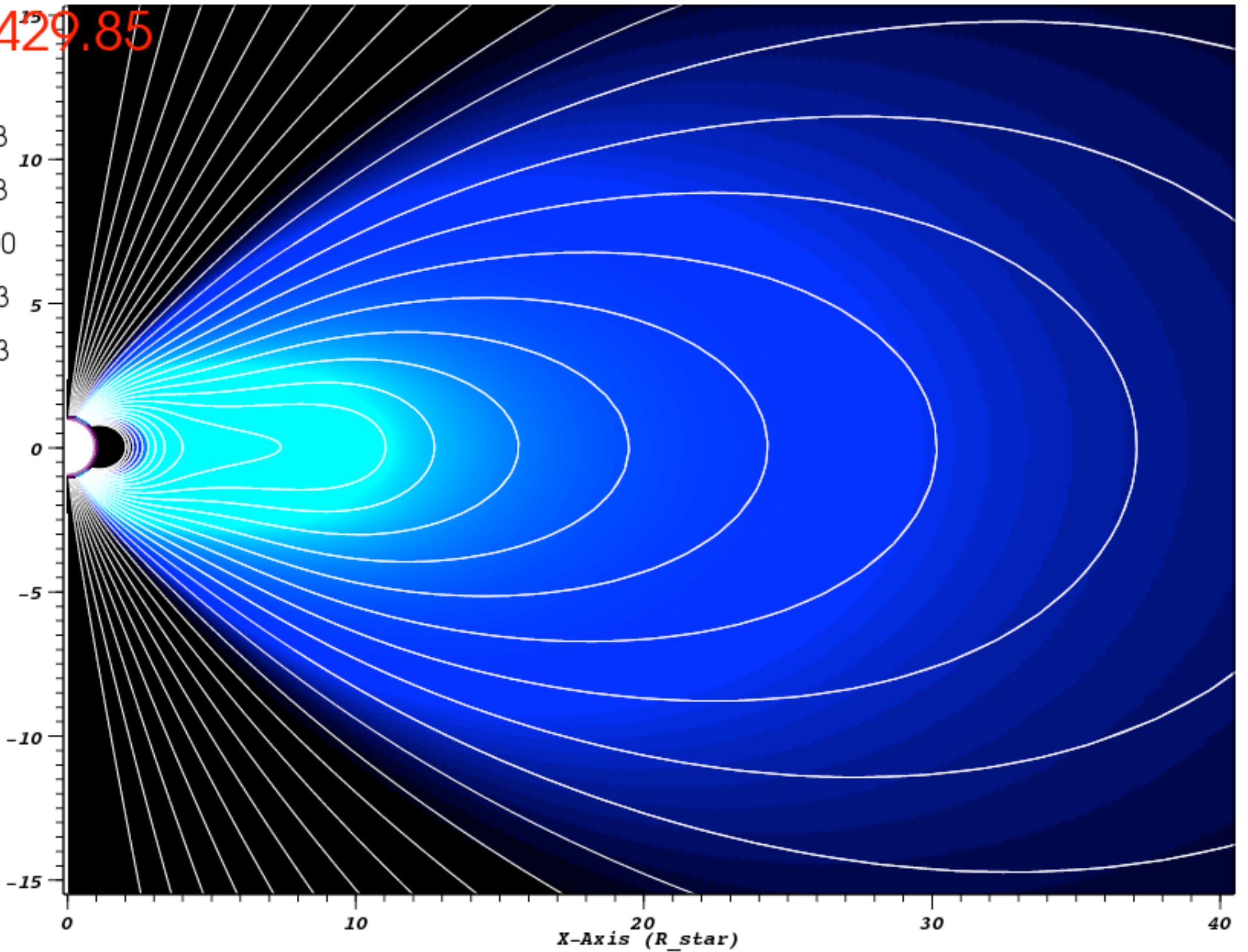
Twisted magnetosphere

Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 3429.85

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

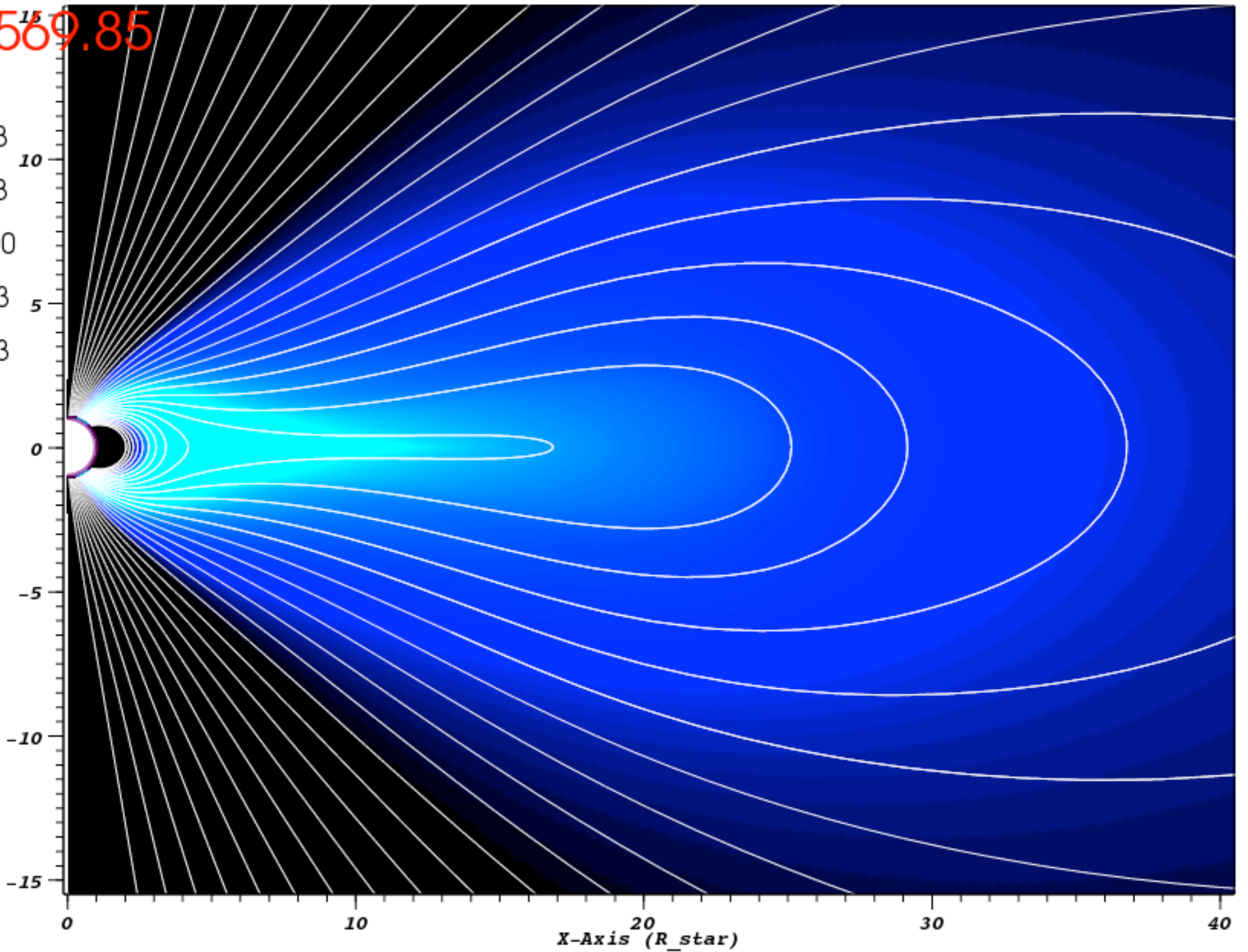


Twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 3569.85

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

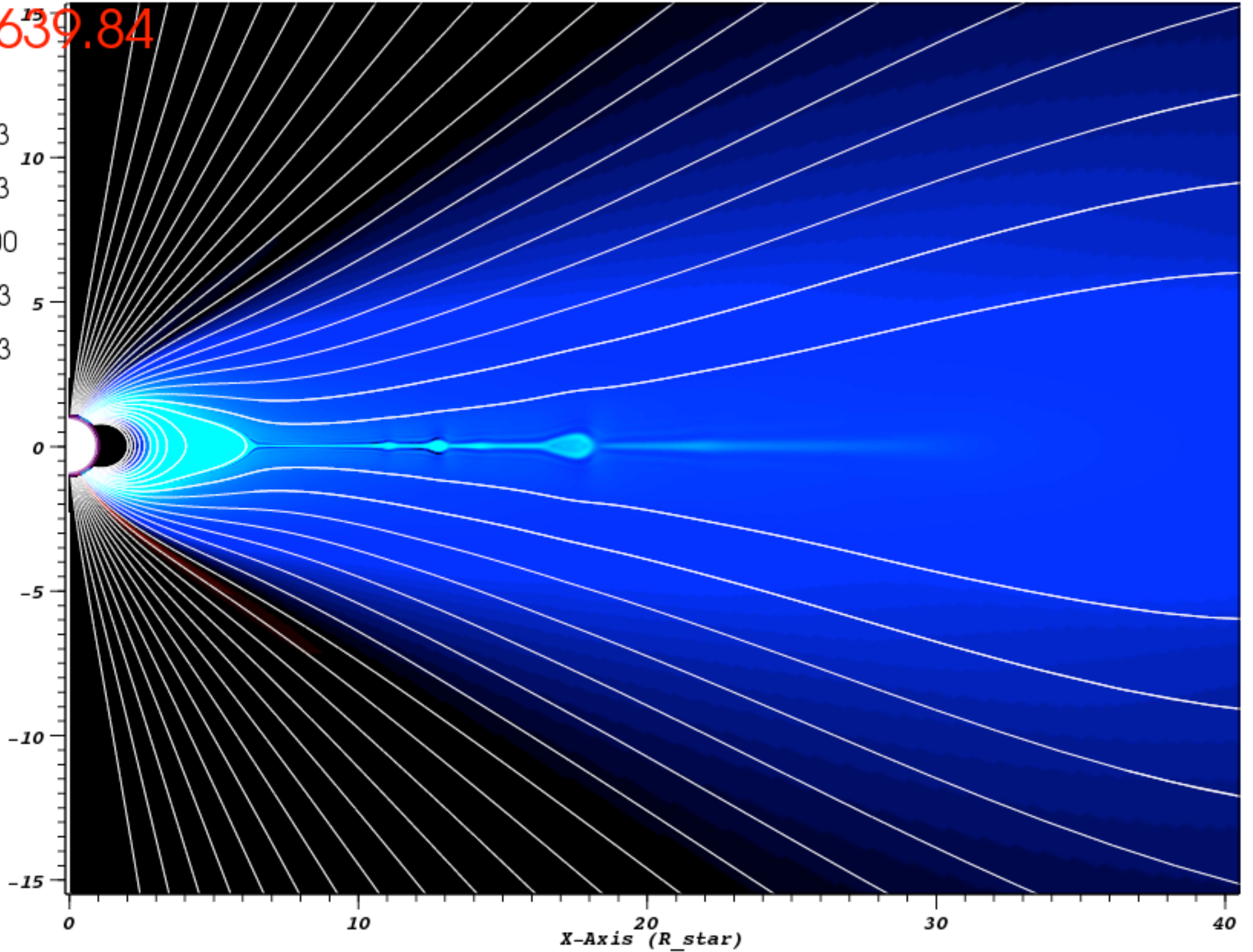


Twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 3639.84

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

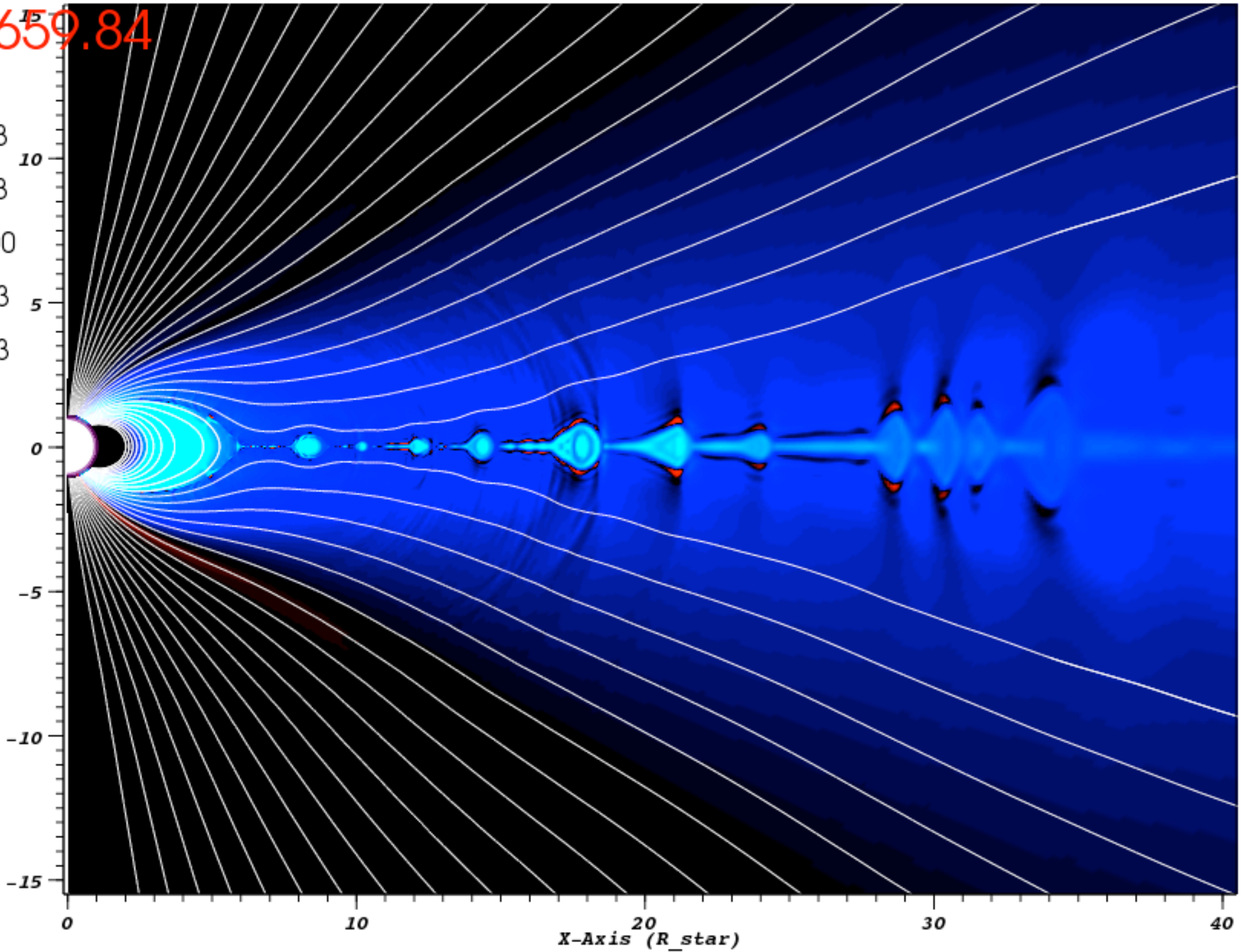


Twisted magnetosphere — second current sheet
Shown: B^ϕ , poloidal field line projections

DB: 2
Time: 3659.84

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



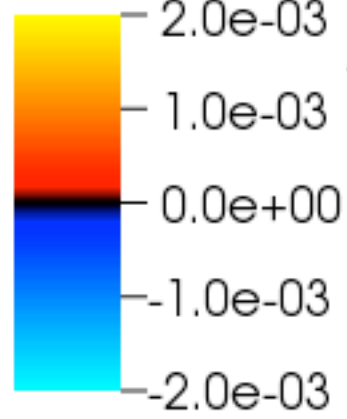
Twisted magnetosphere — second reconnection event
Shown: B^ϕ , poloidal field line projections

DB: 2

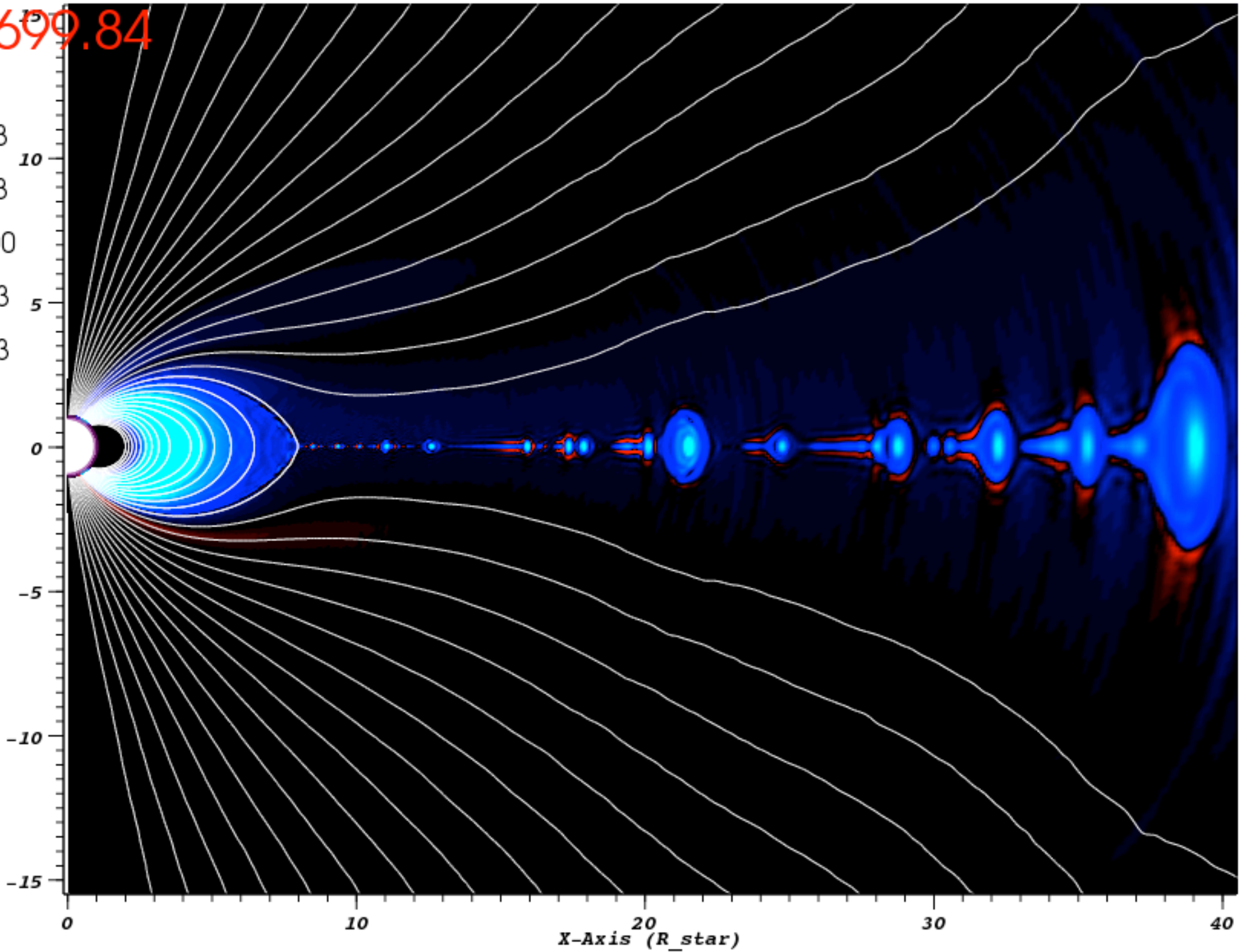
Time: 3699.84

Pseudocolor

Var: B3



Y-Axis
(R_star)

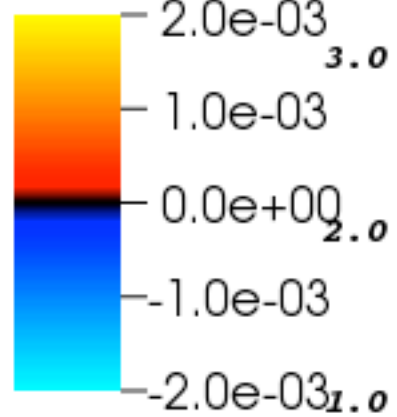


Twisted magnetosphere — giant plasmoids
Shown: B^ϕ , poloidal field line projections

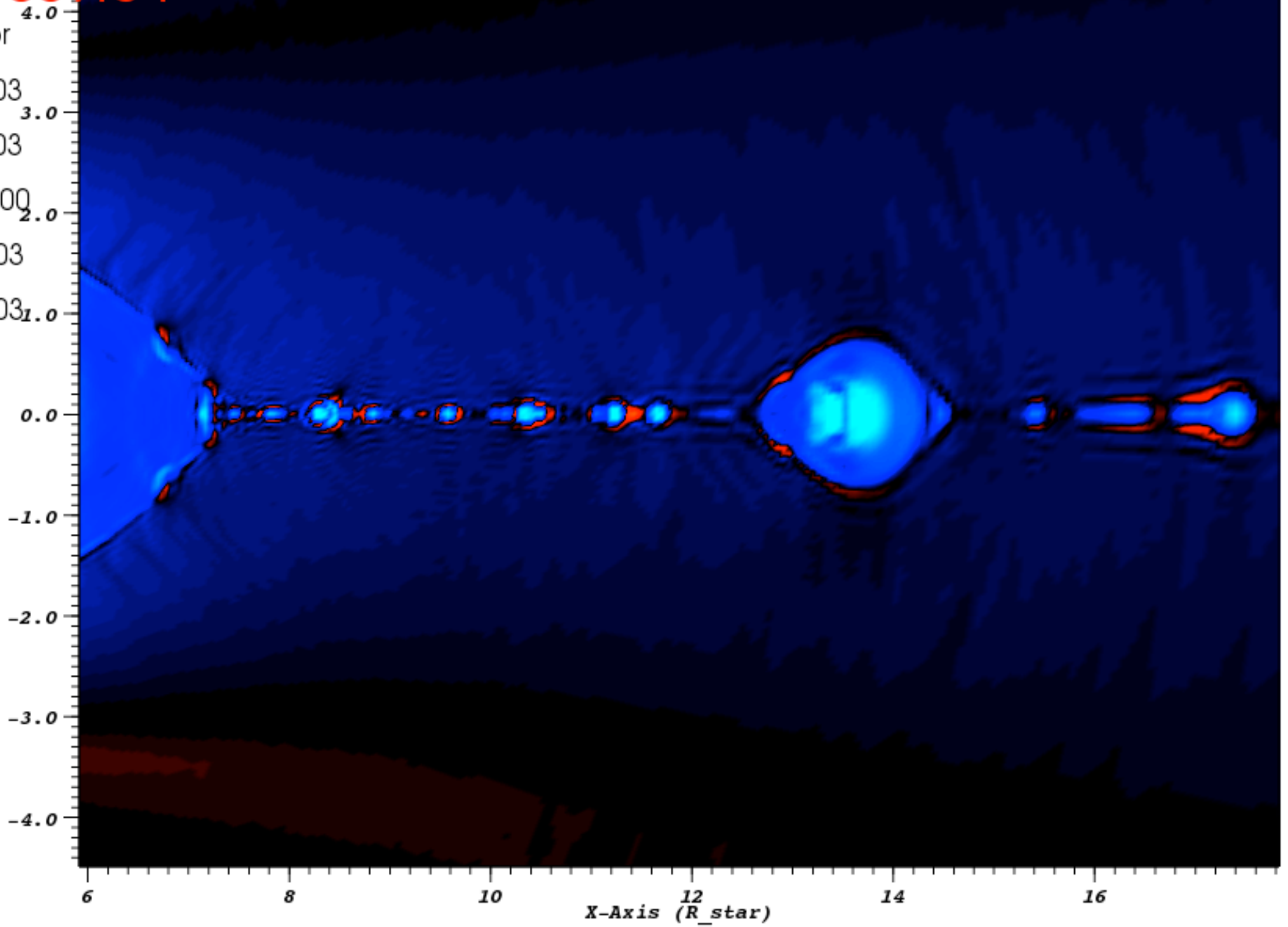


DB: 2
Time: 3689.84

Pseudocolor
Var: B3

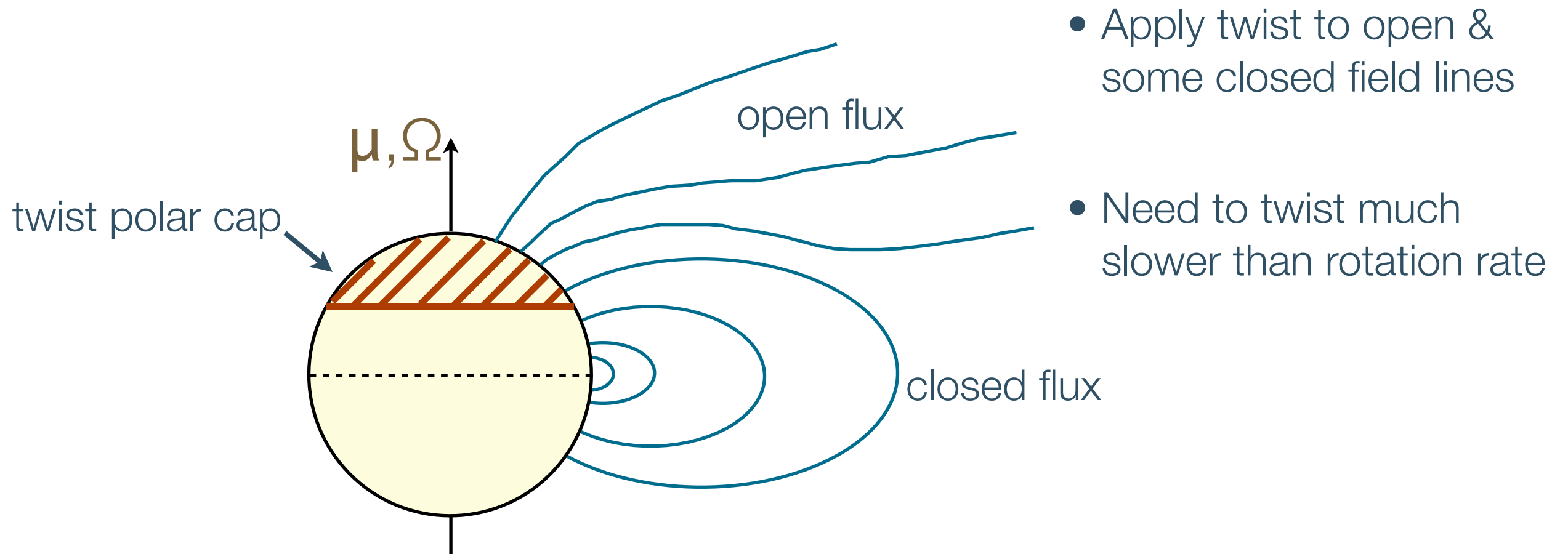


Y-Axis
(R_star)



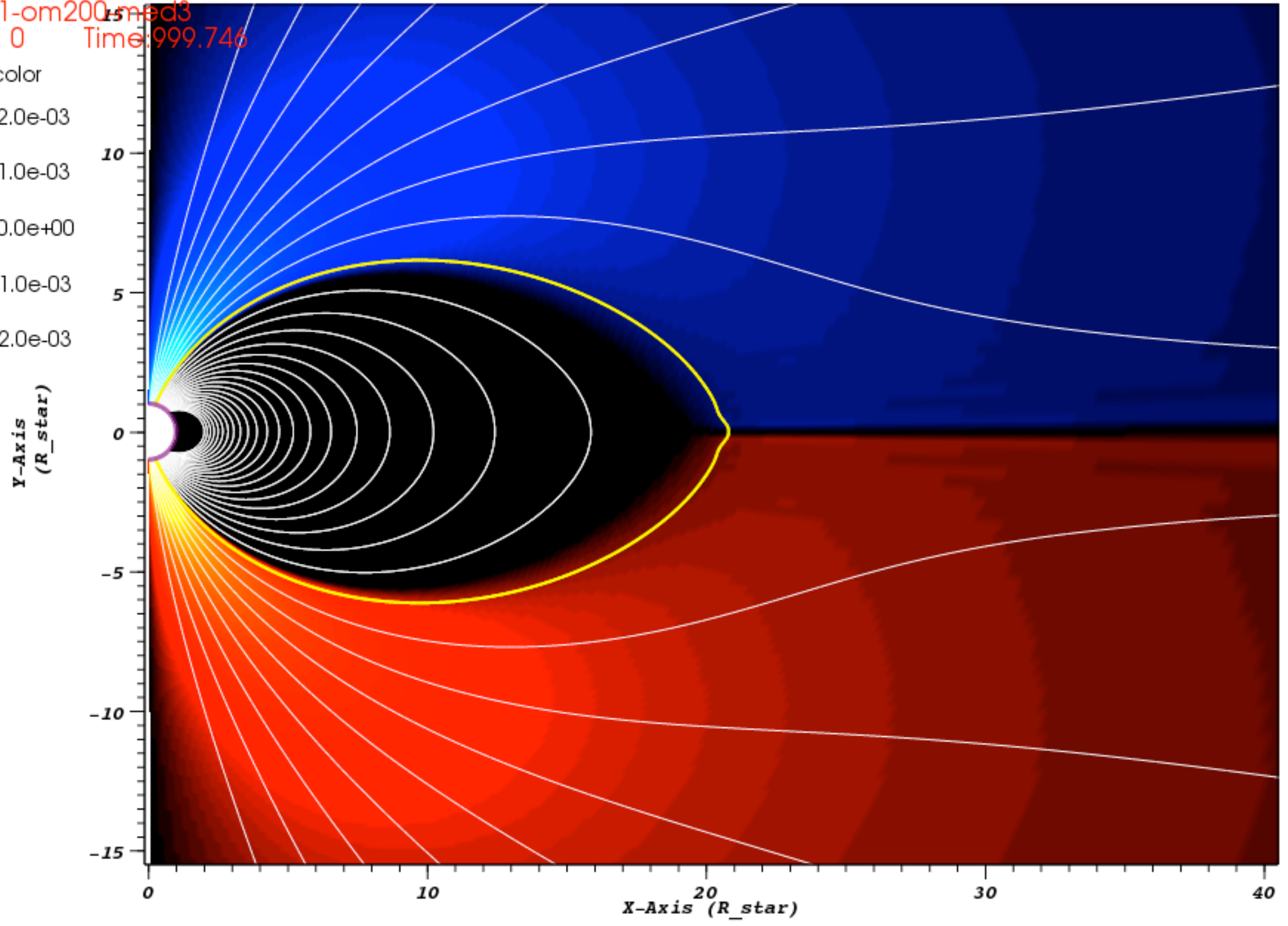
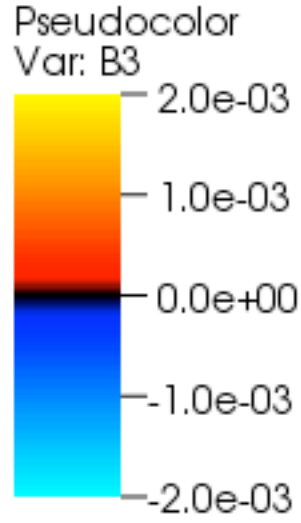
Hierarchical plasmoid formation? (Uzdensky et al 2010)

Rotation + surface differential motion



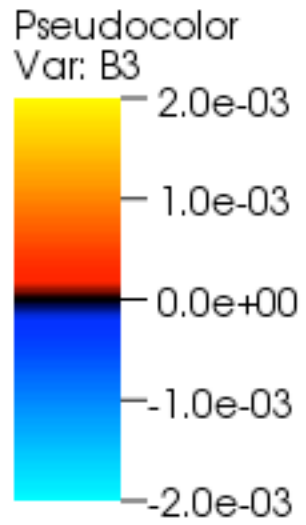
- Simulation: twisted flux = 3 x open flux, $\Omega_{\text{twist}} = \Omega_{\text{rotate}}/200$
- Example object: AXP XTE J1810-197 (Beloborodov 2009)

DB: 3u1-om200.med3
Cycle: 0 Time: 999.746

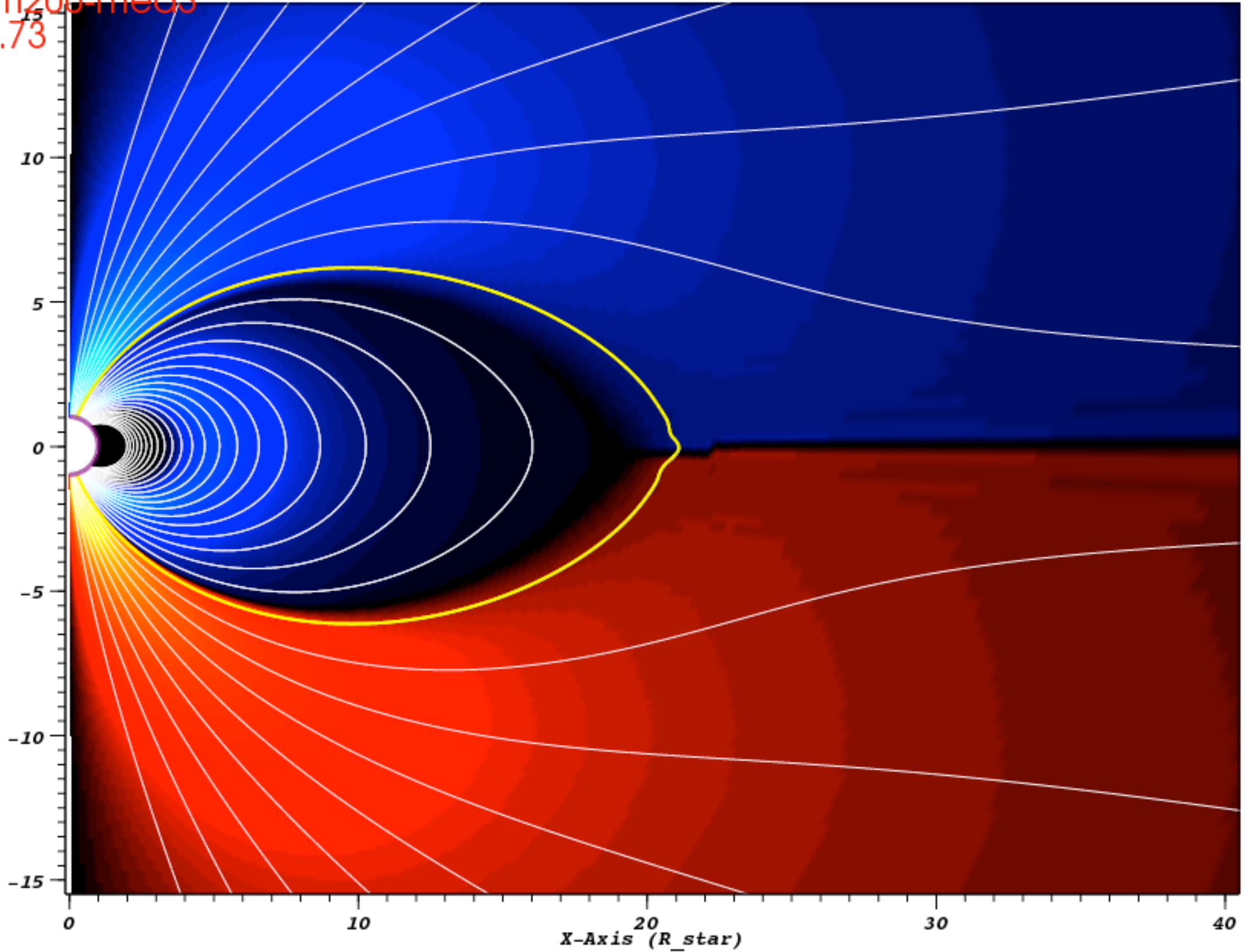


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 1604.73

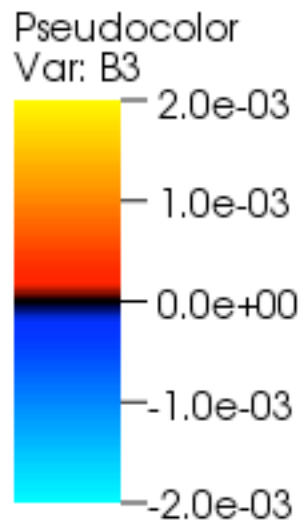


Y-Axis
(R_star)

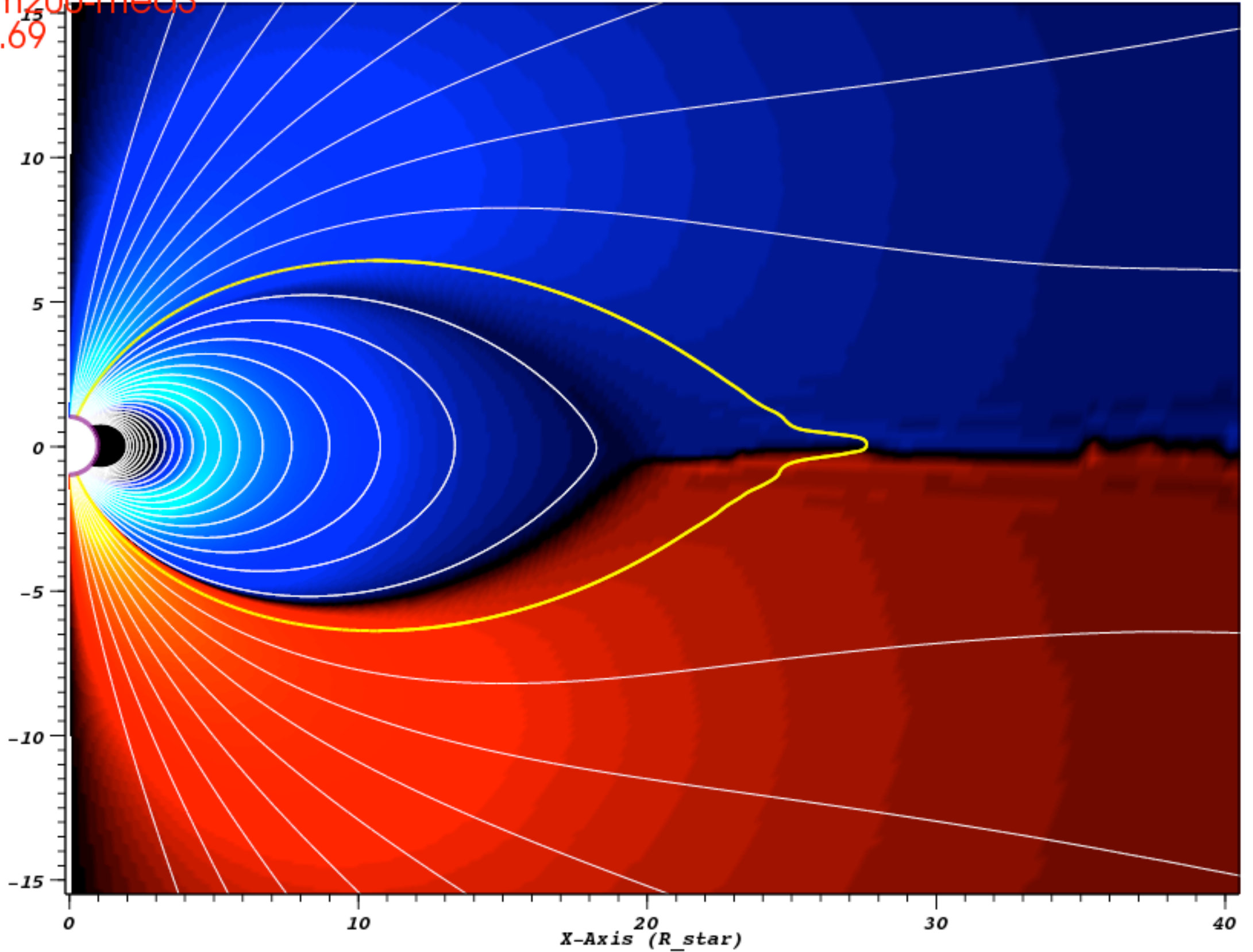


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 2979.69



Y-Axis
(R_star)

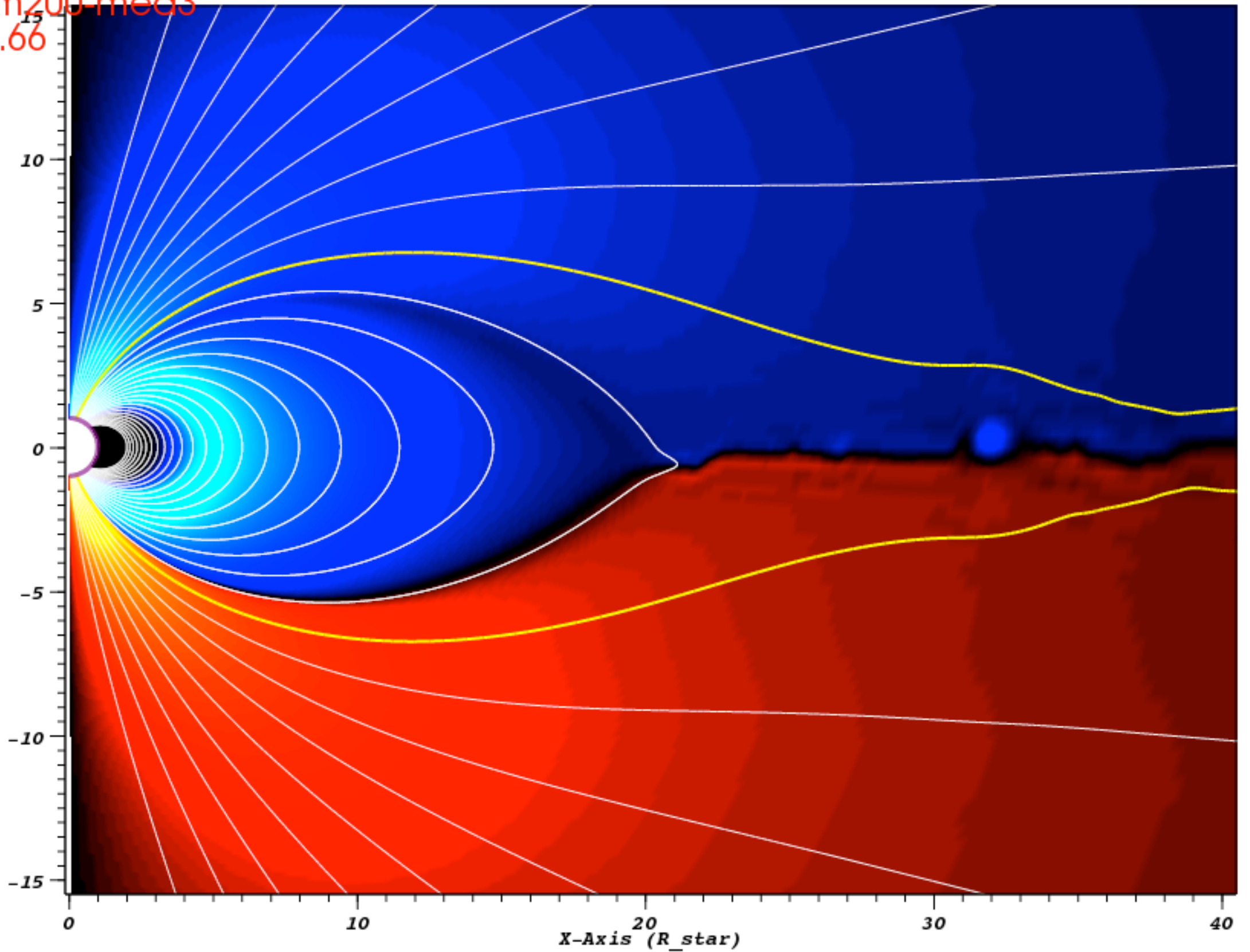


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 3859.66

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

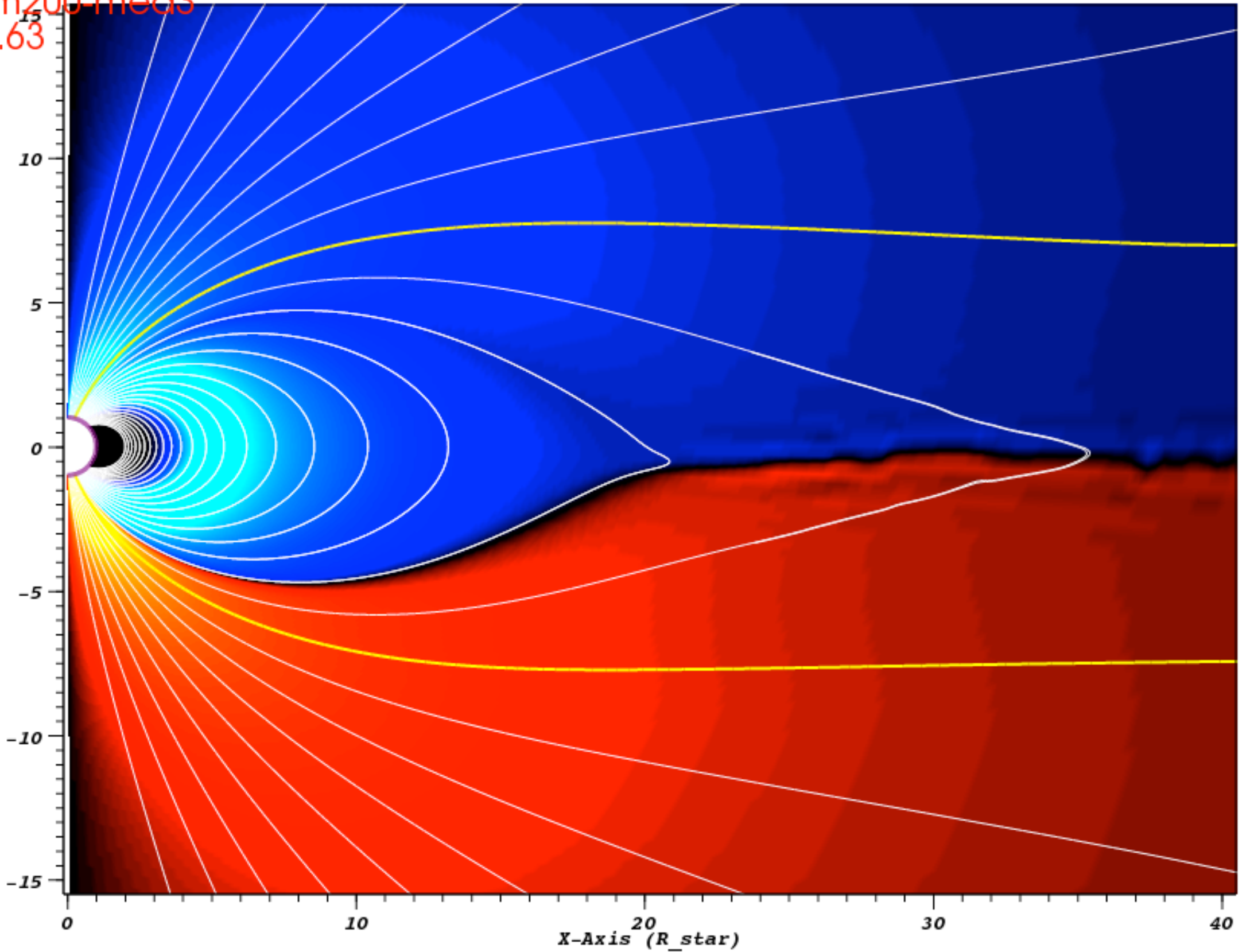


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time:4959.63

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

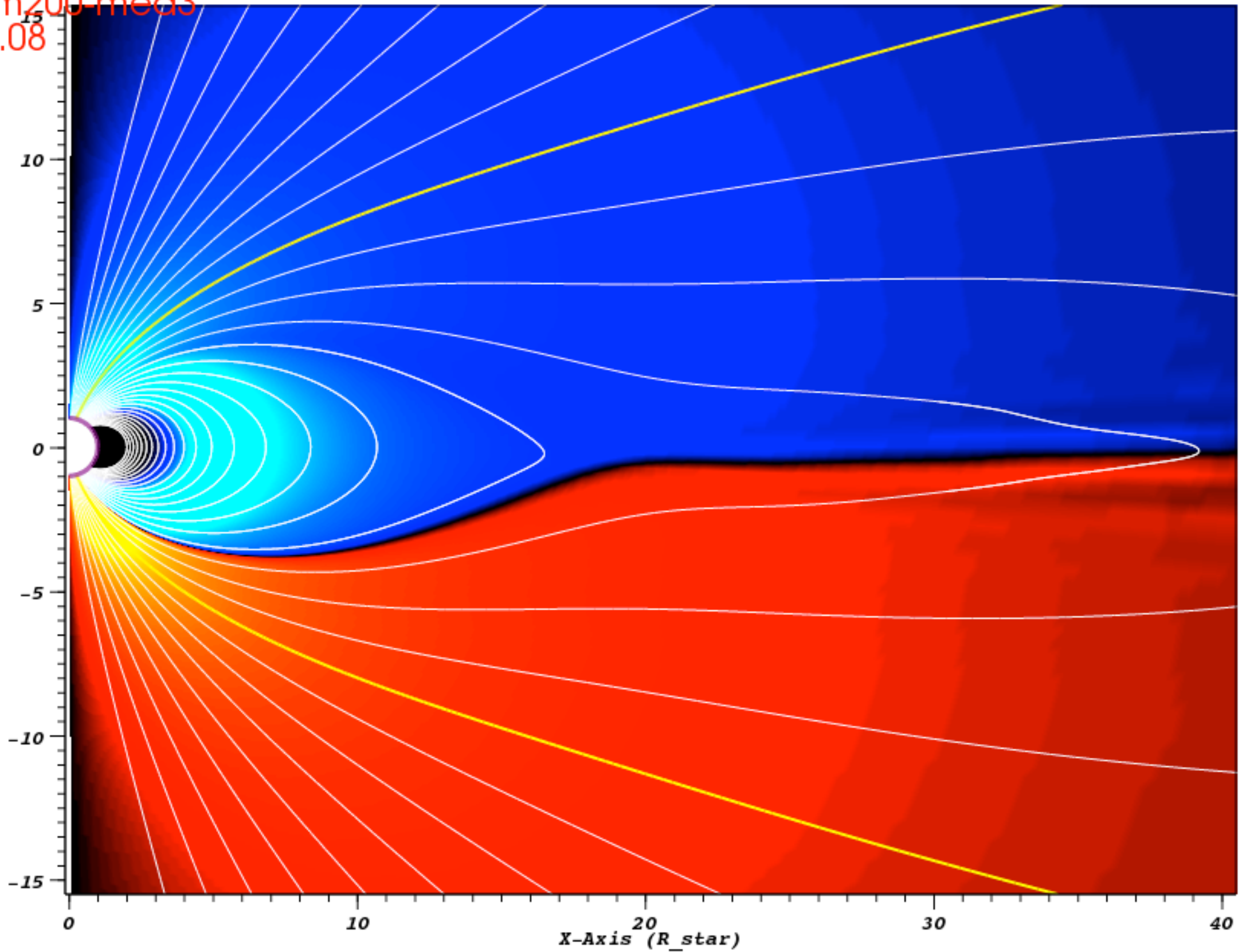


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 6747.08

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

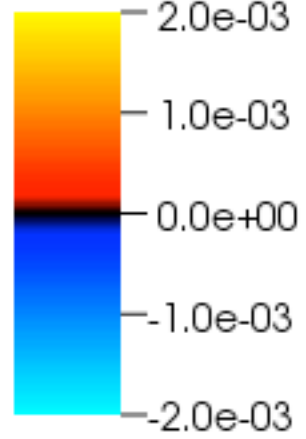
Y-Axis
(R_star)



Rotating, twisted magnetosphere — unstable equilibrium
Shown: B^ϕ , poloidal field line projections

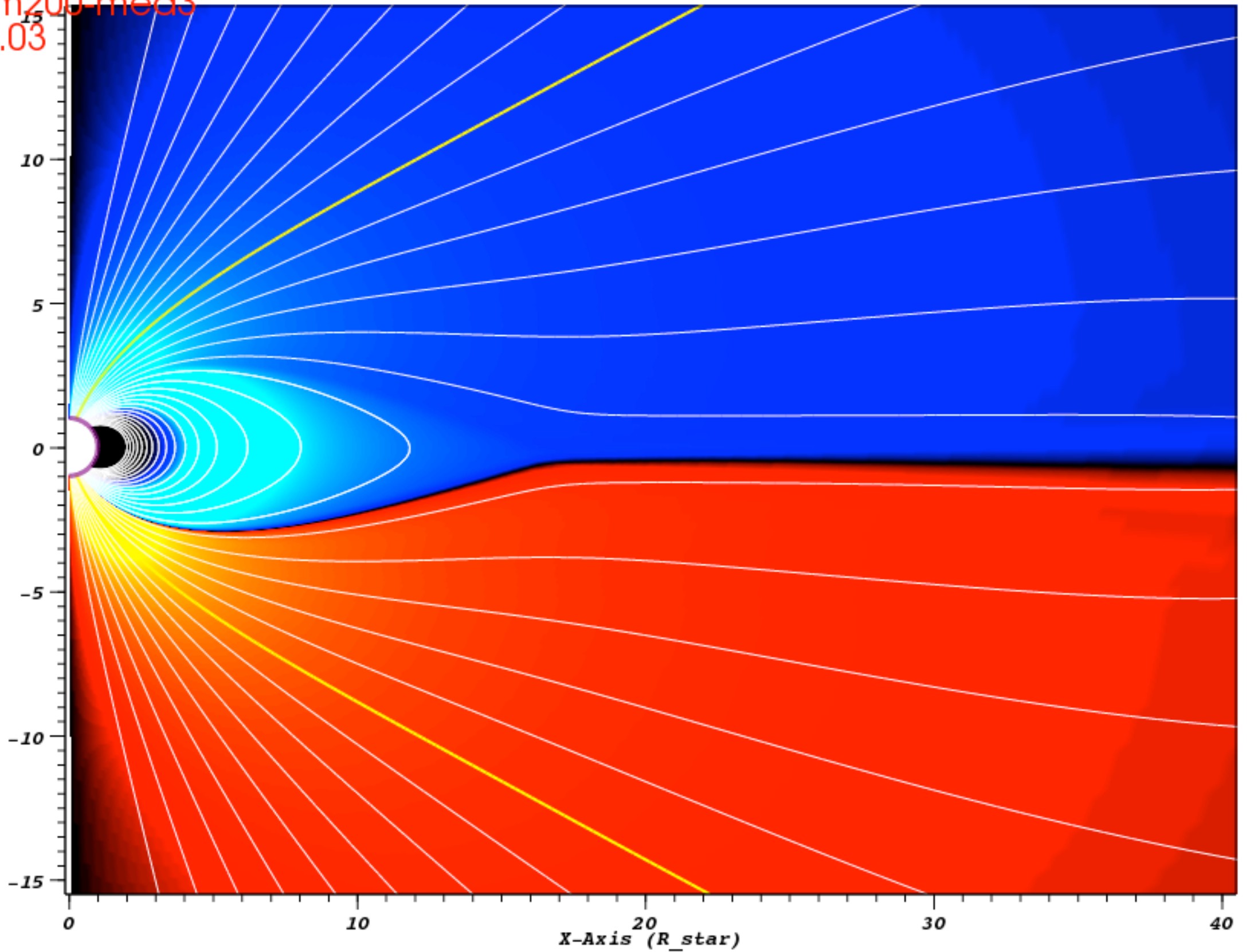
DB: 3u1-om200-med3
Time:8342.03

Pseudocolor
Var: B3



A vertical color bar legend for the variable B3. The scale ranges from -2.0e-03 at the bottom (dark blue) to 2.0e-03 at the top (yellow). Intermediate values are marked at -1.0e-03, 0.0e+00 (black), and 1.0e-03 (red).

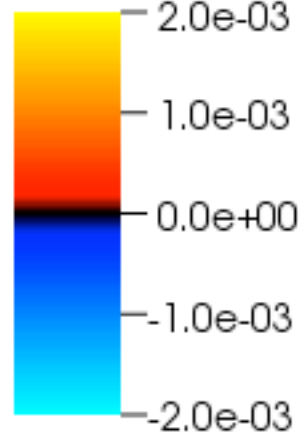
Y-Axis
(R_star)



Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

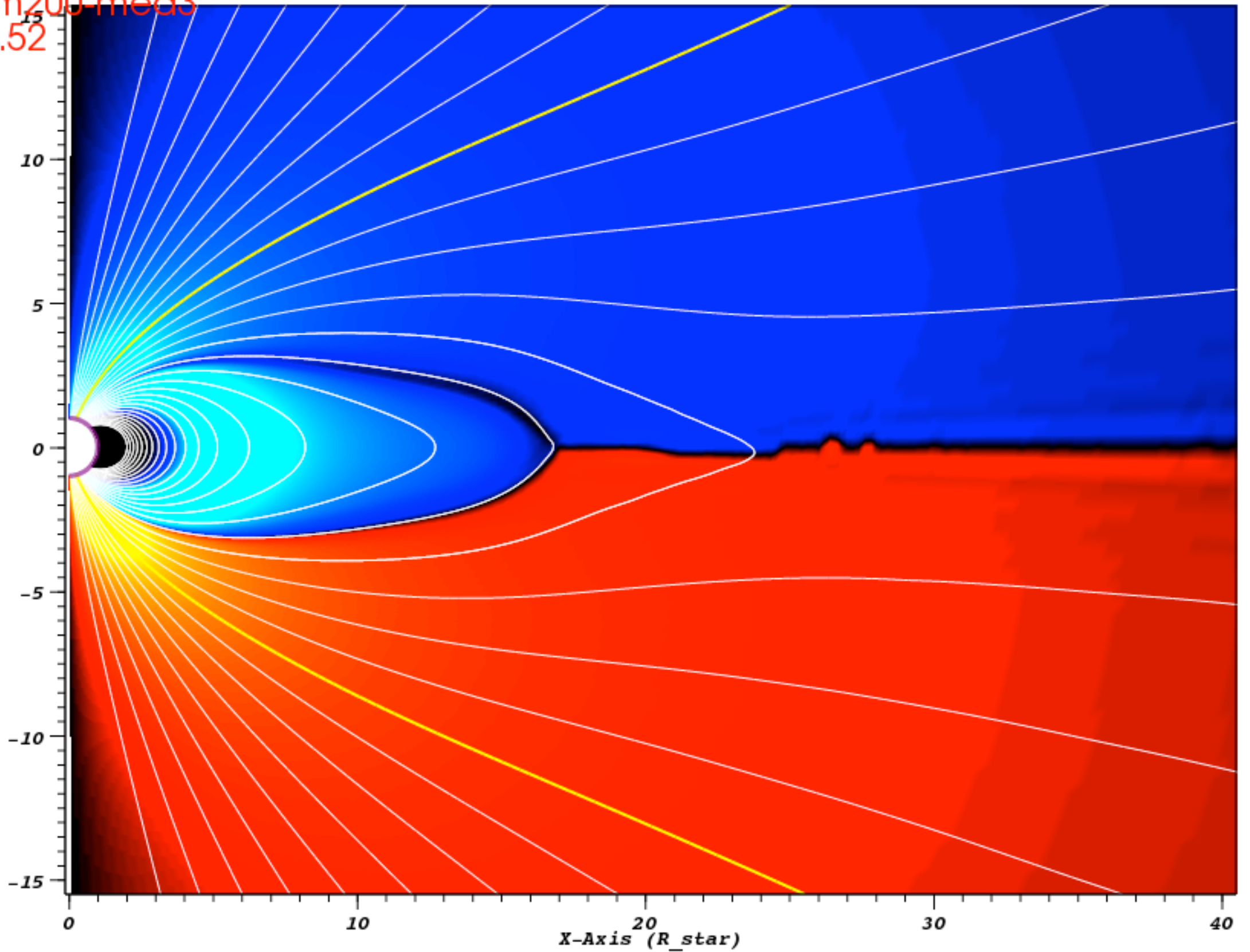
DB: 3u1-om200-med3
Time: 8424.52

Pseudocolor
Var: B3



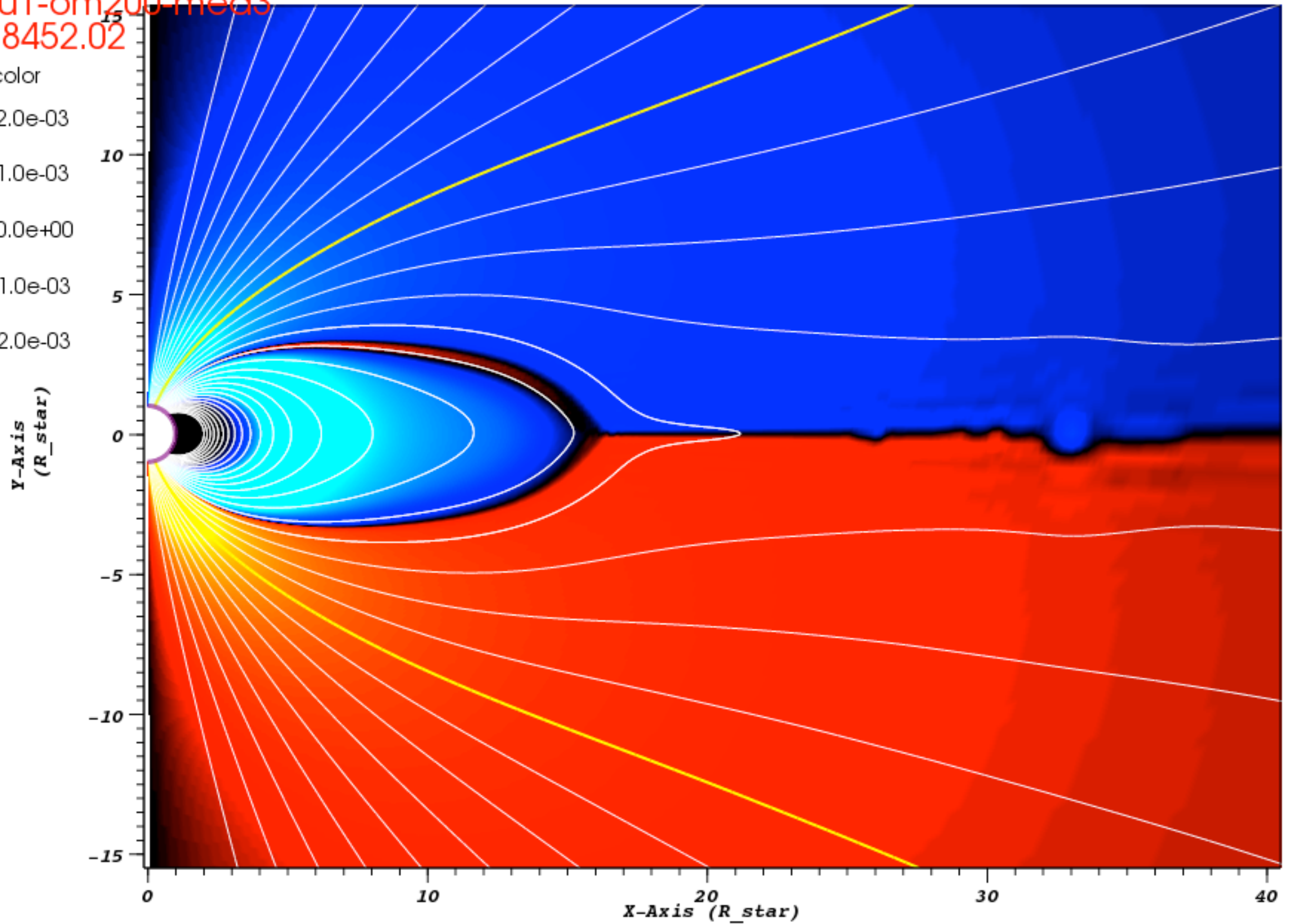
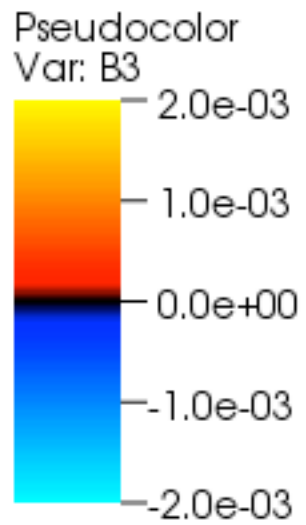
A vertical color bar legend for the variable B3. The scale ranges from -2.0e-03 at the bottom (dark blue) to 2.0e-03 at the top (yellow). Intermediate values are marked at -1.0e-03, 0.0e+00 (black), and 1.0e-03 (red).

Y-Axis
(R_star)



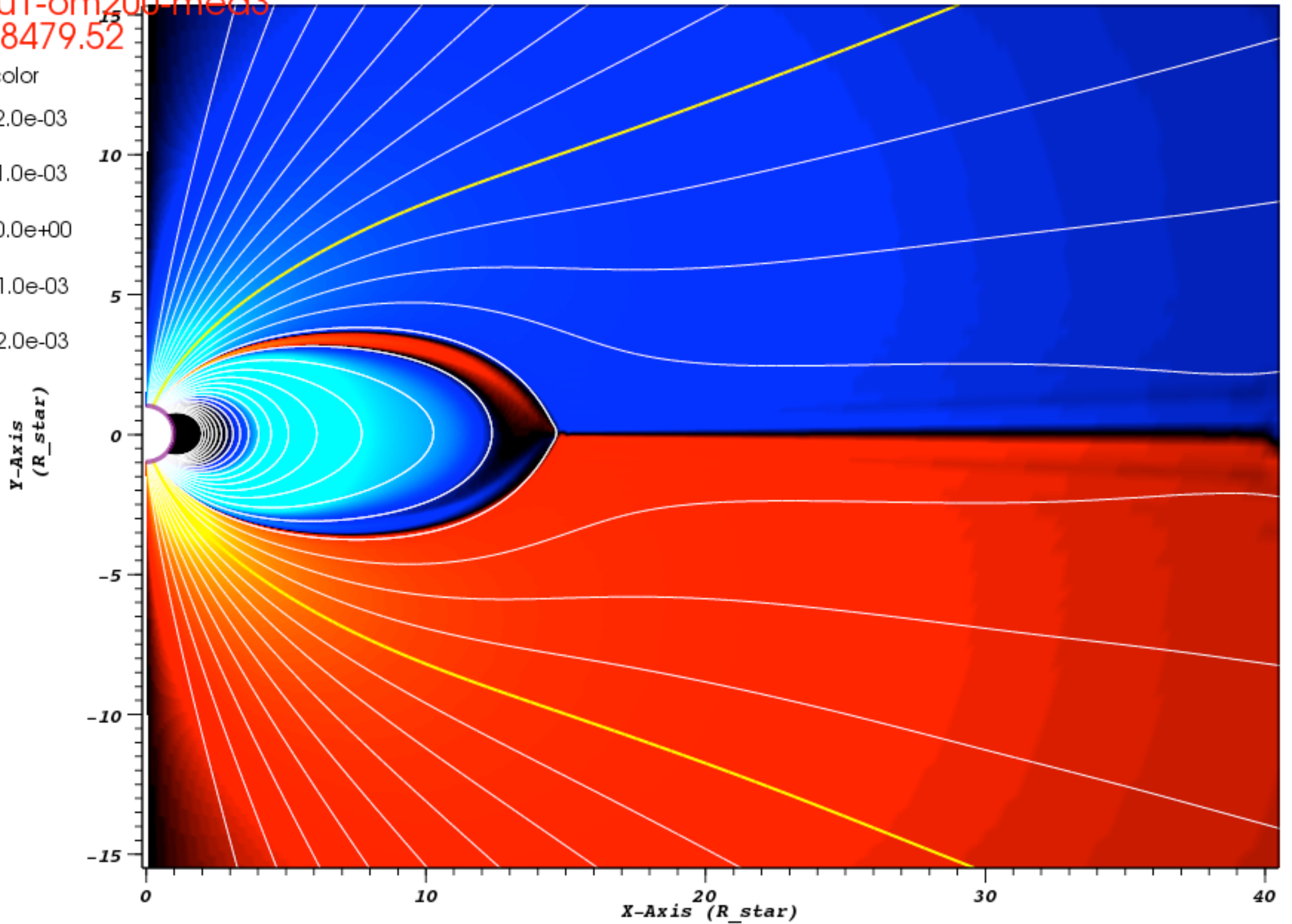
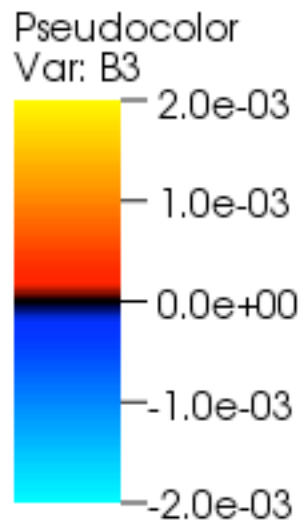
Rotating, twisted magnetosphere — first reconnection
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 8452.02



Rotating, twisted magnetosphere — first reconnection
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 8479.52

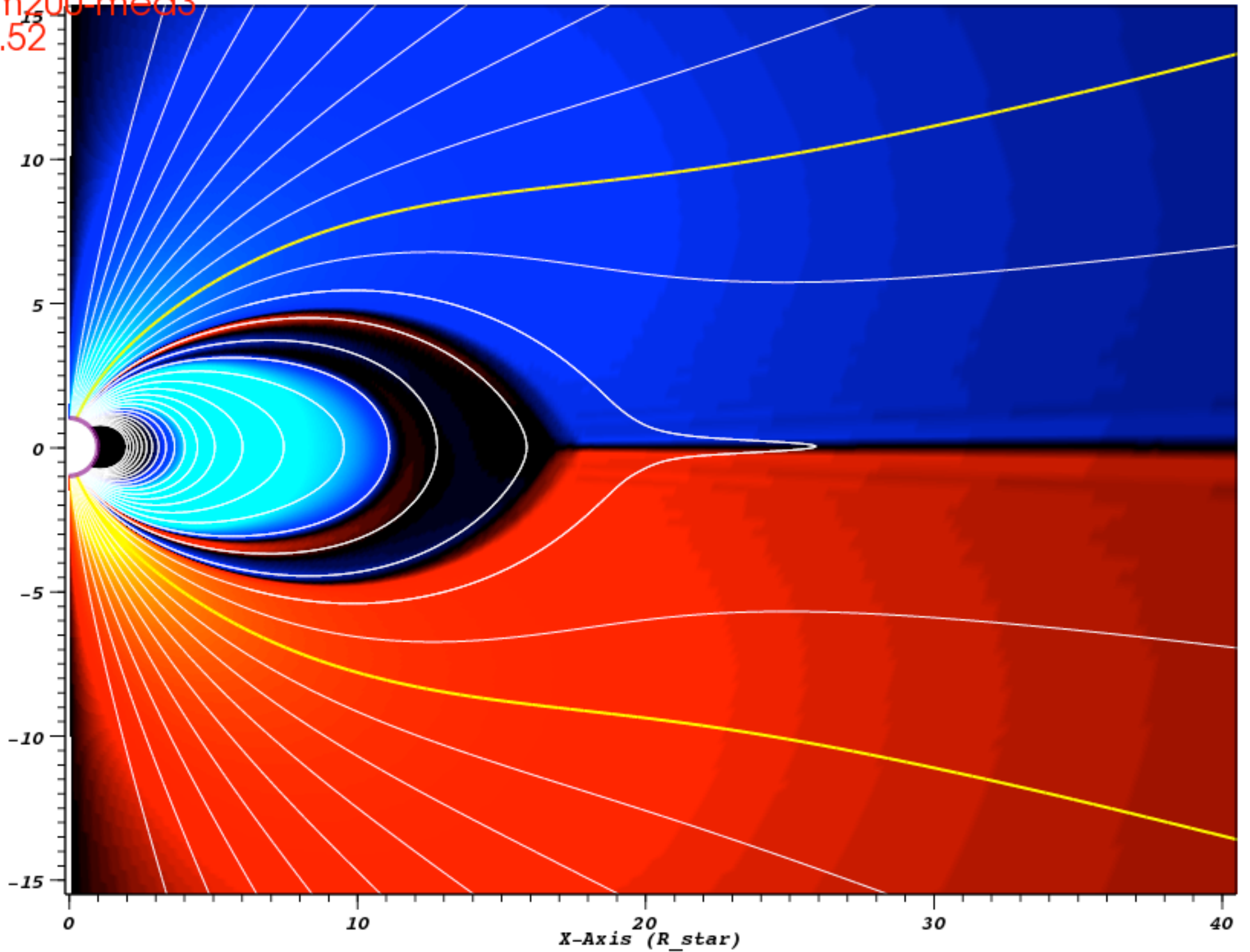


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 8589.52

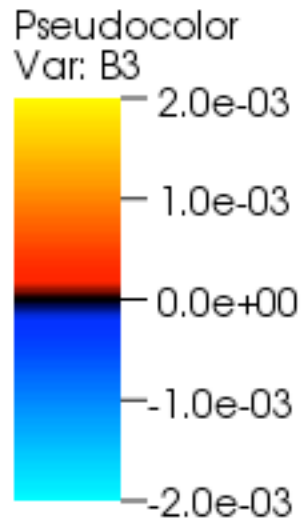
Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

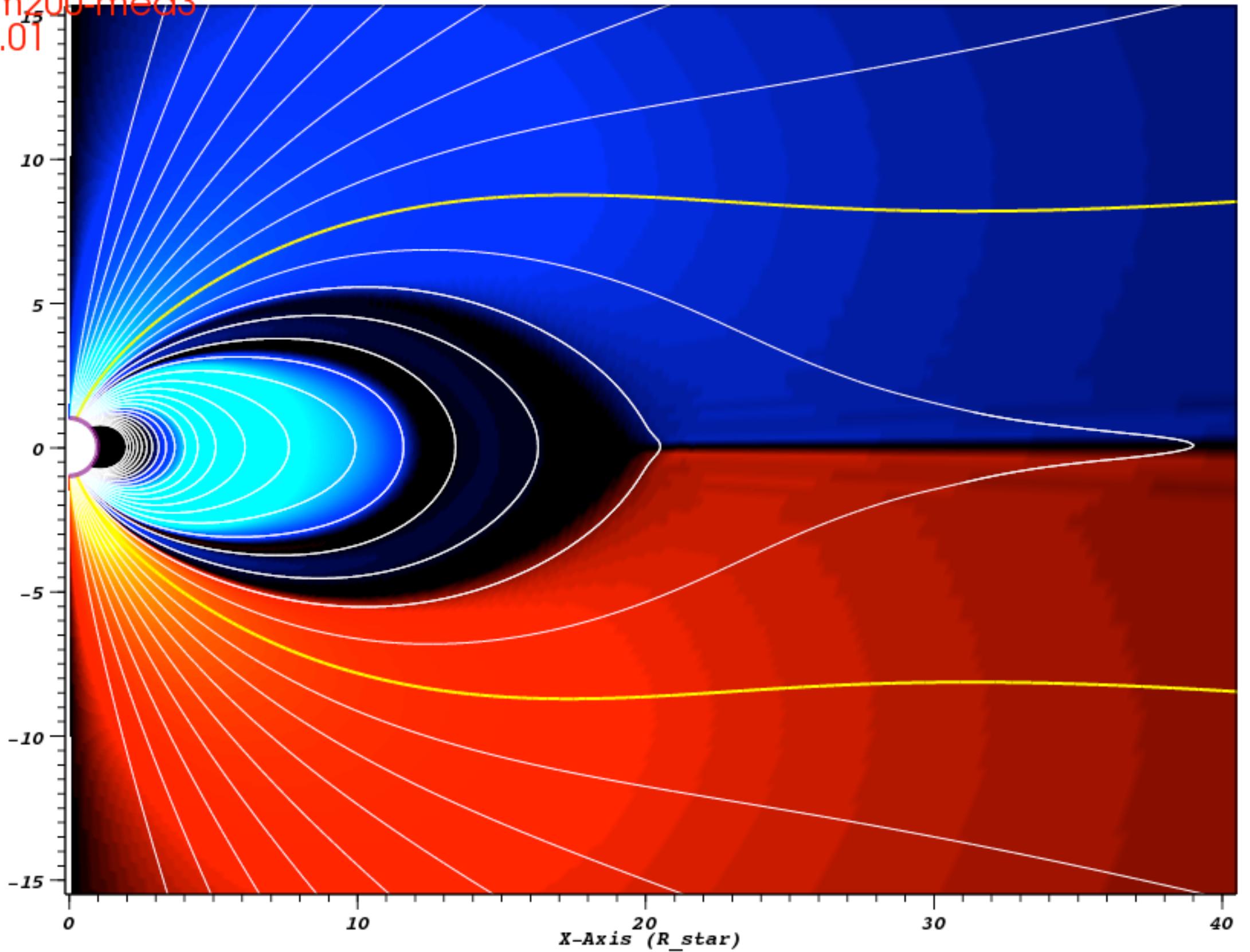


Rotating, twisted magnetosphere — cavity formation
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time:8782.01



Y-Axis
(R_star)

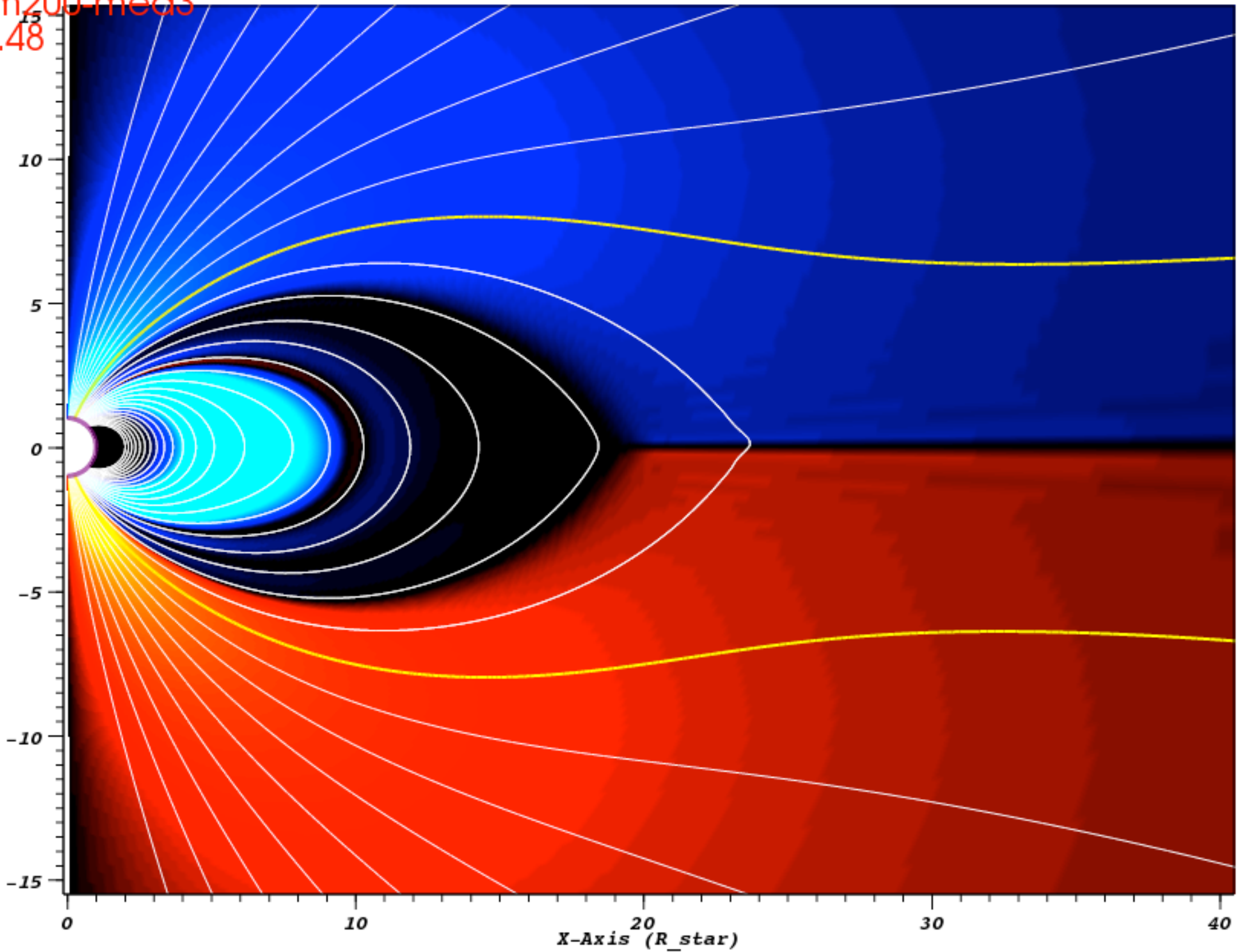


Rotating, twisted magnetosphere — cavity formation
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 9799.48

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

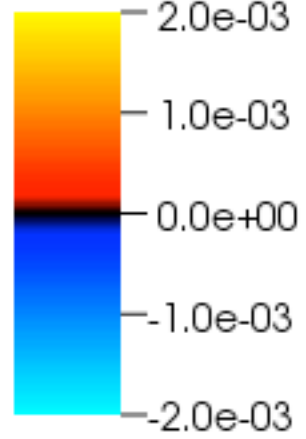
Y-Axis
(R_star)



Rotating, twisted magnetosphere — cavity formation
Shown: B^ϕ , poloidal field line projections

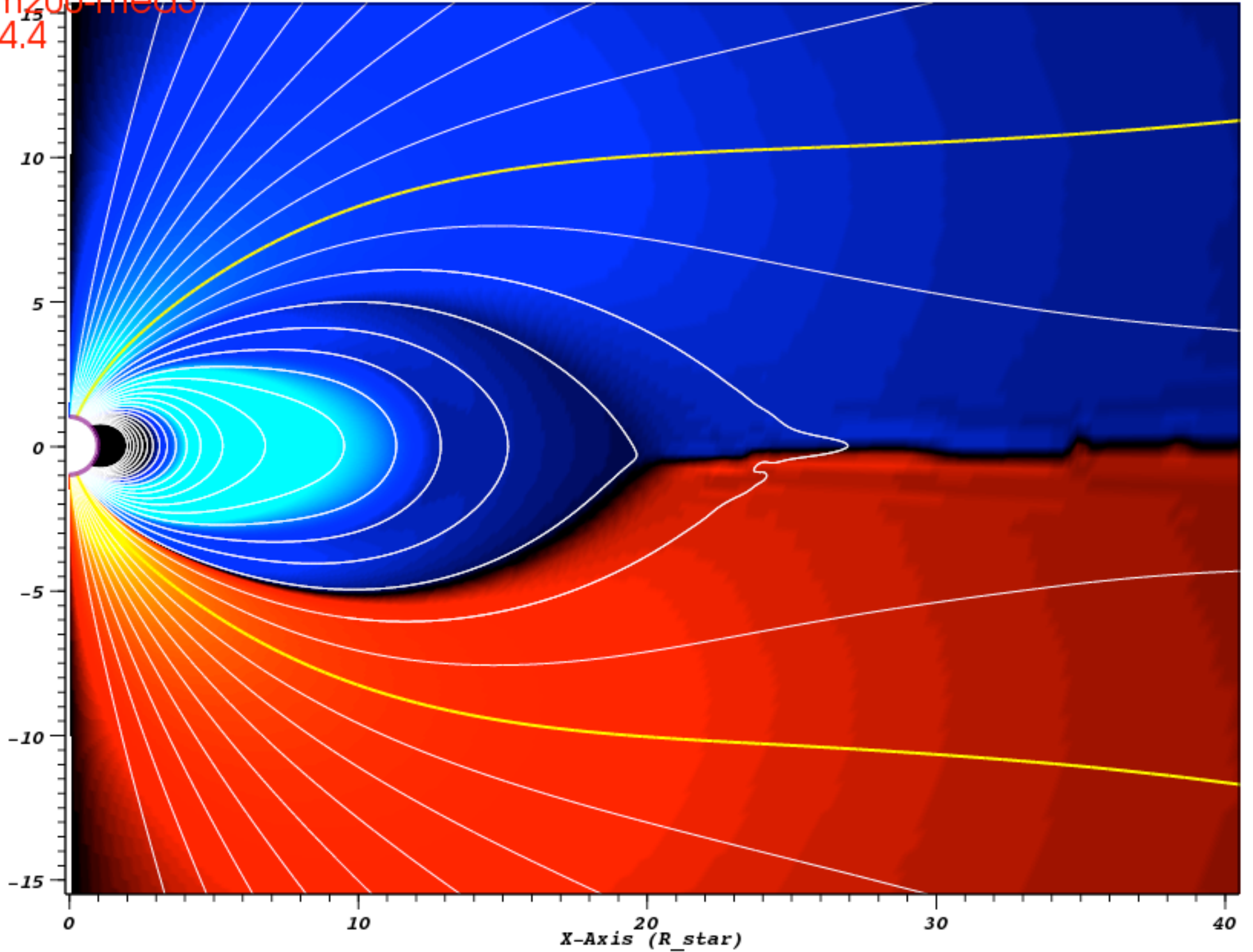
DB: 3u1-om200-med3
Time: 10954.4

Pseudocolor
Var: B3



A vertical color bar legend for the variable B3. The scale ranges from -2.0e-03 (dark blue) at the bottom to 2.0e-03 (yellow) at the top. Intermediate values are marked at -1.0e-03 (light blue), 0.0e+00 (black), and 1.0e-03 (red).

Y-Axis
(R_star)

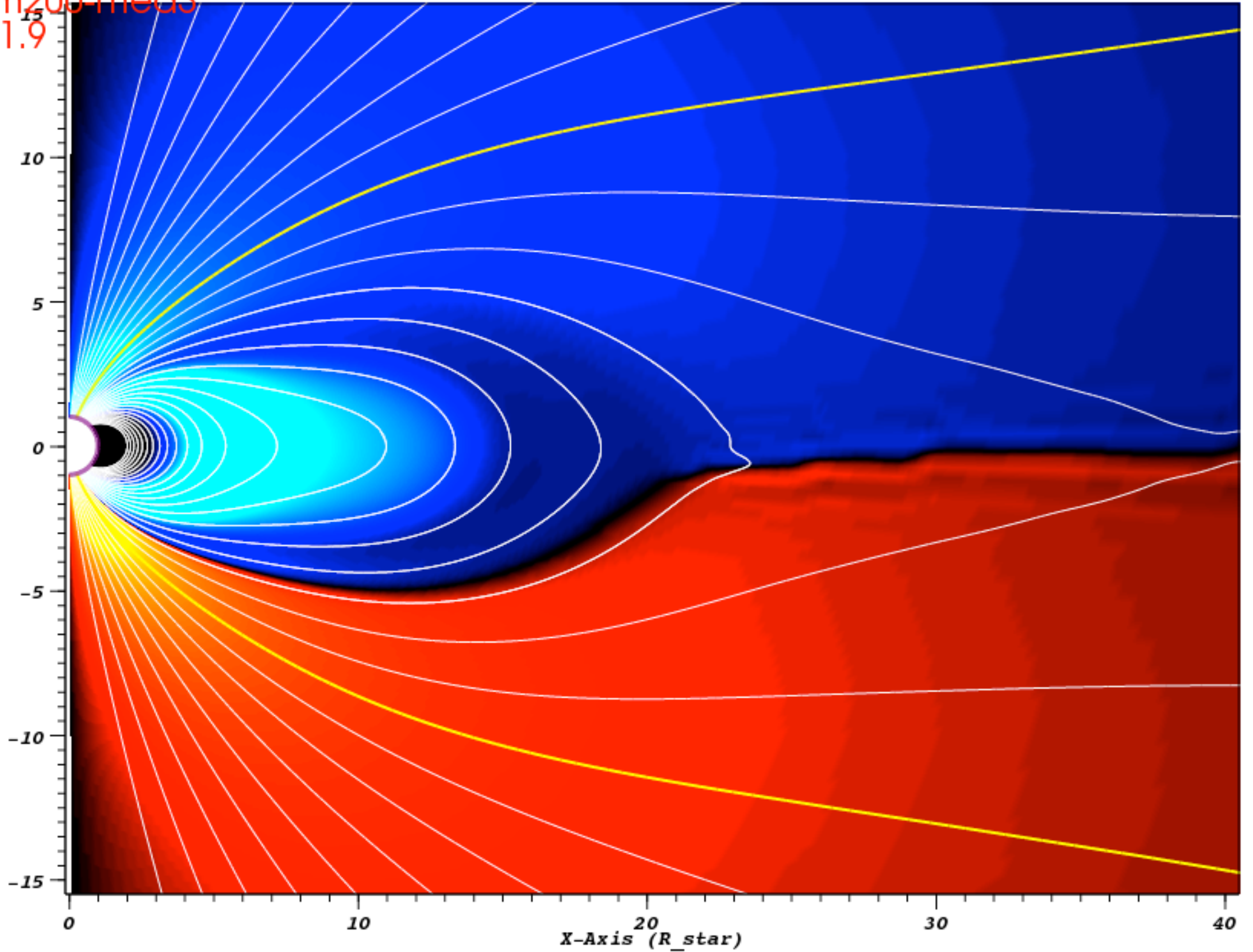


Rotating, twisted magnetosphere
Shown: B^{ϕ} , poloidal field line projections

DB: 3u1-om200-med3
Time: 11201.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

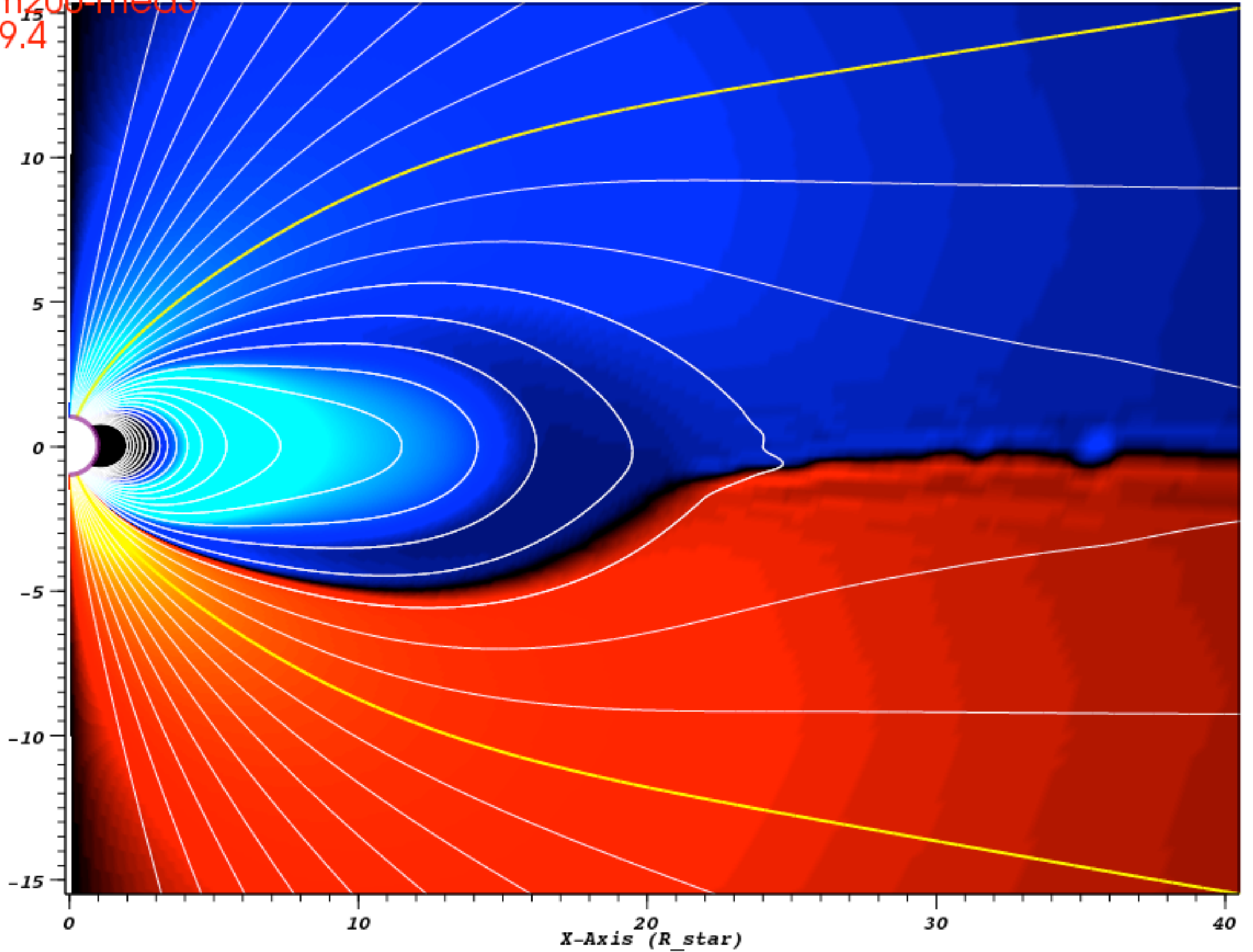


Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11229.4

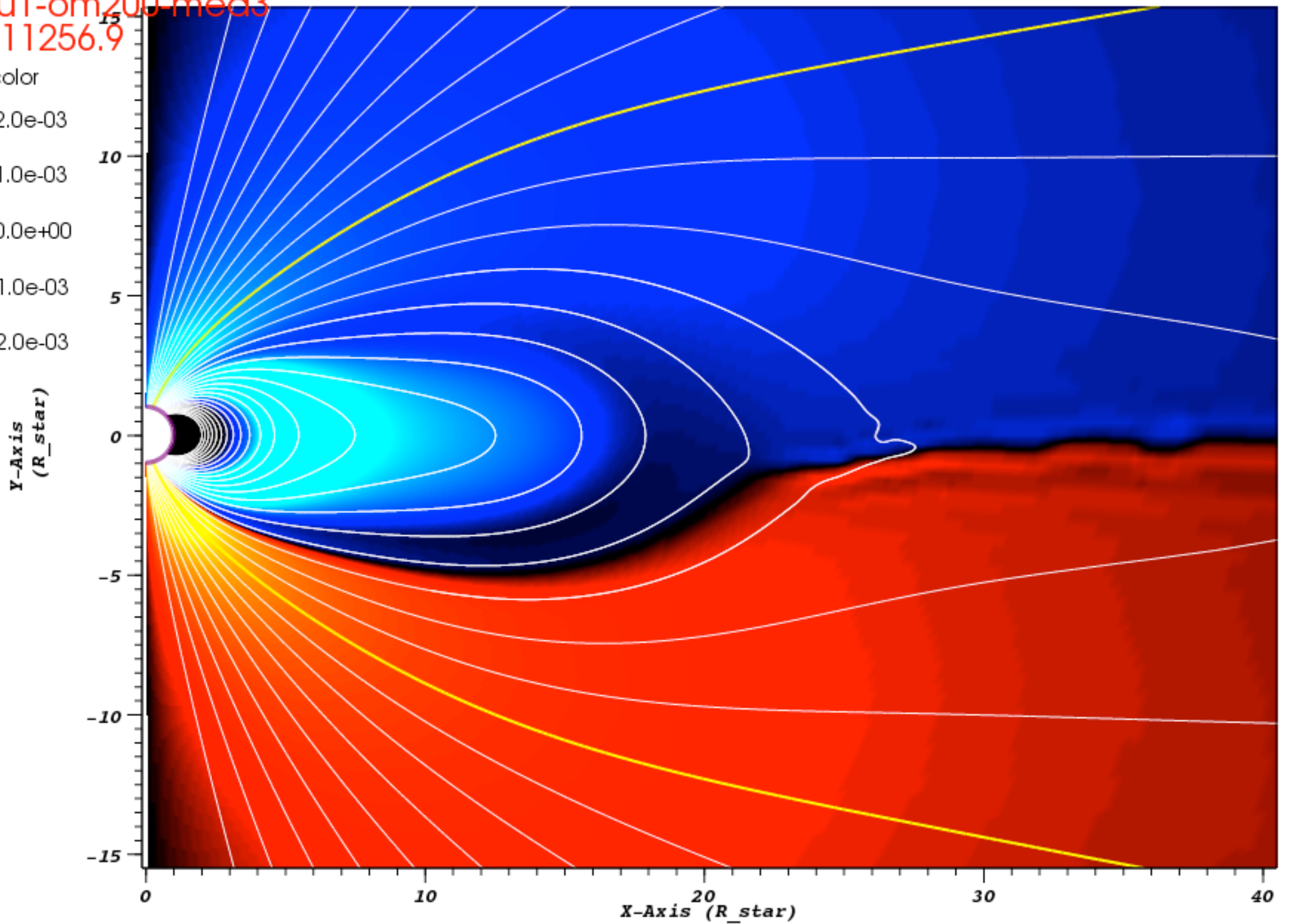
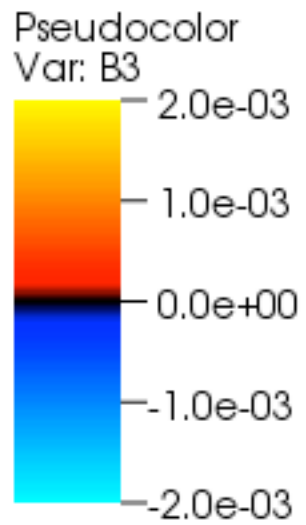
Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



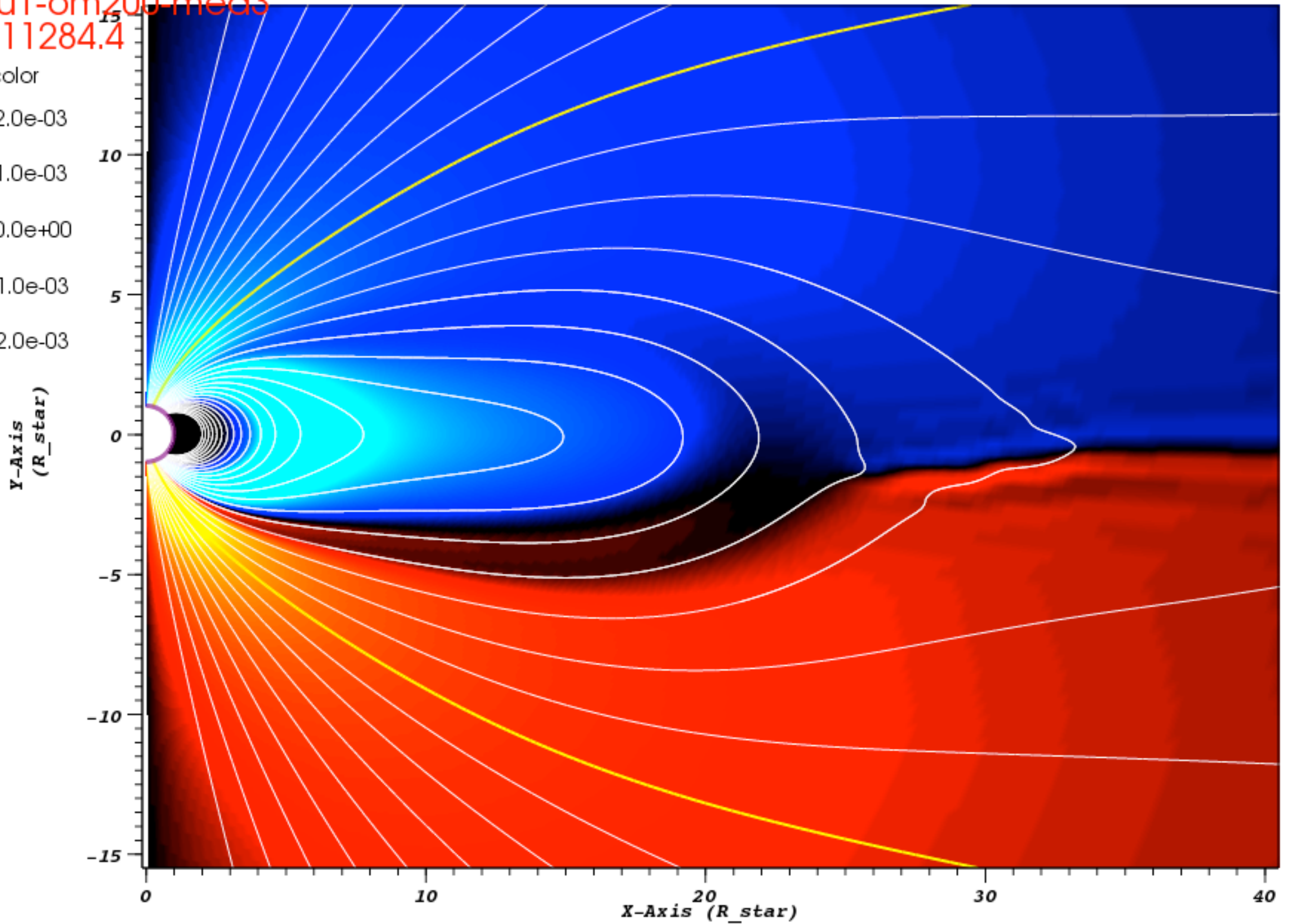
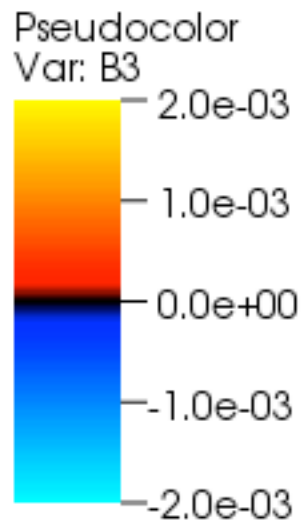
Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11256.9



Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11284.4

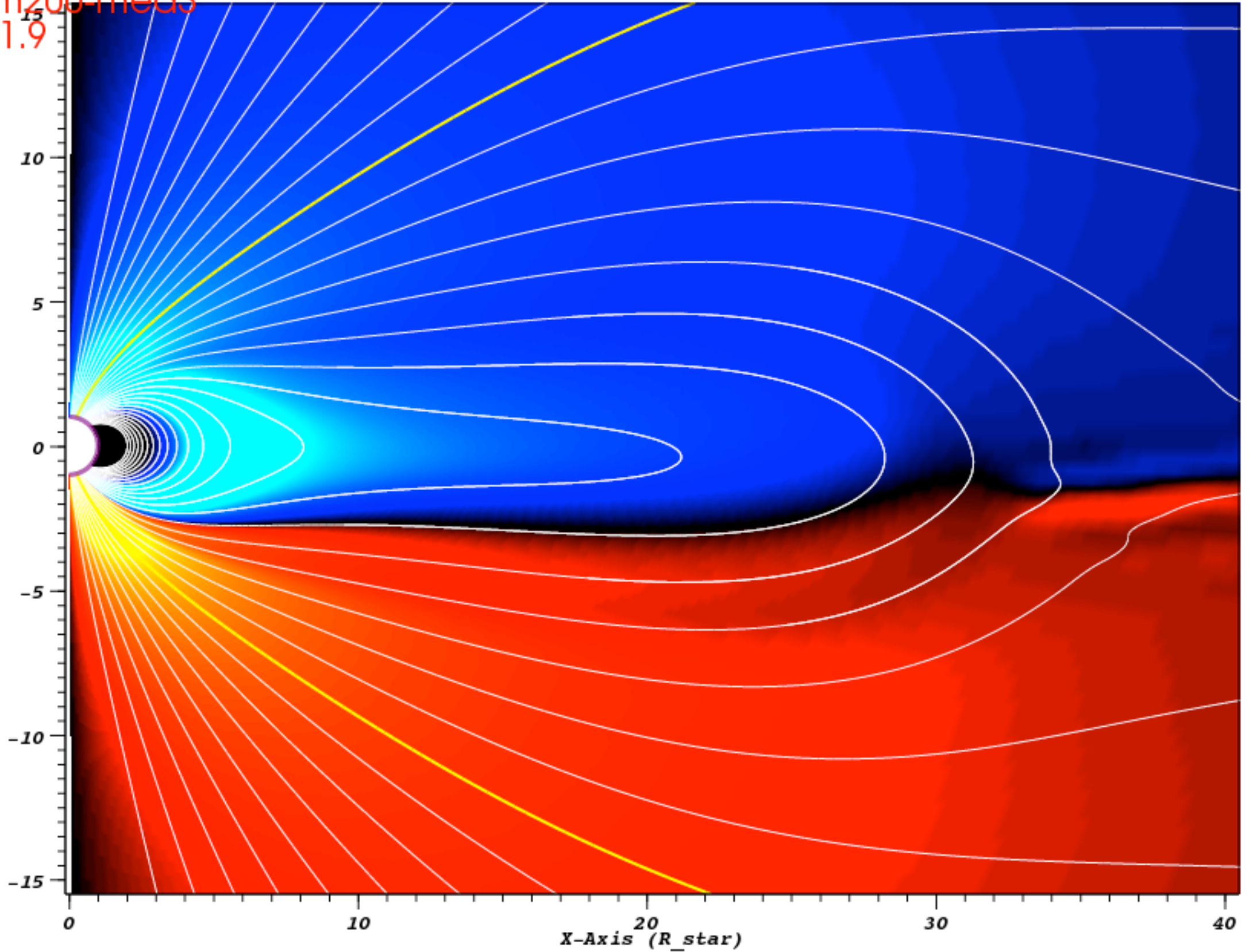


Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11311.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

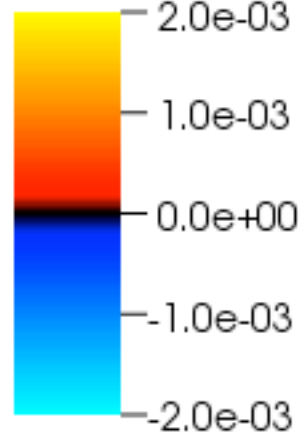
Y-Axis
(R_star)



Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

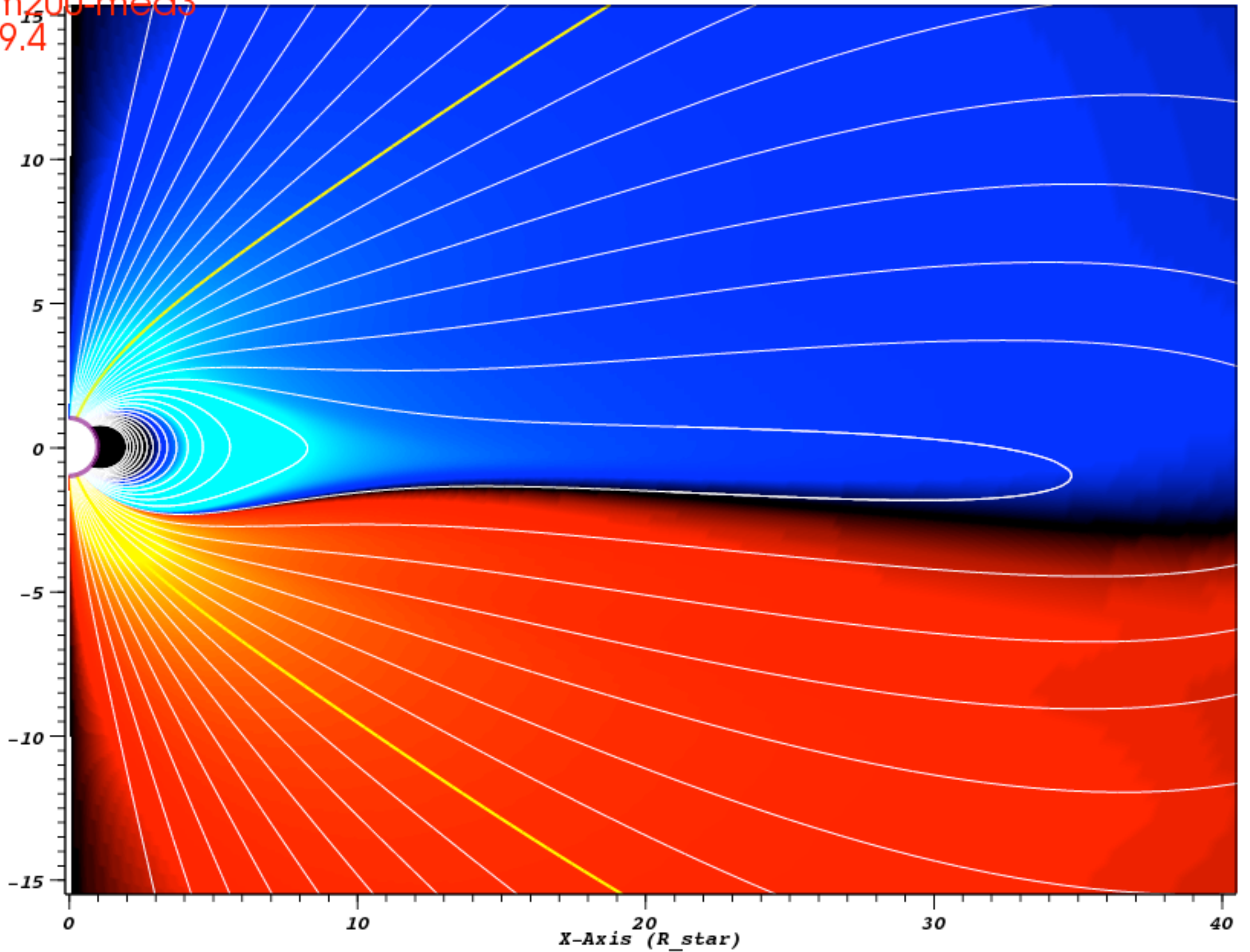
DB: 3u1-om200-med3
Time: 11339.4

Pseudocolor
Var: B3



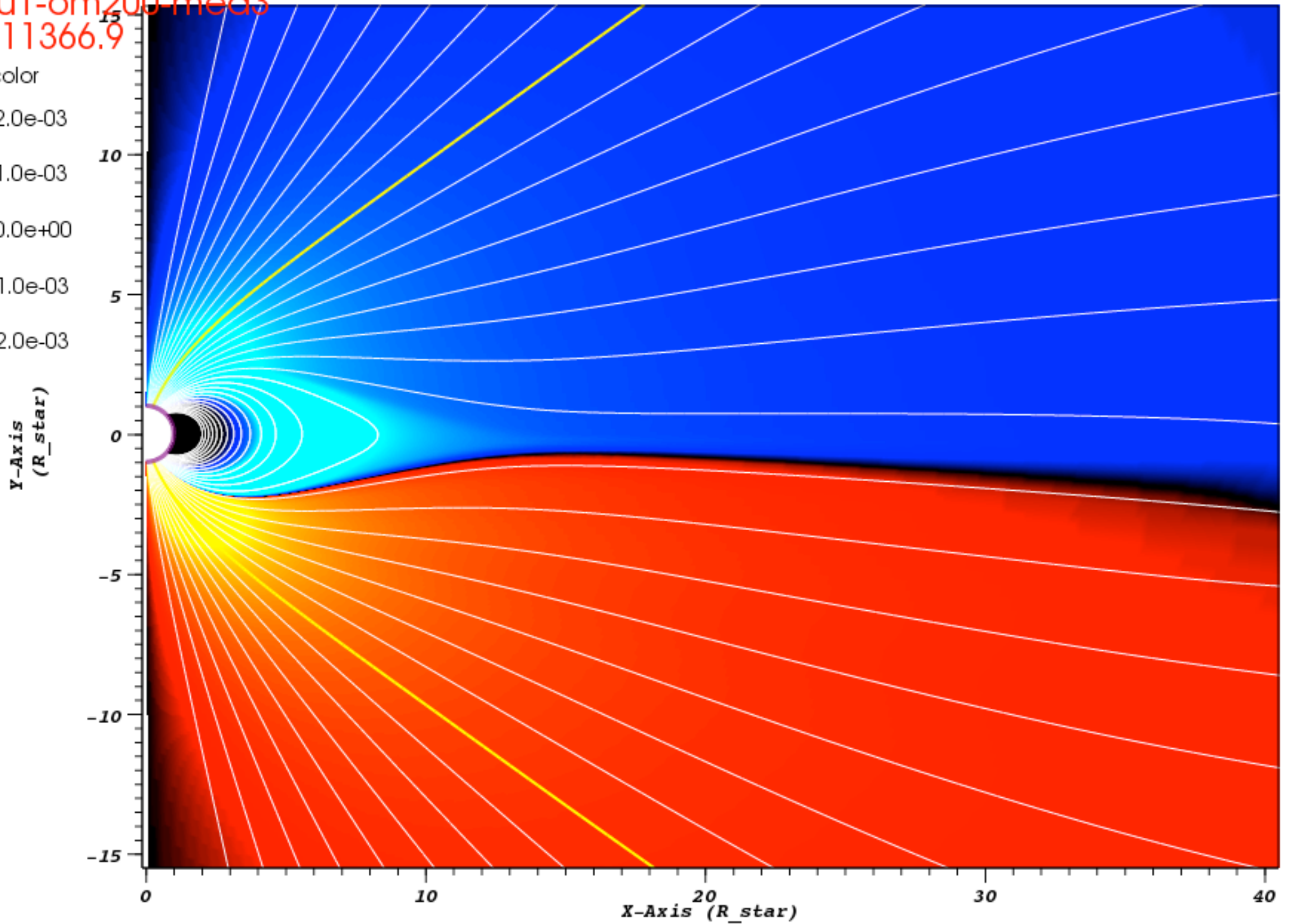
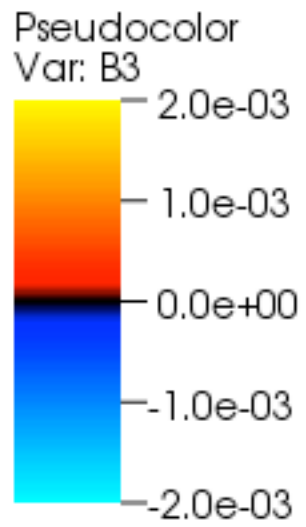
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11366.9

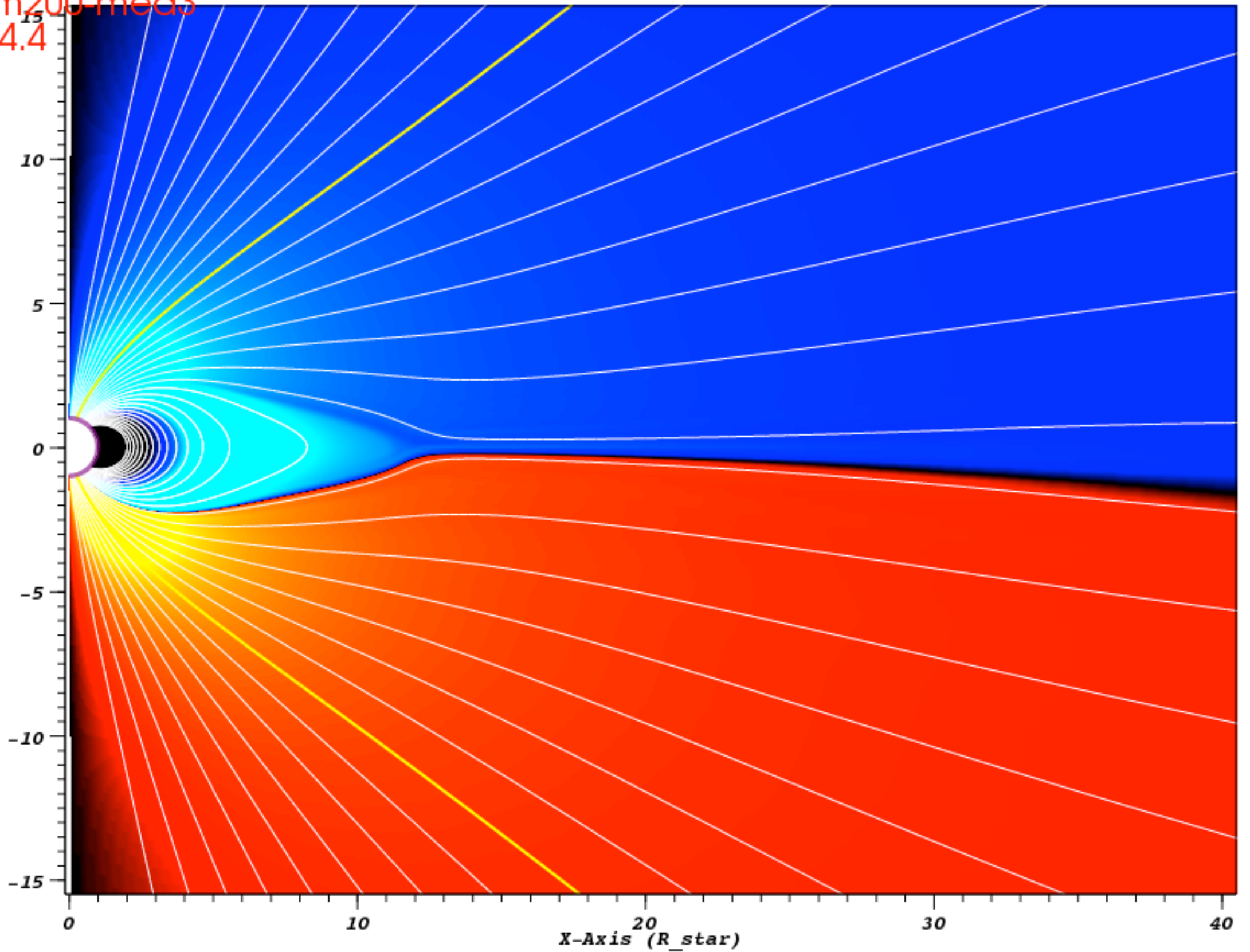


Rotating, twisted magnetosphere — impulsive opening
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11394.4

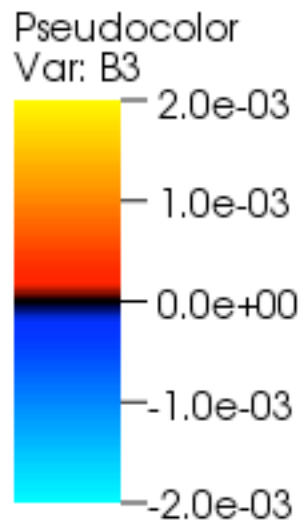
Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

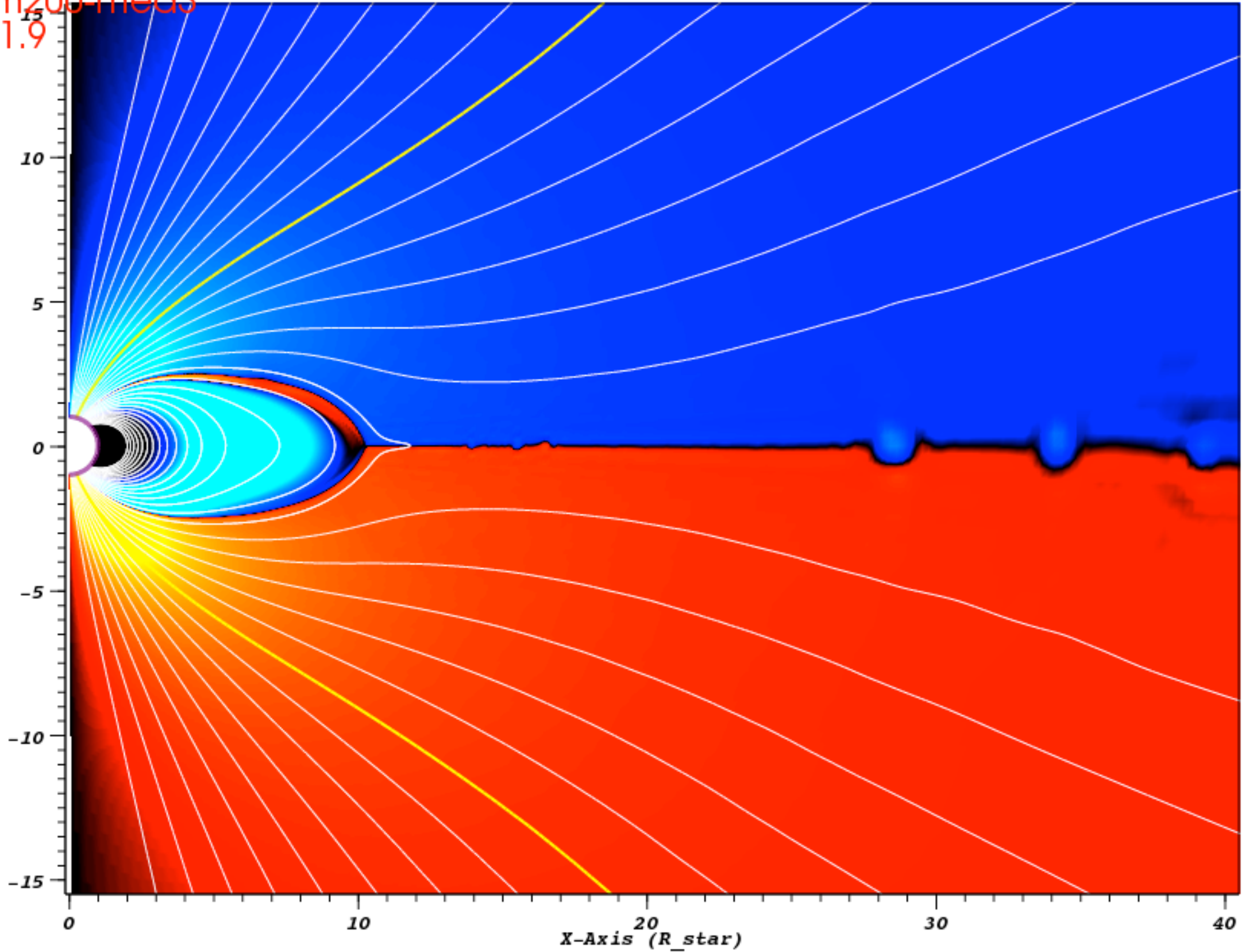


Rotating, twisted magnetosphere — second current sheet
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11421.9



Y-Axis
(R_star)

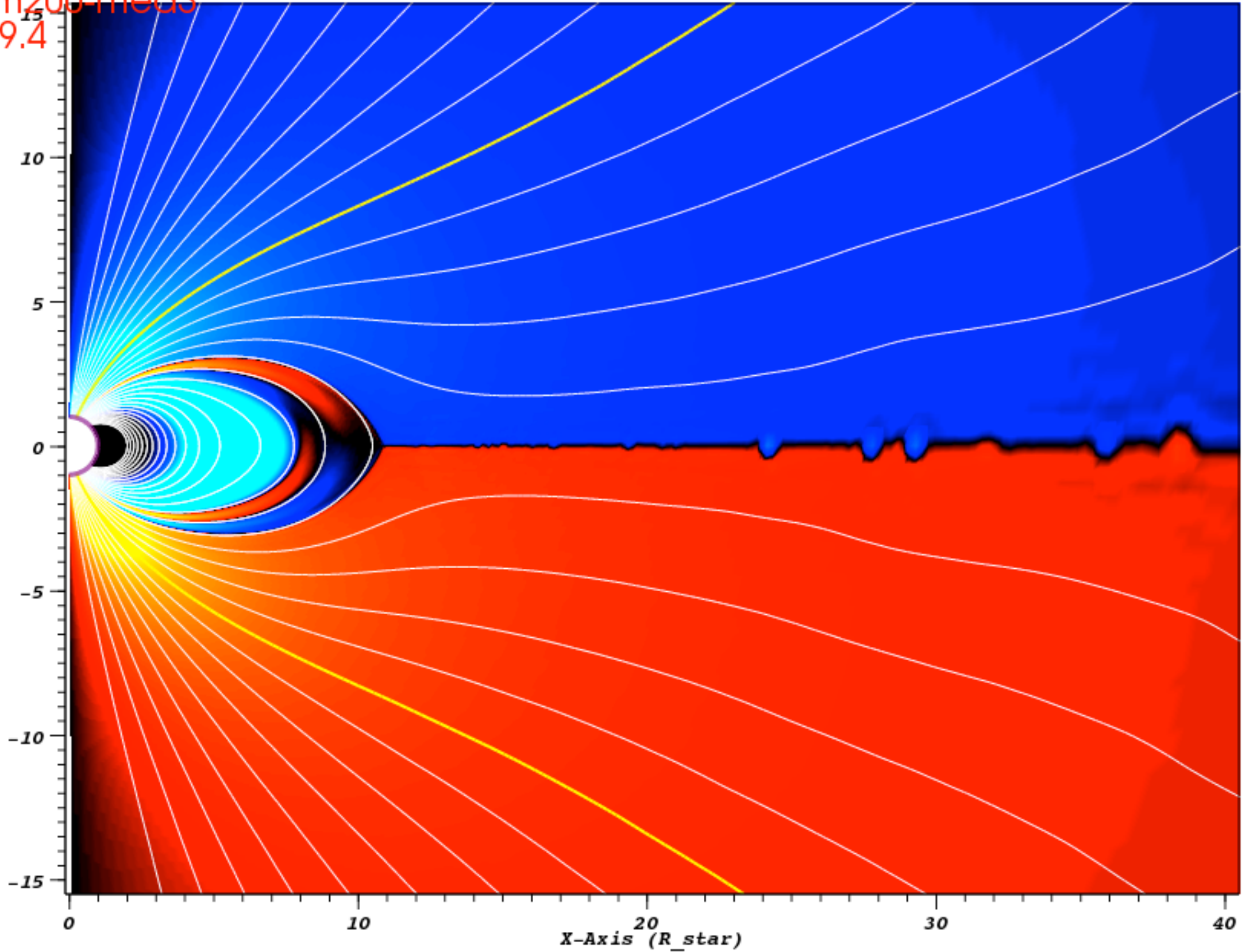


Rotating, twisted magnetosphere — second reconnection
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11449.4

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

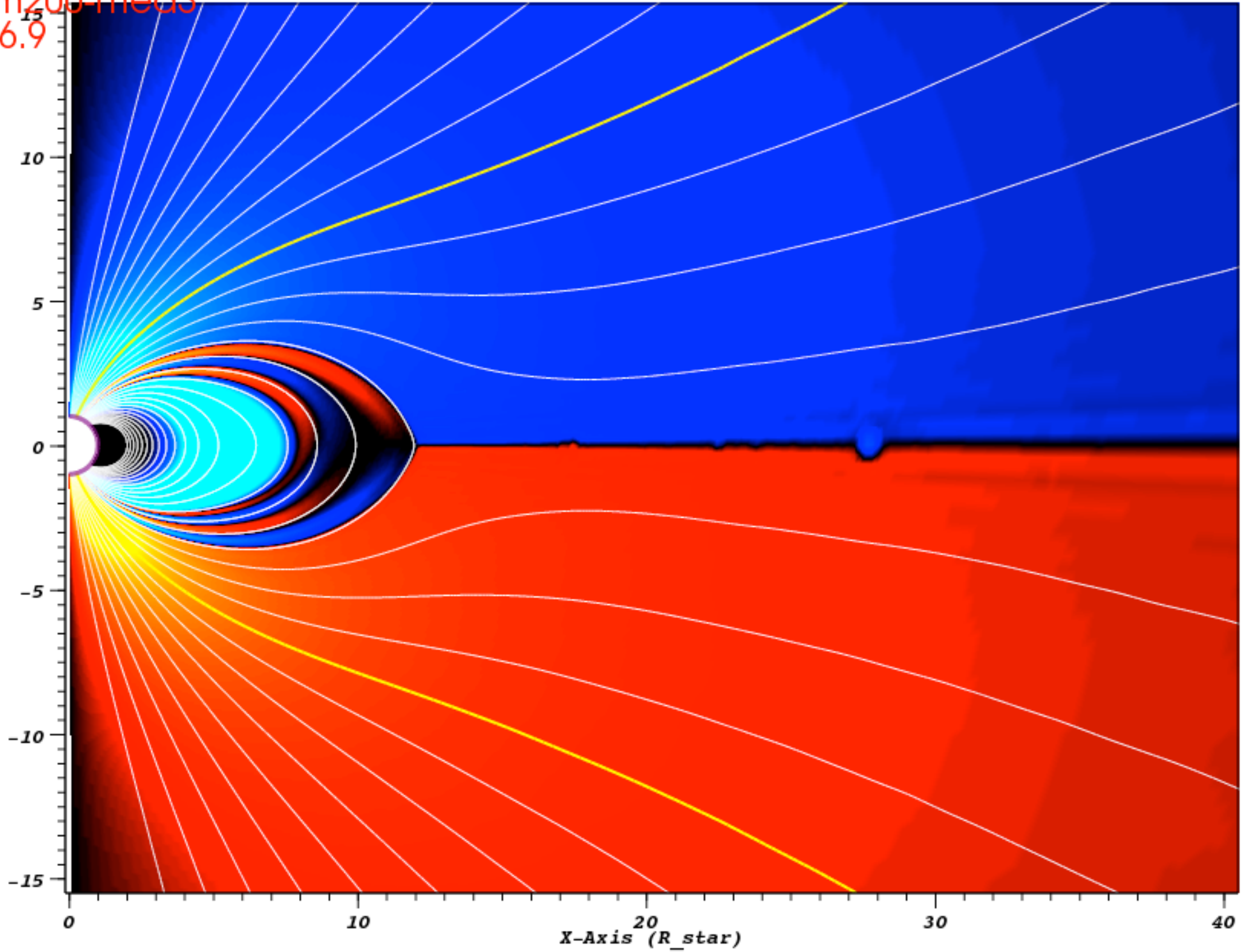


Rotating, twisted magnetosphere — second reconnection
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11476.9

Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)

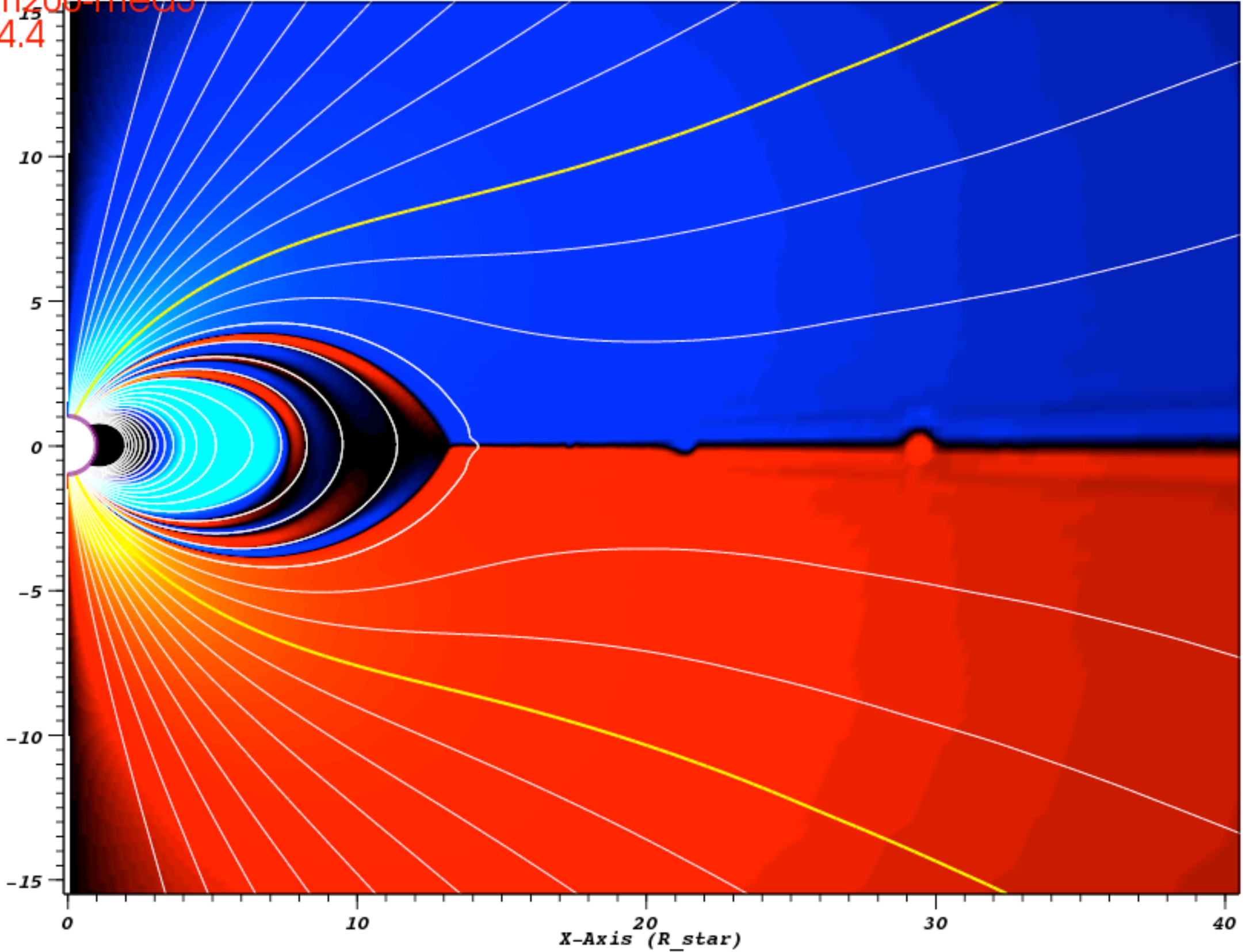


Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11504.4

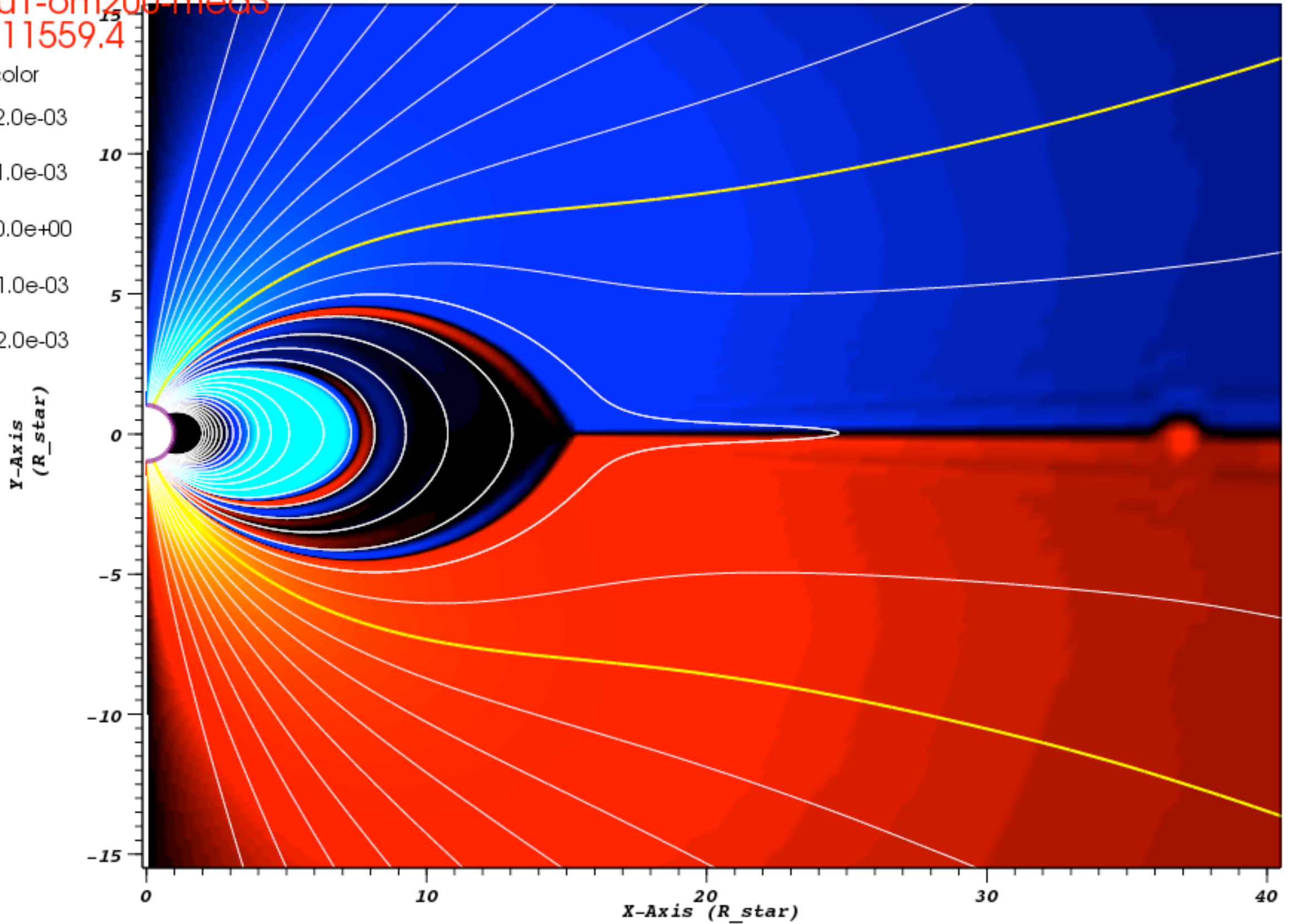
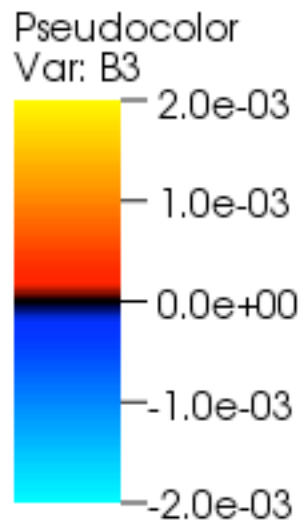
Pseudocolor
Var: B3
2.0e-03
1.0e-03
0.0e+00
-1.0e-03
-2.0e-03

Y-Axis
(R_star)



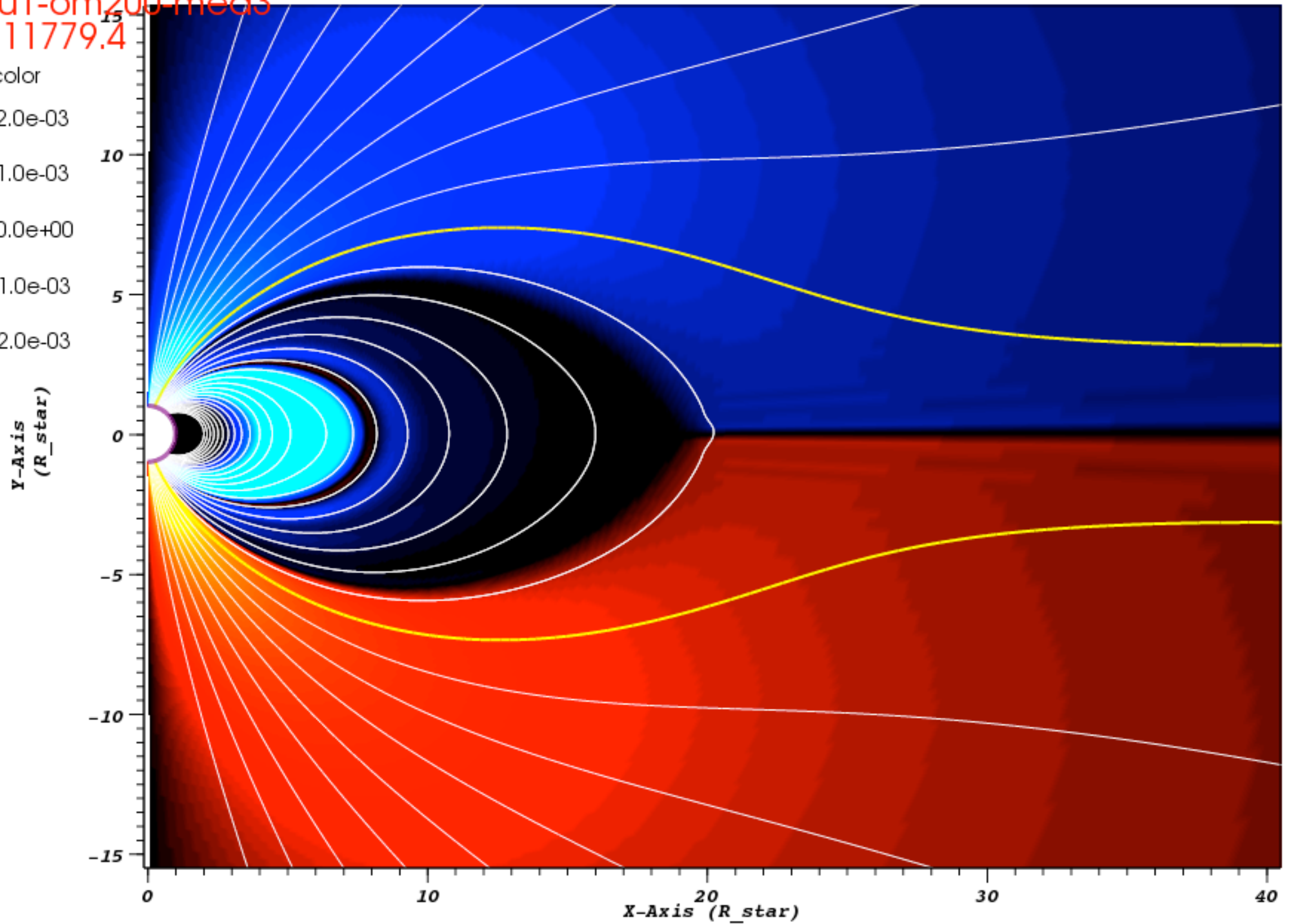
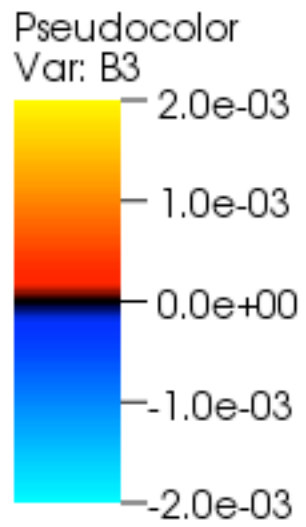
Rotating, twisted magnetosphere
Shown: B^ϕ , poloidal field line projections

DB: 3u1-om200-med3
Time: 11559.4



Rotating, twisted magnetosphere — second cavity
Shown: B^ϕ , poloidal field line projections

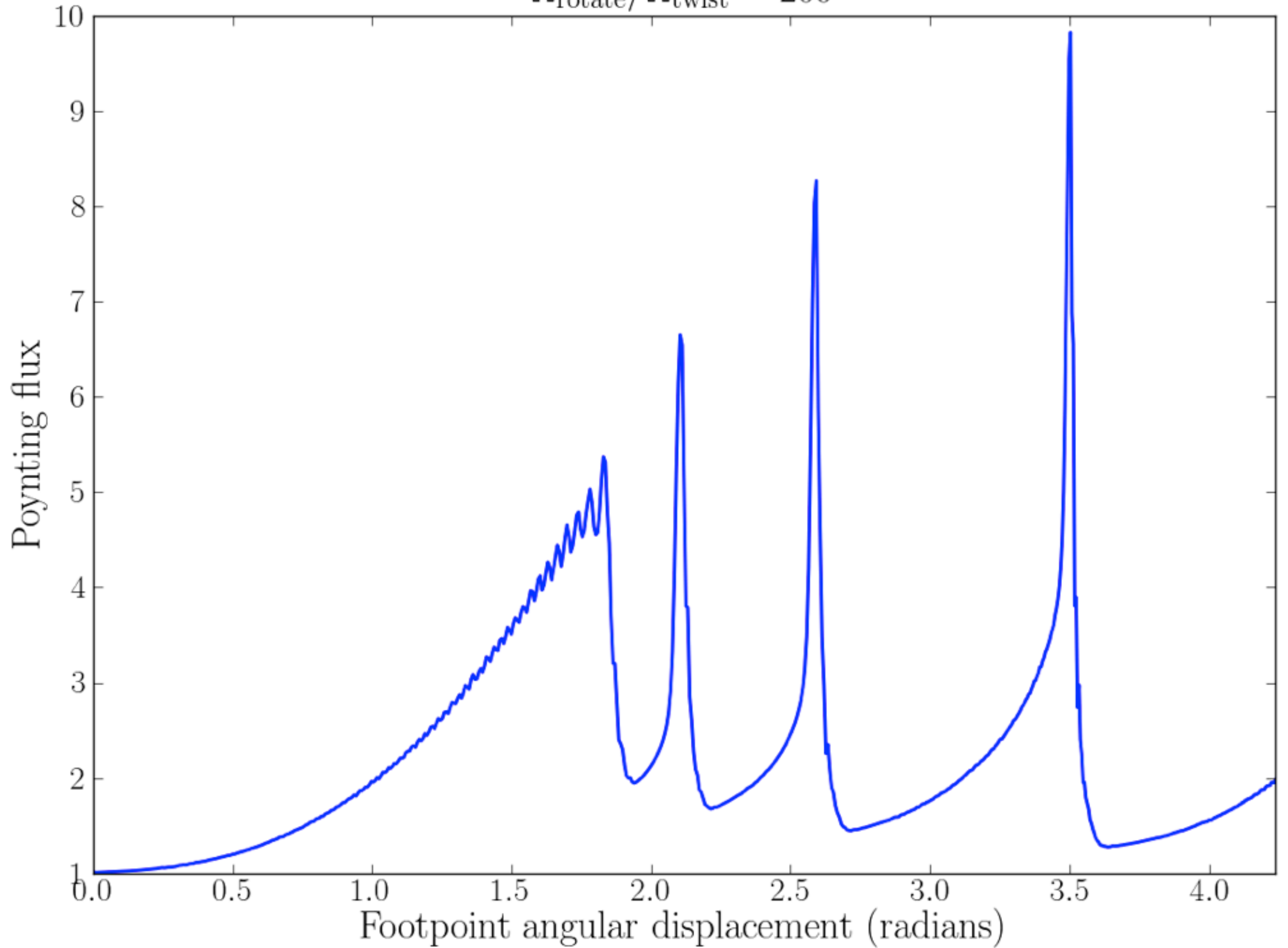
DB: 3u1-om200-med3
Time: 11779.4



Rotating, twisted magnetosphere — second cavity
Shown: B^ϕ , poloidal field line projections



$$\Omega_{\text{rotate}}/\Omega_{\text{twist}} = 200$$



Conclusions

- Pulsar: no plasmoid generation, steady Y-point & current sheet
- Plasmoid-dominated reconnection of twisted configurations
- Twisting of rotating stars can lead to large, sudden ‘anti-glitches’
- Future work
 - Full 3D simulations (spherical harmonics) — kink instability?
 - Resistivity
 - Curved spacetime (Schwarzschild, Kerr) with formalism of Komissarov (2004)

Спасибо!