Constraining the mass and moment of inertia of neutron stars from quasi-periodic oscillations in X-ray binaries

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Outline

1. Very brief reminder on QPOs
   - QPOs around compact objects
   - some aspects of several models
2. Parametric resonance model
   - the idea
   - resonances for test particles
3. Results
   - Newtonian field
   - general relativistic field
   - slow against fast rotator
   - black hole binaries
4. Conclusions & Perspectives

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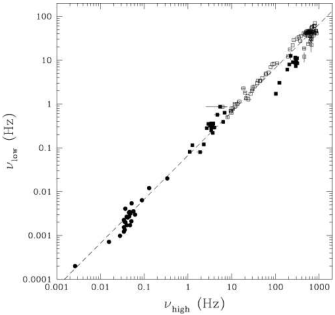
Typical high frequencies

- for white dwarfs (WD), $\nu_{\rm high} \approx 0.1$ Hz
- for neutron stars (NS), $\nu_{\rm high} \approx 1$ kHz
- for black holes (BH), $\nu_{\rm high} \approx 100$ Hz

In any kind of compact object, the relation seems to be

$$\nu_{\rm high} \approx 15\,\nu_{\rm low}$$

Why this relation ?
Do general-relativistic effects matter ?



(Mauche 2002, Warner et al. 2003)

Our basic assumption

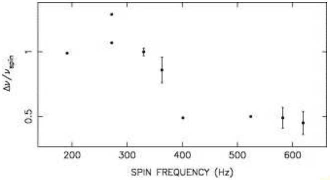
There must be

- one same physical mechanism producing these QPOs
- irrespective of the nature of the compact object.

For accreting neutron stars in LMXBs, observations reveal that they can be divided into two categories

	slow rotator	fast rotator
spin rate ν_* in Hz	≈ 300	≈ 600
QPO frequency difference $\Delta\nu$ between the two peaks	$\approx \nu_*$	$\approx \nu_*/2$
QPO frequency ratio	$\nu_2/\nu_1 \approx 3/2$	

Questions
$\Rightarrow$ how could we explain this segregation ?
$\Rightarrow$ which model can account for this ?



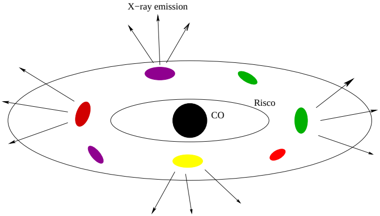
(van der Klis, 2006)

General idea

- Inhomogeneities forming in the disk
- create clumps of matter orbiting around the compact object
- generate a modulation in the intensity of the radiation.
- In the case of interest here, emission occurs mostly in the X-ray range

=> what are these inhomogeneities : density waves, blobs of plasma ?

A cartoon



Aim of this work

- find a new model for the high-frequency quasi-periodic oscillations *(kHz QPOs)*
- observed in accretion disks orbiting around Low Mass X-ray Binaries (LMXBs).

The physical mechanism

To show that an accretion disk evolving in

- either a gravitational potential
- or a magnetic field

which possesses the two following essential properties

- an asymmetry with respect to the rotation axis of the disk
- a rotating motion compared to the disk

will be subject to some resonances.

Three kind of resonances are expected

- a corotation resonance at the radius where the angular velocity of the disk equals the rotation speed of the star, only possible for prograde motion

$$\Omega_k = \Omega_*$$

- an inner and outer Lindblad resonance at the radius where the radial/vertical epicyclic frequency equals the rotation rate of the the gravitational potential perturbation as measured in the frame locally corotating with the disk

$$m\,|\Omega_k - \Omega_*| = \kappa_z$$

- a parametric resonance related to the periodically time-varying radial/vertical epicyclic frequency, (Mathieu equation)

$$m\,|\Omega_k - \Omega_*| = 2\,\frac{\kappa_z}{n}$$

- m, n are integers (m azimuthal mode number)
- Ω_* neutron star rotation rate
- Ω_k keplerian orbital frequency
- κ_z vertical epicyclic frequency

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Characteristic frequencies

For a test particle, the rotation is Keplerian and

$$\kappa_r = \kappa_z = \Omega_k = \sqrt{\frac{G\,M_*}{r^3}}$$

Parametric resonance conditions

$$\frac{\Omega_k}{\Omega_*} = \frac{m}{m \pm 2/n} \Rightarrow \frac{\Omega_*}{3} \leq \Omega_k \leq 3\,\Omega_*$$

The two highest frequencies are $\nu_1 = 2\nu_*, \nu_2 = 3\nu_*$ thus $\Delta\nu/\nu_* = 1$.

Orbital frequency at resonance, ν in Hz

Mode m	$\nu_* = 600$ Hz		$\nu_* = 300$ Hz	
	$n = 1$	$n = 2$	$n = 1$	$n = 2$
1	-600 / 200	—- / 300	-300 / 100	— / 150
2	—- / 300	1200 / 400	— / 150	600 / 200
3	1800 / 360	900 / 450	900 / 180	450 / 225

Application to a neutron star with spin ν_*

$$\Omega(r, a_*) \pm 2 \, \frac{\kappa_z(r, a_*)}{m\, n} = \Omega_*$$

For a given angular momentum a_*, we have to solve these equations for the radius r. For a neutron star, we adopt the typical parameters

- mass $M_* = 1.4 \, M_\odot$
- spin frequency $\nu_* = \Omega_*/2\pi = 300 - 600$ Hz
- moment of inertia $I_* = 10^{38} \mathrm{kg\, m}^2$
- angular momentum $a_* = \frac{c\, I_*}{G\, M_*^2} \, \Omega_* = 5.79 * 10^{-5} \, \Omega_*$

Orbital frequency at vertical resonance, $\nu(r, a_*)$ in Hz

Mode m	$\nu_* = 600$ Hz		$\nu_* = 300$ Hz	
	$n = 1$	$n = 2$	$n = 1$	$n = 2$
1	—- / 200	—- / 300	— / 100	— / 150
2	—- / 300	1198 / 400	— / 150	599 / 200
3	1790 / 360	899 / 450	898 / 180	450 / 225

General relativistic motion

Application to a neutron star with spin ν_*

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For a given angular momentum a_*, we have to solve these equations for the radius r.
For a neutron star, we adopt the typical parameters

- mass $M_* = 1.4\,M_\odot \Rightarrow \nu_{\rm QPO} \leq \nu_{\rm isco} = 1571$ Hz ($a_* \ll 1$)
- spin frequency $\nu_* = \Omega_*/2\pi = 300 - 600$ Hz
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Model prediction

- accounts for the segregation if the ISCO is taken into account
- for a neutron star with a typical mass of $M_* = 1.4 M_\odot$ the orbital frequency at the ISCO is $\nu_{\rm isco} = 1571$ Hz

=> upper limit for any QPO frequency ($a_* \ll 1$)

$$\boxed{\nu_{\rm QPO} \leq \nu_{\rm isco} = 1571\ {\rm Hz}} \tag{1}$$

Discarding the resonance frequencies in the relativistic disk which are higher than $\nu_{\rm isco}$

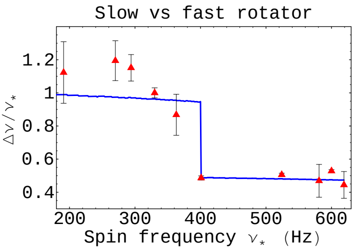
	slow rotator (300 Hz)	fast rotator (600 Hz)
highest frequencies	$< \nu_{\rm isco}$	not stable
ν_1, ν_2 in Hz	599,898	899,1198
$\Delta\nu$ in Hz	299	299
$\Delta\nu / \nu_*$	1	1/2
ν_2 / ν_1	$\approx 3/2$	

These conclusions apply to magnetized as well as to hydrodynamical accretion disks. (Pétri, 2005, A&A Letter, 439, 27)

Can we do better ?

Where is the frontier between slow and fast rotator, if any ?

- sample of a dozen of LMXBs showing this properties
- transition around $\nu_* \approx 400$ Hz
- if interpreted as due to ISCO, constraints on
  - => neutron star mass
  - => moment of inertia



(Pétri, Ap&SS, 2011)

Constraint on M_* and I_*

Four constraints

1. for slow rotator, $\nu_2^s = 3\nu_*$ and $\nu_1^s = 2\nu_*$, thus $\nu_{\rm isco} \geq 3\nu_*$

$$\nu_{\rm isco}(363\ {\rm Hz}) \geq 1089\ {\rm Hz}$$

2. for fast rotator, $\nu_2^s = 2\nu_*$ and $\nu_1^s = 1.5\nu_*$, thus $\nu_{\rm isco} \leq 3\nu_*$

$$\nu_{\rm isco}(401\ {\rm Hz}) \leq 1203\ {\rm Hz}$$

3. fastest rotator at 619 Hz but still $\Delta\nu/\nu_* \approx 0.5$

$$\nu_{\rm isco}(619\ {\rm Hz}) \geq 1238\ {\rm Hz}$$

4. no naked singularity in Kerr space-time => $|a_*| \leq 1$

$$\left(\frac{I_*}{10^{38}\ {\rm kg\ m^2}}\right) \leq 2.26 \left(\frac{M_*}{M_\odot}\right)^2$$

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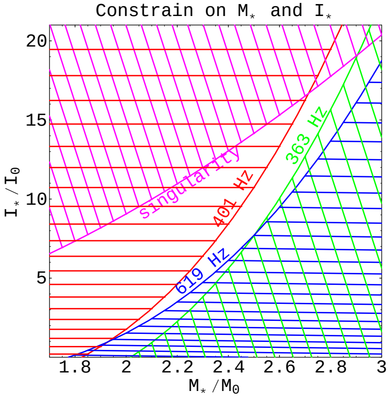
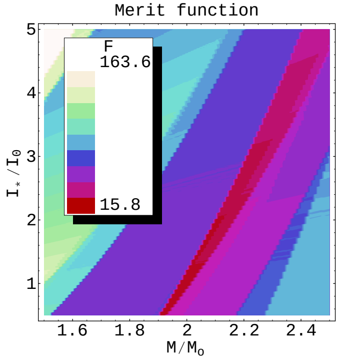
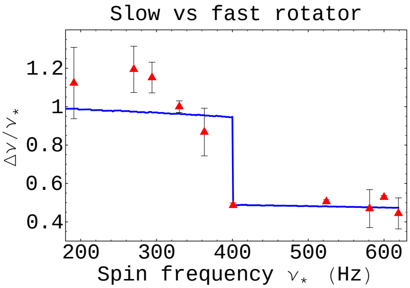
Assumes mass independent of rotation



(Pétri, Ap&SS, 2011)

Assumes mass/rotation relation





- average mass of $M_* \approx 2.0\, M_\odot$
- average moment of inertia of $I_* \approx 1.2 \times 10^{38}\ \mathrm{kg\,m^2}$.

(Pétri, Ap&SS, 2011)

Application to black hole binaries

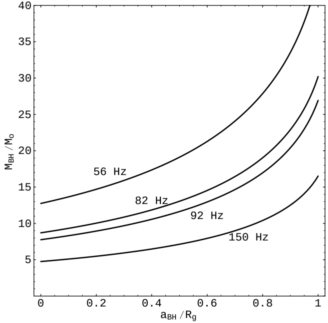
QPO frequencies predicted for GRS 1915+105.

ν_{QPO} (in Hz) for GRS 1915+105				
m	n (−)			
	1	2	3	4
1	-56.0	—	168.0	112.0
2	—	56.0	42.0	37.3
3	56.0	28.0	24.0	22.4

Observations have detected QPOs at 168/113/56/42/28 Hz.

$\Rightarrow$ deduce a mass-spin relation knowing the fundamental frequency.

Mass-spin relation for four BHB



- GRS1915+105 with fundamental at 56 Hz
- H1743-322 with 82 Hz
- XTE J1550-564 with 92 Hz
- GRO J1655-40 with 150 Hz

(Pétri, Ap&SS, 2008)

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Conclusions

- resonance model can account for HF-QPOs in NS and BHC
- for NS $\Rightarrow$ constraints on $M_{\rm NS}$ and $I_{\rm NS}$
- for BHBs $\Rightarrow$ mass-spin relation $(a_{\rm BH}, M_{\rm BH})$

Perspectives

- from linear to (weakly/strongly) non-linear oscillations

=> kHz-QPO frequency variation

- from test particle to more realistic fluid description

=> radiation processes, light-curves, curved space-time effects ?