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Numerical solutions of force-free twisted magnetosphere in NSs

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Abstract

The study of the magnetosphere in magnetars is important to understand the properties of observed spectra. Since the sixties, different force-free magnetosphere models have been proposed for the pulsar scenario. These models are particular solutions of the ideal MHD equations, in which inertial, collisional, and gravitational terms are neglected, due to the overwhelming electromagnetic forces. We present new solutions obtained with a numerical relaxation approach that provides stable axisymmetric configurations. The geometry of the current distribution and toroidal magnetic field is generally different from self-similar solutions. Finally, we estimate the effect on the observed spectra of resonant Compton scattering through magnetospheres corresponding to different geometries of force-free twisted configurations.

Introduction

The configuration of a neutron star (NS) magnetosphere is of particular interest for magnetars (see the review by Mereghetti (2008)), which are believed to be NSs with a strong magnetic field (MF), whose decay powers their emission and explains their high luminosity. In this respect, the toroidal field plays a crucial role, because it implies the presence of strong currents threading the magnetosphere. In a twisted magnetosphere, the density of the charge carriers can be of several orders of magnitude higher than the Goldreich-Julian density, resulting in a non-negligible effect in the magnetospheric radiation processes. The theoretical study of such twisted configurations has been subject of many recent works, that simulate the radiation processes through the magnetosphere (Lyutikov & Gavriil 2006, Fernandez & Thompson 2007, Nobili et al. 2008). The observed spectra, that typically consist of a blackbody plus a high-energy power-law, are generally well fitted by the models. All these works, however, rely on a particular class of twisted configurations: the semi-analytical self-similar models proposed by Thompson et al. (2002). Our aim in this work is to find more general solutions to the ideal MHD equations in a magnetar magnetosphere end explore the influence of alternative solutions in the emerging spectra.

1. The pulsar equation

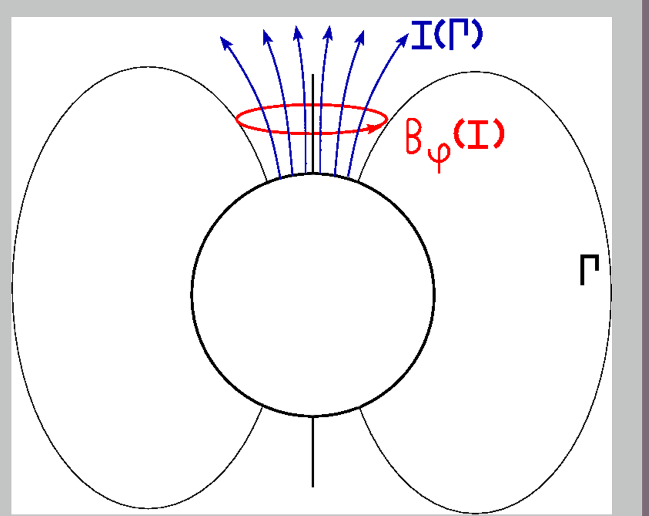
We consider a non-rotating NS with a MF $\vec{B}$. Neglecting inertia, pressure, gravity, the ideal MHD equilibrium conditions are reduced to the requirement of vanishing Lorentz force:

$$\vec{J}\times\vec{B}=0$$

where $\vec{J}=(c/4\pi)\vec{\nabla}\times\vec{B}$ is the current density. Assuming axial symmetry, we can express the poloidal MF with the magnetic flux scalar function Γ:

$$\vec{B}_p=\frac{\vec{\nabla}\Gamma(r,\theta)\times\hat{\phi}}{r\sin\theta}$$

where we have used spherical coordinates, (r,θ,ϕ). The function Γ actually labels the MF lines (i.e. surfaces S_Γ in axial symmetry). An integrability condition implies that the toroidal field is related with the poloidal field by the relation:



$$B_\phi=\frac{2}{cr\sin\theta}I(\Gamma)$$

where $I(\Gamma)$ is the current flowing between the axis and the poloidal surface S_Γ. The choice of this function is free, and this leads to a degeneracy of possible models with the same poloidal field. $I(\Gamma)$ enters as a source term in the equilibrium equation, which can be written as the so-called pulsar equation:

$$(\Gamma_{zz}+\Gamma_{\rho\rho})-\frac{1}{\rho}\Gamma_\rho=-\frac{4}{c^2}I(\Gamma)\frac{dI}{d\Gamma}$$

where subscripts indicate the partial derivatives with respect to the cylindrical coordinates $z=r\cos\theta$ and $\rho=r\sin\theta$. In literature, the most popular models are self-similar solutions, in which the radial dependence of all three MF components are given, by construction, by a power law $B_i\sim r^{-p}$, where the value of p is related to the amount of line twist. Self-similar models are very easy to implement, but they lack of generality since they are based on a peculiar choice of functions Γ and $I(\Gamma)$. Finding more general analytical solutions, not restricted to particular choices of the two scalar functions is, in general, not possible. As a matter of fact, one has to face the built-in freedom of choice with no indication of which is the most reasonable option from a physical point of view.

2. A Numerical approach

Because of the limitations of the analytical approach, we have developed a numerical code, based on the magnetofrictional method (Roumeliotis et al. 1994). In this approach, we introduce a fictitious velocity field proportional to the Lorenz force $\vec{v}_f=\vec{J}\times\vec{B}/B^2$. The resulting electric field $\vec{E}_f=-\vec{v}_f\times\vec{B}=\vec{J}-(\vec{J}\cdot\vec{B})\vec{B}/B^2$ mimics a frictional term and is responsible for the MF (fictitious) evolution by means of the induction equation:

$$\partial_t\vec{B}=-\nabla\times\vec{E}_f\ ,$$

that leads the MF configuration to a stationary force-free solution. We employ a fully explicit, finite-difference time-domain scheme, that conserves divergence and helicity by construction. A magnetic field configuration must be provided as initial guess. With appropriate boundary conditions imposed on the fictitious electric field, the code is able to relax to a completely stationary solution, in which the Lorentz force is zero at round-off level everywhere. We have tested the code against the self-similar solutions described above and some perturbed solutions of the dipole vacuum (Timokhin et al. 2008).

3. Numerical solutions

We have obtained solutions for different initial profiles. Even if the code conserves the helicity, this parameter is not enough to uniquely characterize the static configuration found after convergence. As a matter of fact, the solution depends on the starting point, since convergence is easier towards nearby solutions in a multi-parameter space. Here we compare two different configurations obtained when we employ as initial guess a MF consisting of a vacuum dipole plus a dipolar toroidal field with different radial dependences (more details in Viganò et al. (2011)). Even if the models are normalized to have the same magnetic helicity, the geometry of the toroidal field and the current pattern is different. Furthermore, the decay with distance of the toroidal field can be faster or slower than the poloidal components. This implies a different distribution of current and charge across the magnetosphere, which in turn has implications on radiative processes involving scattering with the magnetospheric charged particles.

4. Resonant optical depth

The Thomson scattering cross-section is modified in presence of a strong MF. For a NS magnetosphere, the dominant contribution in the X-ray band comes from the resonance at $\omega=\omega_{res}=ZeB/mc$, where Ze and m are the charge and mass of the scattering particle. For electrons, the resonant energy is $11.6\,B[10^{12}\,G]$ keV. The scattering between photons (in the X-ray band) and electrons takes place typically at tens of kilometers from the surface, depending on the MF geometry and the photon energy. Neglecting the natural cyclotron width, the resonant cross-section (for incoming unpolarized photons) can be approximated by (Ventura, 1979; Canuto et al., 1971):

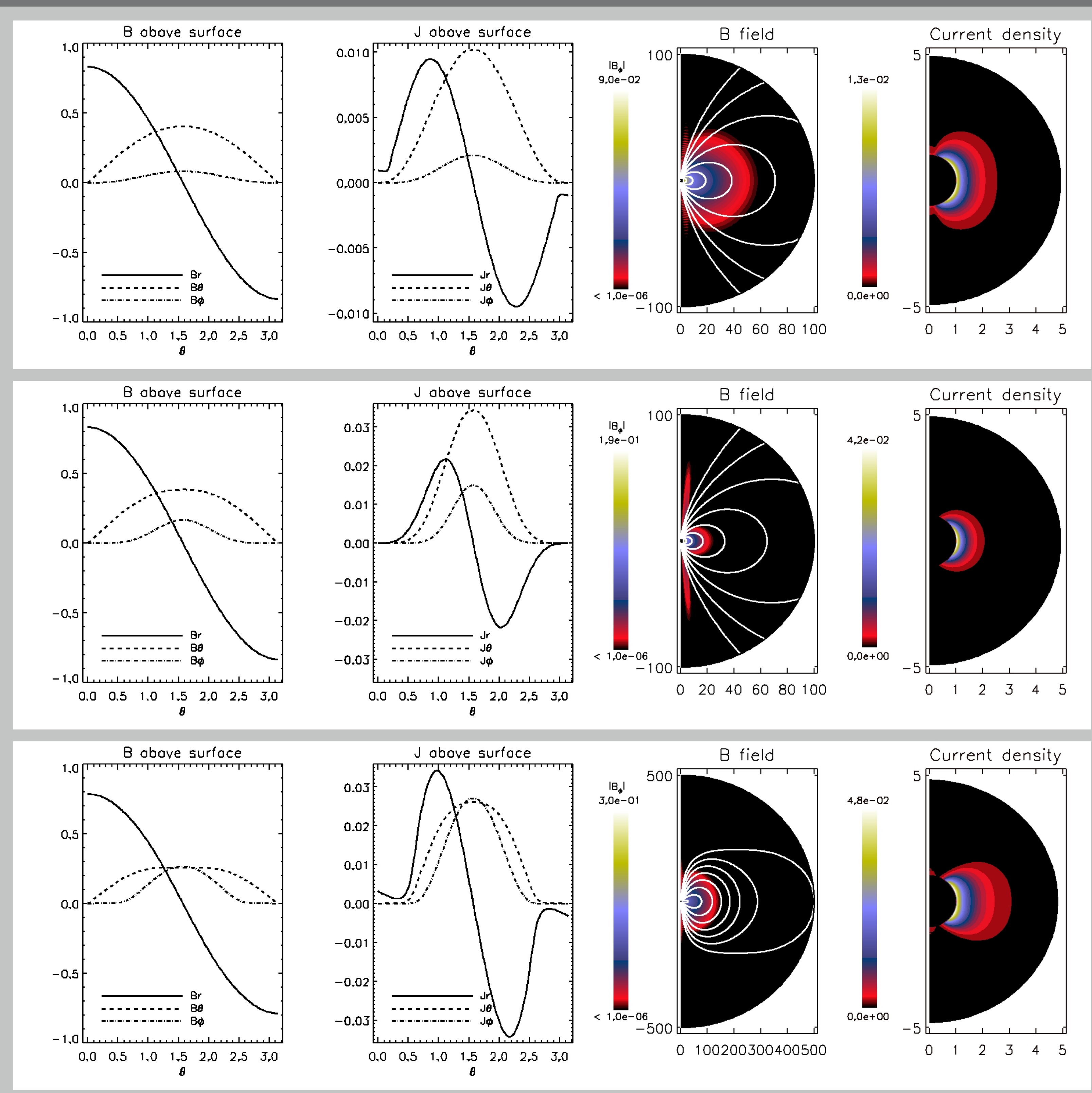
$$\sigma_{res}=\pi^2(1+\cos^2\theta_{kB})\frac{(Ze)^2}{mc}\delta(\omega-\omega_{res})$$

After integrating along the line of sight, the resonant optical depth can be estimated as:

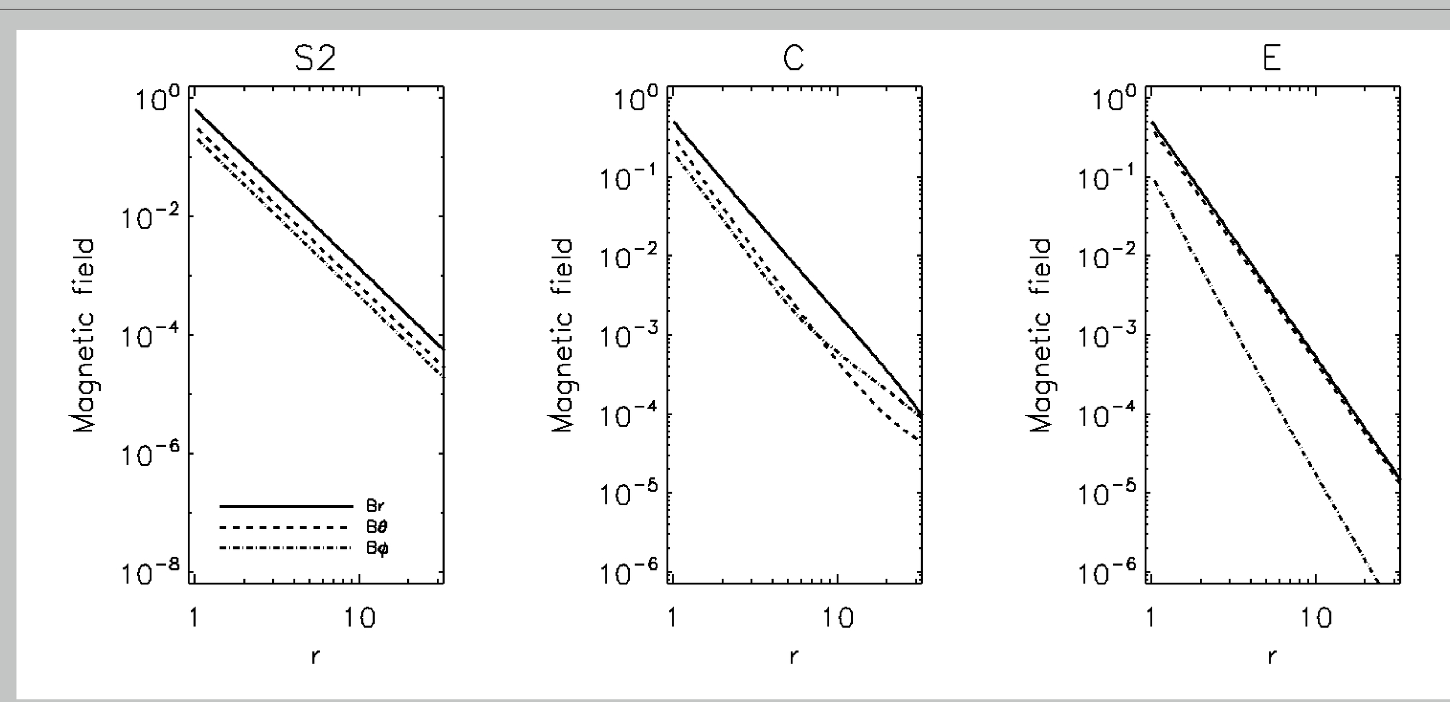
$$\kappa\tau_{res}(\theta)=\pi^2\frac{J}{c}\left(1+\frac{B_r^2}{B^2}\right)\left|\frac{dB}{dr}\right|^{-1}$$

where the factor $\kappa=J/cn_ZZe$ links the current intensity with the scatterers density and can be estimated of order $\sim\ O(1)$. Its value depends locally on which kind of particle (electrons, ions, positrons) are the scatterers, and on their bulk velocity. In the formula above κ is taken constant along the line of sight, for simplicity. Furthermore, thermal broadening, polarization effects and non-resonant contributions are not included in the cross-section: we underline that this is a very crude estimate of the resonant optical depth.

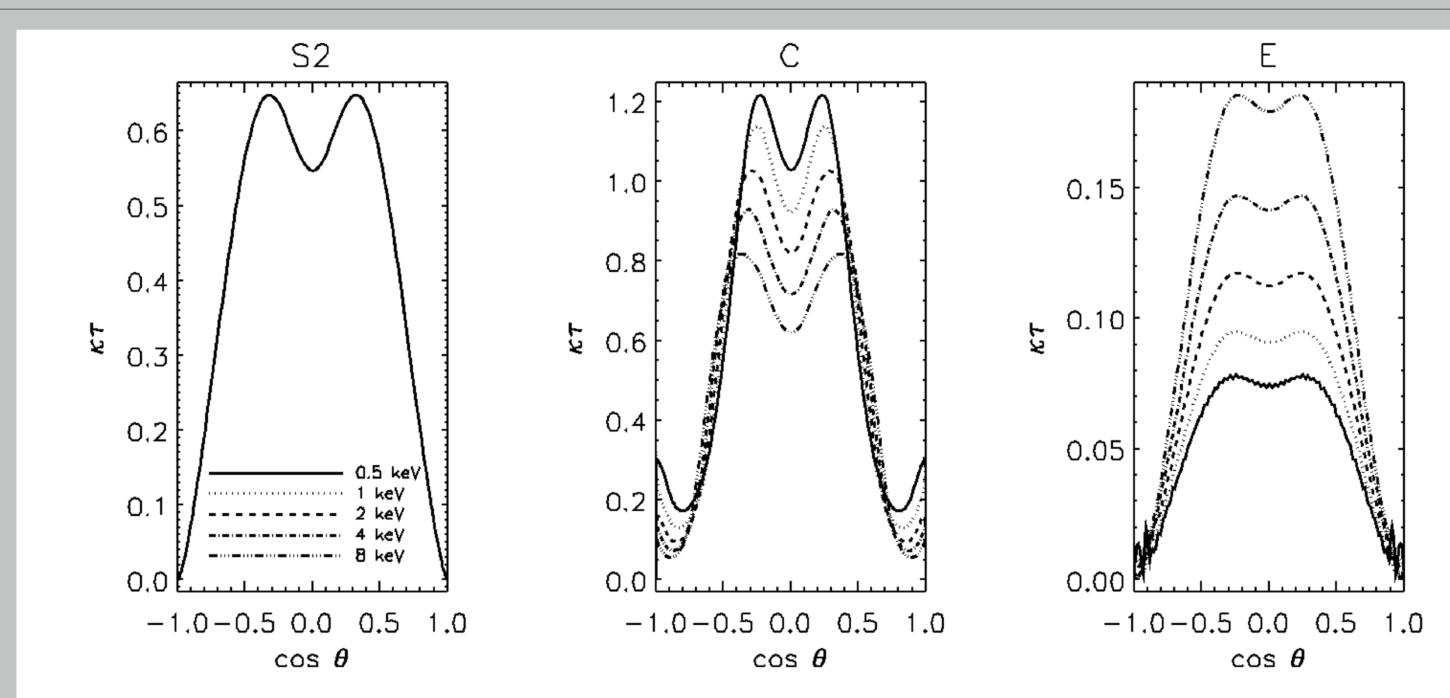
However, our aim is simply to explore the sensitivity of the optical depth to the MF geometry. Below we compare the estimated resonant optical depth for a self-similar solution and two numerical configurations. Note that in a self-similar model the ratio $J/|dB/dr|$ does not depend on the on the photon frequency, nor on the resonant radius (for each component i: $B_i\sim r^{-p}$, $J_i\sim r^{-(p+1)}$). On the contrary, the current patterns in our models are generally non-trivial. As a consequence we deal now with a dependence on the resonant radius, i.e., on frequency. Note that the optical depth can increase or decrease with photon energy, depending on models, and on the magnetic colatitude θ within the same configuration (see model C).



Three numerical models (B, E and C of Viganò et al. (2011)). On each row, from left: (1) angular profile of MF and (2) current components just above NS surface; (3) toroidal MF (colour scale) plus poloidal lines (white); (4) current density J.



Comparison between radial profile at $\theta=\pi/3$ of MF components in a self-similar (left) and numerical models (center and right).



Estimate of resonant optical depth (multiplied by the microphysical factor κ), as a function of magnetic co-latitude, for a self-similar model (left) and two numerical solutions (center and right).

Conclusions

In this work we have numerically solved the pulsar equation in the limit of no rotation. The solutions obtained differ significantly from the (semi-)analytical solutions usually found and used in literature. In particular, the different, non-trivial radial dependence implies that the resonant optical depth is dependent on the photon energy, unlike the self-similar models. We expect these solutions to be realistic near the surface of slowly rotating NSs (such as magnetars), since the rotationally-induced electric field is negligible in this case.

The following step is to implement these new models in a Monte Carlo code to study how the surface radiation is reprocessed in the magnetosphere.