MAXIMALLY MODEL INDEPENDENT EQUATION OF STATE FOR NEUTRON STAR MATTER

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Content

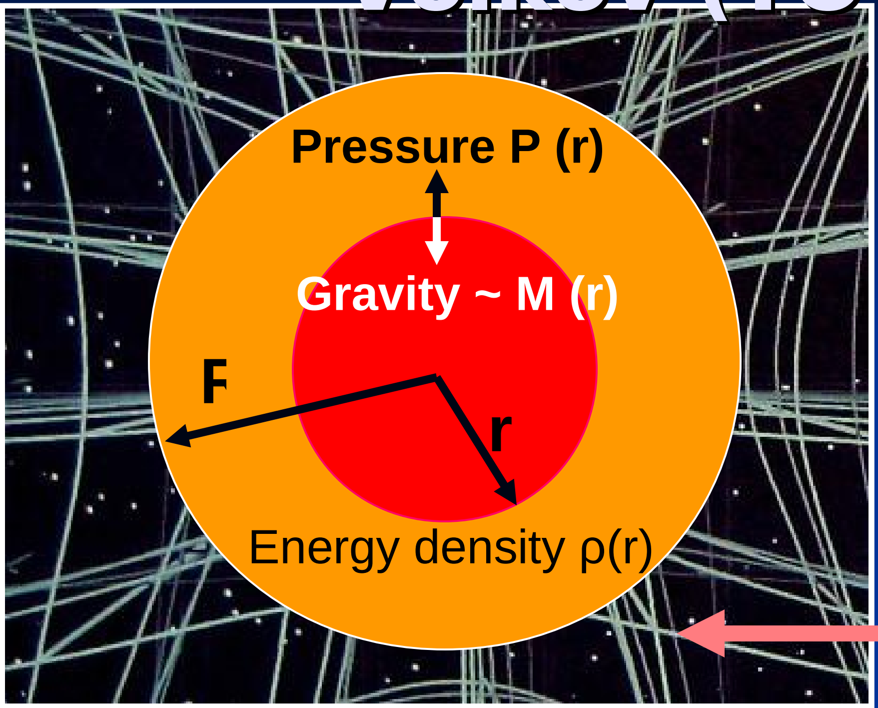
- Objectives.
- The Tolman-Oppenheimer-Volkov (TOV) equations.
- Potentially observable parameters of a NS.
- Observational data with errors and their effects on the inferred Equation of State (EOS).
- Argument for smoothness of EOS.
- Schemes to generate wide class of EOS.
- Simplest inference from 4 stars using X-ray bursts data.
- Sequential Bayesian analysis.
- The TOOL for astronomers.
- **Taking steps further.**
- **Summary.**

Objectives

1. Use observations of masses and radii (binding energy, moment of inertia, Love number, period and etc.) of several individual NS stars to determine the dense nuclear matter EOS.

2. To provide a benchmark maximally model-independent dense matter EOS for ongoing microscopic studies.

Neutron Star in hydrostatic equilibrium Tolman-Oppenheimer-Volkov (TOV) equations



1) Equations of stellar structure connects the matter pressure P(r) and the enclosed gravitational mass M(r) at the macrophysical level

2) Equation of State (EOS): $P \equiv P(\rho)$ connects pressure and energy density through microphysics of dense matter

3) Due to compactness, gravitational force is large and General Relativity (GR) must be considered

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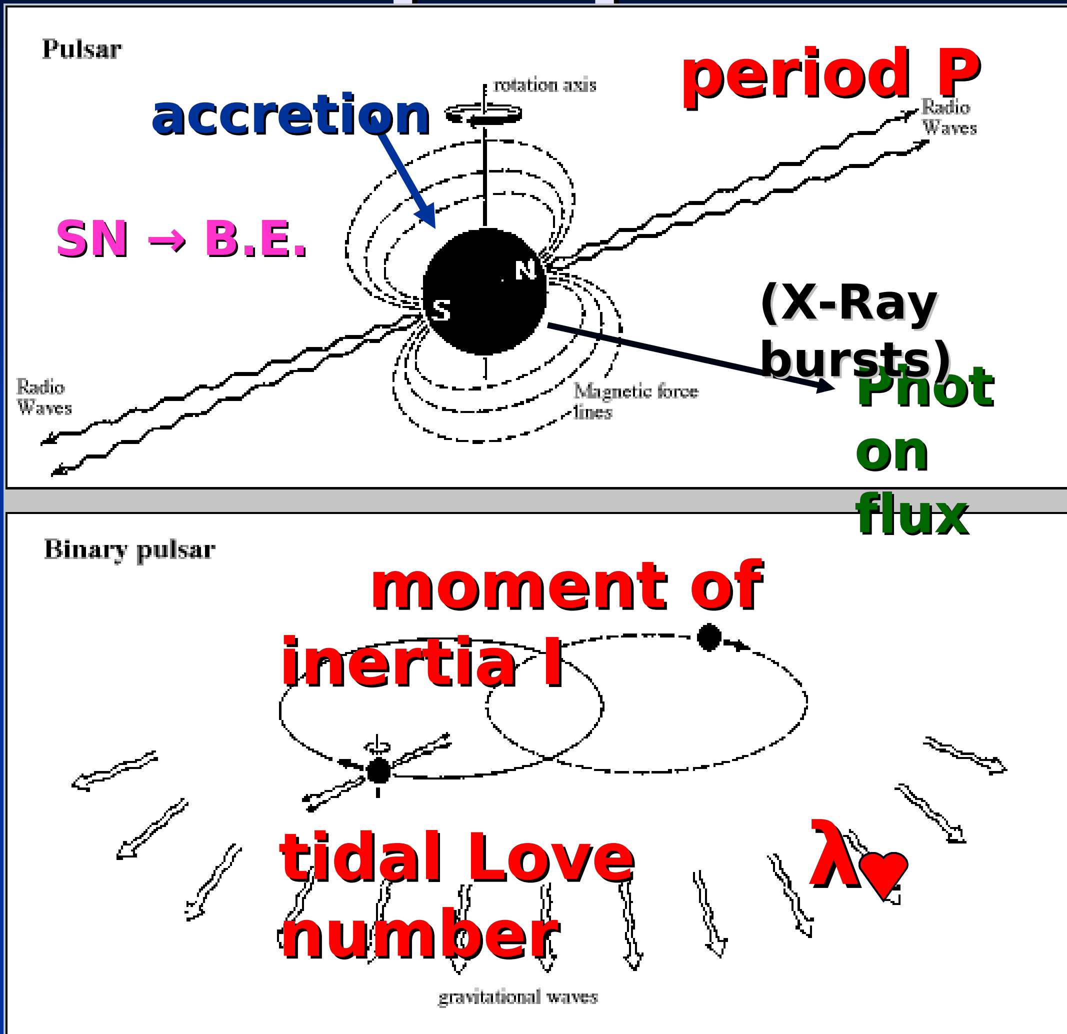
Center: $\begin{cases} \rho(0) = \rho_c \\ P(0) = P_c = P(\rho_c) \end{cases}$ $\xleftrightarrow[\text{EOS}]{\text{TOV}}$ Surface: $\begin{cases} M(R) \text{ total mass} \\ P(R) = 0 \end{cases}$

microphysics **the bridge** **macrophysics**

Potentially observed properties of a NS



$$\beta \equiv GM/Rc^2$$

Radiation radius R

∞

and effective

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_c^2 T_\infty^2}$$

Redshift z can be measured from lines or peak curvature of spectra.

∞

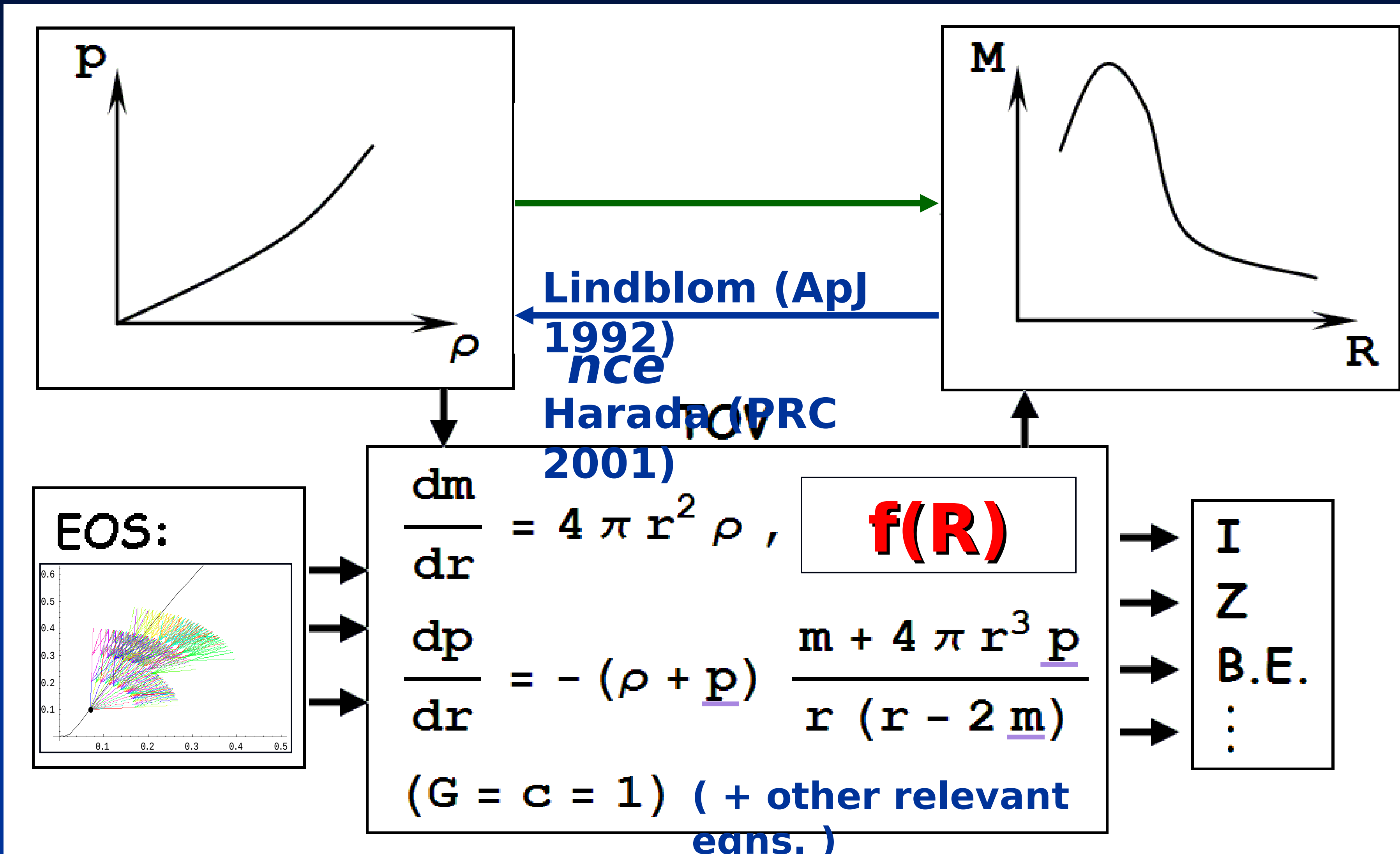
measured from photon flux spectra.

$$z = (1-2\beta)^{-1/2} - 1$$

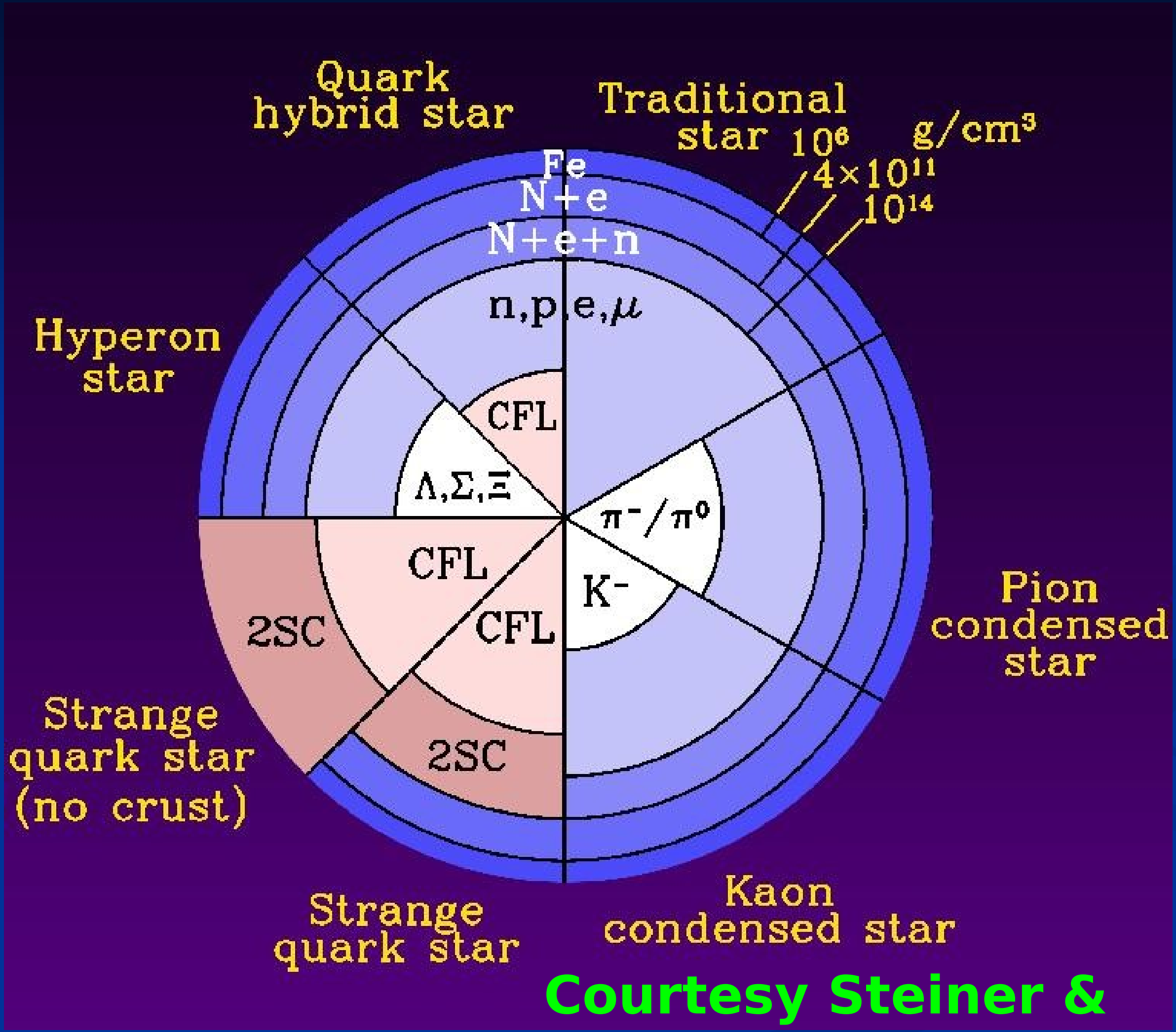
And the other ways and parameters…

(http://nobelprize.org/nobel_prizes/physics/laureates/1993/press.html)

Mass and radius of a NS



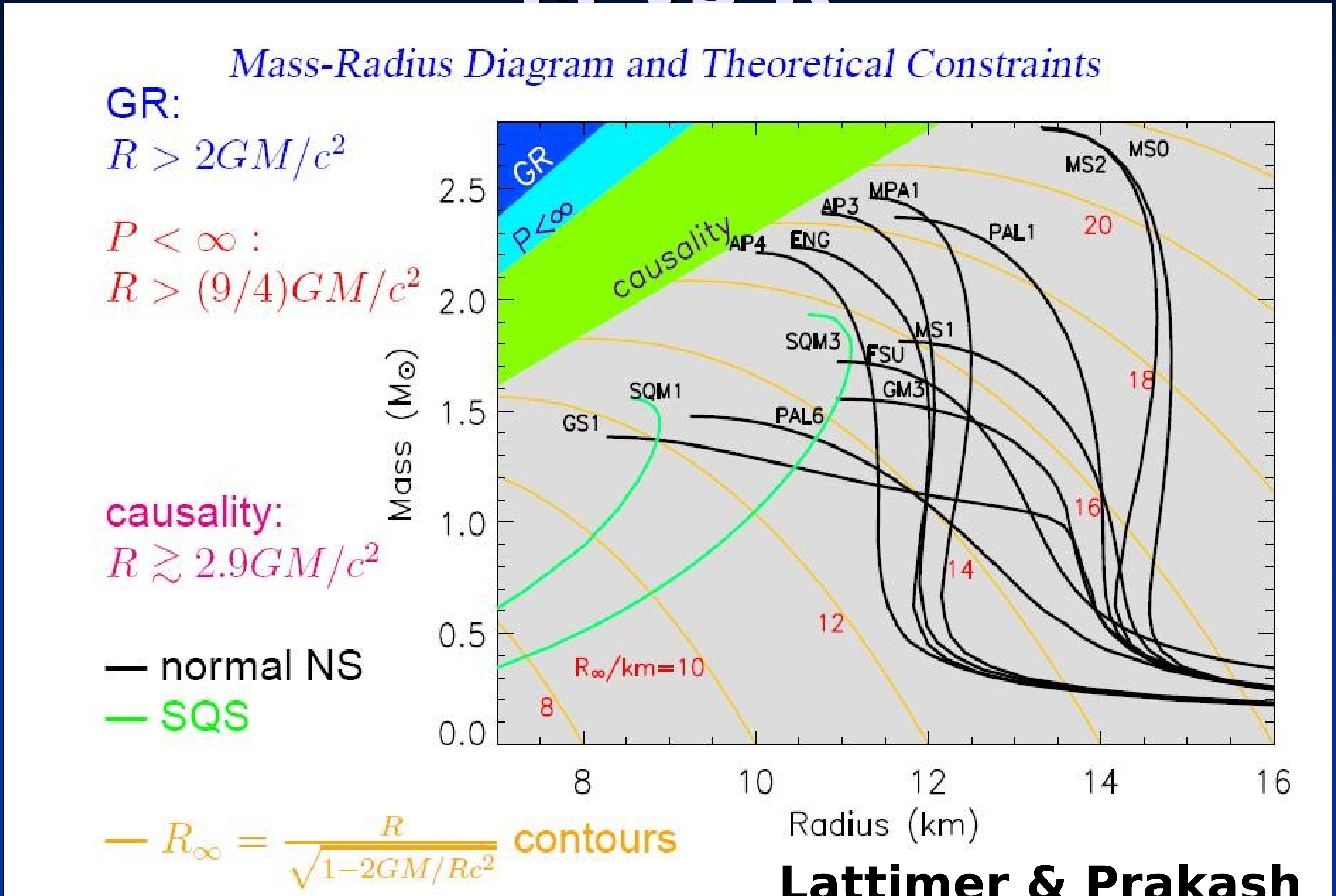
"The American Pie"



Courtesy Steiner & Prakash

Many exotic scenarios for interior composition, but none confirmed!

Direct approach: from EOS to M vs R



Lattimer & Prakash (`05)

Note: Diverse predictions for masses and radii; not satisfactory!

Neutron Star Matter Pressure and the Radius

$p \simeq K\epsilon^{1+1/n}$ **- polytrope**

$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto p_*^{1/2} \epsilon_*^{-1} M^0$

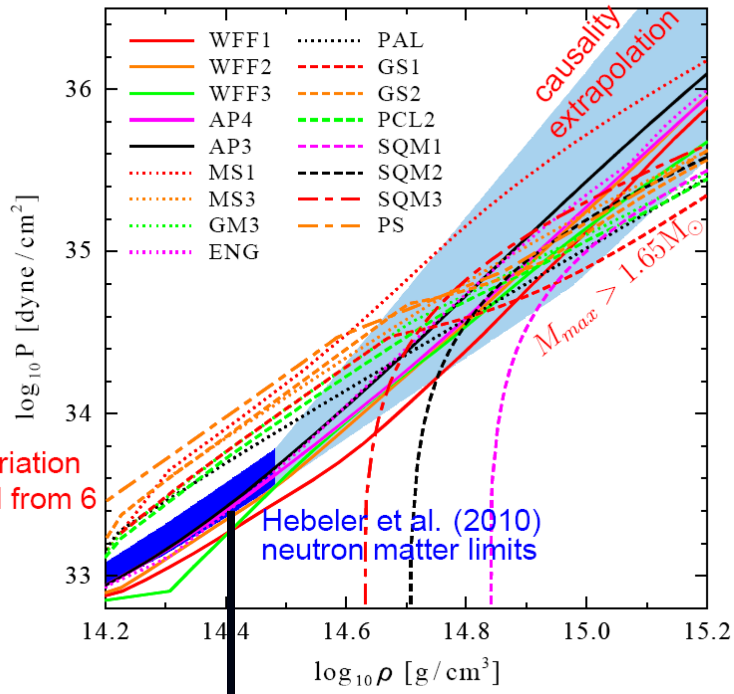
$(1 < \epsilon_*/\epsilon_0 < 2)$

Wide variation:

$$1.2 < \frac{p(\epsilon_0)}{\mathrm{MeV\ fm^{-3}}} < 7$$

(Lattimer & Prakash 2001)

$p(\rho_0)$ variation reduced from 6 to 1.5

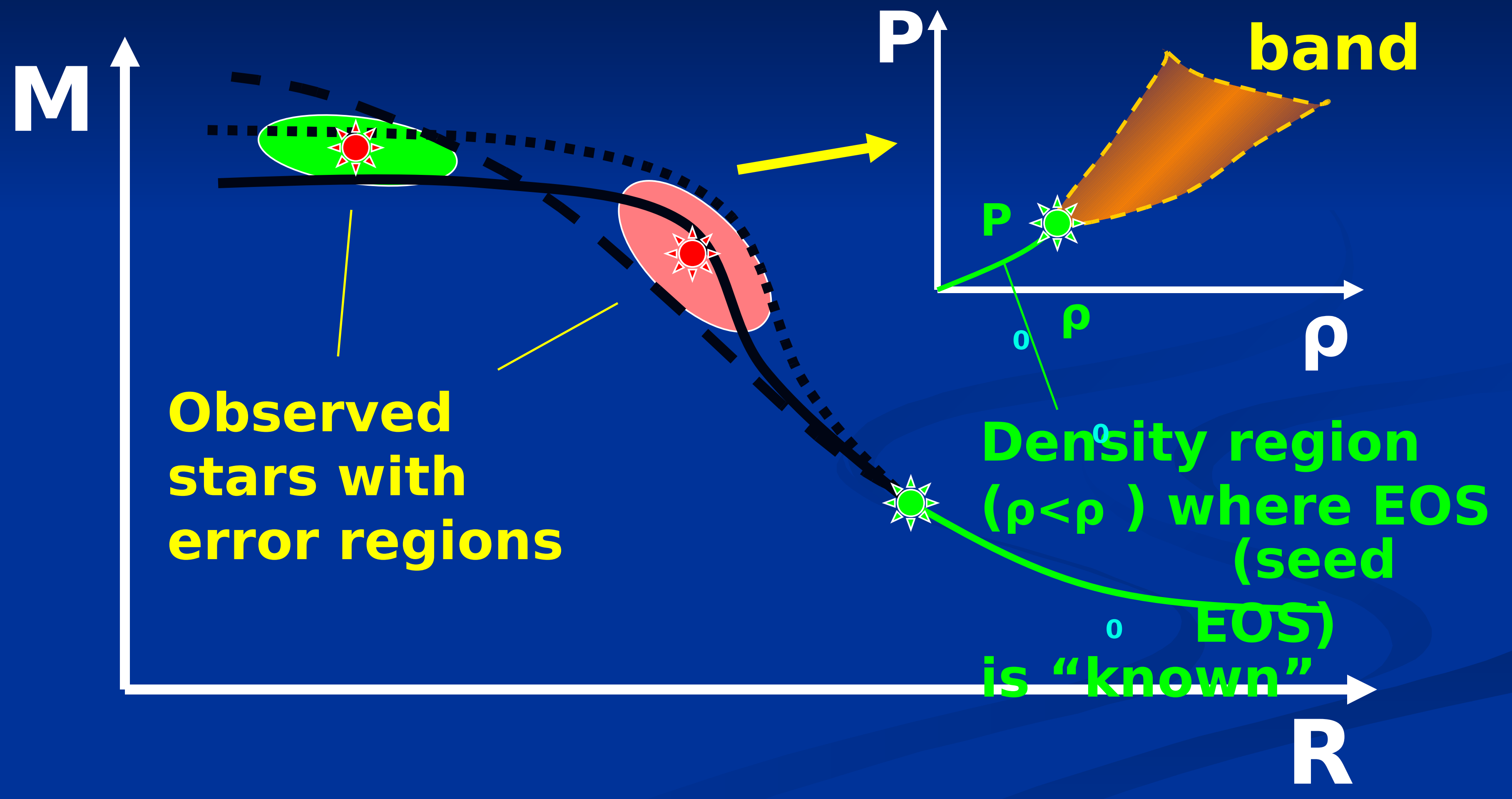


Note: Even at nuclear density, predicted pressures vary up to a factor of 5 - 6 !

Experiment with Nuclei

Nuclear Symmetry Energy

How several observed individual stars determine the EOS.



Smoothness of the EOS in NS matter

Phase transitions with more than one conserved charge

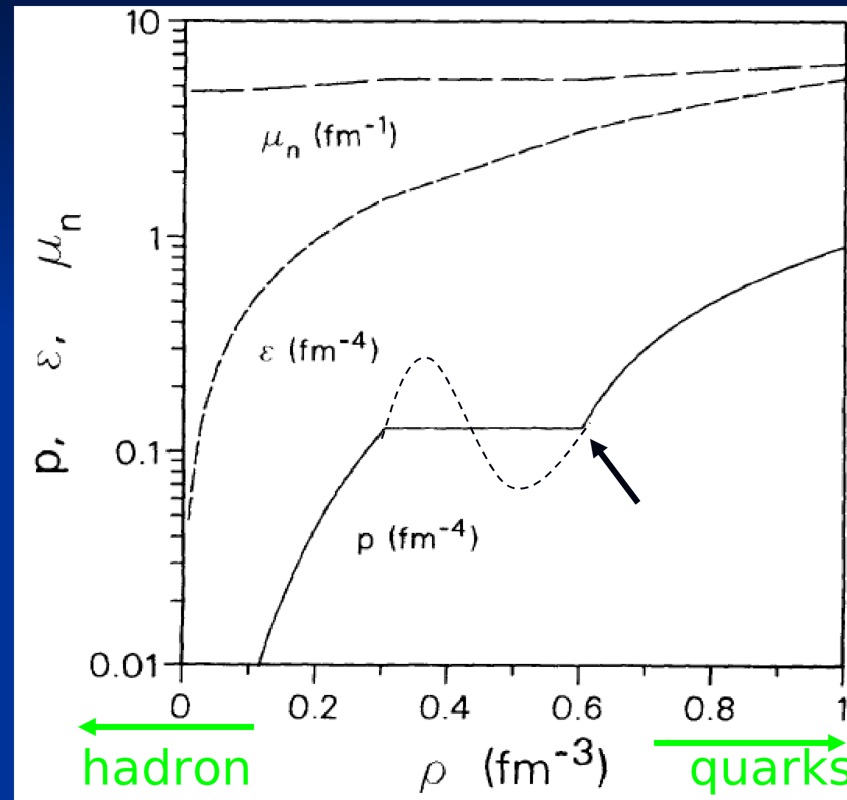
Glendenning Phys Rev D 46 (1992) 1274

Gibbs' rules

$P_1 = P_2$ (mechanical equilibrium)

E.g. , $\mu_n = \mu_u + \mu_d$ (chemical eq.)

- System with one conserved charge
  - Maxwell construction
  - NS profile ρ**(r)** has discontinuity
  - dP/dρ discontinuities are large
- Charge and baryon number conservation
  - Chemical beta equilibrium: E.g. : d(*or* s) ↔ u + e^- + $\bar{\nu}_e$
  - Global charge neutrality
  - ρ**(r)** is continuous
  - dP/dρ discontinuities are small



"Pressure, energy density, and chemical potentials a[redacted] function of weighted b[redacted] density when there is more than one conserved charge." (Glendenning 1992)

In NS matter we can start with smooth EOS's and later correct for discontinuities (phase transitions)

Scheme for generating EOS's

0. Use low density "known" EOS to set starting point
1. Recast the hypothetical EOS as speed of sound squared **$c_s^2(h)=dP/d\rho$** vs **h** (where **$dh=dp/(p+\rho)$**) in the unknown region

1. Since **$c_s^2(h)$** is chosen to be smooth and varies slowly, at each next small step in **h** we can generate a linear piece of curve with slight deviation in its slope from the previous step: "tree"

$$\alpha_{\text{next slope}} = \alpha_{\text{previous slope}}(1+\delta_\alpha j/N_\alpha)$$

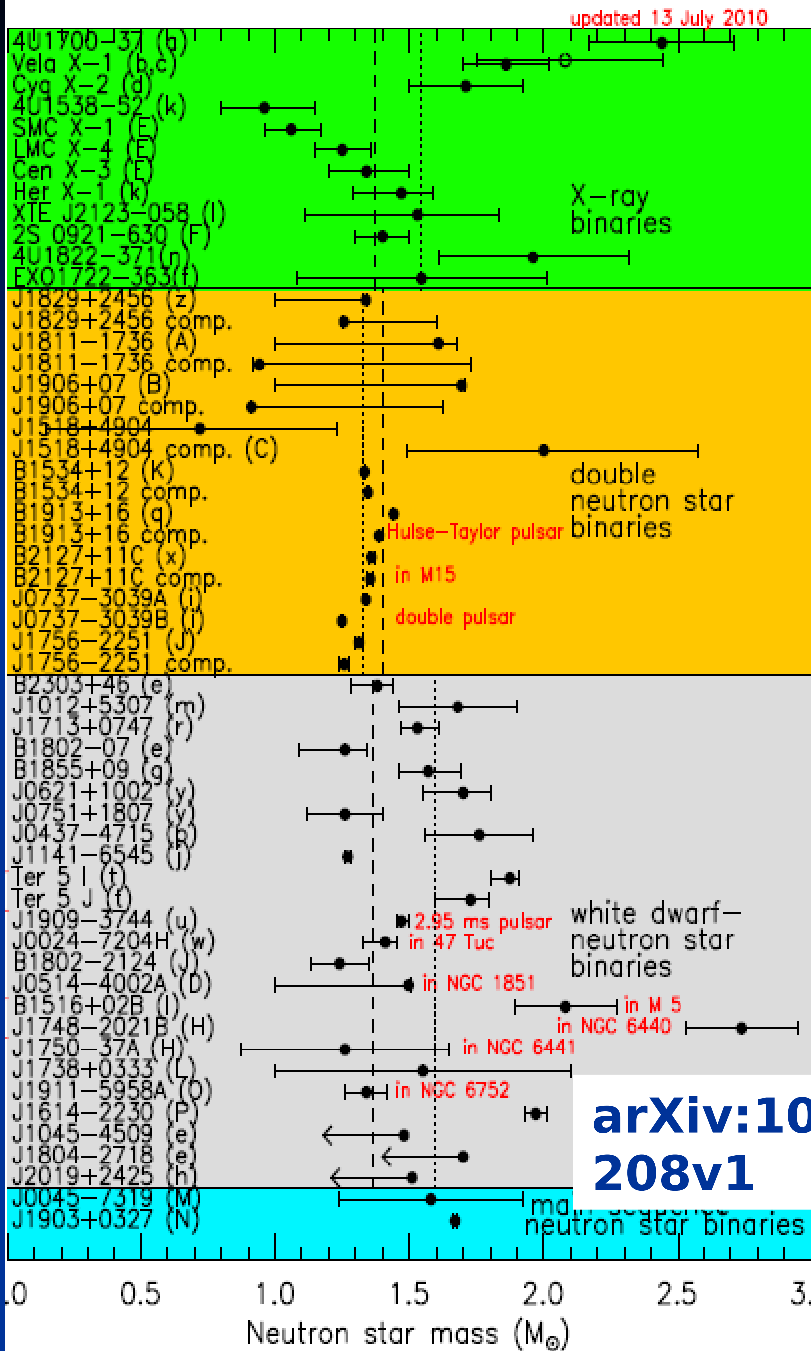
w
δ
j

$$\alpha(h_{[0,0]\ldots[i,j]}, c^2_{s\,[0,0]\ldots[i,j]}) = \frac{\alpha(h_{[0,0]\ldots[i-1,j_{i-1}]}, c^2_{s\,[0,0]\ldots[i-1,j_{i-1}]}) + \tan\left(\delta_i \frac{j}{N_\alpha}\right)}{1-\alpha(h_{[0,0]\ldots[i-1,j_{i-1}]}, c^2_{s\,[0,0]\ldots[i-1,j_{i-1}]})\tan\left(\delta_i \frac{j}{N_\alpha}\right)},$$

2

1. For every piecewise **$c_s^2(h)$**, calculate corresponding **$P(\rho)$**

2. **Now I am also using faster Monte Carlo scheme**



J.M. Lattimer, NIC XI, Heidelberg, 20/07/10

Masses are well measured in binary systems but radii are not precisely known.

The other systems, in which the radiation radius R is measured, do not

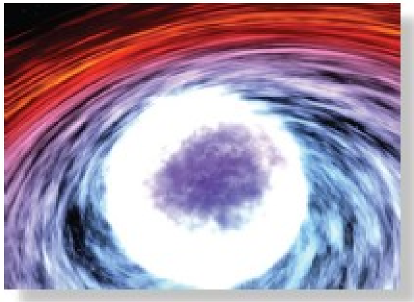
Object	R_∞	D	$kT_{eff,\infty}$	Ref.
	(km)	(kpc)	(eV)	
Omega Cen (Chandra)	13.5 ± 2.1	$5.36 \pm 6\%$	66^{+4}_{-5}	Rutledge et al. ('02)
Omega Cen (XMM)	13.6 ± 0.3	$5.36 \pm 6\%$	67 ± 2	Gendre et al. ('02)
M13 (XMM)	12.6 ± 0.4	$7.80 \pm 2\%$	76 ± 3	Gendre et al. ('02)
47 Tuc X7 (Chandra)	$14.5^{+1.6}_{-1.4}$ $(1.4\ M_\odot)$	$5.13 \pm 4\%$		Rybicki et al. ('05)
M28 (C andra)	$14.5^{+6.9}_{-3.8}$	$5.5 \pm 10\%$	90^{+30}_{-10}	Becker et al. ('03)
0748-676 andra)	13.8 ± 1.8 $(2.10 \pm 0.28\ M_\odot)$	9.2 ± 1.0		Ozel ('06)

$$R_\infty = R/\sqrt{1-(2GM/c^2R)}\ ; \qquad F = 4\pi T_{eff}^4 (R_\infty/D)^2$$

Atmospheric (sometimes magnetic) modeling required.

arXiv:1012.3208v1

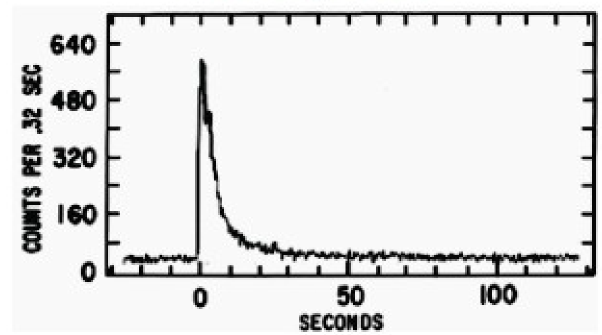
Thermonuclear X-ray Bursts



Accretion on neutron star

Unstable nuclear burning of accreted matter on the neutron star surface causes type I (thermonuclear) X-ray bursts.

Burst light curve



Rise time ≈ 0.5 - 5 seconds
Decay time ≈ 10 - 100 seconds
Recurrence time ≈ hours to day
Energy release in 10 seconds
≈ 10^{39} ergs
↓

Sun takes more than a week to release this energy.

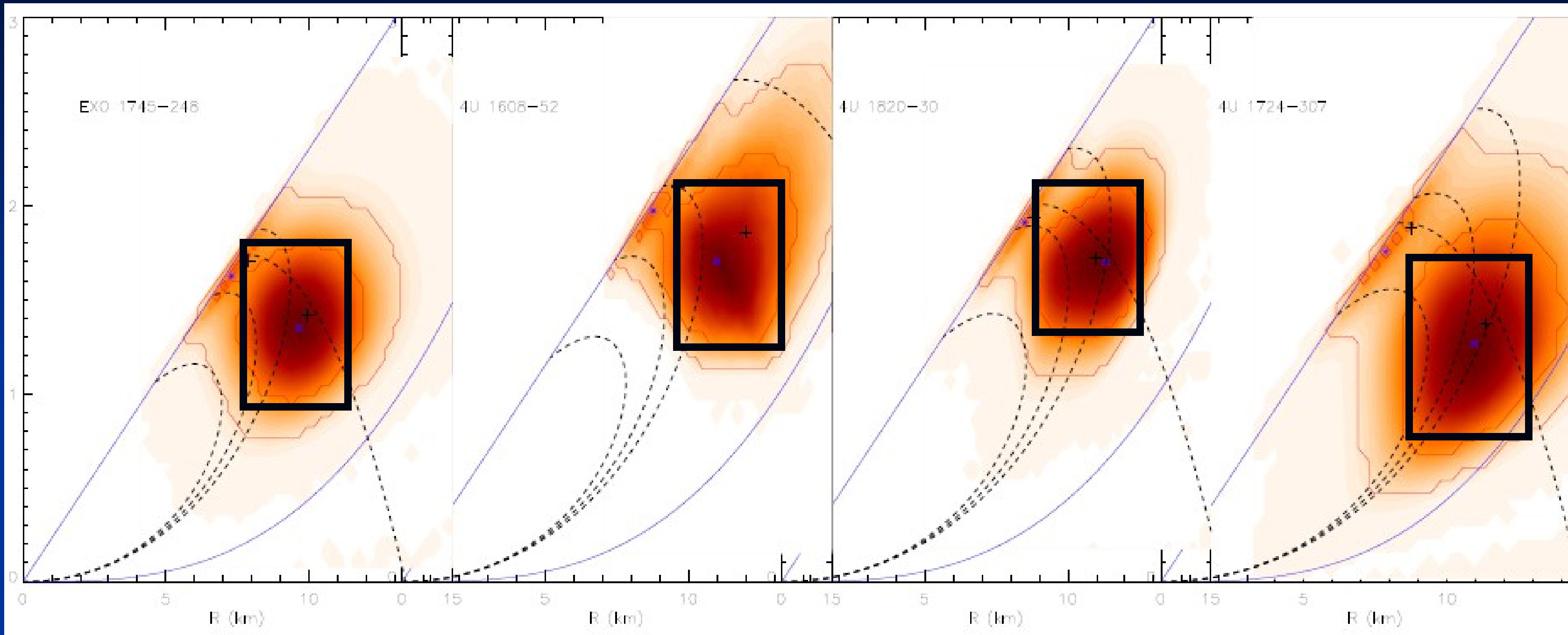
Why is *unstable* burning needed?

Energy release:
Gravitational ≈ 200 MeV / nucleon
Nuclear ≈ 7 MeV / nucleon

Accumulation of accreted matter for hours → Unstable nuclear burning for seconds ⇒ Thermonuclear X-ray burst.

Sudip Bhattacharyya
NASA's Goddard Space Flight Center

M & R from X-Ray Bursts

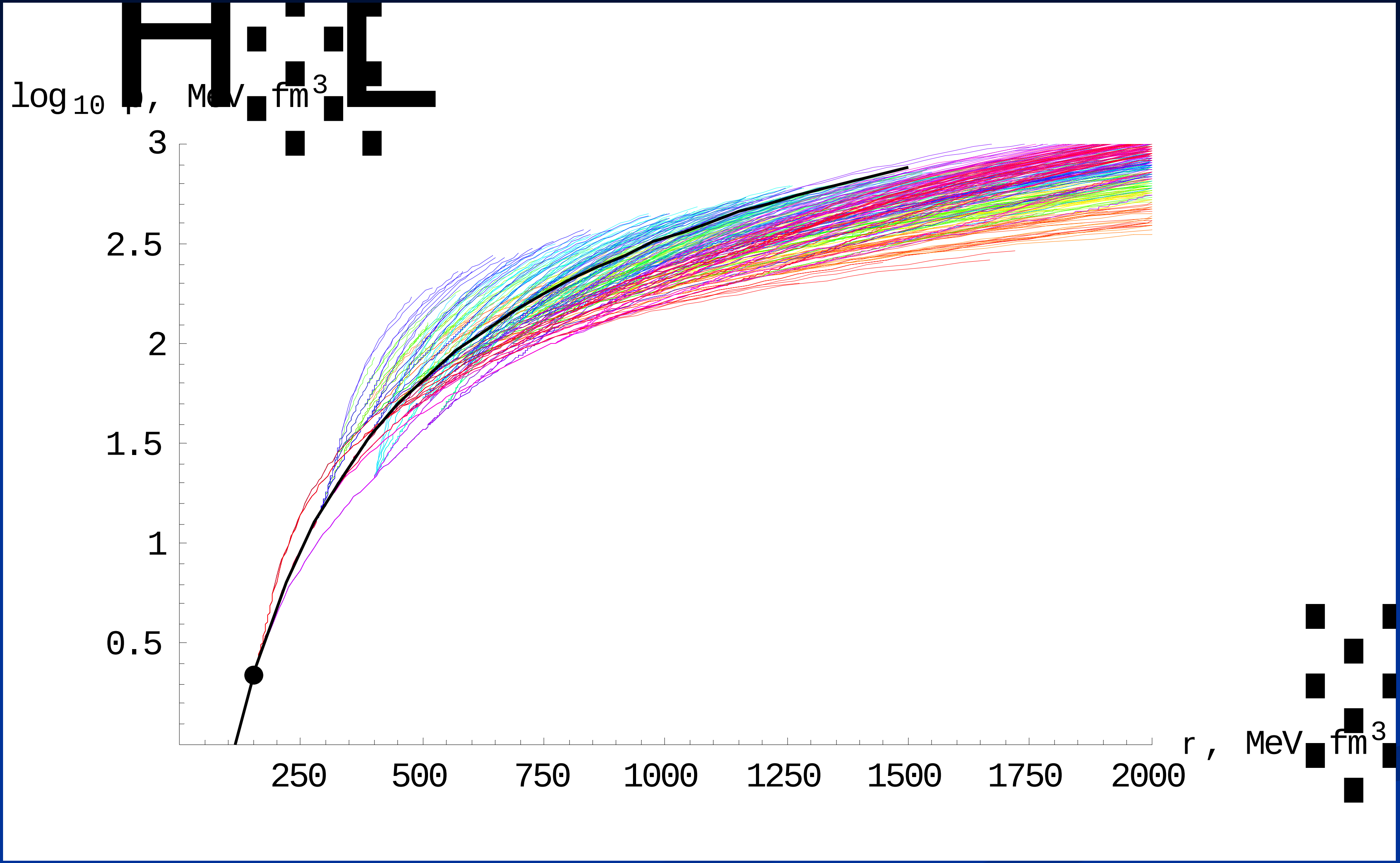


Sources of observational data:

4U 1745-248: Özel et al. (2009)
4U 1608-522: Güver et al. (2010)
4U 1820-30: Güver et al. (2010)
4U 1724-307: Suleimanov et al. (2010)

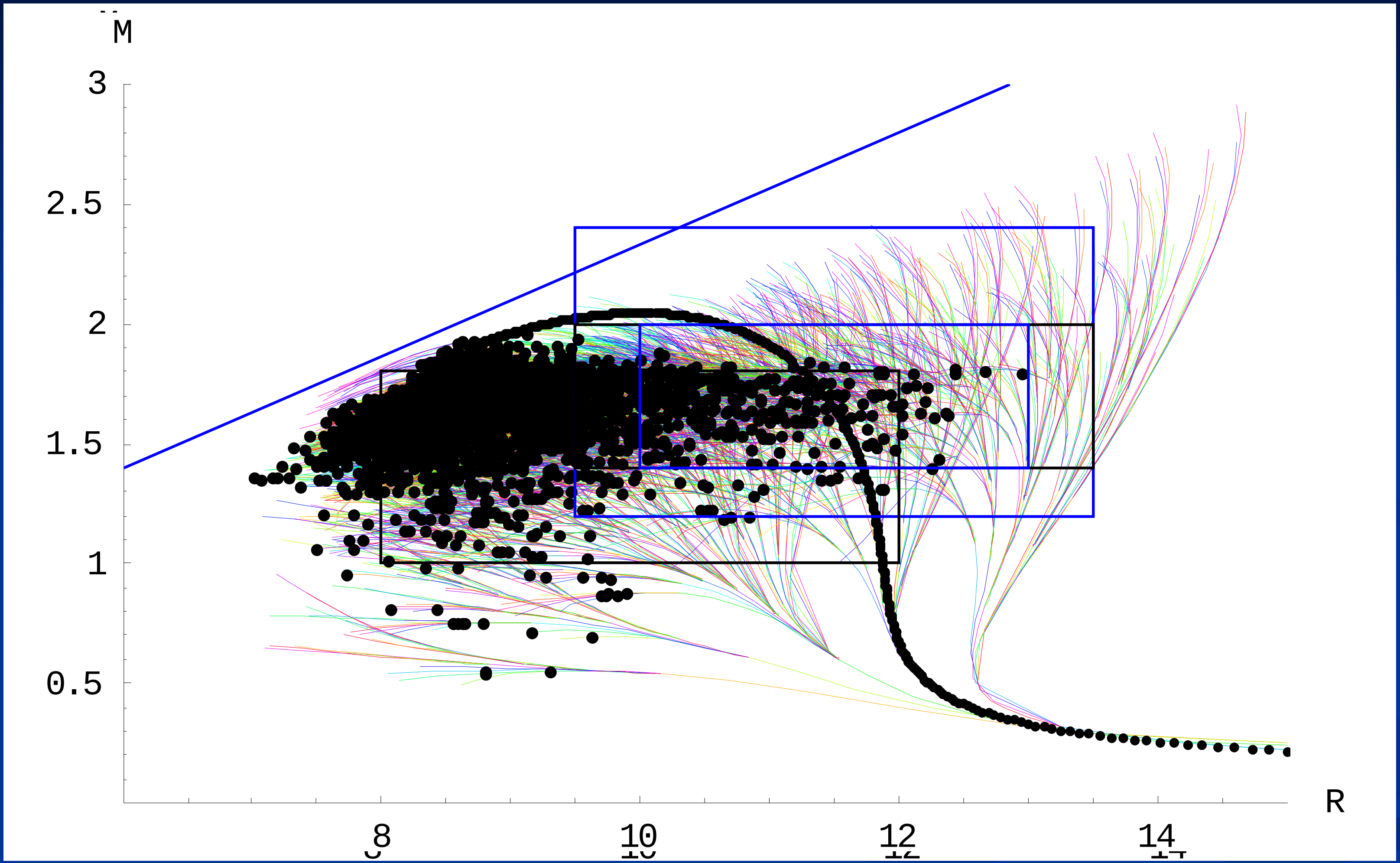
Jim Lattimer (2010)

n =0.16 fm)



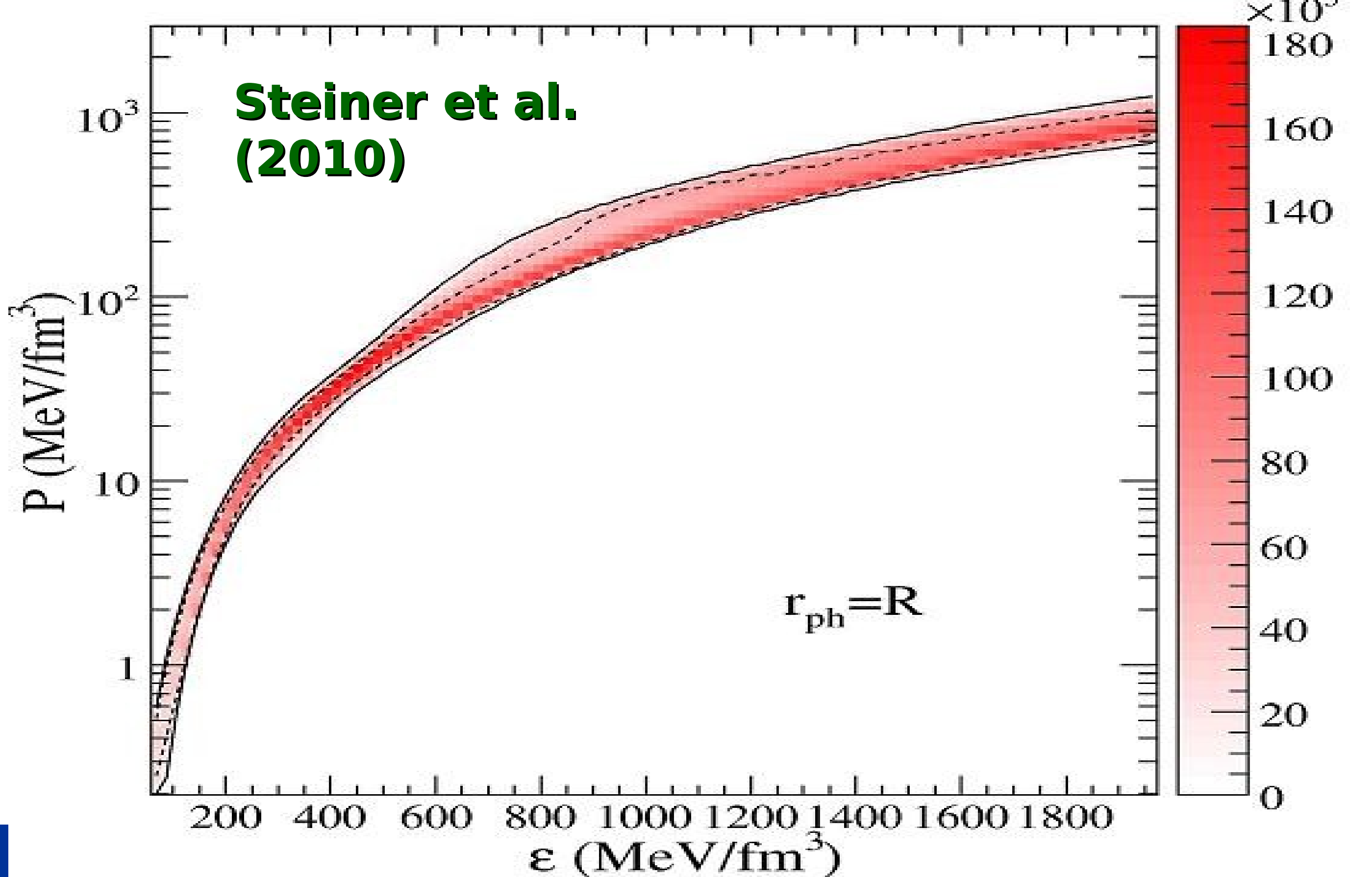
[movie]

$< \rho = 0.16$)



The band vs model EOSs and Andrew’s work.

lg P, MeV fm^3

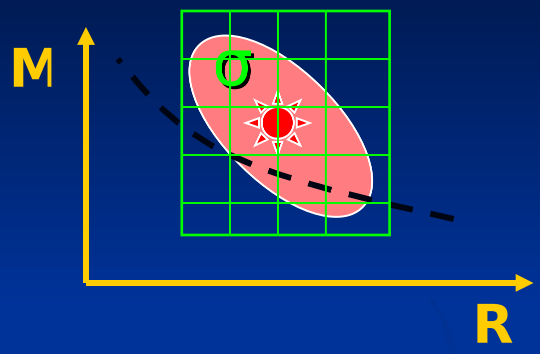


AP1c
AP3c
FPSc
WFF4c
FF1e
FF2e
FF3e
Gc
Y4e
32e
L11c

fm^3

Sequential Bayesian analysis

1. $P(H_i)$=1/(total number of curves)
2. *Likelihood:* $P(D|H_i) = ?$
   1. **integral of σ along the curve**
   2. **maximum[**crossed σ along the curve**]**
   3. division into smaller boxes and then proper curve counting
3. *Normalization:* $P(D) = \sum_i P(D|H_i)\, P(H_i)$
4. *Prior:* $P(H_i) \leftarrow P(H_i|D) = P(D|H_i)\, P(H_i)/P(D)$
5. If several (M & R)'s (e.g. data D_i) available, go to step 2.



6. Every curve acquires a weight $P(H)$

The TOOL

EOS space discretization [speed vs resolution]
(step size, branching, slope ...ance)

refinement (genetic algorithm)

☑ Data with errors
(contours)

☐ Nuclear physics
(low ρ EOS)

☐ Other observational constrai...
(Pmin, ...
...)

as more data comes

Post-processing:
(Bayesian, Mmax, Rmin...)

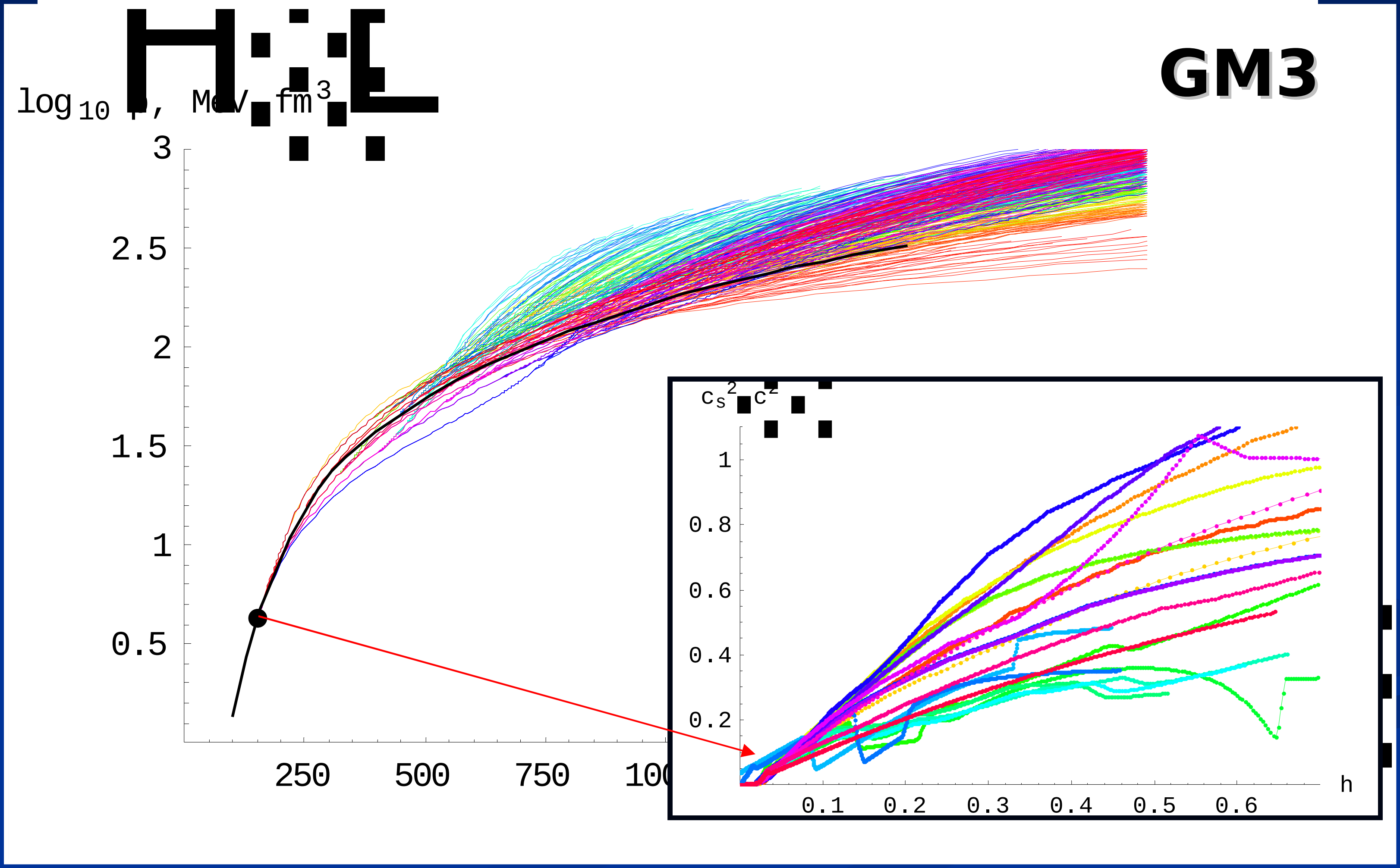
- EOS band
- Statistically weighted EOSs
- Benchmark EOS

- Seed 'low' density EOS (ρ < ρ0)
- Phase transitions, 'jumps' (ρj and Δcsj2)

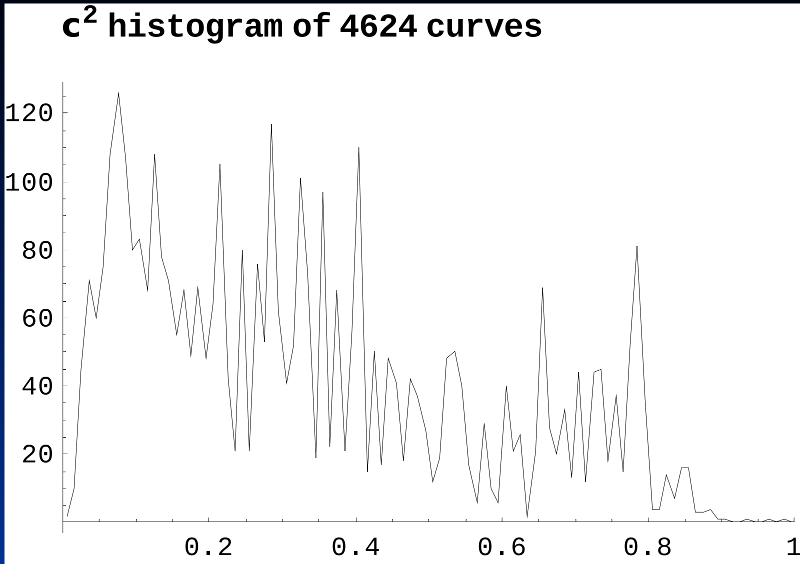
model dependence

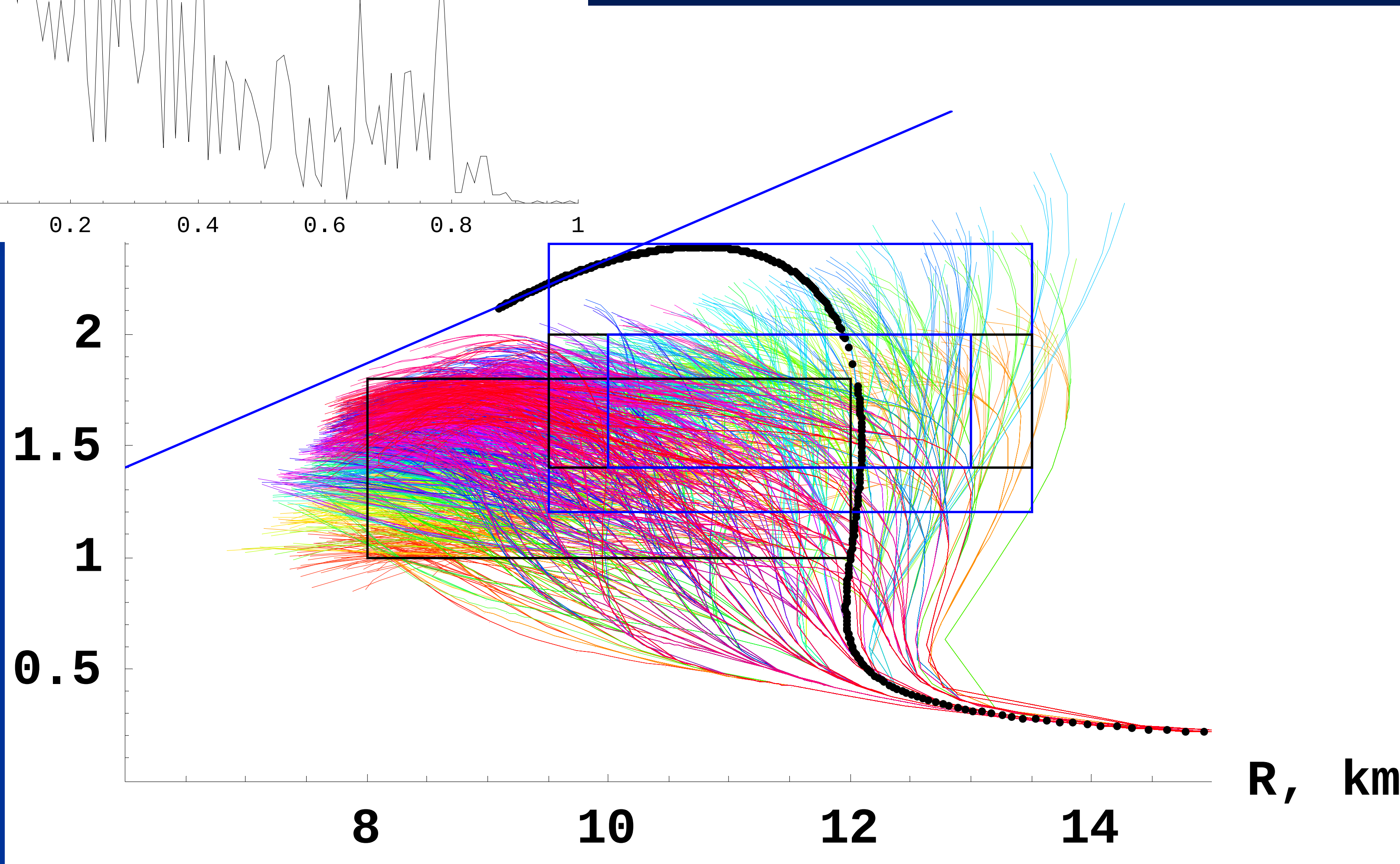
Dependence on seed EOS

$$\delta p(\rho) \approx \delta p(\rho_o) + \int_{\rho_o}^{\rho} c_s^2 d\rho \approx \begin{cases} \delta p(\rho_o) & \text{for } \rho \geq \rho_o, \\ \int_{\rho_o}^{\rho} c_s^2 d\rho \gg \delta p(\rho_o) & \text{for } \rho \gg \rho_o. \end{cases}$$

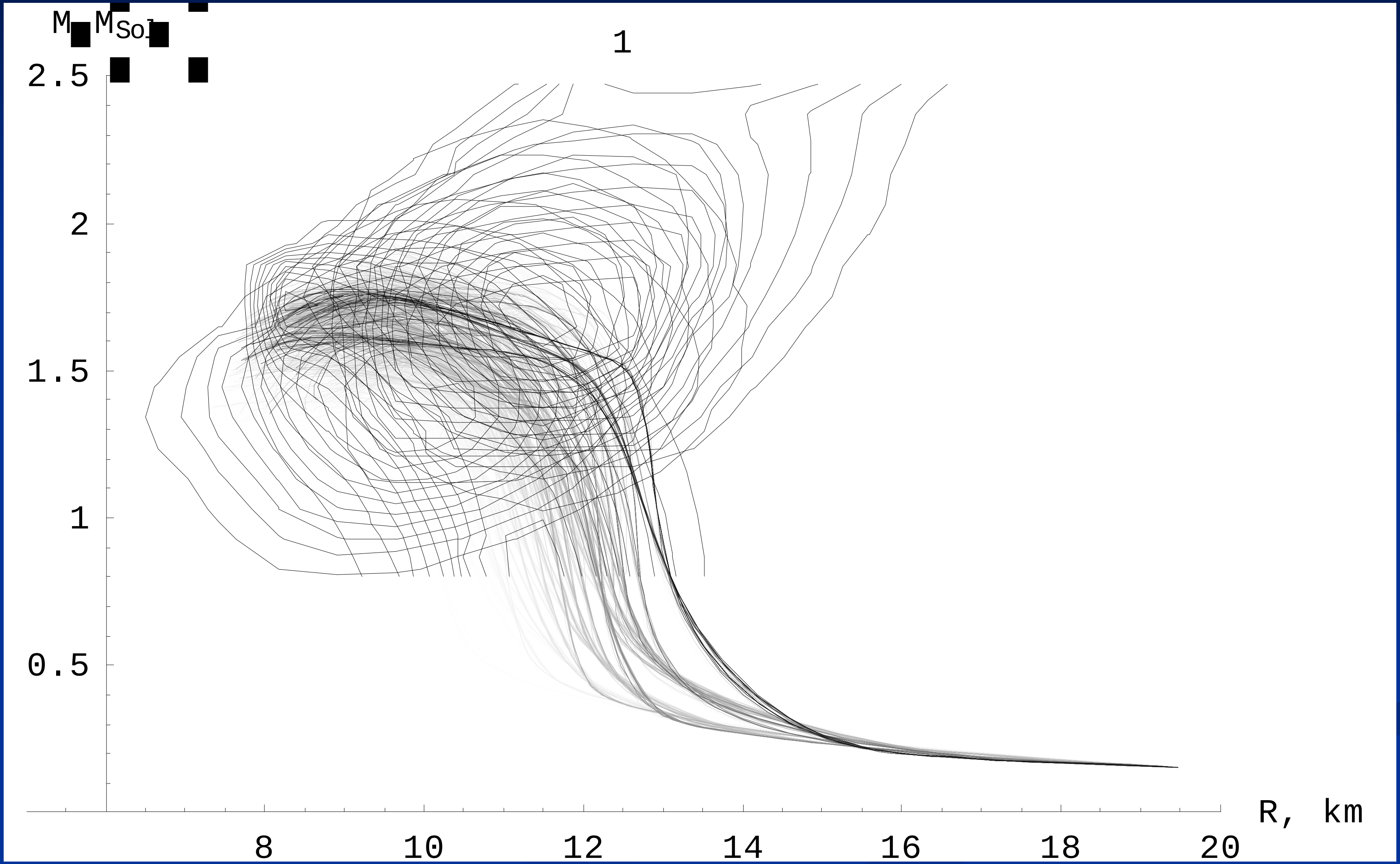


ted tree with
es (AP3)

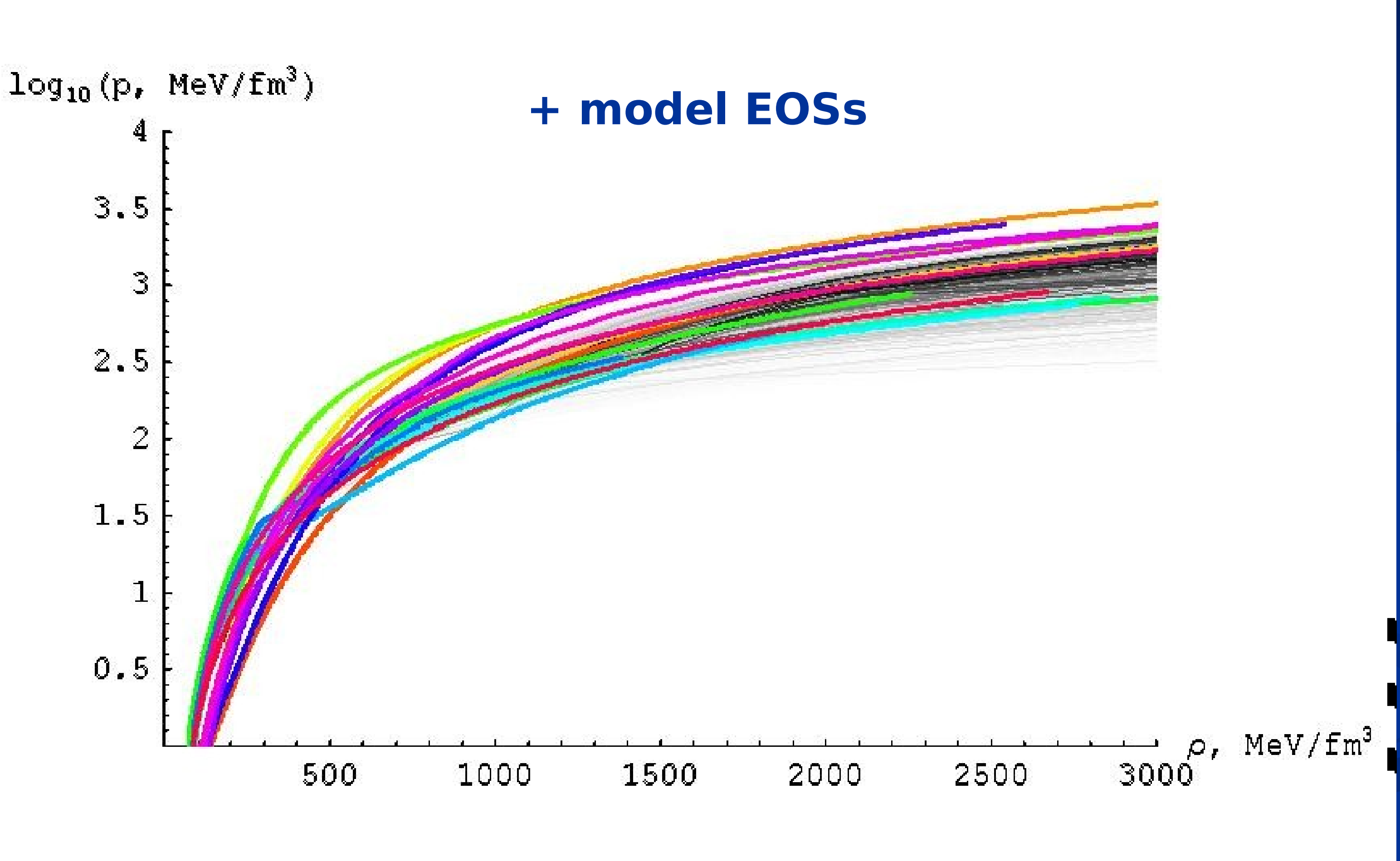




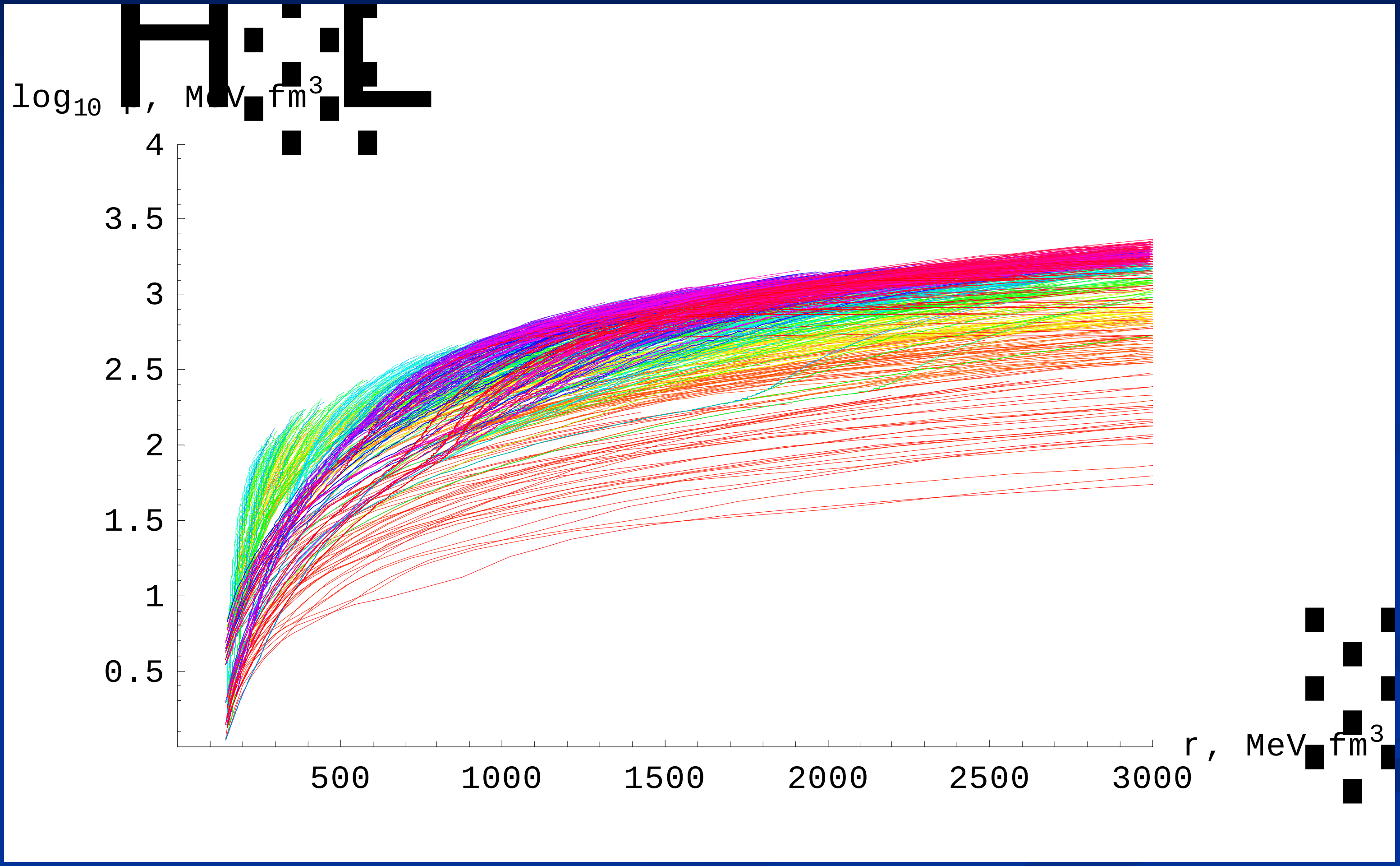
Bayesian analysis with contours (AP3)



Resulting EOS band



Many seed EOSs



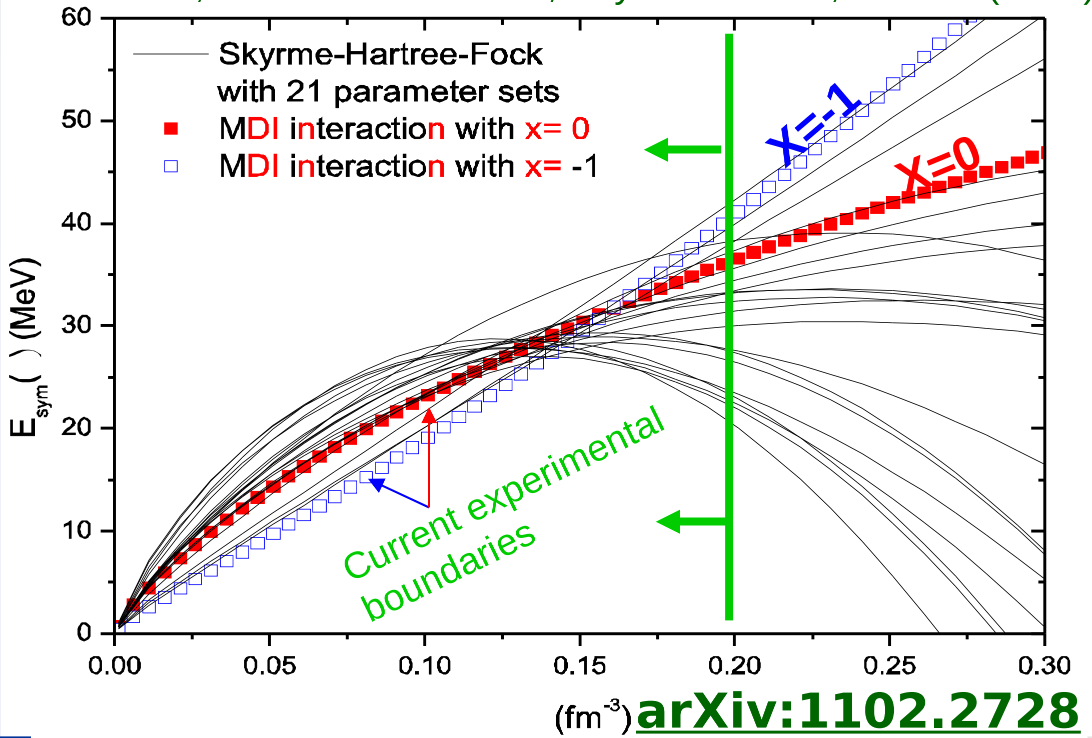
Recent 2 solar mass star

Next steps being taken

- **Generation of complementary EOSs from TOV inversion schemes (M-R plane). Schemes are developed and ready for use.**
- Inclusion of more constraints.
  - Rotational period, redshift, B.E., Love number…
  - Data from nuclear experiments
  - Cooling and bursting data
- Sequential data analysis as more individual stars available.
- Strange quark matter stars, very deferent seed EOS.

The Nuclear (A)Symmetry Energy

L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C72, 064309 (2005).



arXiv:1102.2728

- Energy cost to create an asymmetry (δ) between neutrons and protons

$$E_{sym} = \frac{1}{2n}\frac{d^2\epsilon}{d\delta^2}$$

$$\delta = 1 - 2\frac{n_p}{n_p + n_n} = 1 - 2x$$

- Structure of nuclei & neutron stars determined by the energy & pressure of beta-stable nucleonic matter

y

$$E(n,x) \simeq E(n,0.5) + E_{sym}(n)\boxed{(1-2x)^2} + \cdots$$

$$P(n,x) \simeq n^2[E'(n,0.5) + E'_{sym}(n)(1-2x)^2] + \cdots$$

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Courtesy M. Prakash

Summary

- The way to use available observations of several stars to determine a maximally model-independent EOS is being developed.
- Scheme to generate EOS with incorporation of observational errors is constructed on the basis of sequential Bayesian analysis and 'smoothness' assumption. It uses EOS expressed as the speed of sound *c (h)* and the variable *h* and

  s

  complementary inversion scheme from M-R into EOS.
- Scheme is tested on 4 stars from X-ray burst data and produced reasonable and consistent band of EOSs.
- Additional theoretical constrains and phase transitions can be easily implemented.
- New measurements of several individual stars is expected to get us closer and closer to pinpointing benchmark EOS.

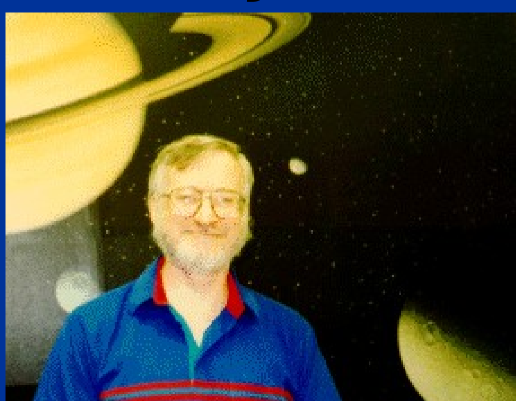
Acknowledgments

- Prof. Madappa Prakash

Ohio University

- Prof. Jim Lattimer

Stony Brook University



- Dr. Andrew Steiner

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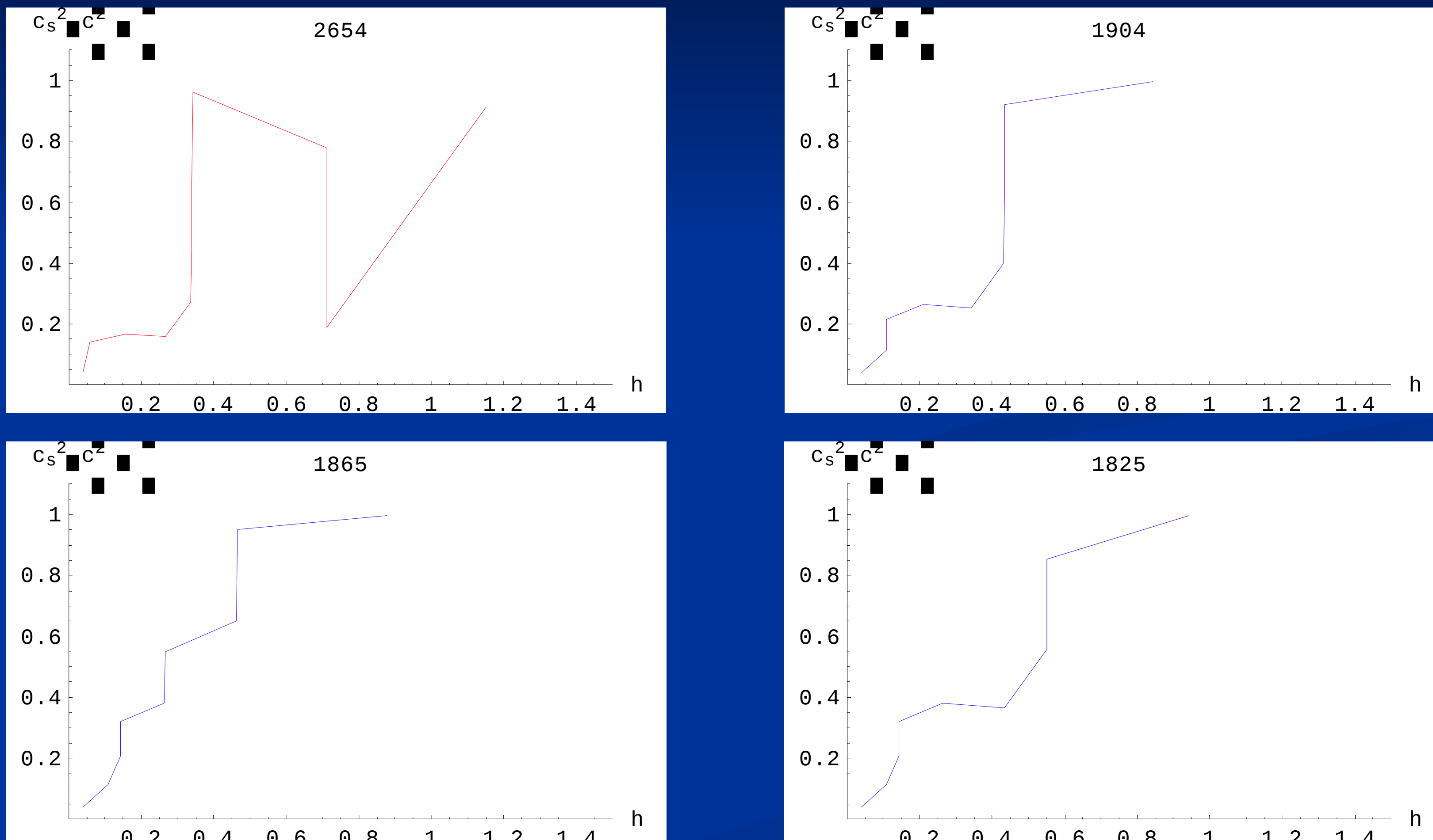
The end

Thank you!

Questions?

Examples of generated “phase transition”



Upper Limit from LIGOII

(150000 km5)

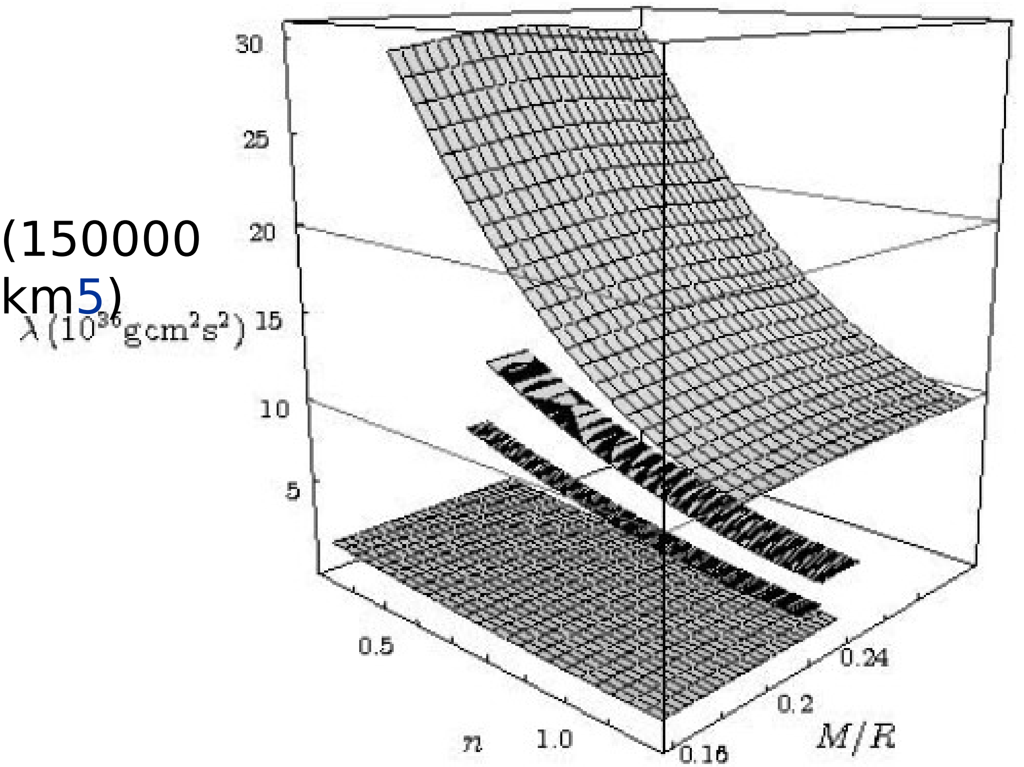


FIG. 3.—Range of Love numbers for the estimated NS parameters from X-ray observations. *Top to bottom sheets*: EXO 0748-676, ωCen, M13, and NGC 2808. For an inspiral of two 1.4 $M_\odot$ NSs at a distance of 50 Mpc, LIGO II detectors will be able to constrain λ to $\lambda \leq 20.1 \times 10^{36}$ g cm^2 s^2 with 90% confidence (Flanagan & Hinderer 2008).

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ Rhoades & Ruffini (1974), Hartle (1978)
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_{central} < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3}

  Lattimer & Prakash (2005)
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  (empirical) Lattimer & Prakash (2004)
- $\epsilon_{central} > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

J.M. Lattimer, NIC XI, Heidelberg, 20/07/10 – p. 6/31

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

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Remarks

- $\chi 2$ histogram to analyze array of generated EOS