

***New advances
in spectral modeling of X-ray bursts:
constraining neutron star parameters***

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Plan

- Advances in the atmosphere models
- X-ray burst spectral evolution
- Theory vs observations → neutron star M, R
- Conclusions

Model atmospheres of X-ray bursts accounting for Compton scattering

- Using Kompaneets equation: London et al. 1984, 1986; Lapidus et al. 1986; Ebisuzaki 1987; Pavlov et al. 1991, Suleimanov et al. 2006, 2011
- Using approximate Compton redistribution function (Guilbert 1981): Madej 1991; Madej et al. 2004; Majczyna et al. 2005
- Using exact relativistic Compton redistribution function (Aharonian & Atoyan 1981, Nagirner & Poutanen 1993, Poutanen & Svensson 1996): our ongoing work

Basic equations

Hydrostatic equilibrium

$$\frac{1}{\rho} \frac{dP_{gas}}{dr} = -\frac{GM_{NS}}{R_{NS}^2 (1 - R_g / R_{NS})^{1/2}} + \frac{4\pi}{c} \int H_\nu (k_{ff} + \sigma_e) d\nu$$

Radiation transfer

$$\frac{\partial^2 (f_\nu J_\nu)}{\partial \tau_\nu^2} = \frac{k_{ff}}{k_{ff} + \sigma_e} (J_\nu - B_\nu) - \frac{\sigma_e}{k_{ff} + \sigma_e} \frac{kT}{m_e c^2} x \frac{\partial}{\partial x} \left(\frac{\partial J_\nu}{\partial x} - 3J_\nu + \frac{T_{eff}}{T} x J_\nu \left(1 + C \frac{J_\nu}{x^3} \right) \right)$$

$$x = \frac{h\nu}{kT_{eff}} \quad C = c^2 h^2 / 2(kT_{eff})^3$$

Compton scattering
(Kompaneets equation)

Radiation equilibrium

$$\int k_{ff} (J_\nu - B_\nu) dx - \sigma_e \frac{kT}{m_e c^2} \int \left(4J_\nu - \frac{T_{eff}}{T} x J_\nu \left(1 + \frac{C J_\nu}{x^3} \right) \right) dx = 0$$

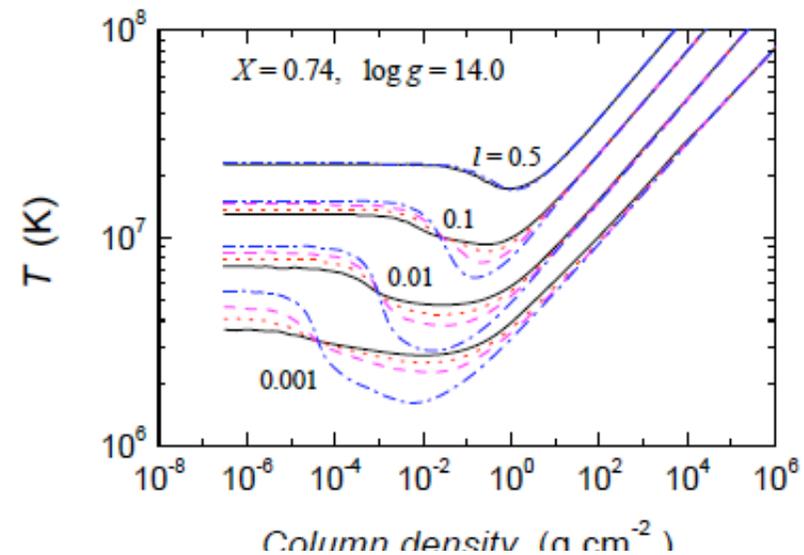
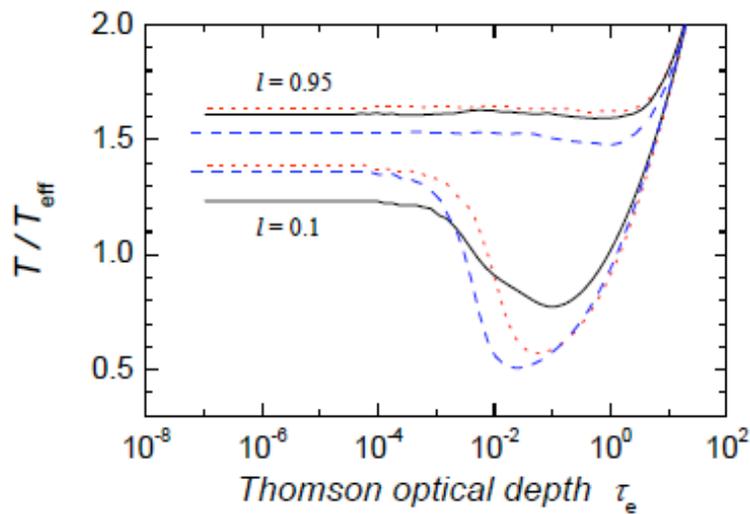
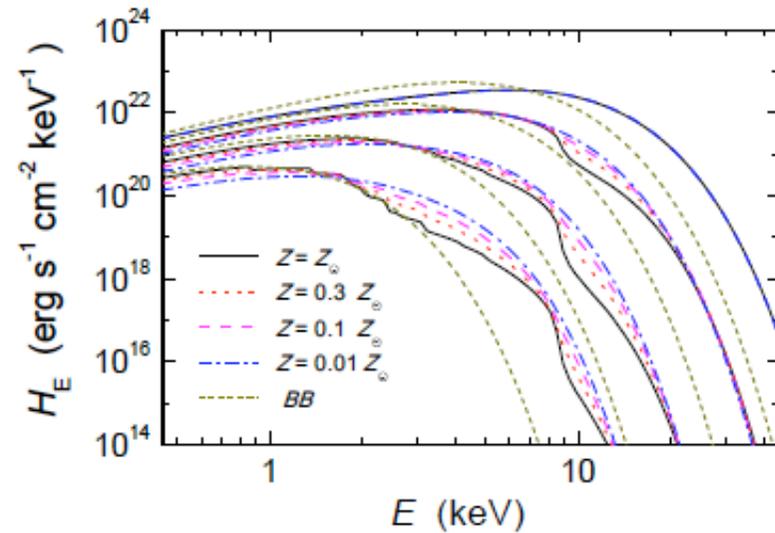
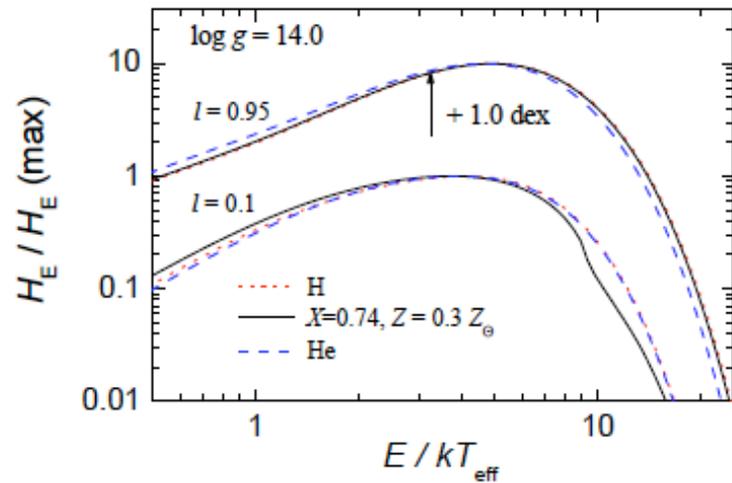
k_{ff} - true absorption opacity (mainly free-free transitions)

σ_e - Thomson electron scattering opacity

Model atmosphere calculations

(Suleimanov, Poutanen, Werner 2011, A&A 527, A139)

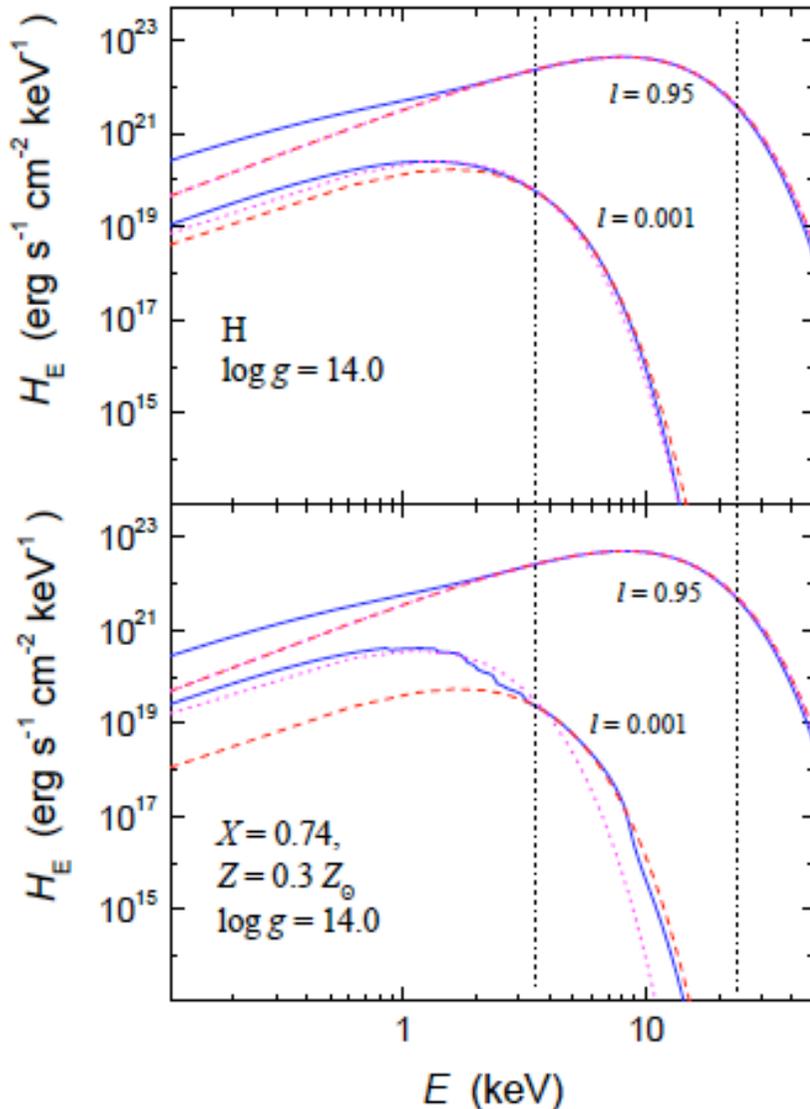
- 6 chemical compositions: H, He, solar H/He with $Z = 1, 0.3, 0.1, 0.01 Z_{\text{sun}}$
- 3 surface gravities: $\log g = 14.0, 14.3$ and 14.6
- 20 relative luminosities $l = L / L_{\text{Edd}}$: from 0.001 to 0.98



Color correction f_c calculations

Calculated spectra are redshifted and fitted by diluted blackbody (one- and two-parameters functions) (assuming $M = 1.4 M_{\text{sun}}$) in the *PCA/RXTE* energy band (3-20) keV

$$F_E = w B_E(f_c T_{\text{eff}})$$



5 fitting procedures

1) Minimizing deviations in flux

$$\sum_{n=1}^N (F_{E_n} - w_1 B_{E_n}(f_{c,1} T_{\text{eff}}))^2$$

2) Minimizing deviations in photon number flux

$$\sum_{n=1}^N \frac{(F_{E_n} - w_2 B_{E_n}(f_{c,2} T_{\text{eff}}))^2}{E_n^2}$$

3) Minimizing integral

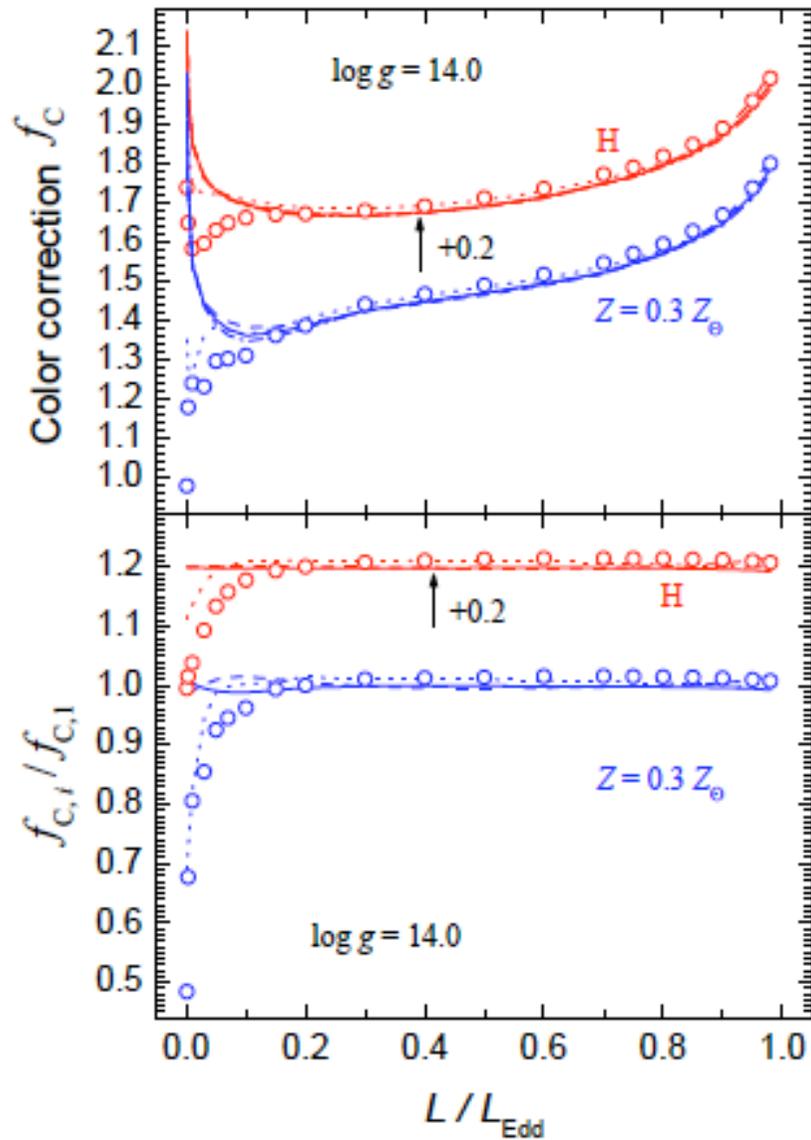
$$\int_{E_{\text{min}}}^{E_{\text{max}}} (F_E - w_3 B_E(f_{c,3} T_{\text{eff}}))^2 dE$$

4) = 1), but $w = f_c^{-1/4}$

5) Ratio of peak position (as in Madej et al.)

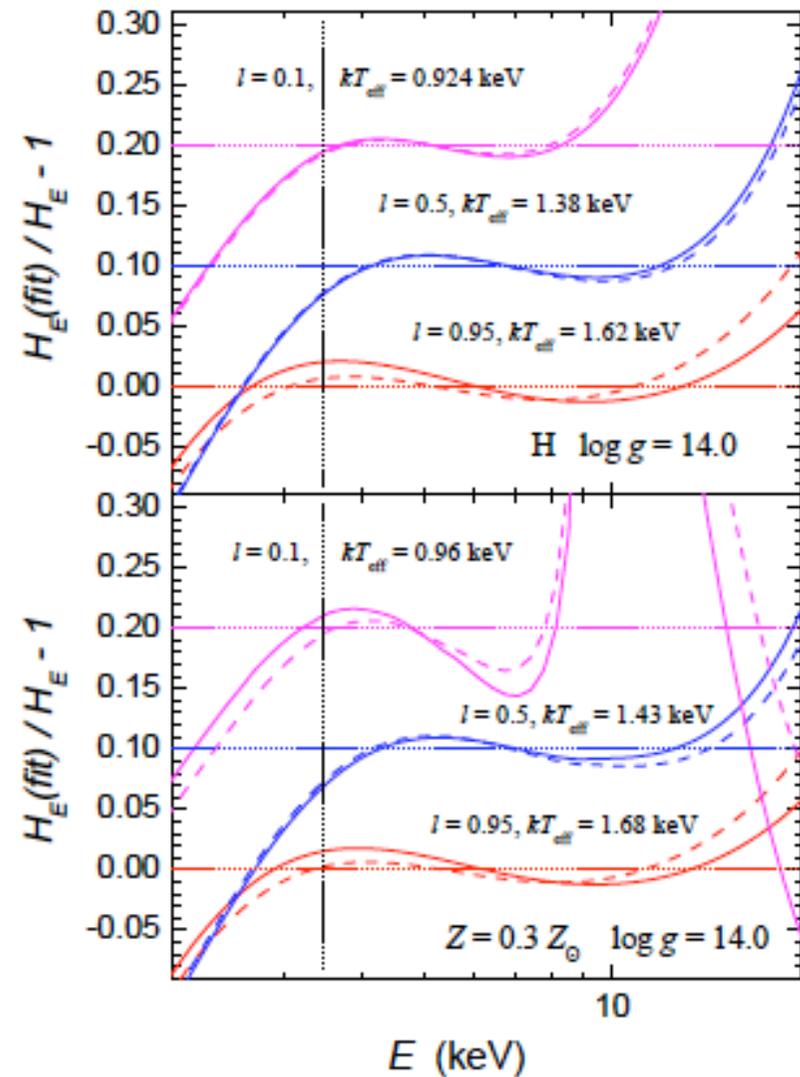
Color correction f_c calculations

Color correction for various fitting procedures



differences are small at $L/L_{\text{Edd}} > 0.3$

Residuals of the blackbody fits



Solid curves – procedure 1)

Dashed curves – procedure 2)

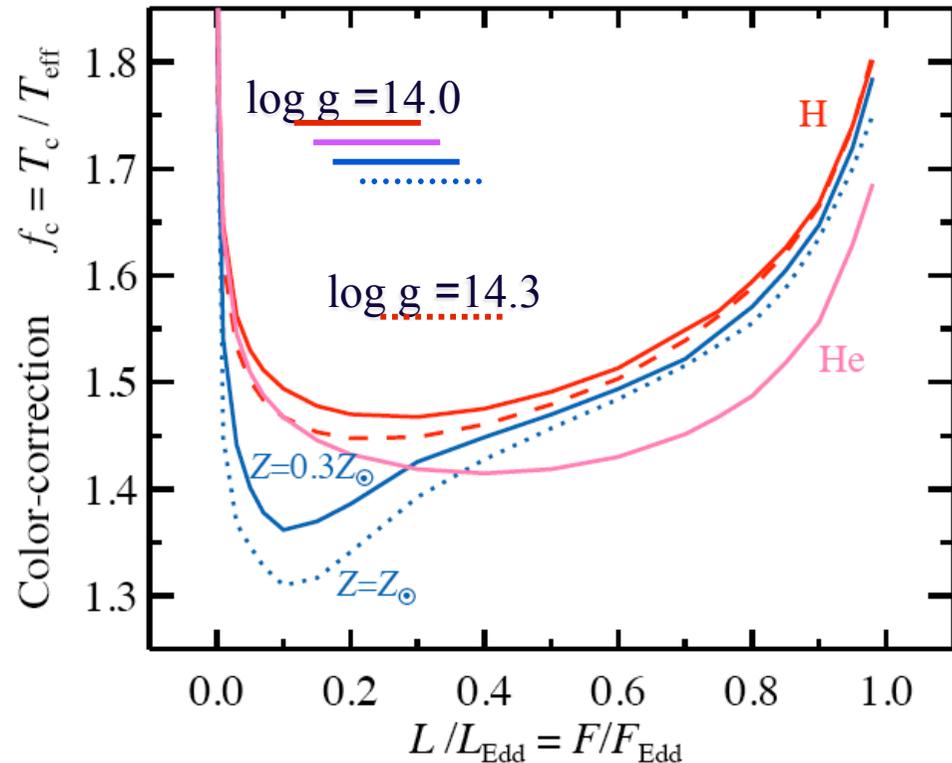
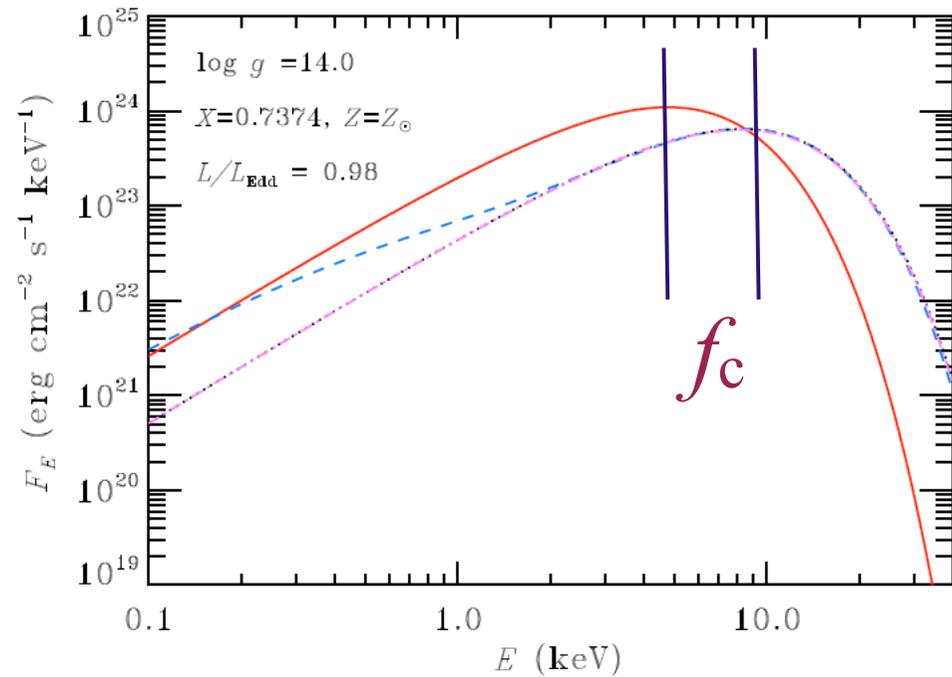
A large set of atmosphere models
is computed

(Suleimanov et al. 2011)

- 1) $\log g = 14.0, 14.3, 14.6$
- 2) $L/L_{\text{Edd}} = 10^{-3}, \dots, 0.98$
- 3) Pure hydrogen, pure helium,
solar composition, solar He/H
+ subsolar metals.

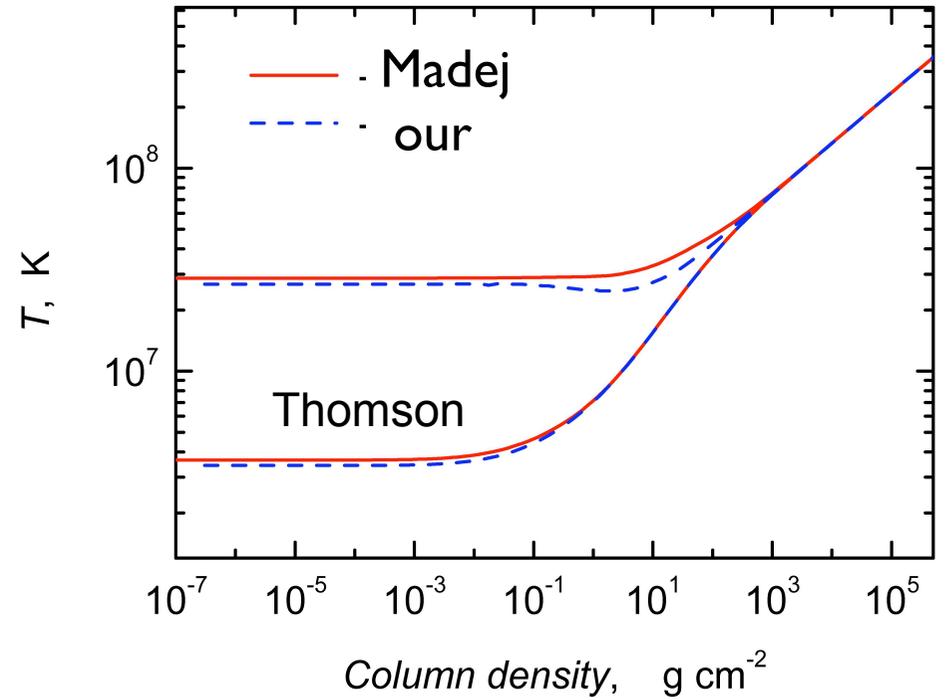
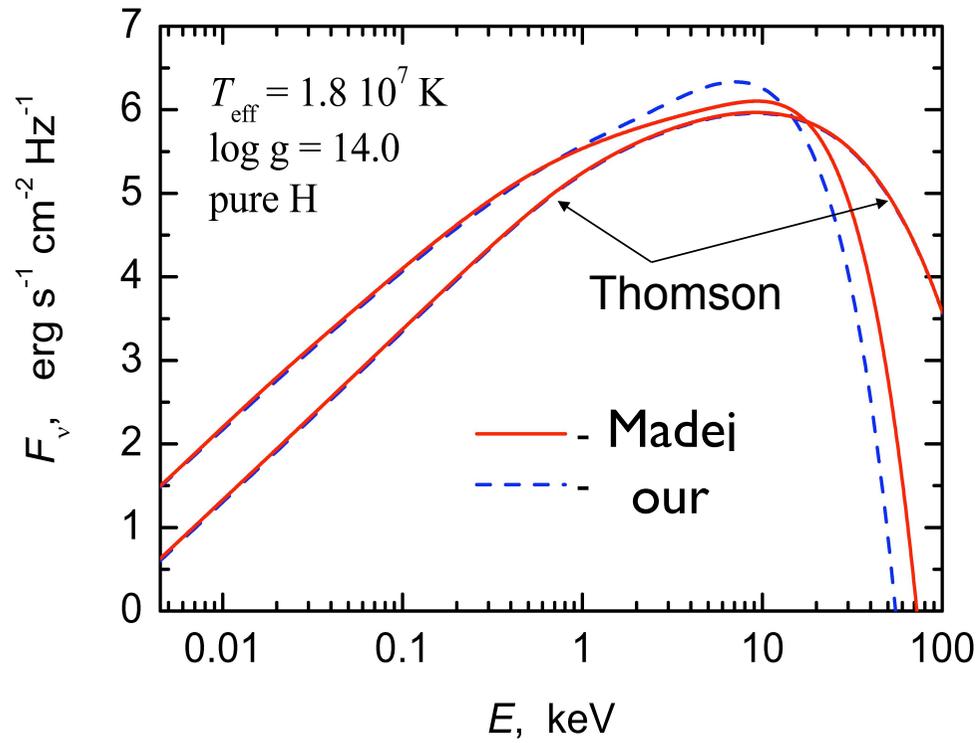
Atmosphere spectra are fitted
with a diluted blackbody and the
color-correction factor is
determined.

Our spectral results differ
significantly from previous
calculations by Madej et al. 2004
and Majczyna et al. 2005.



Comparison with Madej 's code

(calculations by Agata Rozanska)



Accurate treatment with relativistic Compton redistribution function

Radiation transfer equation (RTE)

$$\mu \frac{dI(x, \mu)}{d\tau_x} = I(x, \mu) - S(x, \mu), \quad d\tau_x = -(\sigma(x, \mu) + k(x)) \rho(z) dz$$

Electron scattering opacity

$$\sigma(x, \mu) = \frac{\sigma_e}{x} \int_0^\infty x_1 dx_1 \int_{-1}^1 d\mu_1 R(x, \mu, x_1, \mu_1) \exp\left(-\frac{x_1 - x}{\Theta(z)}\right) \left(1 + \frac{C I(x_1, \mu_1)}{x_1^3}\right)$$

$$\sigma_T = 6.65 \cdot 10^{-25} \text{ cm}^2$$

$$\sigma_e = \sigma_T \frac{n_e}{\rho}, \quad C = \frac{h^2}{2m_e^3 c^4}$$

$$x_1 = \frac{h\nu_1}{m_e c^2}, \quad x = \frac{h\nu}{m_e c^2},$$

$$\Theta(z) = \frac{kT(z)}{m_e c^2}$$

Source function

$$S(x, \mu) = \frac{k(x)}{\sigma(x, \mu) + k(x)} B(x) +$$

$$\frac{x^2}{\sigma(x, \mu) + k(x)} \left(1 + \frac{C I(x, \mu)}{x^3}\right) \int_0^\infty \frac{dx_1}{x_1^2} \int_{-1}^1 d\mu_1 R(x, \mu, x_1, \mu_1) I(x_1, \mu_1),$$

Redistribution function (RF)

$$R(x, x_1, \mu, \mu_1) = \int_0^{2\pi} R(x, x_1, \eta) d\varphi, \quad \eta = \mu\mu_1 + \sqrt{1 - \mu^2} \sqrt{1 - \mu_1^2} \cos \varphi$$

Accurate treatment with relativistic Compton redistribution function

We use two RFs:

1) exact fully relativistic (Aharonian & Atoyan 1981, Nagirner & Poutanen 1993, Poutanen & Svensson 1996)

2) approximate, isotropic in the electron rest-frame (Aharonian & Atoyan 1981, Poutanen 1994)

$$R(x, x_1, \eta) = \frac{1}{8\pi Q} \frac{e^{-\gamma_*/\Theta(z)}}{K_2(1/\Theta(z))},$$

where:

$$\gamma_* = \left(x - x_1 + Q \sqrt{1 + 2/q} \right) / 2, \quad Q^2 = (x - x_1)^2 + 2q, \quad q = xx_1(1 - \eta),$$

We also use angle-averaged RFs

$$R(x, x_1) = \frac{1}{2} \int_{-1}^{+1} d\eta R(x, x_1, \eta).$$

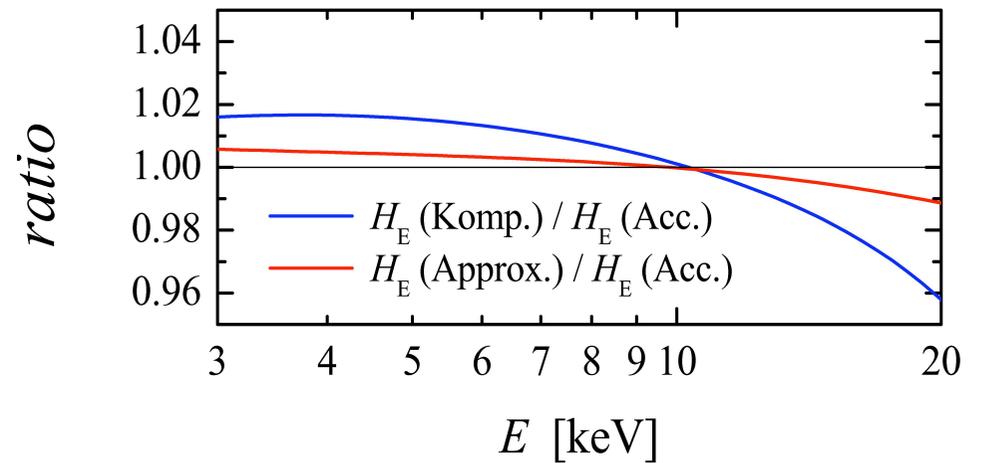
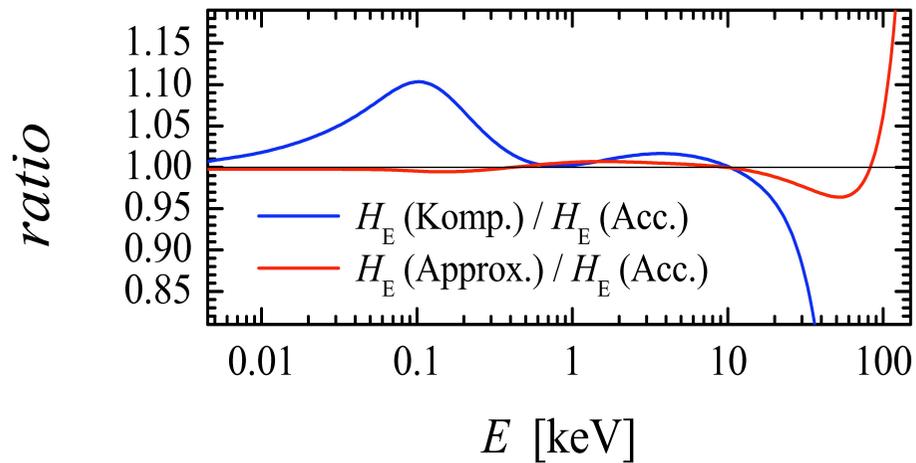
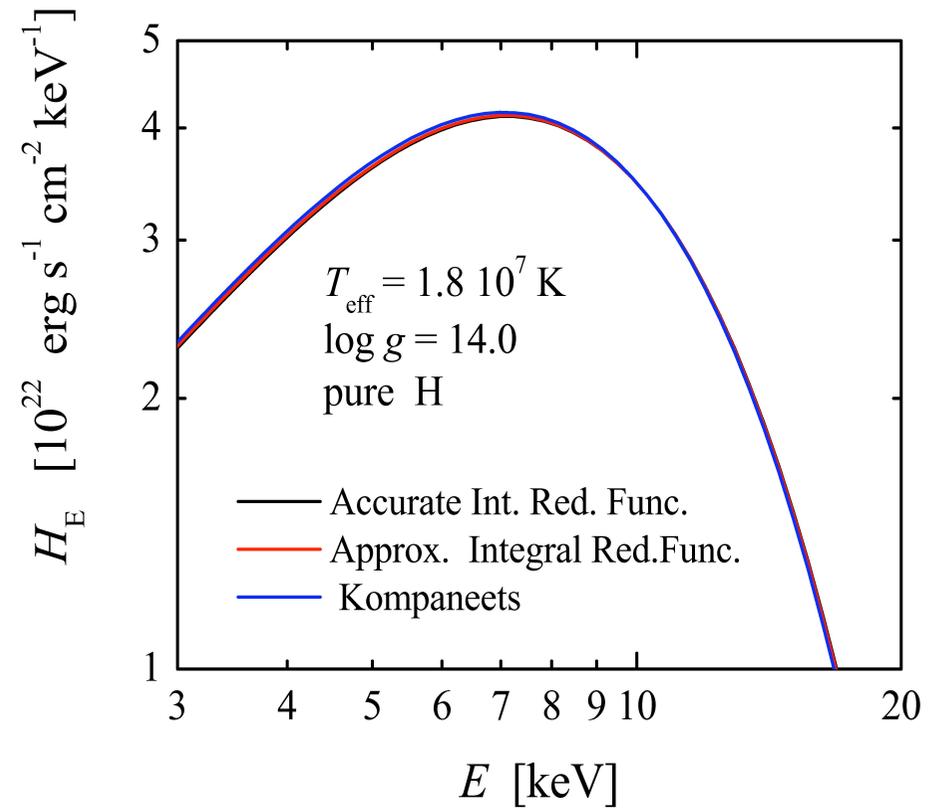
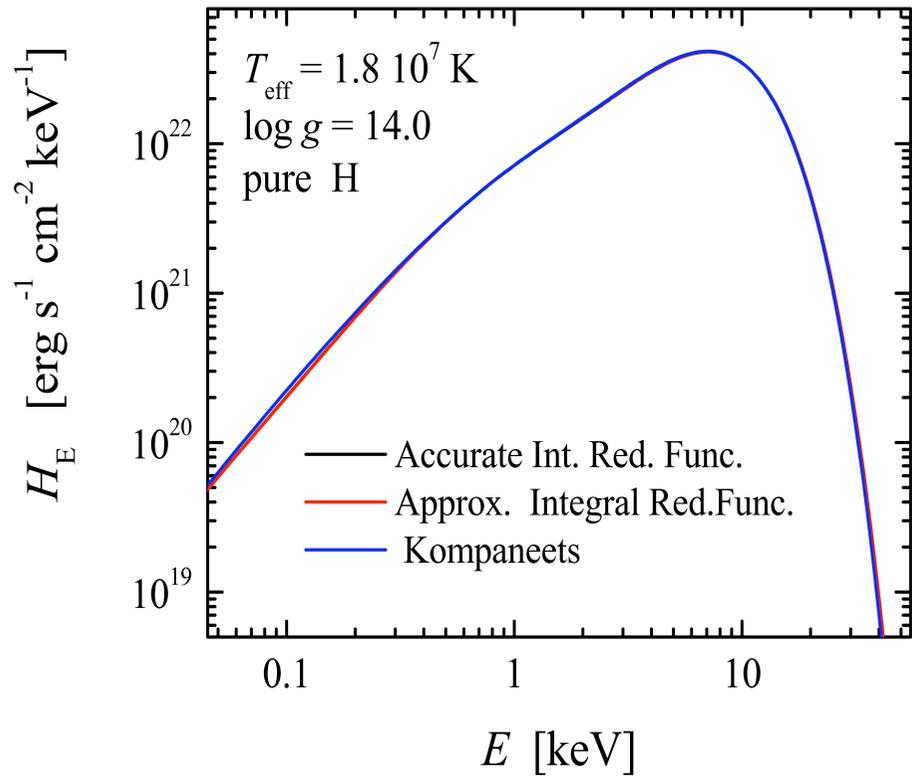
Standard boundary conditions

$$I^-(\mu, 0) = 0 \quad I^+(\mu, x, z_{\max}) = I^-(\mu, x, z_{\max}) + 2 \frac{dB(x, z_{\max})}{d\tau_x}$$

RTE is solved using accelerated Lambda-iterations reaching accuracy

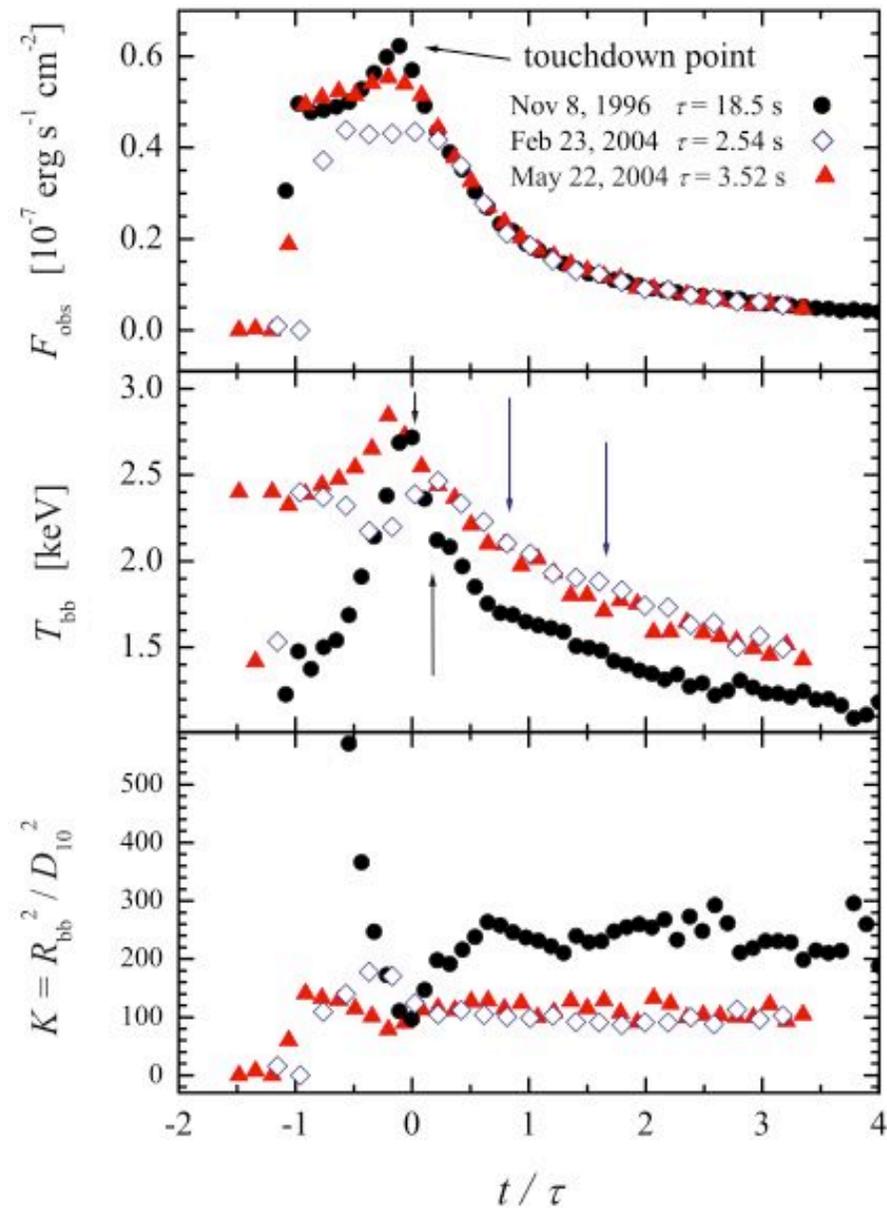
$$\max \left[\frac{J^{n+1}(x, z) - J^n(x, z)}{J^n(x, z)} \right] < 10^{-4}$$

Comparison of emergent spectra



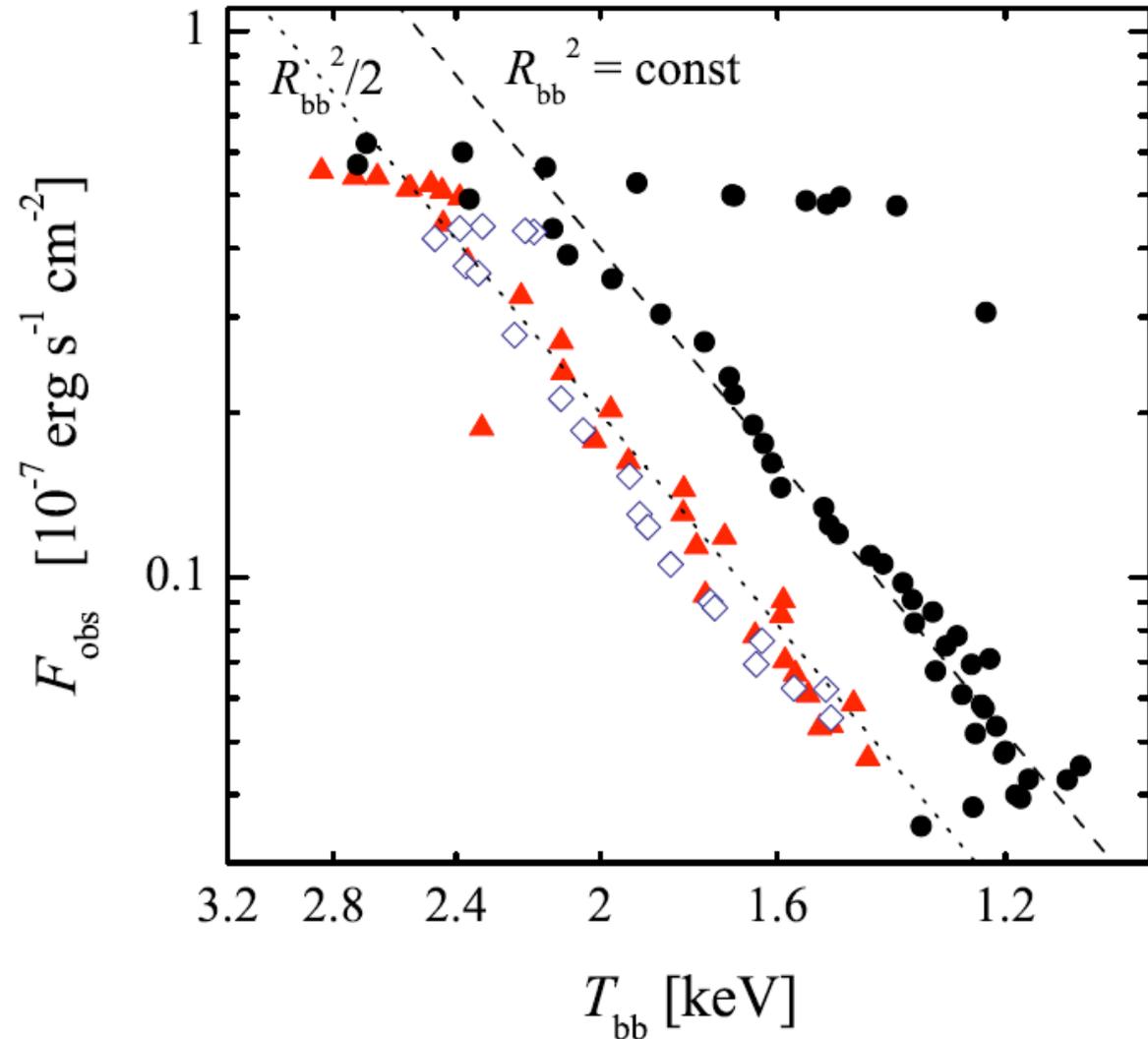
Spectral evolution of
photospheric radius
expansion (PRE)
X-ray bursts

X-ray bursts from 4U 1724-307

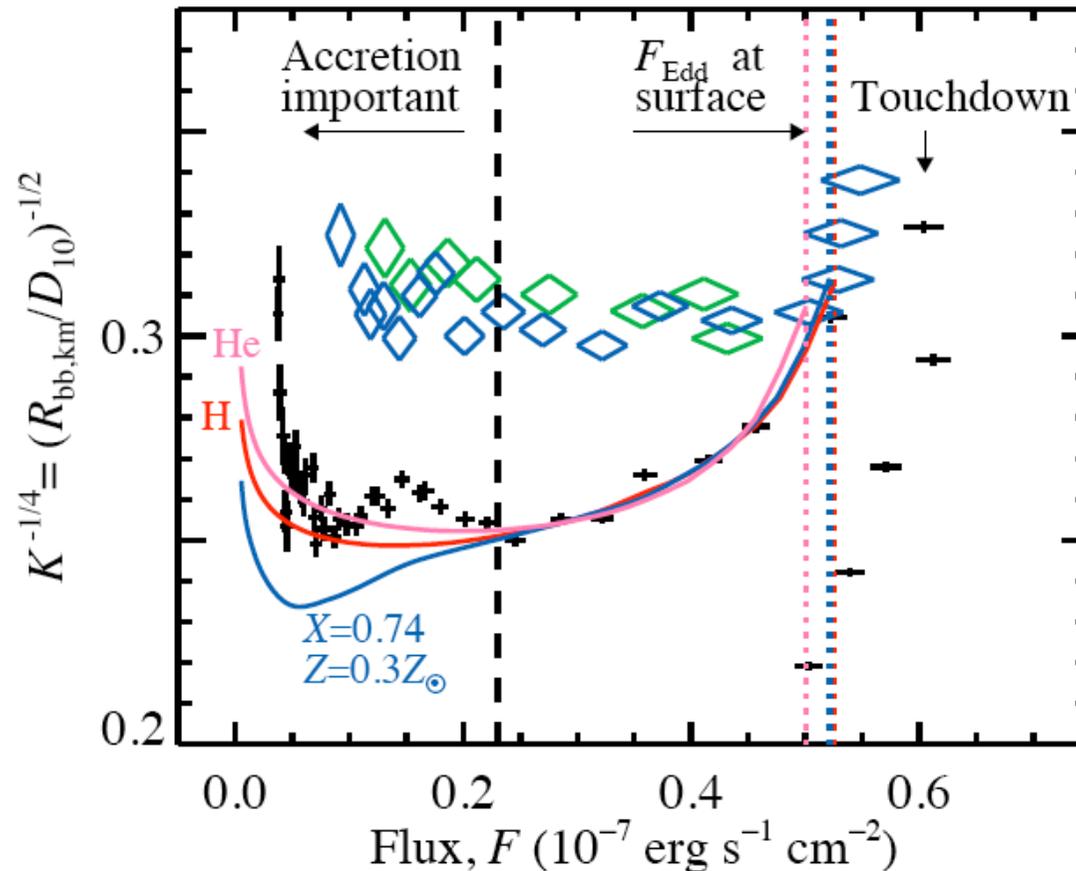


Spectral evolution of two types of bursts

Apparent area in short bursts is 2 times smaller than that for the long burst.



Cooling tails of PRE bursts from 4U 1724-307



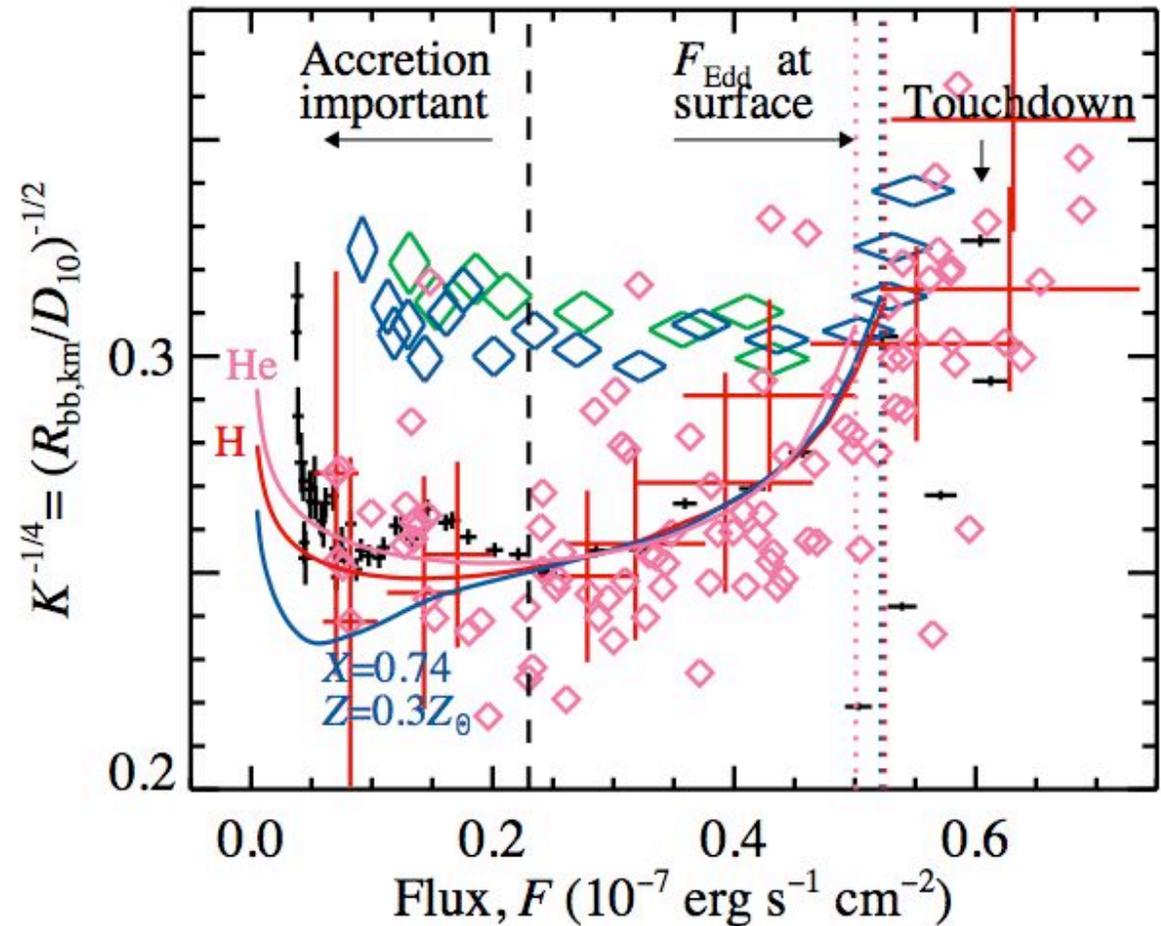
Crosses: Long, >150 sec, PRE burst during **hard/low state** on Nov 8, 1996.

Diamonds: two short PRE bursts on Feb 23 and May 22, 2004 during **soft state**.

Spectral evolution is spectacularly different!

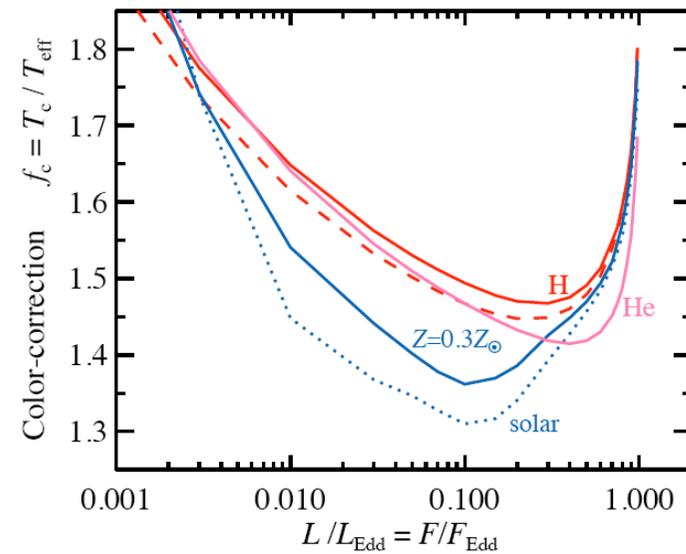
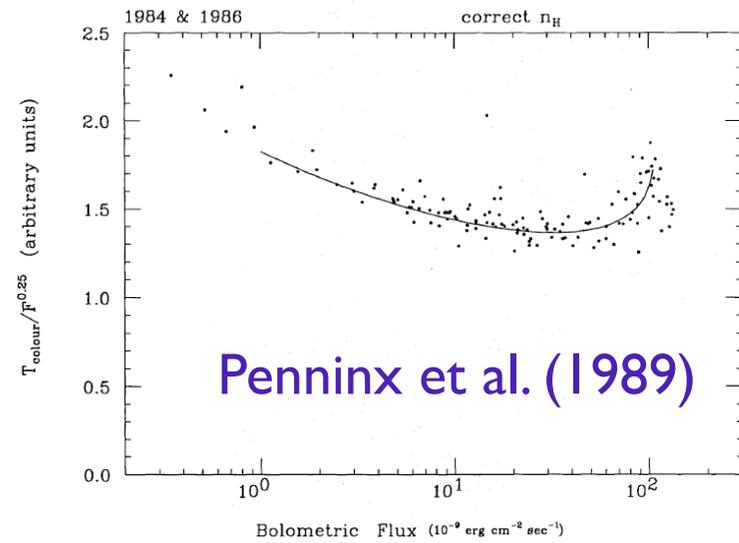
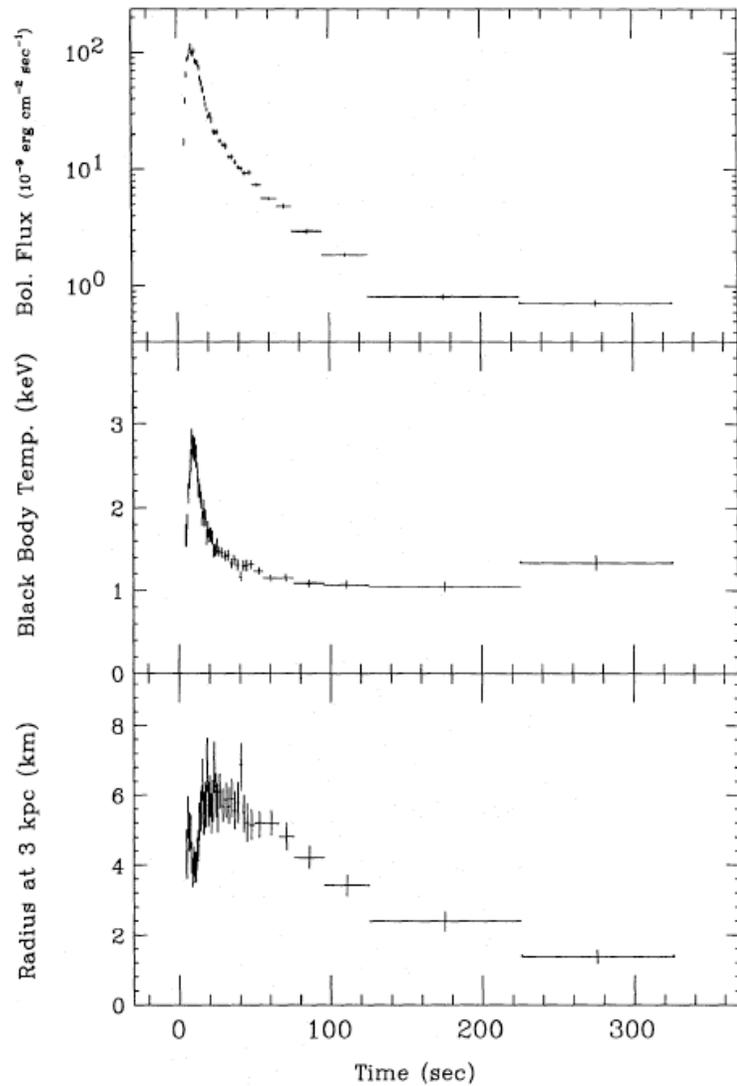
4U 1724-307 as seen by WFC @BeppoSAX

1. 24 long PRE X-ray bursts have been detected by WFC at BeppoSAX (Kuulkers et al. 2003).
2. It has 50 times less effective area than RXTE.
3. Spectral evolution is consistent with that seen by RXTE from a long bursts.

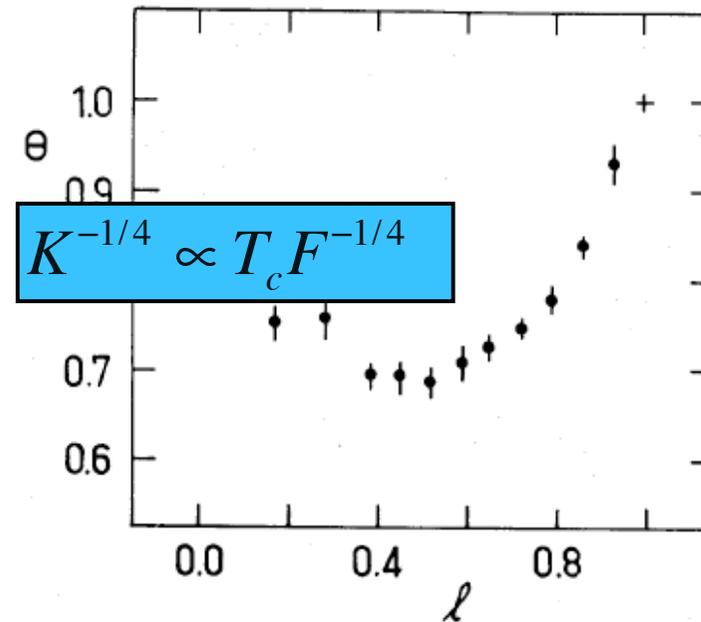
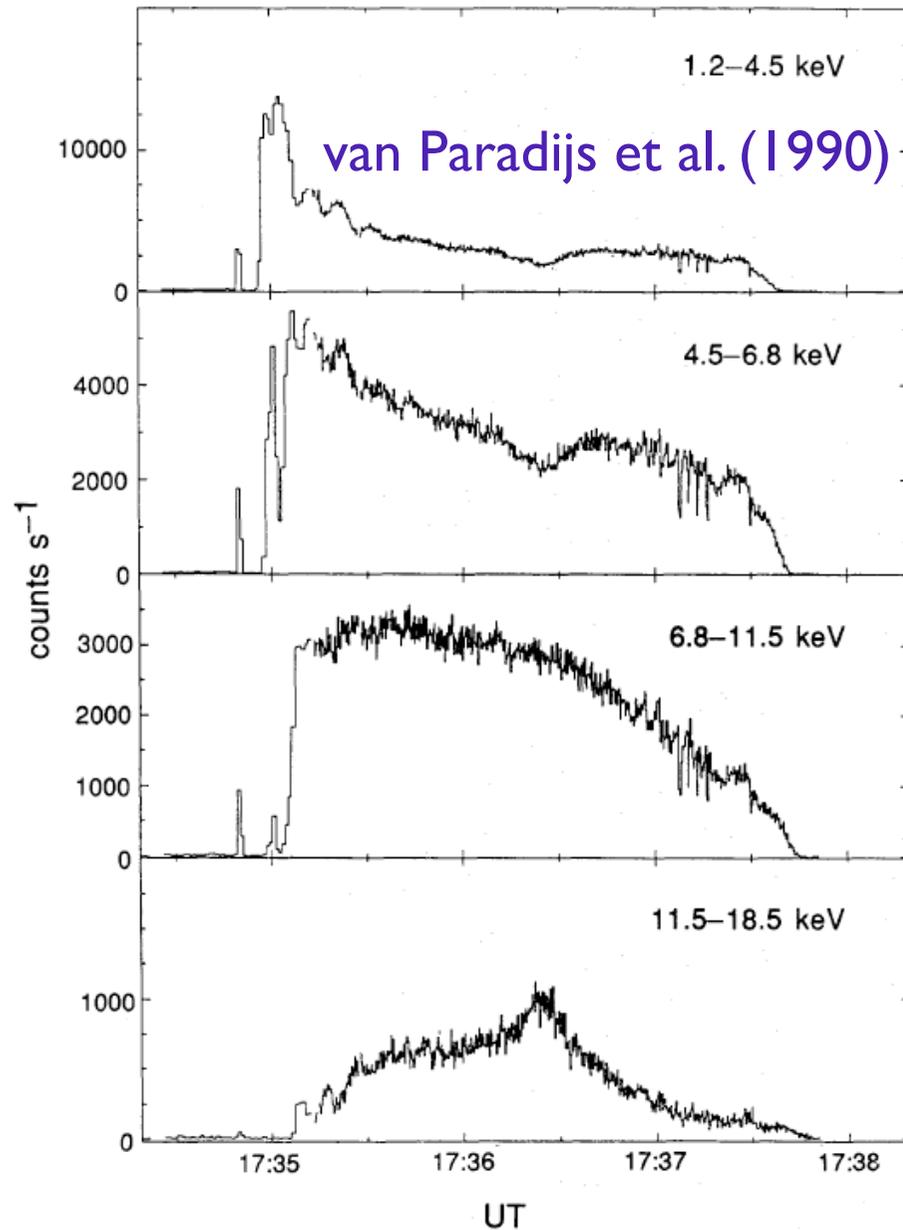


data by Jean in 't Zand

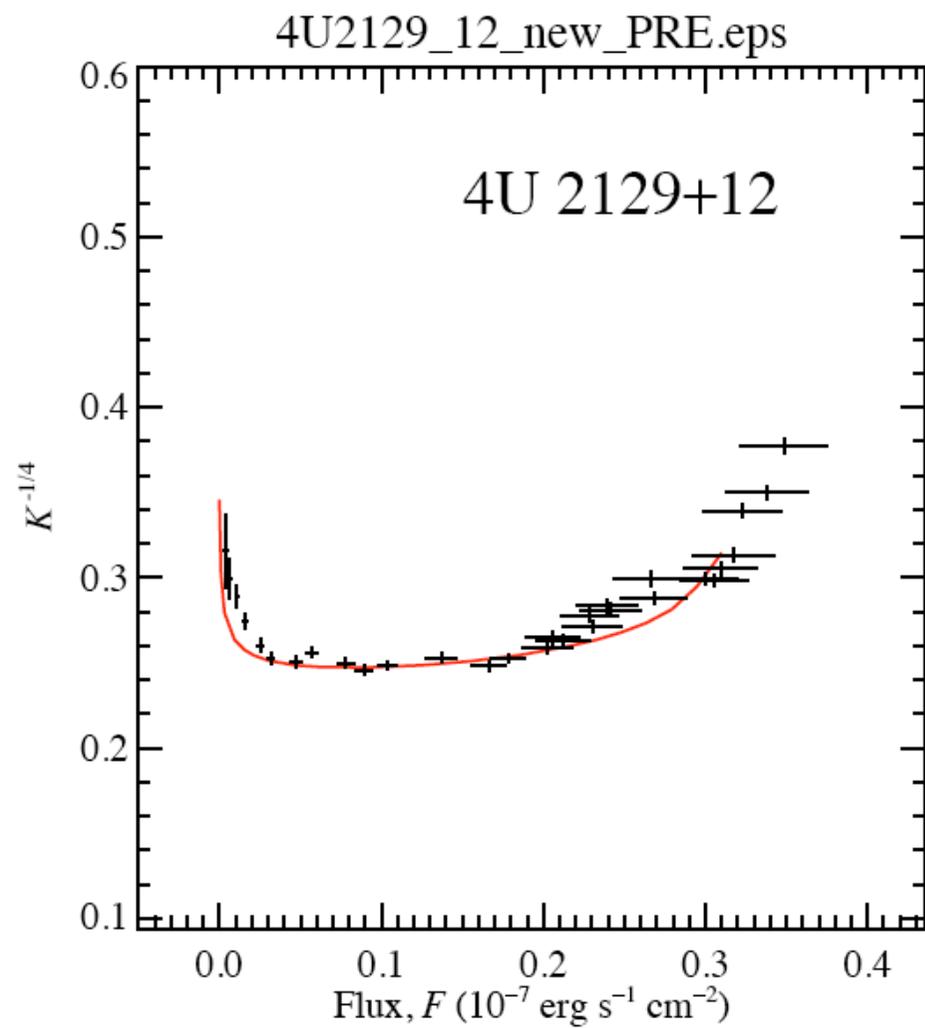
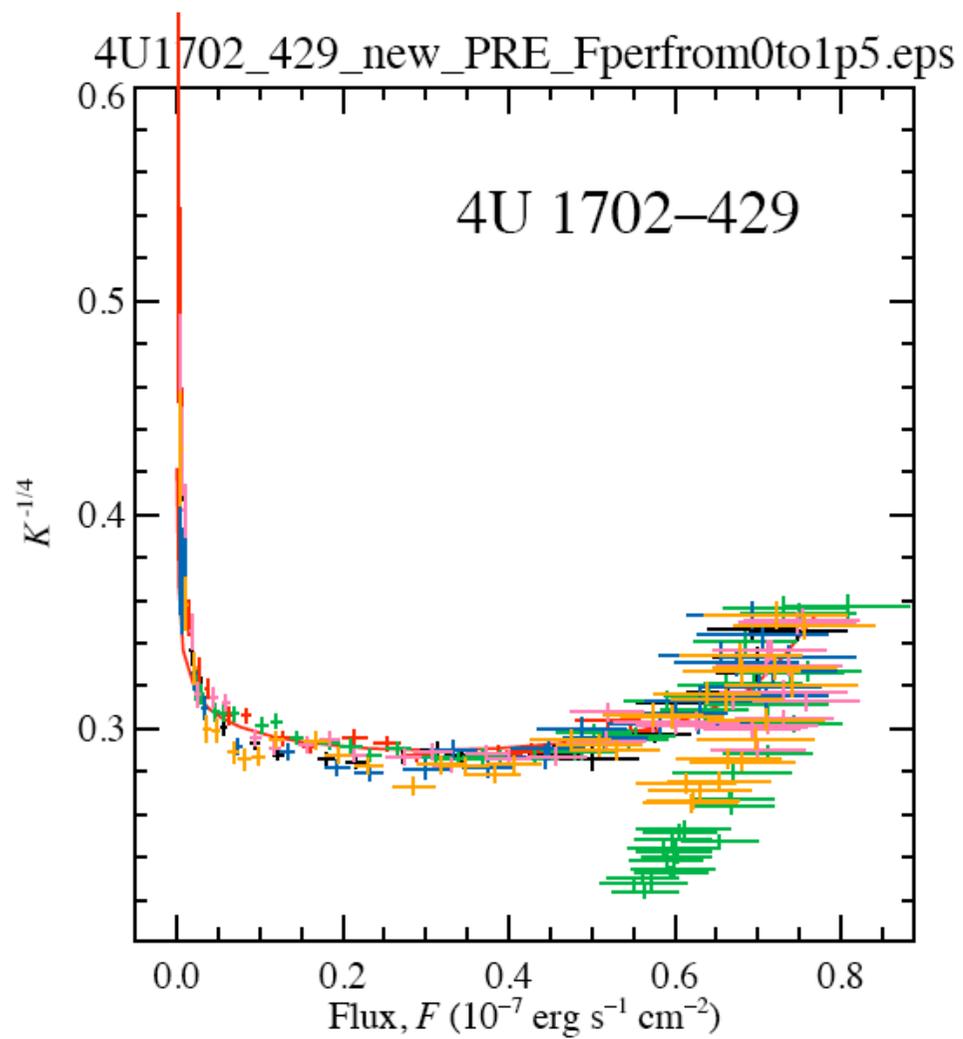
Long bursts from 4U 1608-52 seen by EXOSAT



Long bursts from 4U 2129+11 in M15 seen by Ginga

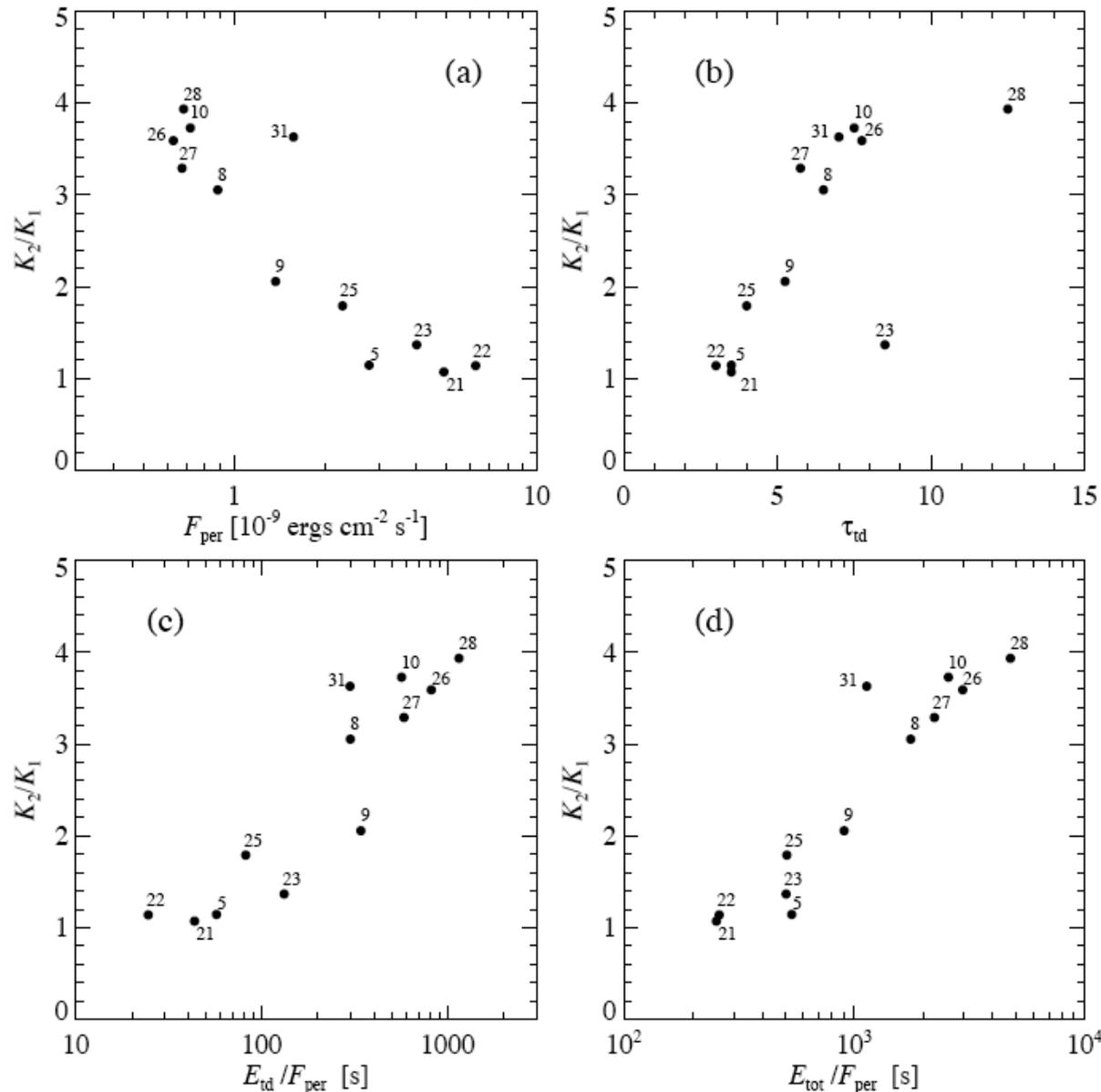


Evolution identical to
the long burst from
4U 1724-307



Bursts at various
persistent luminosities in
4U 1608-52

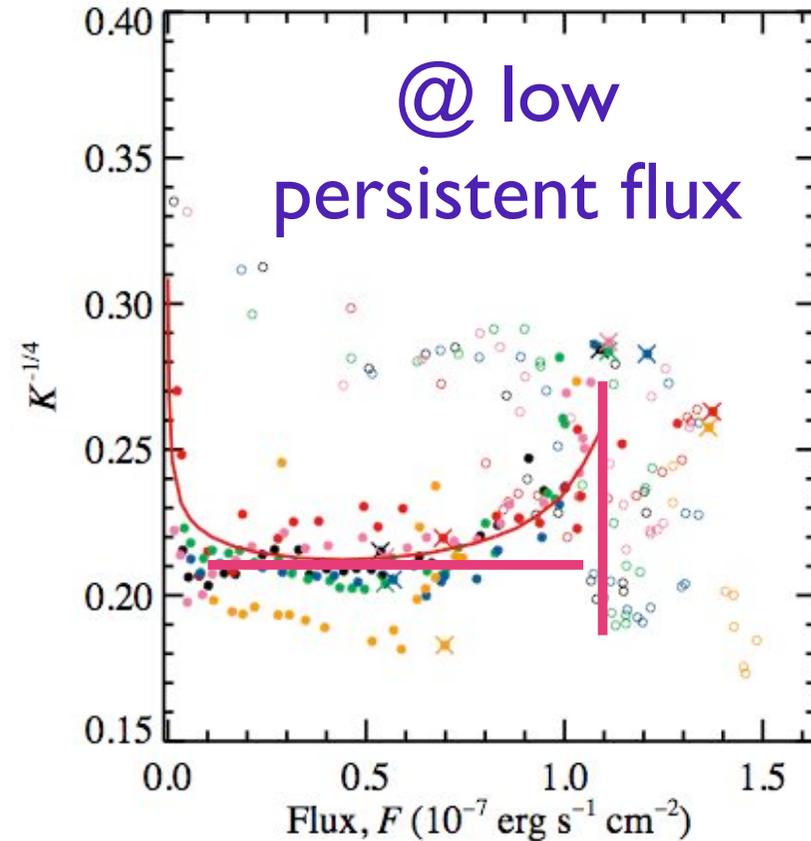
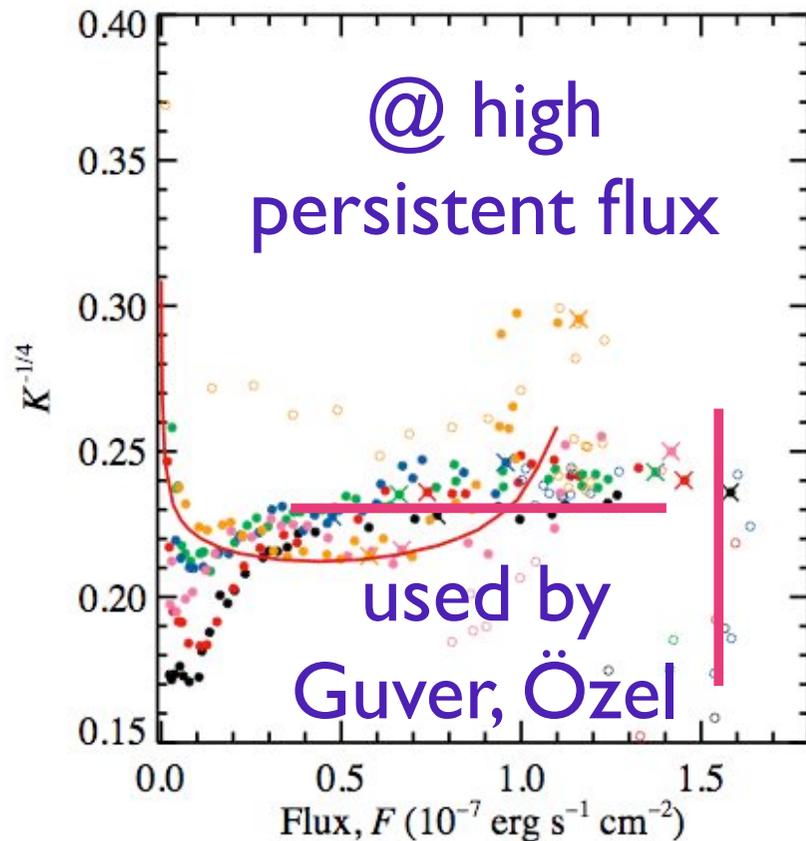
bb normalizations ratio at =1/2 touchdown flux to the touchdown



Evolution of
blackbody
normalization
depends strongly
on persistent flux

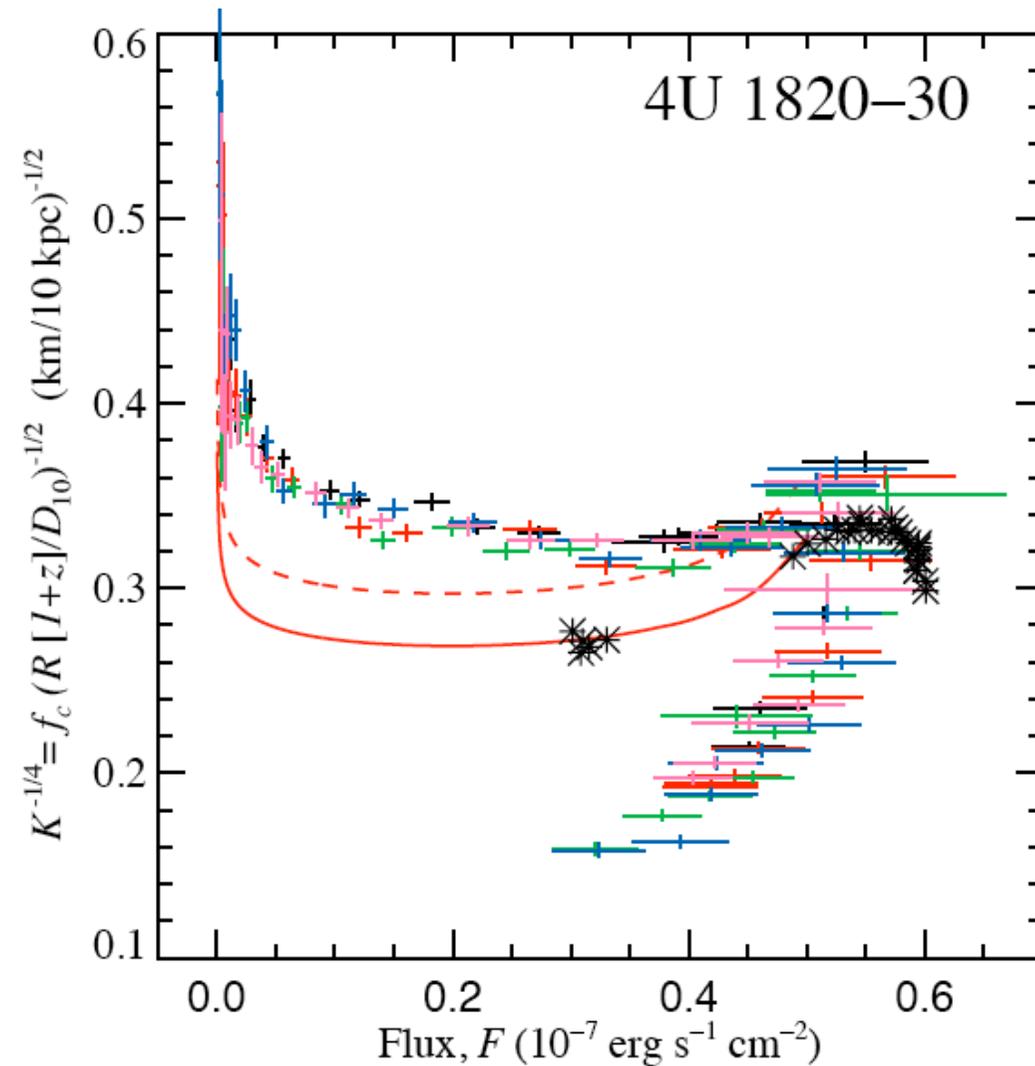
4U 1608-52

Bursts from 4U 1608-52 at different accretion rates



Poutanen et al. (2011, in preparation)

PRE bursts from 4U 1820-30



Crosses: short bursts in the soft state.

Stars: superburst

Measuring neutron star
parameters from
the PRE bursts
using “touchdown” and
“cooling tail” methods

- Photospheric radius expansion (PRE) bursts = Super-Eddington fluxes
- Measurements of the **Eddington flux** and the **blackbody radius** at the cooling tail for sources with known distances allow us to get **two constraints** on the neutron star mass and radius.

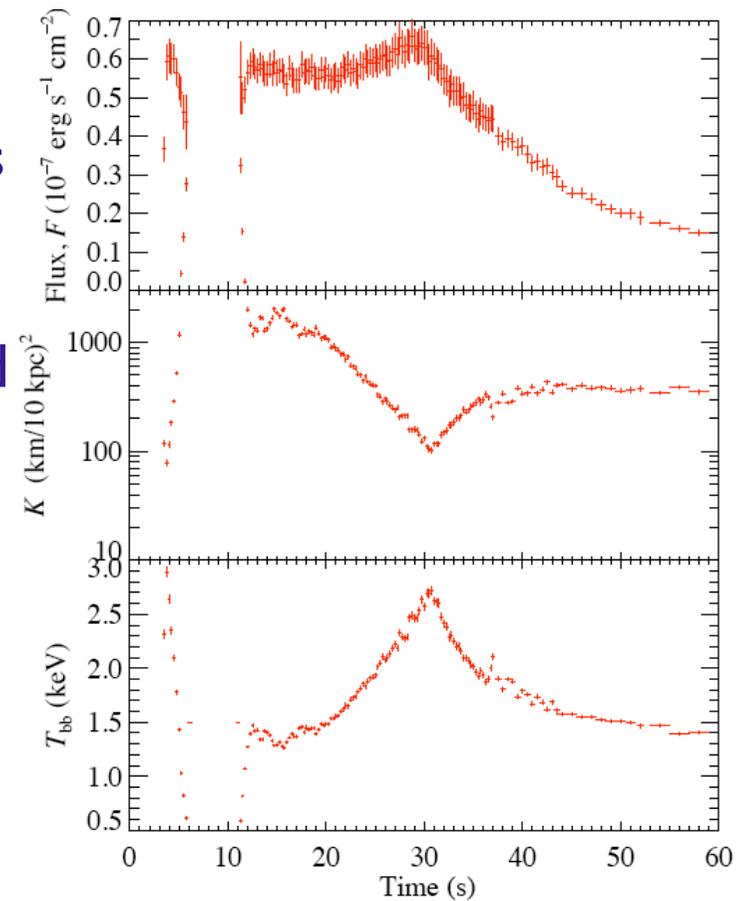
$$F_{\text{Edd}} = \frac{L_{\text{Edd}}}{4\pi D^2} = \frac{GMc}{D^2 \kappa_e (1+z)}$$

$$F = \sigma T_c^4 \left(\frac{R_{bb}}{D} \right)^2 = \sigma T_{\text{eff},\infty}^4 \left(\frac{R_\infty}{D} \right)^2 \longrightarrow K = \left(\frac{R_{bb}}{D_{10}} \right)^2 = \frac{1}{f_c^4} \left(\frac{R_\infty}{D_{10}} \right)^2$$

$$R_\infty = R(1+z) \quad f_c = T_c/T_{\text{eff},\infty} \quad D = D_{10} \times 10 \text{ kpc}$$

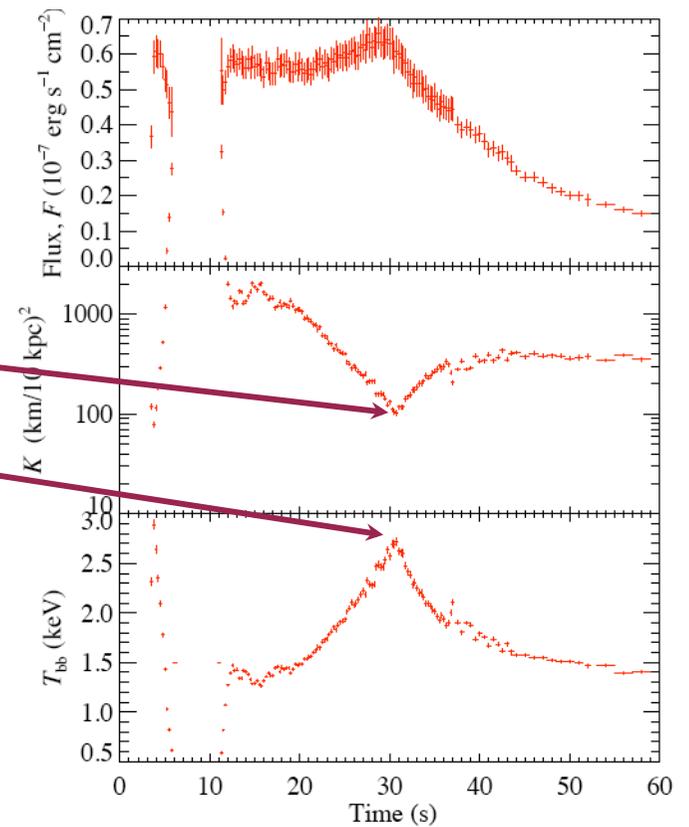
$$1+z = (1-u)^{-1/2} \quad u = 2GM/Rc^2$$

$\kappa_e = 0.2(1+X) \text{ cm}^2 \text{ g}^{-1}$ is the electron scattering opacity.



Often it is assumed that the Eddington flux is reached during the “touchdown” (when blackbody normalization reaches minimum and color temperature maximum).

In addition to the blackbody radius at the cooling tail, one needs the color-correction to get the apparent radius at infinity. Often it is assumed that $f_c = 1.4$

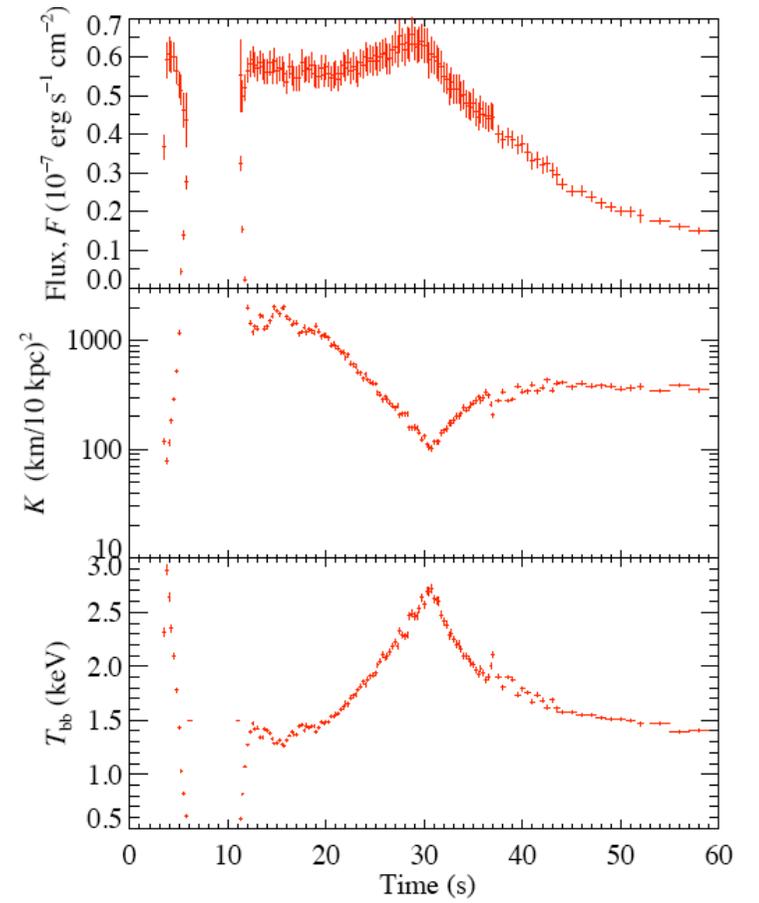
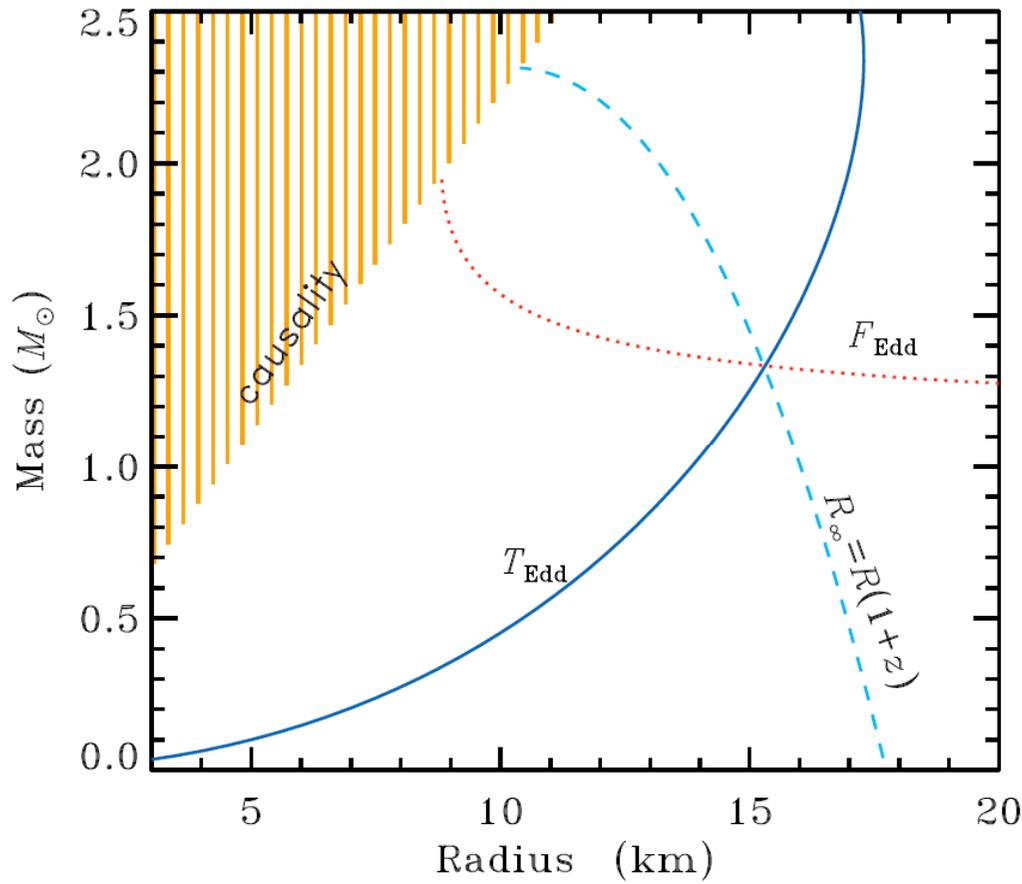


$$F_{\text{Edd}} = \frac{L_{\text{Edd}}}{4\pi D^2} = \frac{GMc}{D^2 \kappa_e (1+z)} \rightarrow R = 14.138 \text{ km} \frac{(1+X) D_{10}^2 F_{-7}}{u \sqrt{1-u}}$$

$$R_{\infty} = R_{bb} f_c^2 = D_{10} \sqrt{K} f_c^2 \rightarrow R = D_{10} \sqrt{K} f_c^2 \sqrt{1-u}$$

Solution exists (two curves cross) only if

$$D \leq D_{\text{max}} = \frac{\sqrt{K} f_c^2 c^3}{8 F_{\text{Edd}} \kappa_e} = \frac{0.177}{(1+X) A^2 F_{\text{Edd},-7}} \text{ kpc}$$



$$F_{\text{Edd}} = \frac{L_{\text{Edd}}}{4\pi D^2} = \frac{GMc}{D^2 \kappa_e (1+z)} \rightarrow R = 14.138 \text{ km} \frac{(1+X)D_{10}^2 F_{-7}}{u\sqrt{1-u}}$$

$$R_{\infty} = R_{bb} f_c^2 = D_{10} \sqrt{K} f_c^2 \rightarrow R = D_{10} \sqrt{K} f_c^2 \sqrt{1-u}$$

$$T_{\text{Edd},\infty} = \left(\frac{gc}{\sigma_{\text{SB}} \kappa_e} \right)^{1/4} \frac{1}{1+z} = 6.4 \times 10^9 A F_{\text{Edd}}^{1/4} \text{ K}, \quad A = (R_{\infty}/D_{10})^{-1/2} = K^{-1/4}/f_c$$

“Touchdown method”

Assumption: Eddington flux = touchdown flux

However,

- The relation between touchdown flux and Eddington flux is not clear (e.g. electron opacity is assumed to be Thomson, while at 3 keV it is 93% of Thomson)
- Color correction in the tail is not a unique number.
- Measurements of the Eddington flux and the apparent area in the tail are decoupled. Not clear whether they are consistent with each other.

Cooling tail method

The observed evolution of $K^{-1/4}$ vs. F should look similar to the theoretical relation f_c vs. F/F_{Edd}

$$K = \left(\frac{R_{bb}}{D_{10}} \right)^2 = \frac{1}{f_c^4} \left(\frac{R_\infty}{D_{10}} \right)^2 \longrightarrow K^{-1/4} = A f_c (F / F_{\text{Edd}})$$

From the fits a more reliable estimate $A = (R_\infty[\text{km}]/D_{10})^{-1/2}$ and apparent radius can be obtained.

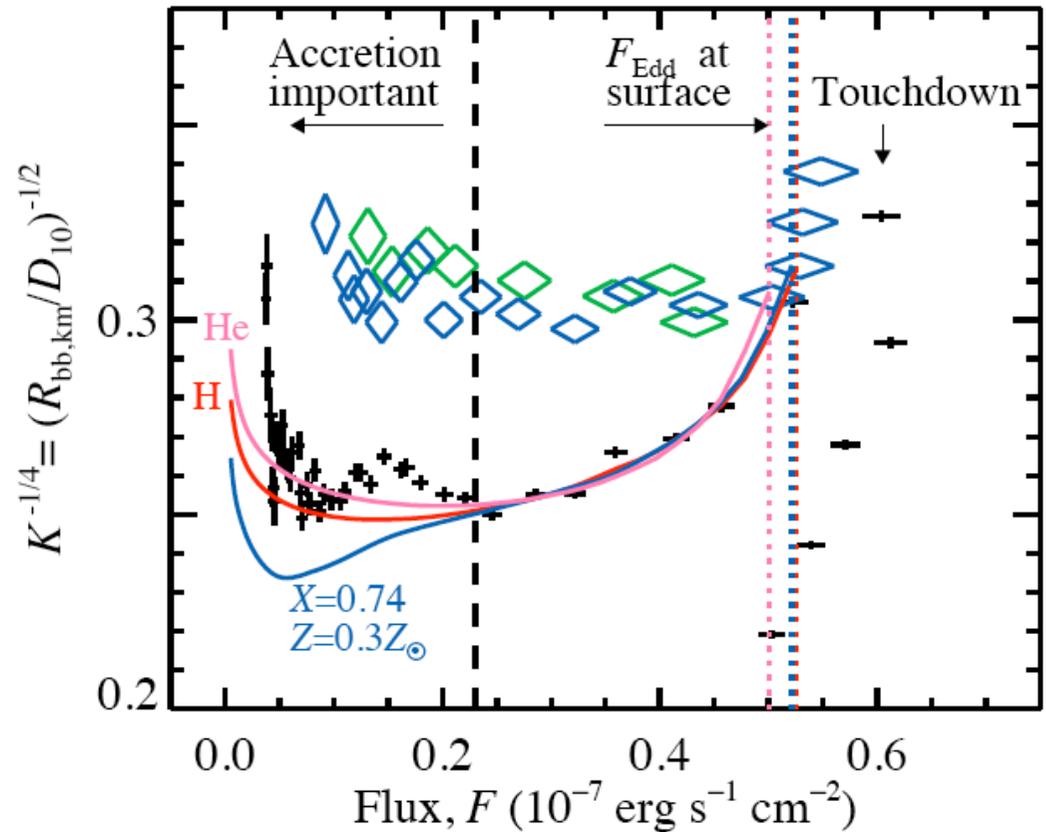
Two free parameters: A and F_{Edd} .

and we use now our theoretical dependences

$$f_c \text{ vs. } F/F_{\text{Edd}}$$

PRE bursts in 4U1724-307 (Terzan 2)

1. Distance of 5.3-7.7 kpc.
2. Long, >150 sec, PRE burst during **hard/low state** on Nov 8, 1996 follows the theory, while
3. short bursts in the soft state do not!



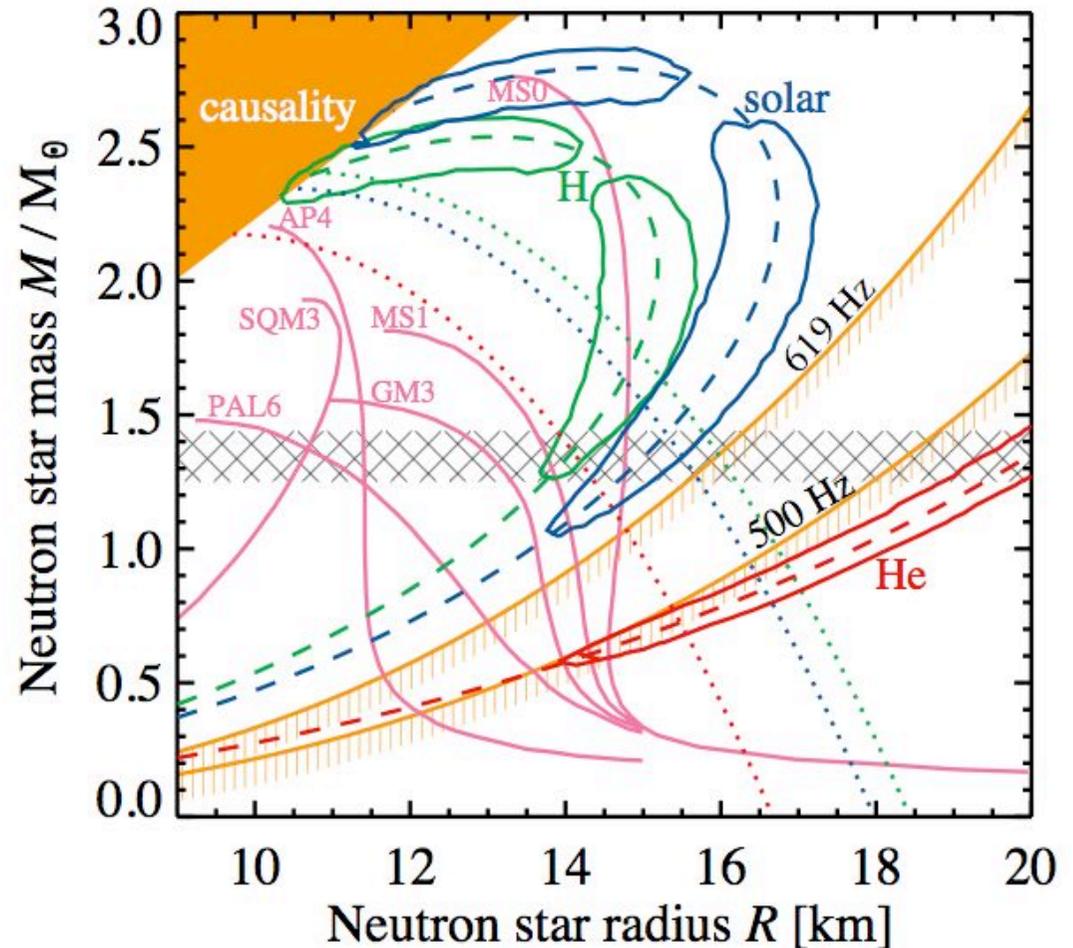
$$K^{-1/4} = f_c A, \quad A = (R_{\infty}[\text{km}]/D_{10})^{-1/2}.$$

Two free parameters: **A** and **F_{Edd}**.

M-R relation

From the best-fit A and F_{Edd} , we can get constraints on M and R if we assume some distance distribution (we take flat in 5.3-7.7 kpc with gaussian tails).

1. Radius > 13.5 km at 90% confidence for any solar composition for $M < 2.3$ solar.
2. Hydrogen-rich atmosphere is preferred.
3. Stiff EoS is preferred.



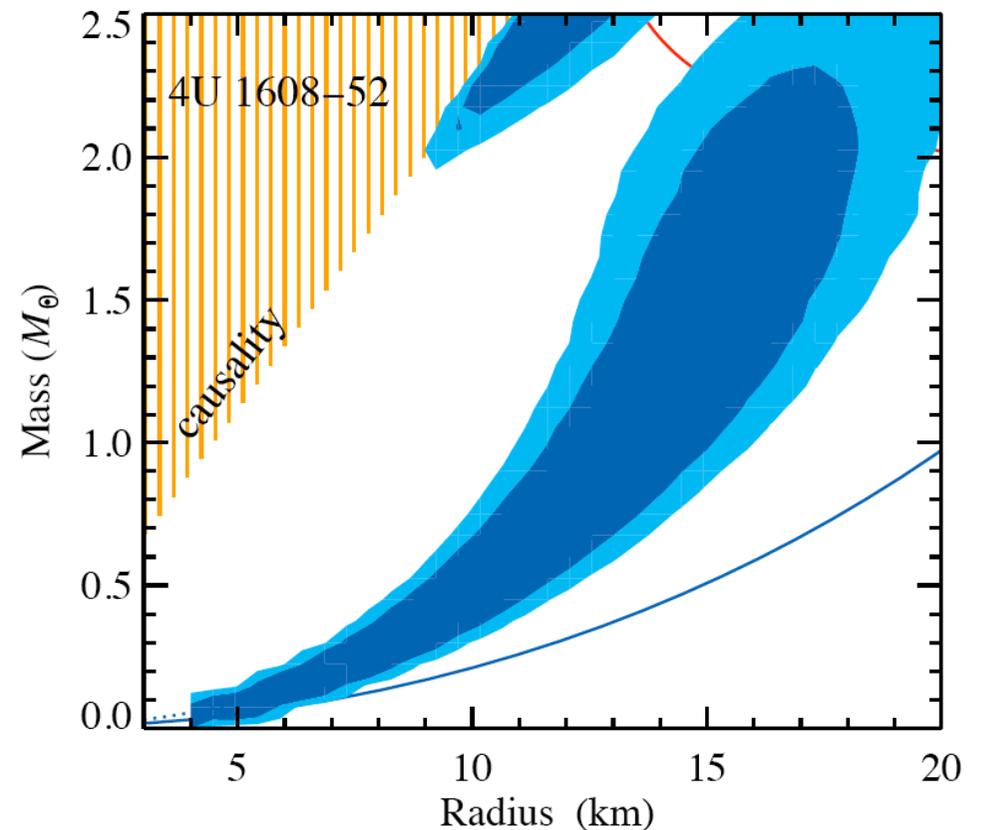
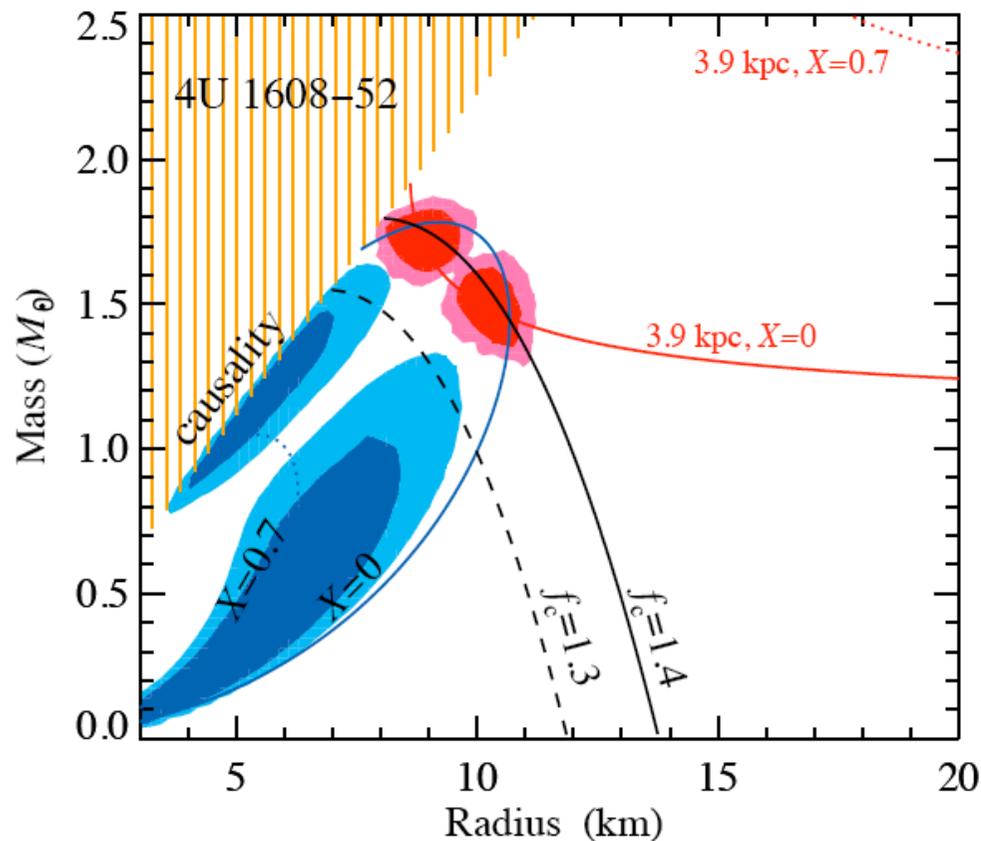
Contours are elongated along $T_{\text{Edd}} = \text{const}$ track

$$T_{\text{Edd},\infty} = \left(\frac{gc}{\sigma_{\text{SB}} \kappa_{\text{e}}} \right)^{1/4} \frac{1}{1+z} = 6.4 \times 10^9 A F_{\text{Edd}}^{1/4} \text{ K.}$$

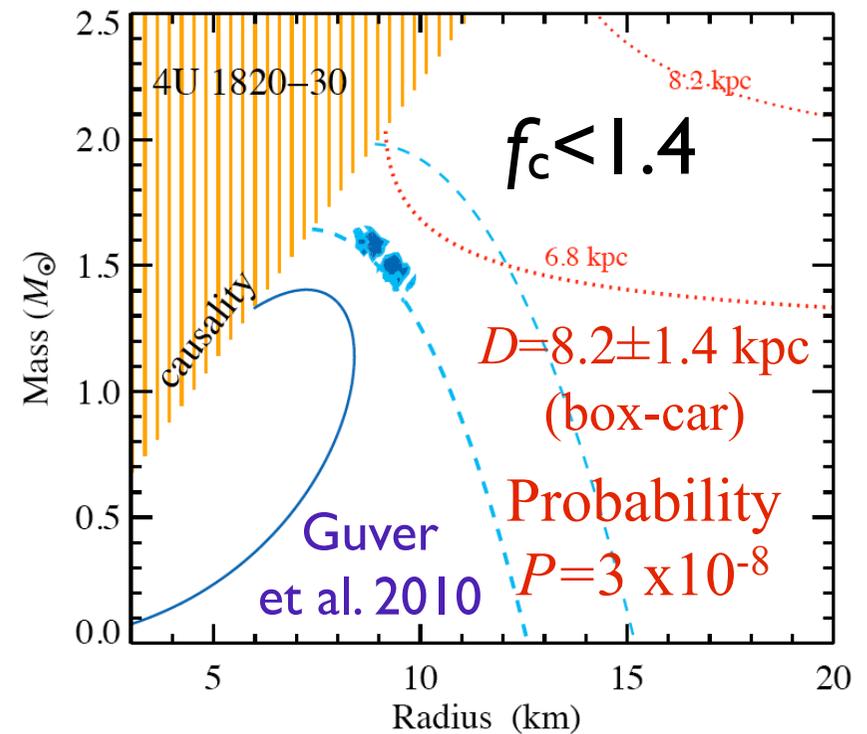
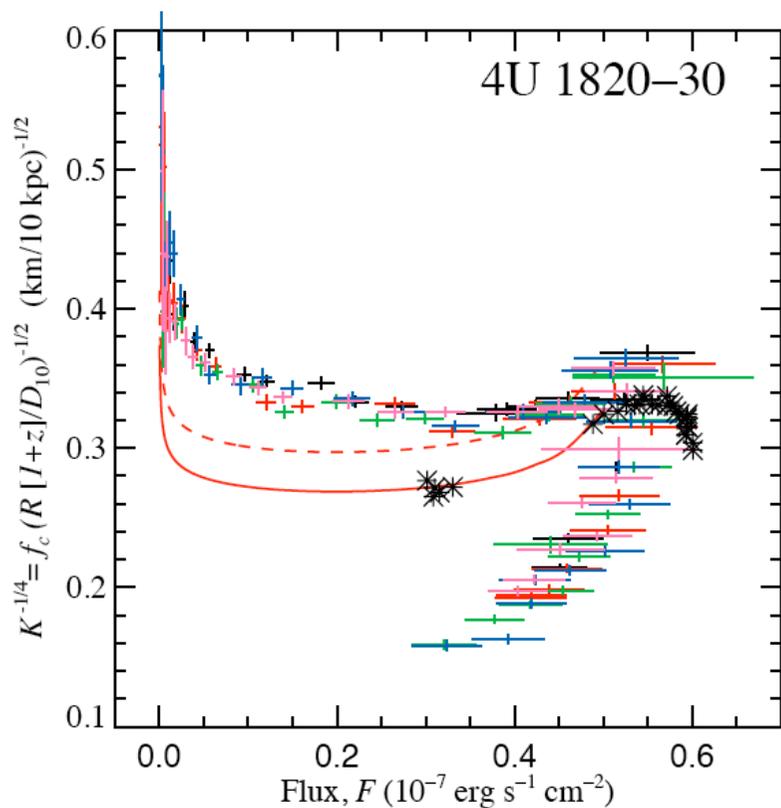
4U 1608-52

short bursts @ high persistent flux
and constraints using touchdown
method (Guver et al. 2010)

longer bursts @ low persistent flux
and constraints using the
cooling tail method.

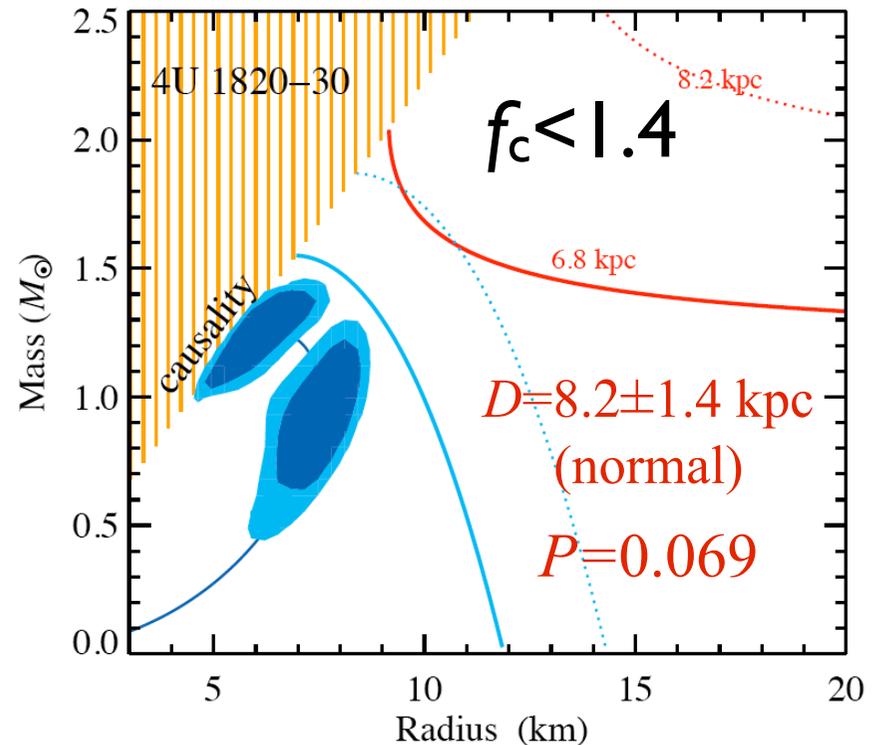


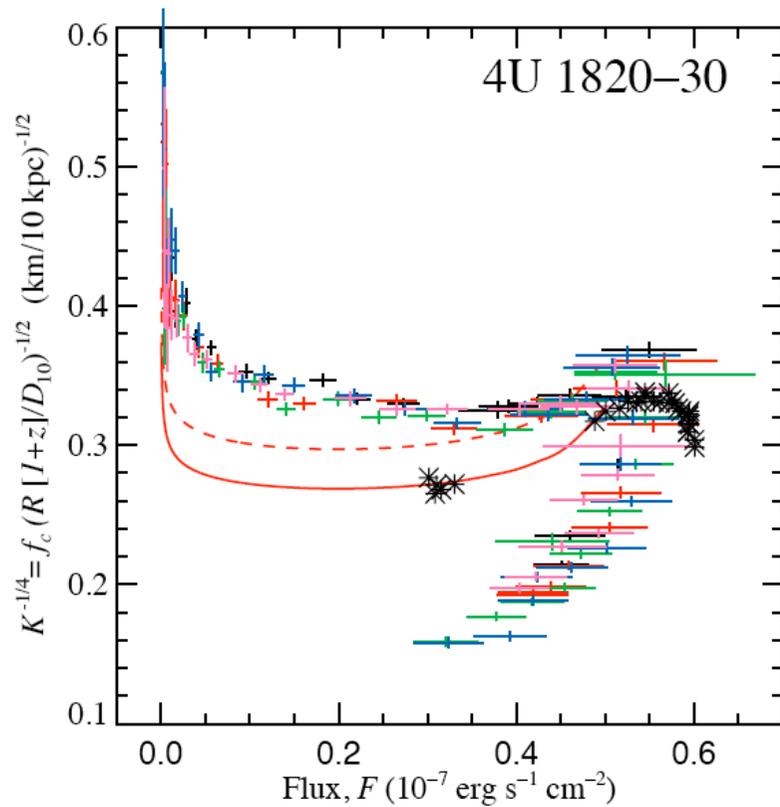
Neutron star radii determined from short bursts are
underestimated by >50%



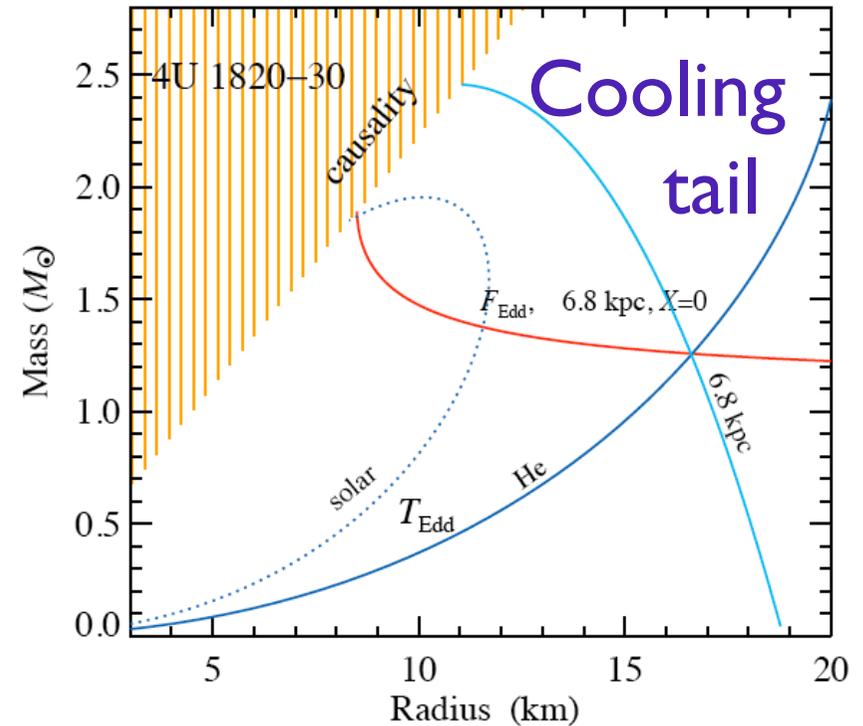
“Standard” analysis assuming $f_c < 1.4$
and taking $D > 6.8$ kpc gives
 $R = 9.0 \pm 1.0$ km.

Assuming a normal distribution of
the distance, gives $R < 8.5$ km.





The apparent area during the superburst is twice as large as the area for short bursts!
 The cooling tail method for the superburst gives $R > 11$ km (solar) and $R > 14$ km (He).

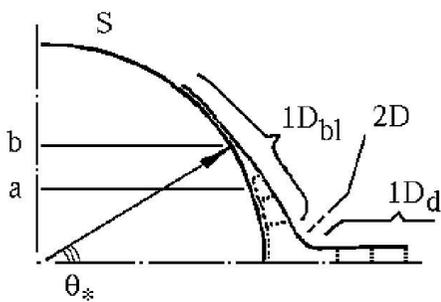
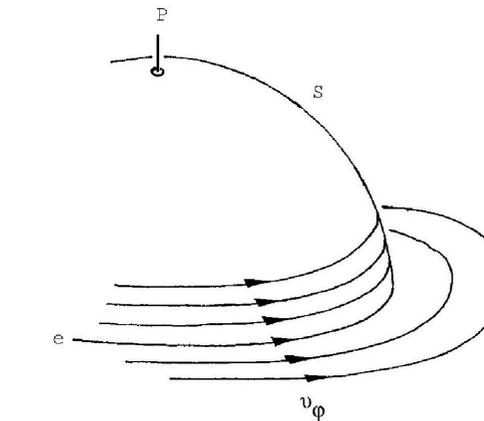


Why the apparent area
is different in different
bursts?

Influence of accretion on the burst
apparent area and the spectra

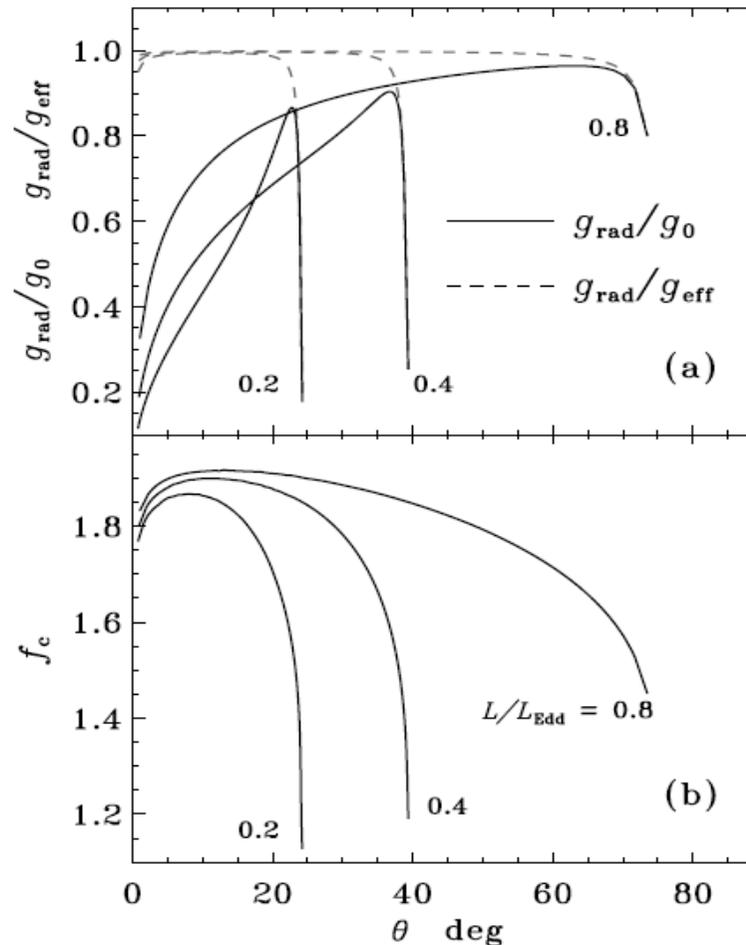
1. Accretion disk can blocks nearly 1/2 of the star.
2. Spreading of matter on NS surface affects the atmosphere structure increasing f_c

Inogamov & Sunyaev (1999)



(a)

Suleimanov & Poutanen (2006)



radiative acceleration/
gravitational
radiative / effective

Spectra are nearly
diluted blackbodies
with color correction

$$f_c = T_c / T_{\text{eff}} = 1.8$$

Conclusions

1. We have computed a new set of model atmospheres for X-ray bursts covering large range of luminosities, various $\log g$ and chemical composition.
2. Burst properties depend on persistent flux. Optically thick **accretion disk** blocks nearly 1/2 of the star and possibly affects the short burst (soft state) spectra. In the long bursts, accretion is not important (optically thin).
3. Evolution of blackbody normalization with flux $K^{-1/4}$ vs. F in long bursts is well described by the theory. Short PRE bursts from 4U 1724-307, 4U 1608-52 and 4U 1820-30 do not show the evolution of $K^{-1/4}$ vs. F predicted for a passively cooling neutron star, therefore they should not be used for M/R determination.
4. Neutron star radii are constrained at $R > 13.5$ km favoring stiff equation of state (consistent with existence of the $2M_{\odot}$ pulsar).