Radio pulsars (thirty years after)

V.S.Beskin, A.V.Gurevich, Ya.N.Istomin

(R.Rafikov, A.Tchekhovskoy, A.Philippov, A.Zheltoukhov, E.Nokhrina, H.Hakobyan, L.Arzamasskiy, V.Prokofev)

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Radio pulsars

30 years – first pulsar publication

25 years – theory of the pulsar radio emission

20 years – book

Three points

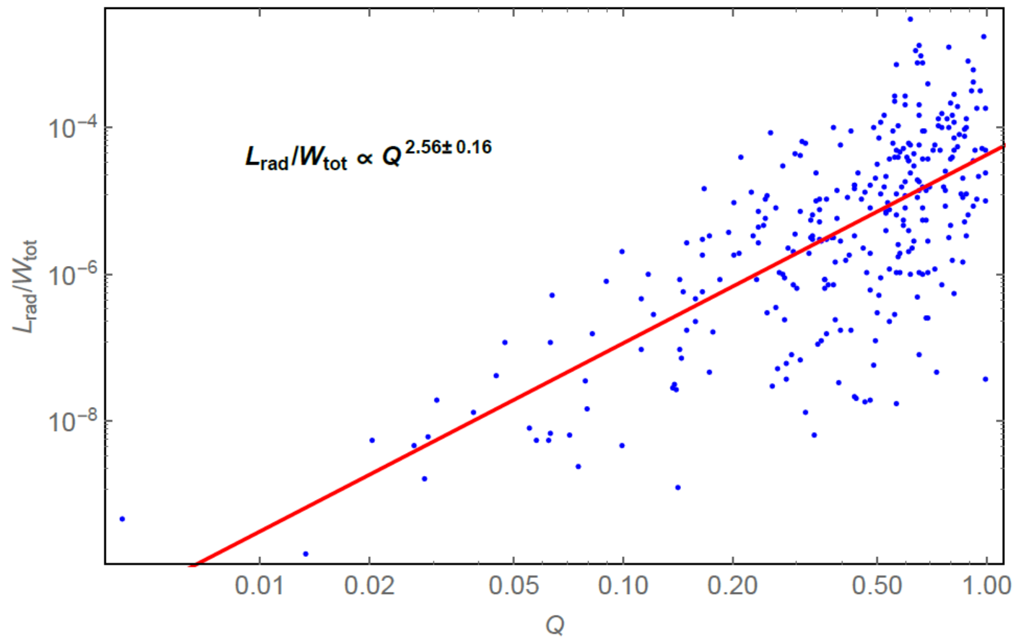
- Current losses
- Asymptotic behavior
- Anomalous torque

One more prediction

$$Q = 2\,P^{11/10}\,\dot{P}_{-15}^{\;-4/10} \quad \text{(RS – style gap)}$$

- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9}$ for $Q < 1$
- $W_{part}/W_{tot} \sim Q^2$

$\alpha = L_{rad}/W_{tot}$



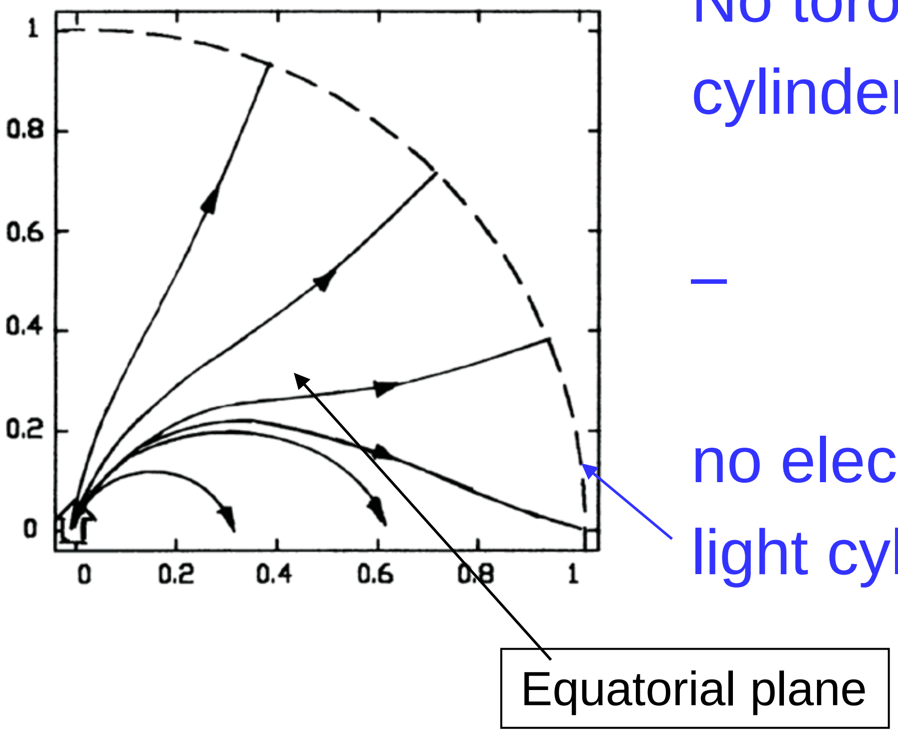
Very briefly

- Modern numerical simulations confirm the current loss mechanism of the radio pulsar braking.
- Current sheet in the pulsar wind asymptotical infinity (we do not beleive in) is essentially time-dependent.
- Anomalous torque acting on the body depends on its internal structure (as it depends on the angular momentum of the electromagnetic field inside the body).

Current losses

NO magnetodipole radiation

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)



No toroidal magnetic field at the light cylinder for zero longitudinal current

– $$B_\varphi \propto (1 - x_r^2)^2$$

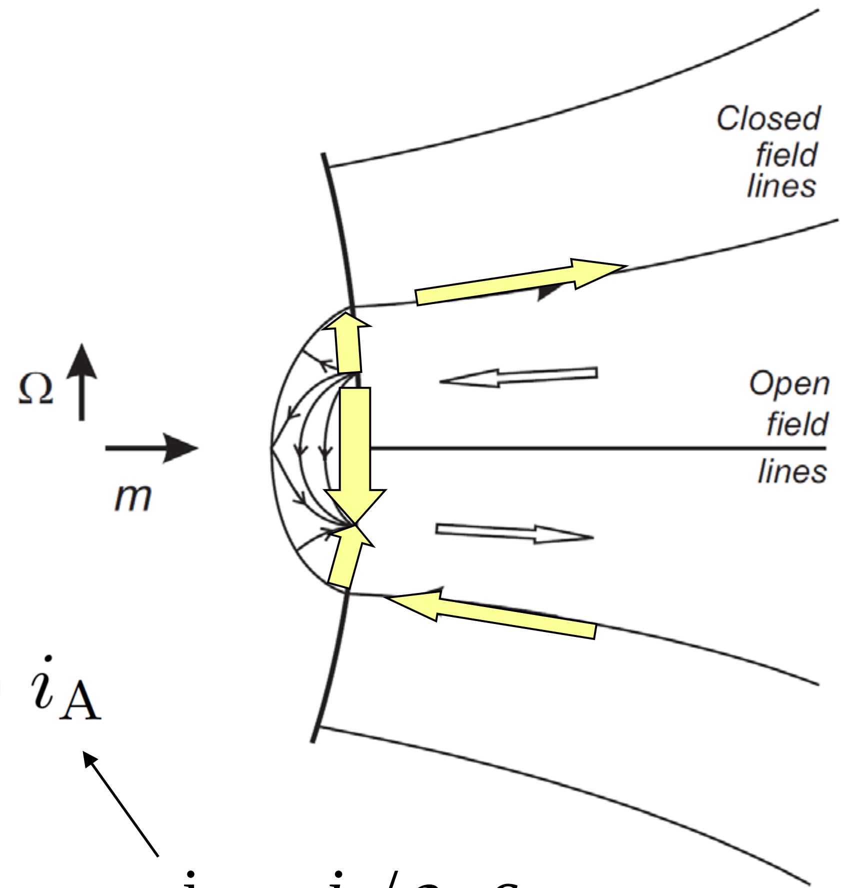
no electromagnetic flux through the light cylinder (dashed line).

Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

$$j_{\mathrm{GJ}} \approx \frac{\Omega B}{2\pi} \cos \theta_{\mathrm{m}}$$

$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]] \mathrm{d}S$$



$$W_{\mathrm{tot}}^{(90)} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c} \right) i_{\mathrm{A}}$$

$$\boxed{W_{\mathrm{tot}} \approx \frac{B_0^2 \Omega^4 R^6}{c^3} \cos^2 \chi}$$

$$\mathrm{i}_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$$

$\mathrm{i}_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$

BGI – main results

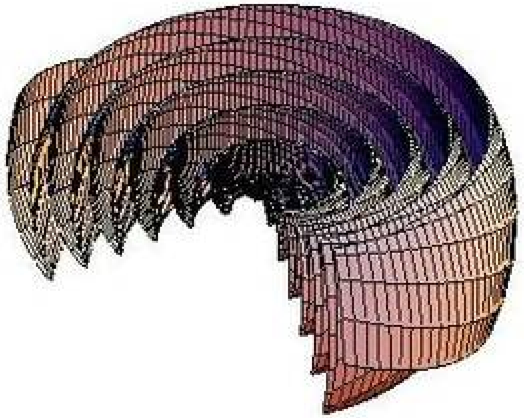
- No magnetodipole radiation
- **<u>All</u>** energy losses are to be connected with current losses

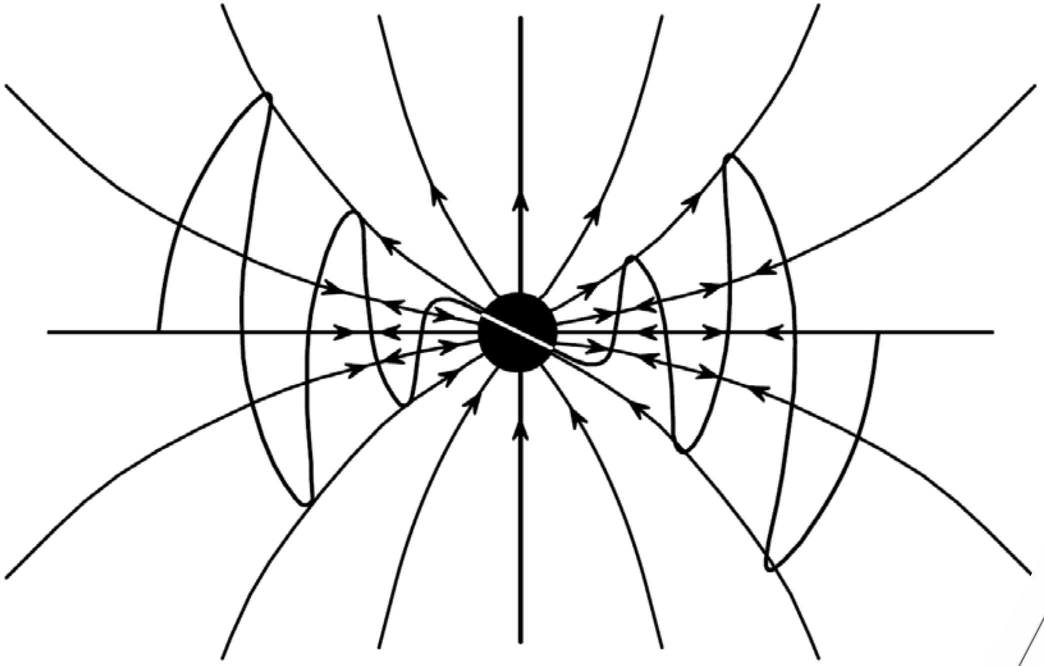
$$\boxed{W_{\rm tot} \approx \frac{B_0^2 \Omega^4 R^6}{c^3} \cos^2 \chi}$$

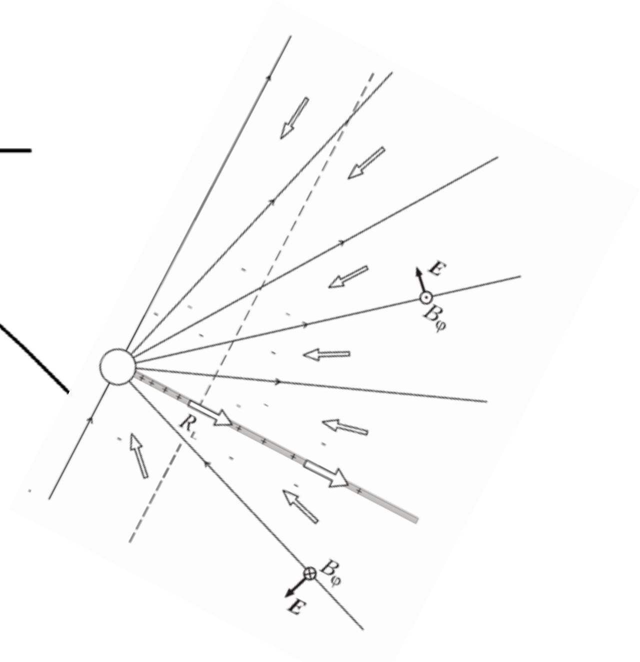
- Smaller energy losses for orthogonal rotator
- Inclination angle evolves to 90 deg.
- Existence of the back electric current along the separatrix

Orthogonal rotator – inclined split monopole

S.V.Bogovalov, A&A, **349**, 1017 (1999)

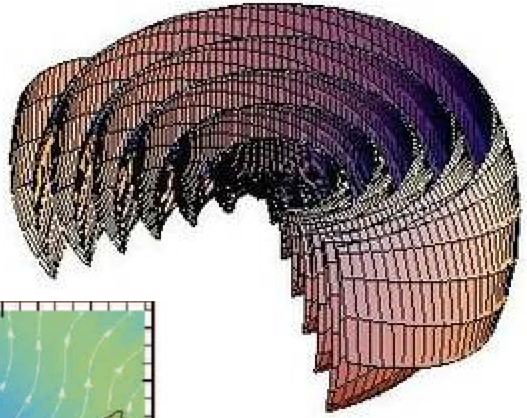


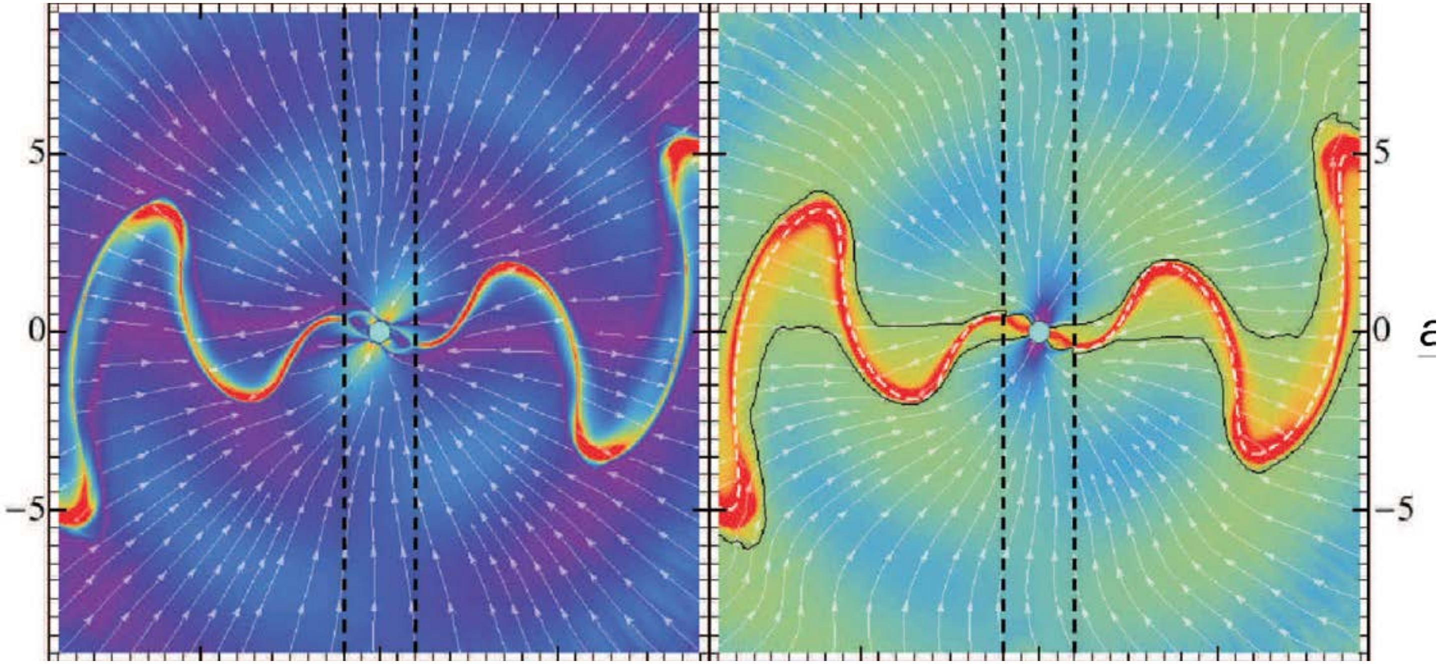




Orthogonal rotator – numerical

I.Contopoulos et al, (2012)





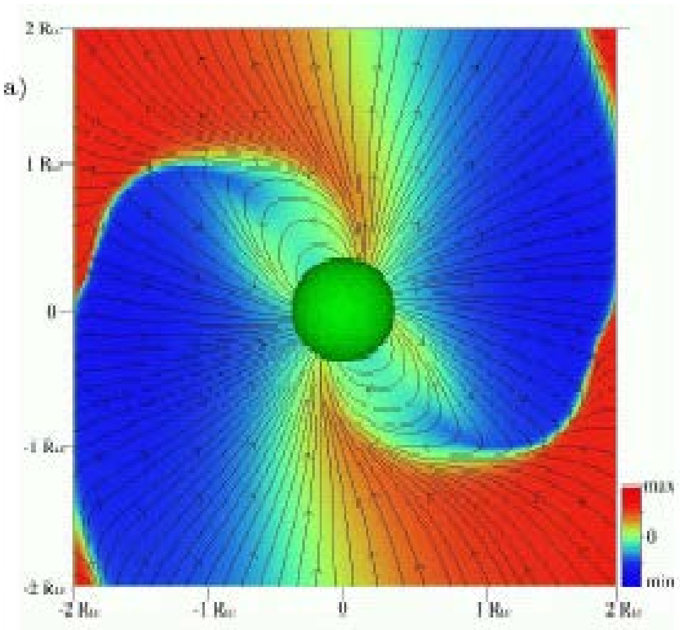
$$\Phi = \sin\alpha \sin\theta \sin(\varphi - \Omega t + \Omega r/c) + \cos\theta \cos\alpha$$

Split monopole – main results

- No magnetodipole radiation
- Energy losses do not depend on the inclination angle
- Existence of the current sheet separating magnetic fluxes
- No information about the inclination angle evolution

Incline rotator – numerical

A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

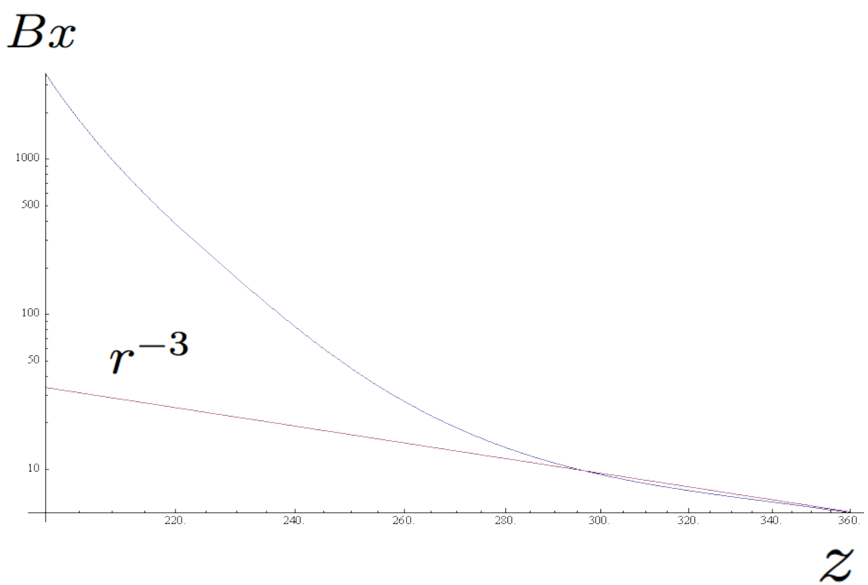


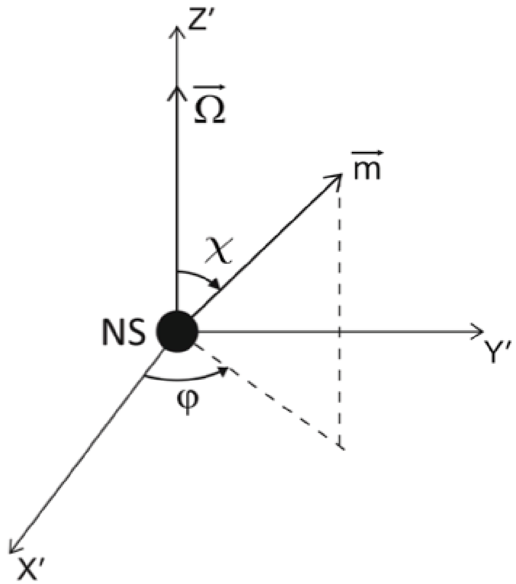
$$W_{\rm tot} \approx \frac{1}{4}\,\frac{B_0^2\Omega^4 R^6}{c^3}\,(1+\sin^2\chi)$$

- No magnetodipole radiation
- Back electric current along the separatrix
- Smaller energy losses for axisymmetric rotator

Spitkovsky solution, $\chi = 60^o$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

Current losses again

$$K_{\|} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[c_{\|}i_{\rm S} + \mu_{\|}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm A}\right],$$

$$K_{\perp} = -\frac{B_0^2\Omega^3R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\rm S} + c_{\perp}\left(\frac{\Omega R}{c}\right) i_{\rm A}\right]$$

$$J_r\frac{{\rm d}\Omega}{{\rm d}t} = K_{\|}\cos\chi + K_{\perp}\sin\chi,$$

$$J_r\Omega\frac{{\rm d}\chi}{{\rm d}t} = K_{\perp}\cos\chi - K_{\|}\sin\chi.$$

VB, A.V.Gurevich, Ya.N.Istomin, JETP **58**, 235 (1983)

Current losses again

$$K_{\|} = -\frac{B_0^2 \Omega^3 R^6}{c^3}\left[c_{\|} i_{\mathrm{S}} + \mu_{\|}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\mathrm{A}}\right],$$

$$K_{\perp} = -\frac{B_0^2 \Omega^3 R^6}{c^3}\left[\mu_{\perp}\left(\frac{\Omega R}{c}\right)^{1/2} i_{\mathrm{S}} + c_{\perp}\left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}\right]$$

$$J_r \frac{\mathrm{d}\Omega}{\mathrm{d}t} = K_{\|}^{(A)} + \left[K_{\perp}^{(A)} - K_{\|}^{(A)}\right]\sin^2\chi,$$

$$J_r \Omega \frac{\mathrm{d}\chi}{\mathrm{d}t} = \left[K_{\perp}^{(A)} - K_{\|}^{(A)}\right]\sin\chi\cos\chi.$$

A.Philippov, A.Tchekhovskoy, J.Li

$$\boxed{K_{\perp}^{(A)} = i_{\mathrm{A}}\left(\frac{\Omega R}{c}\right) K_{\|}^{(A)}}$$

Current losses again

$$J_r \frac{\mathrm{d}\Omega}{\mathrm{d}t} = K_{\parallel}^{(A)} + \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] \sin^2\chi,$$

$$J_r \Omega \frac{\mathrm{d}\chi}{\mathrm{d}t} = \left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] \sin\chi \cos\chi.$$

- One-to-one correspondence

  χ evolves to 90 deg. if $W_{tot}(0) > W_{tot}(90)$

  χ evolves to 0 deg. if $W_{tot}(0) < W_{tot}(90)$

$$K_{\perp}^{(A)} = i_{\mathrm{A}} \left(\frac{\Omega R}{c}\right) K_{\parallel}^{(A)}$$

- For BGI $i_A \sim 1$
- For Michel-Bogovalov $\left[K_{\perp}^{(A)} - K_{\parallel}^{(A)}\right] = 0$
- For Spitkovsky et al low the asymmetrical current is to be (much) larger than GJ one

$$i_A > (\Omega R/c)^{-1}$$

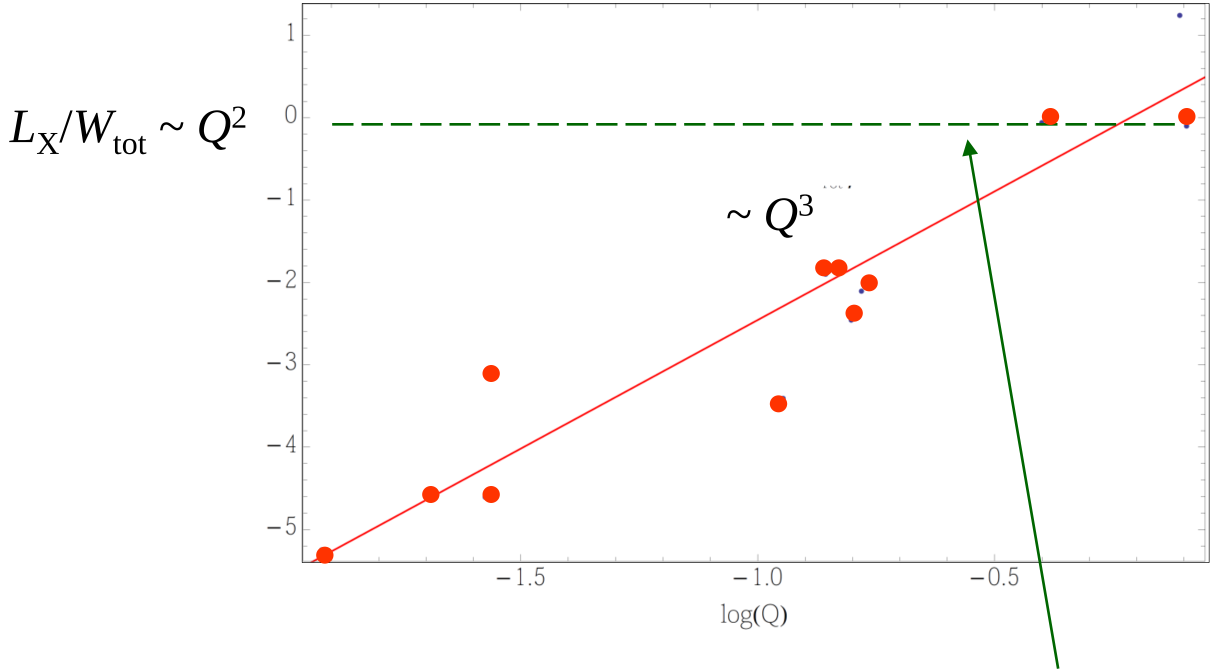
Main problems

- For orthogonal rotator the longitudinal current is to be 10000 (!!!) times larger than local GJ one.
- Heating problem.
- Magnetic field disturbance.

Heating problem

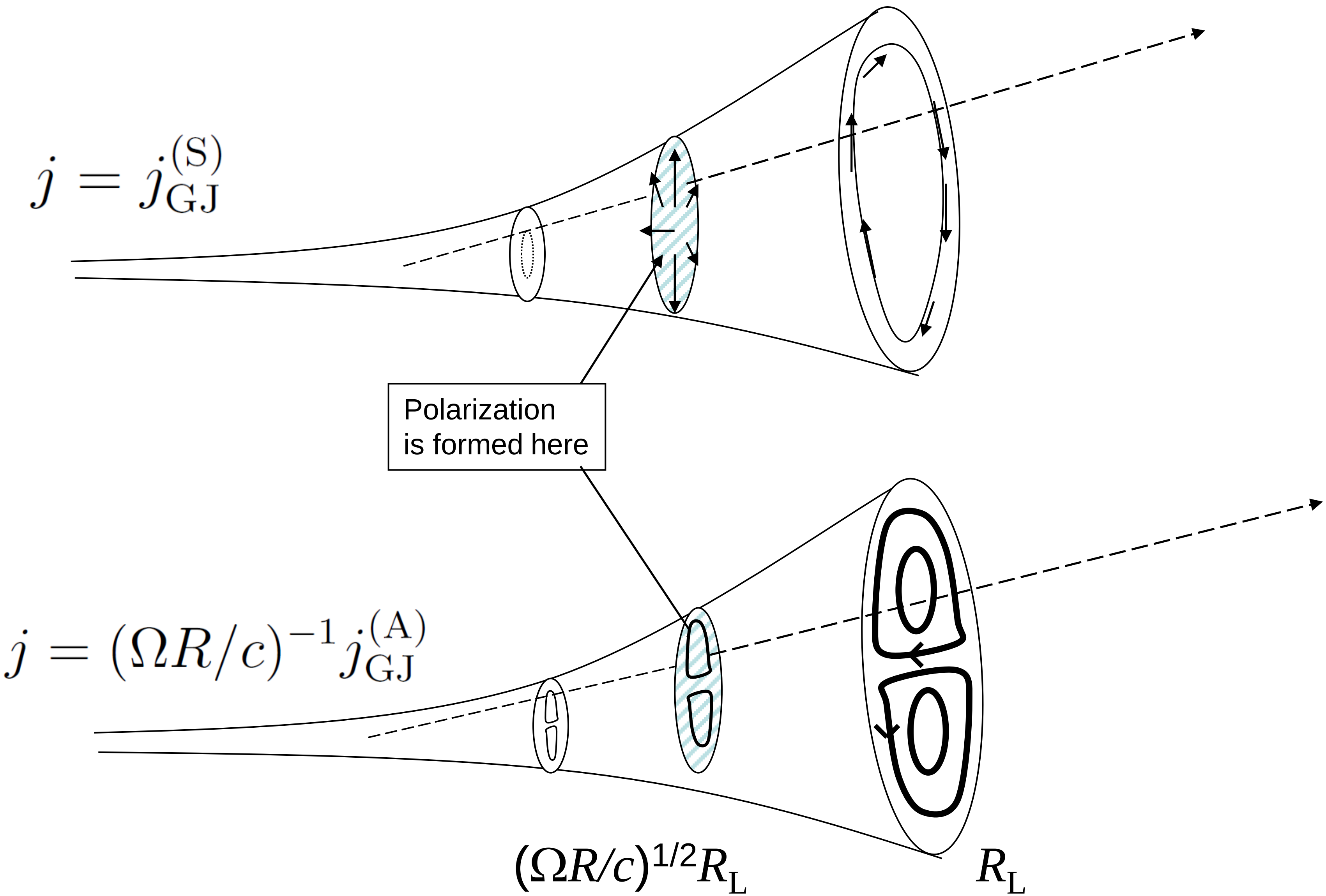
- $Q = 2\, P^{11/10}\, \dot{P}_{-15}{}^{-4/10}$

- $H/R_0 \sim Q\,, \quad r_{in}/R_0 \sim Q^{7/9}$



Istomin & Sobyanin (2010), Timokhin (2010), Timokhin & Arons (2013)

Magnetic field disturbance



A problem

Can the pair creation process generate large enough longitudinal current to avoid the formation of the light surface?

The main point of view – YES.

Our point of view – NO.

If not, the light surface $|\boldsymbol{E}| = |\boldsymbol{B}|$ is to exist.

Prediction

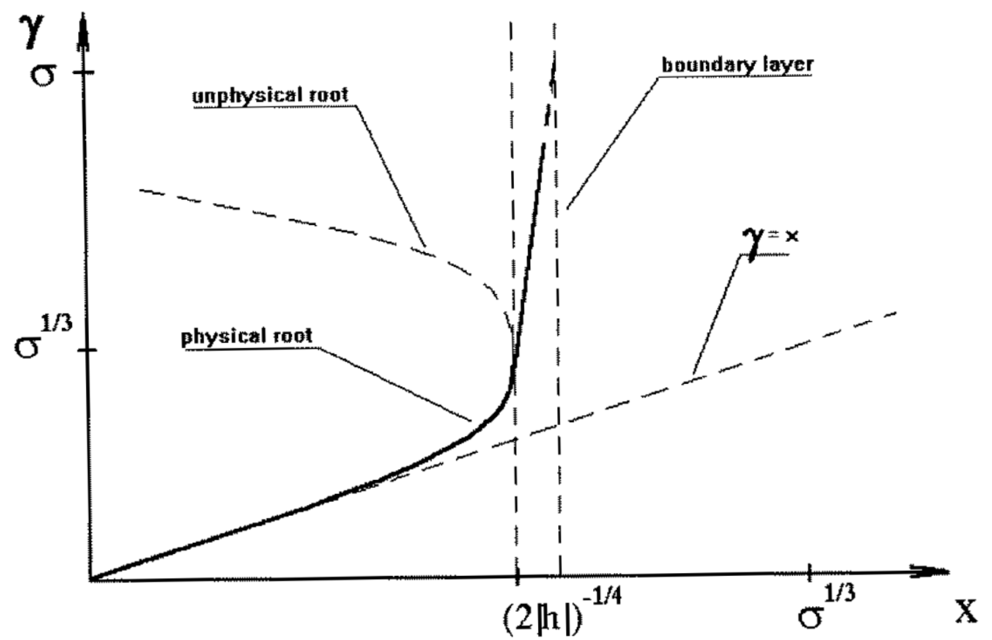
VB, A.V.Gurevich & Ya.N.Istomin, JETP, **85,** 235 (1983)
VB, R.R.Rafikov, MNRAS, **313**, 433 (2000)

- Narrow sheet $\Delta r \sim \mathrm{R_L}/\lambda$
- Effective particle acceleration up to $\gamma \sim \sigma$ (10^6 for Crab)

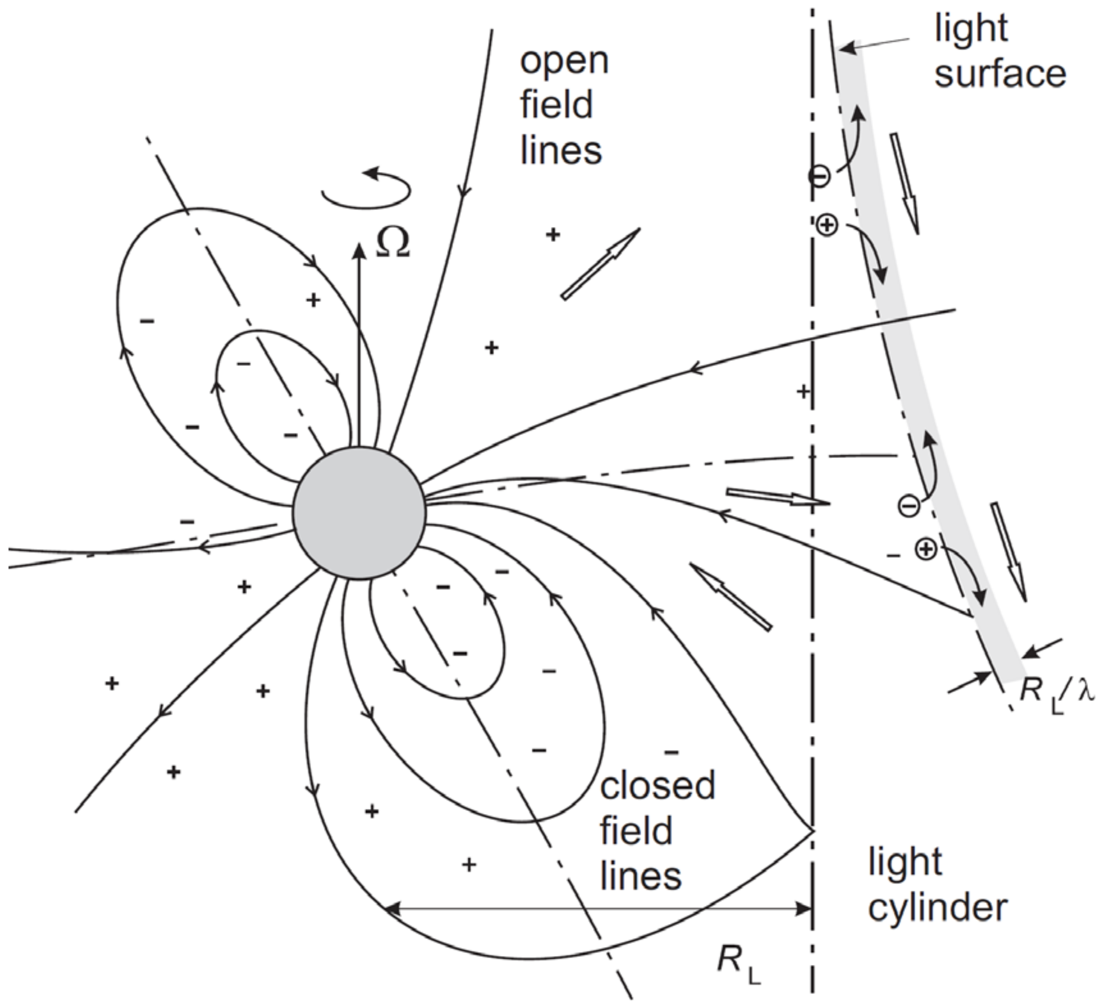
$$\lambda = \frac{n^{(\mathrm{lab})}}{n_{\mathrm{GJ}}}$$

$$\sigma = \frac{\Omega^2 \Psi_{\mathrm{tot}}}{8\pi^2 c^2 \mu \eta}$$

maximum bulk Lorentz-factor



Prediction



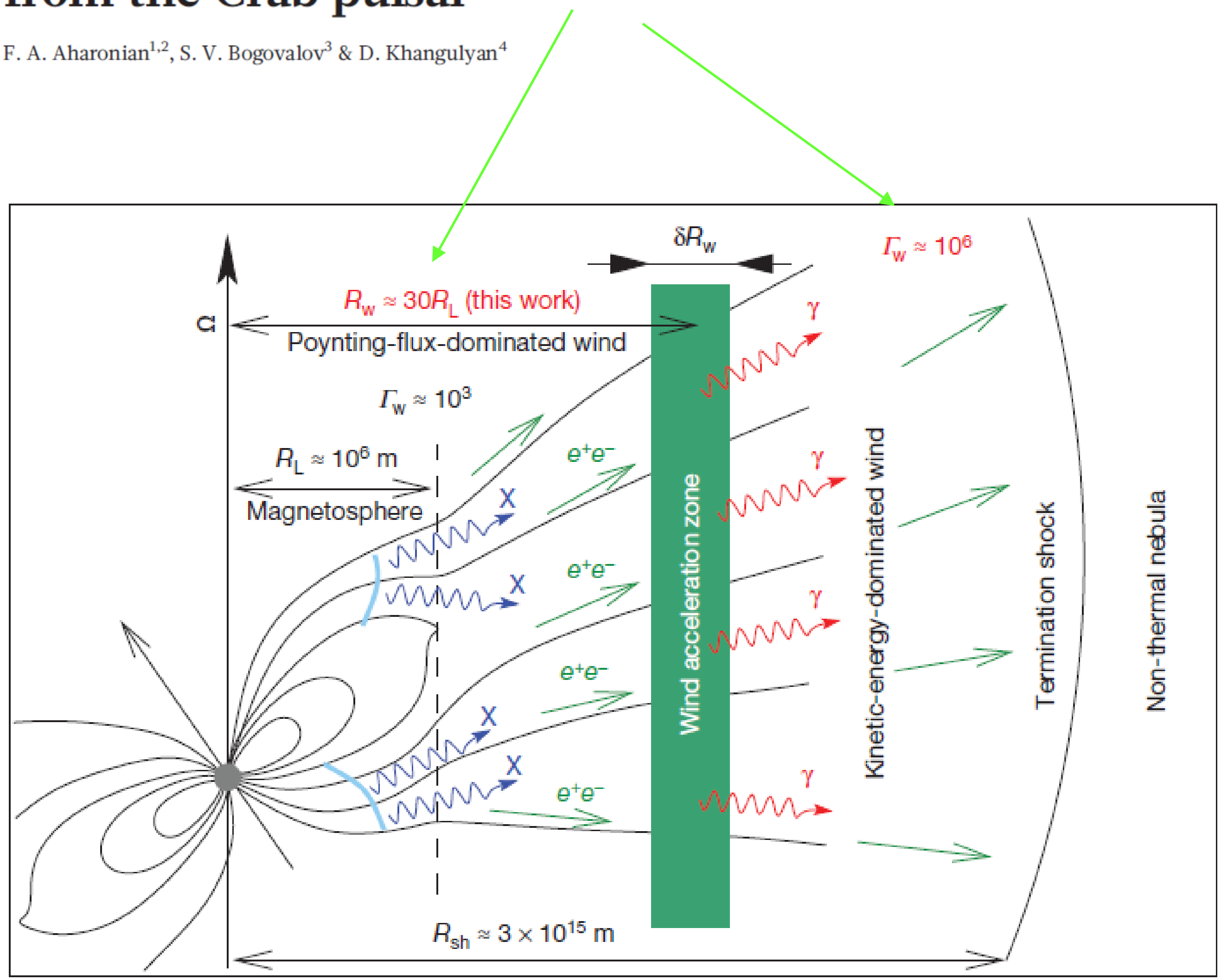
open
field
lines
light
surface
Ω
closed
field
lines
light
cylinder
R_L
R_L/λ

LETTER

doi:10.1038/nature10793

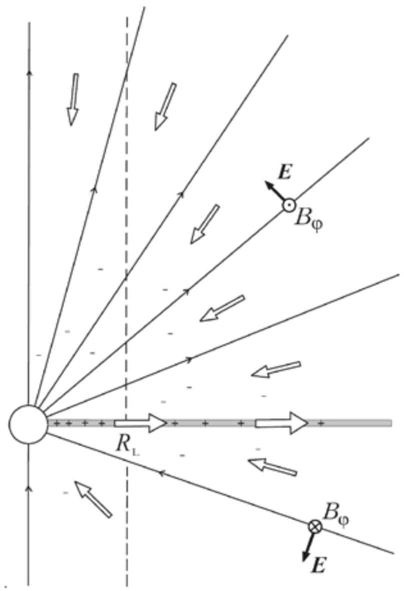
Abrupt acceleration of a 'cold' ultrarelativistic wind from the Crab pulsar

F. A. Aharonian[1,2], S. V. Bogovalov[3] & D. Khangulyan[4]

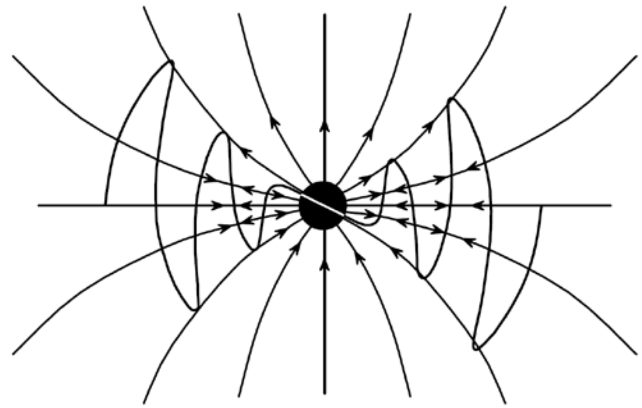


Asymptotic behaviour

Force-free magnetosphere: simple analytical solutions



F.C.Michel (1973)

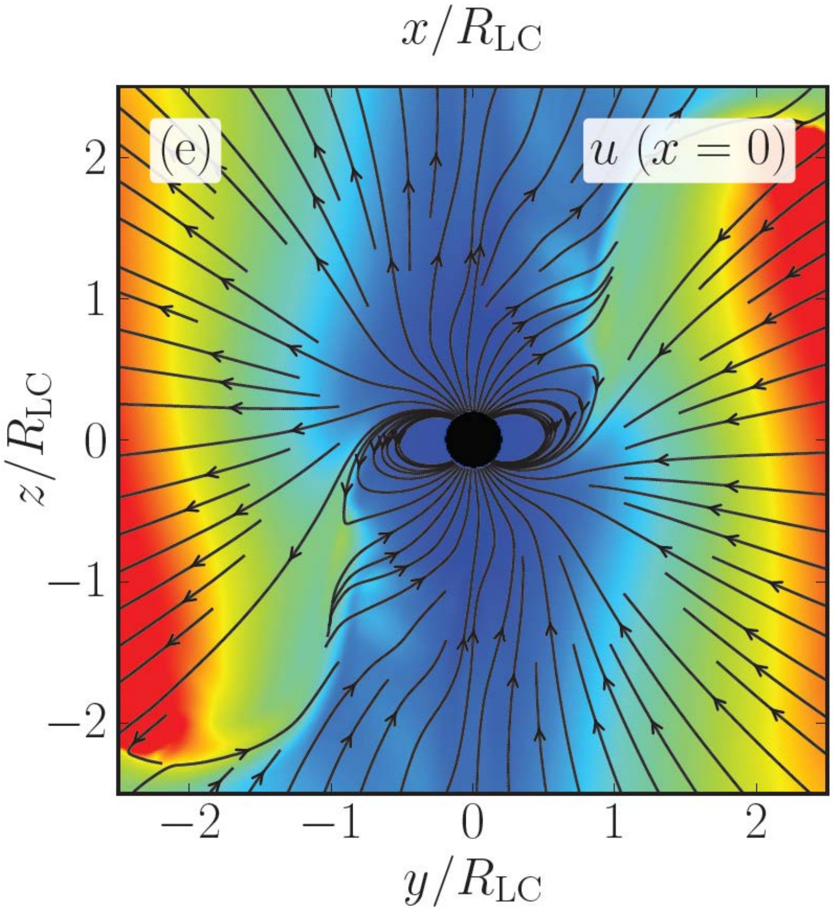


S.V.Bogovalov (1999)

$$B_{\hat{\varphi}} = E_{\hat{\theta}} = -B_0 \left(\frac{\Omega R}{c} \right) \frac{R}{r} \sin\theta$$

$$S \propto \sin^2\theta$$

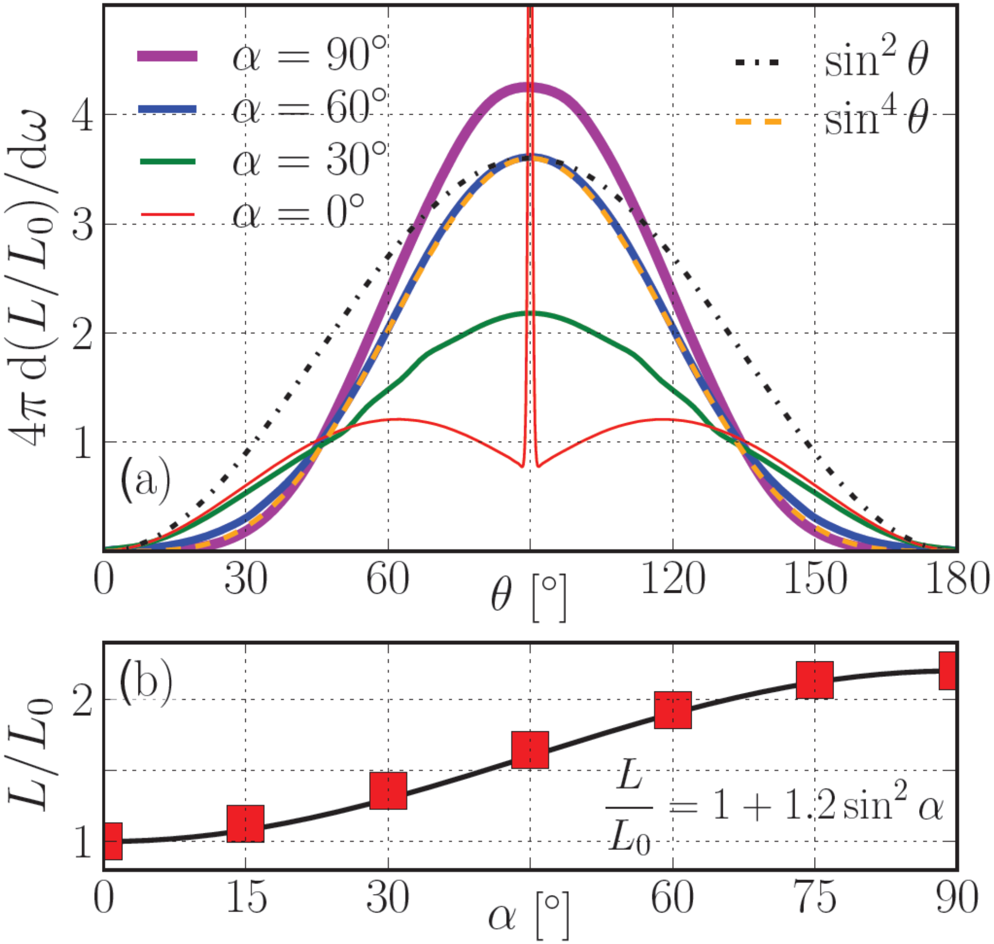
Orthogonal rotator – numerical



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

MHD

Orthogonal rotator – numerical



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
arxiv.org/pdf/1211.2803.pdf

MHD

For larrge enough inclination angle

$B_r \sim \sin\theta$

$E_\theta, B_\varphi \sim \sin^2\theta$

Numerical – main results

- No magnetodipole radiation
- Larger energy losses for orthogonal rotator
- No monopole Michel-Bogovalov poloidal field
- Inclination angle evolves to 0 deg.

Problem 5.2. Show that the relation similar to (5.24) can be obtained for the conical solutions $\Psi = \Psi(\theta)$, but only at large distances $r \gg R_{\mathrm{L}}$ from the compact object. It has the form [Ingraham, 1973, Michel, 1974]

$$4\pi I(\theta) = \Omega_{\mathrm{F}}(\theta) \sin\theta \, \frac{\mathrm{d}\Psi}{\mathrm{d}\theta}. \qquad (5.25)$$

$$\boxed{E_\theta = B_\Pi}$$

S.Gralla,T.Jacobson, G.Menon, C.Dermer ($B_{\mathrm{p}} = 0$)

Orthogonal rotator – numerical

- The shape of the current sheet

$$\Phi = \sin\alpha \sin\theta \sin(\varphi - \Omega t + \Omega r/c) + \cos\theta\cos\alpha$$

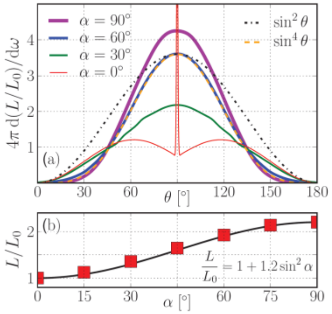
doesn’t depend on the azimuthal structure.

$$\boxed{\begin{array}{l} B_r \sim \sin\theta \\ \\ E_\theta, B_\Pi \sim \sin^2\theta \end{array}}$$

- The general condition is to be fulfilled

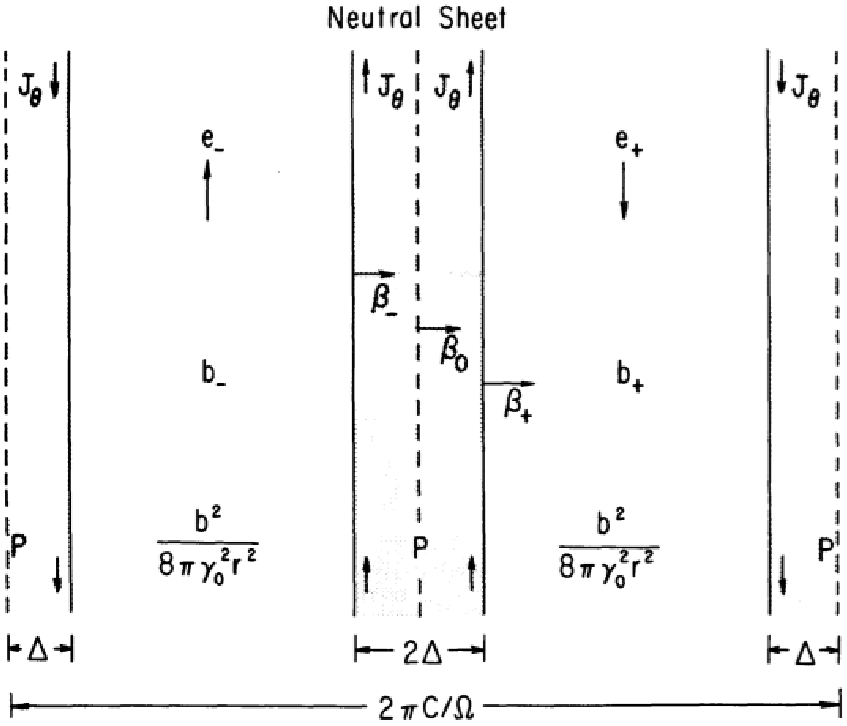
$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

(as Π– average)



Internal structure

F.Coroniti, ApJ, **349**, 538 (1990), F.C.Michel (1994)



$$n \propto 1/r^2 \Rightarrow \Delta \propto r$$

Not a self-consistent solution

Co-moving reference frame

M.Lyutikov, Phys. Rev. D, **83**, 124035 (2011)

$$\partial_t^2 \Omega = \partial_r^2 \Omega$$

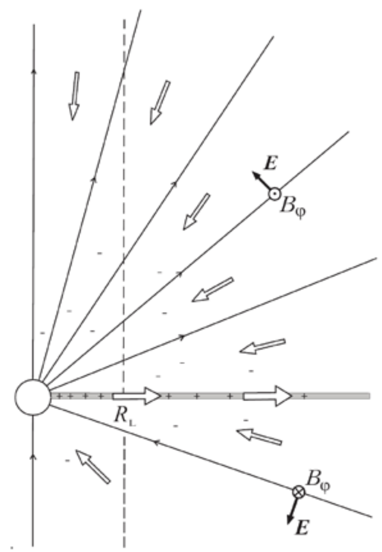
$$B_{\hat\varphi} = E_{\hat\theta} = -B_0 \left(\frac{\Omega R}{c} \right) \frac{R}{r} \sin\theta$$

$$\Omega = \Omega(r \pm ct)$$

Exact force-free solution <u>inside</u> a sheet

For $B_p = 0$ describes internal region

Fields in the laboratory frame



$$B_\varphi = \frac{1}{\beta} \frac{B_L R_L}{r} \sin\theta \tanh\left(\frac{r - \beta ct}{\Delta} \right)$$

$$E_\theta = \frac{B_L R_L}{r} \sin\theta \tanh\left(\frac{r - \beta ct}{\Delta} \right),$$

Force-free limit corresponds to $\beta = 1$

$\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ is valid for any β

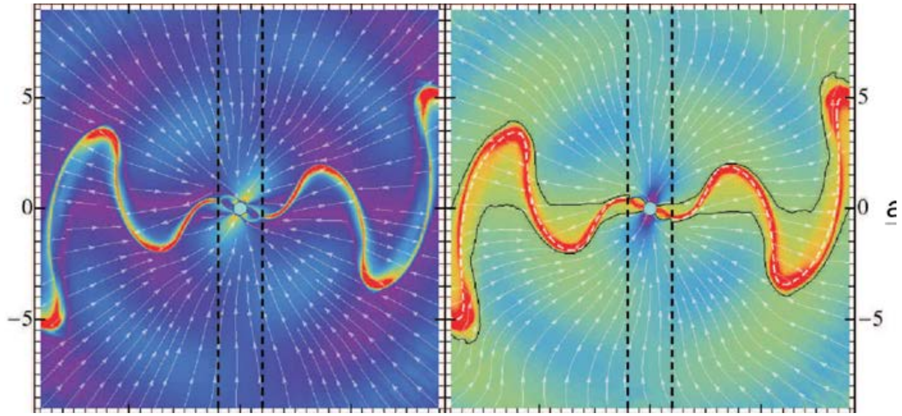
Boost

Due to the finite speed of the sheet it's possible to provide boost into comoving frame. Fields in this frame:

- orthogonal

$$B_y = \frac{B_L R_L \sin^m \theta}{ct\beta^2\gamma^2} \tanh\left(\frac{x}{\gamma\Delta}\right)$$

$$E_x = \frac{z B_L R_L \sin^m \theta}{c^2t^2\beta^2\gamma^2} \tanh\left(\frac{x}{\gamma\Delta}\right)$$



- aligned

$$B_y = \frac{B_{\mathrm{L}} R_{\mathrm{L}}}{\gamma^2\beta^2} \frac{1}{ct} f\left(\frac{z}{\beta ct}\right),$$

$$E_x = \frac{B_{\mathrm{L}} R_{\mathrm{L}}}{\gamma^2\beta^2} \frac{z}{c^2t^2} f\left(\frac{z}{\beta ct}\right)$$

Maxwell equation $\nabla \times \mathbf{E} = -(1/c)\partial \mathbf{B}/\partial t$ is satisfied inside and outside the sheet.

Internal structure

Quasiadiabatic invariant (Zeleniy et al., 2013)

$$L \ll r_{\mathrm{p}} \Rightarrow I_z = \int p_z dz \approx \mathrm{const}$$

$$I_z = \frac{1}{\pi} p(t) \sqrt{\frac{cp(t)}{eB(t)} L(t)} \; \psi(s) \qquad s = p_x / p$$

$$\psi(s) = \int_{-\sqrt{s+1}} \sqrt{1 - (s - \xi^2)^2} d\xi$$

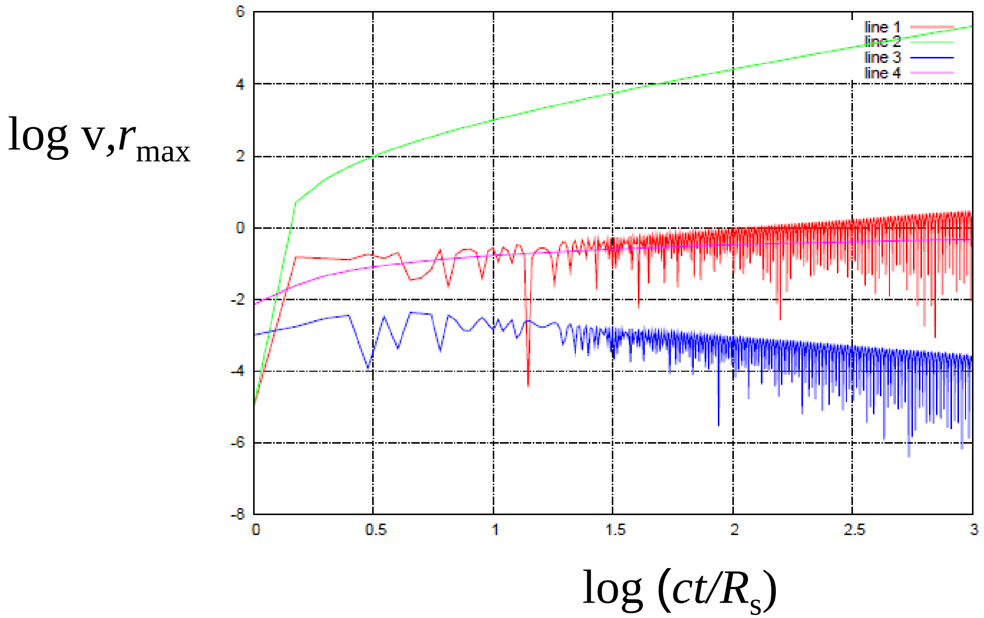
Internal structure

$B(t) \sim 1/t$

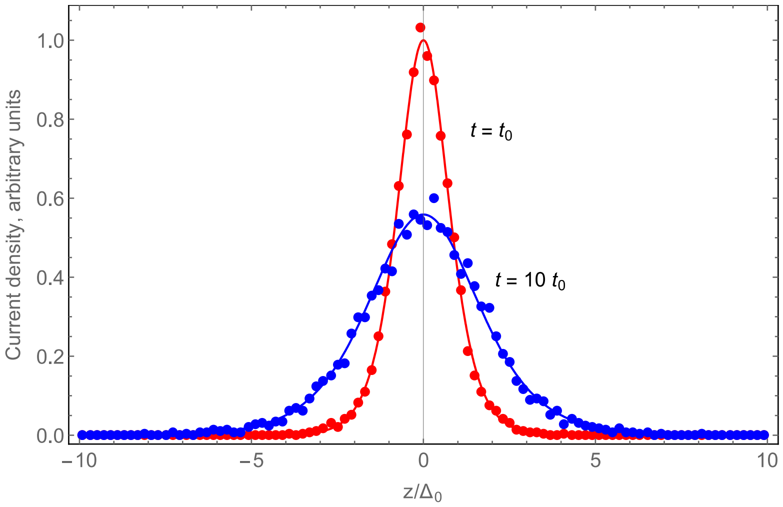
$L(t) \sim t$

$\mathrm{v}(t) \sim t^{-1/4}$

$r_{\max} \sim t^{1/4}$



Simple solution

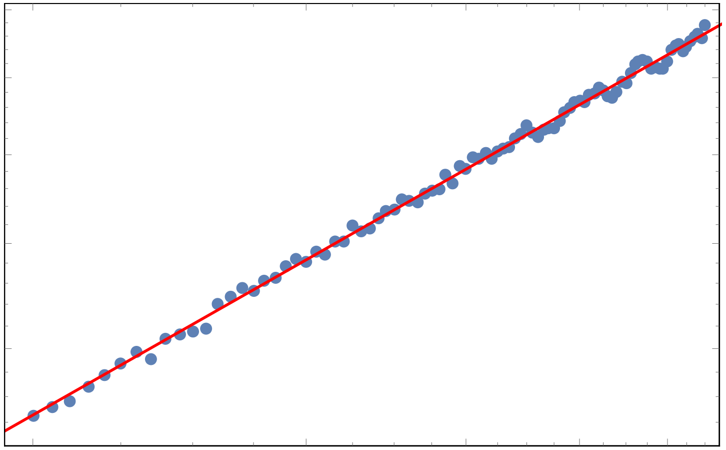


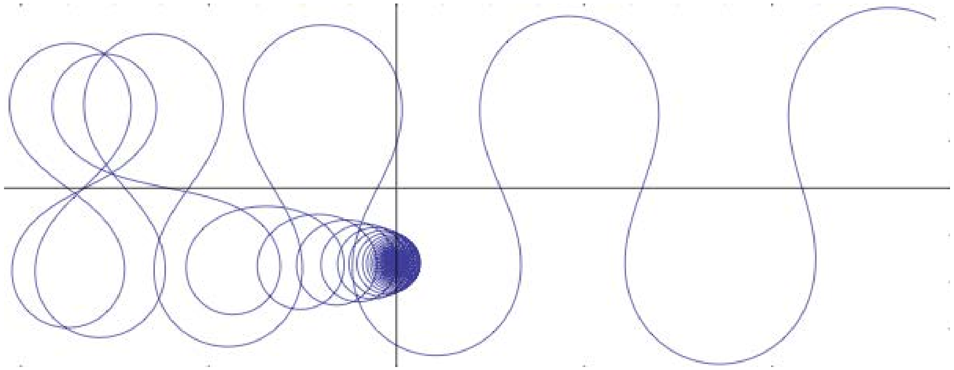
In special a case of $p_x = 0$ one can obtain a simple self-consistent solution:

$$L \sim r_{\max} \sim t^{1/2}$$

$$n \sim t^{-1}$$

$$p \sim t^{-1/2}$$

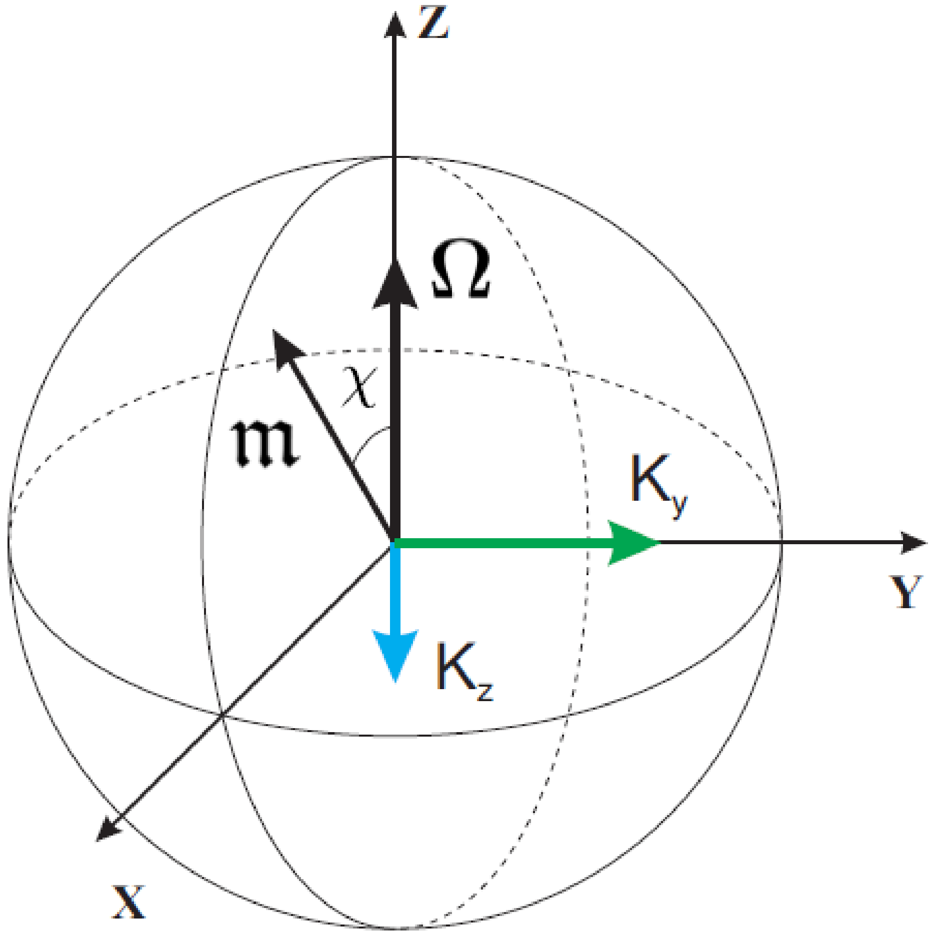




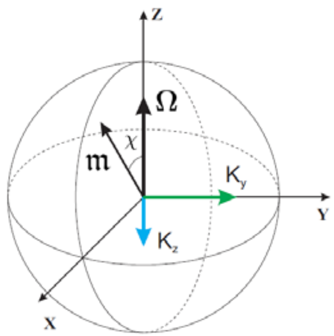
Results

- The shape of the current sheet does not depend on the asymptotic field behavior.
- In the comoving reference frame the current sheet is essentially time-dependent.
- The particle motion inside the sheet was described using quasi-adiabatic invariant.
- Our approach allows one to obtain simple self-consistent solution.

Anomalous torque



Anomalous torque



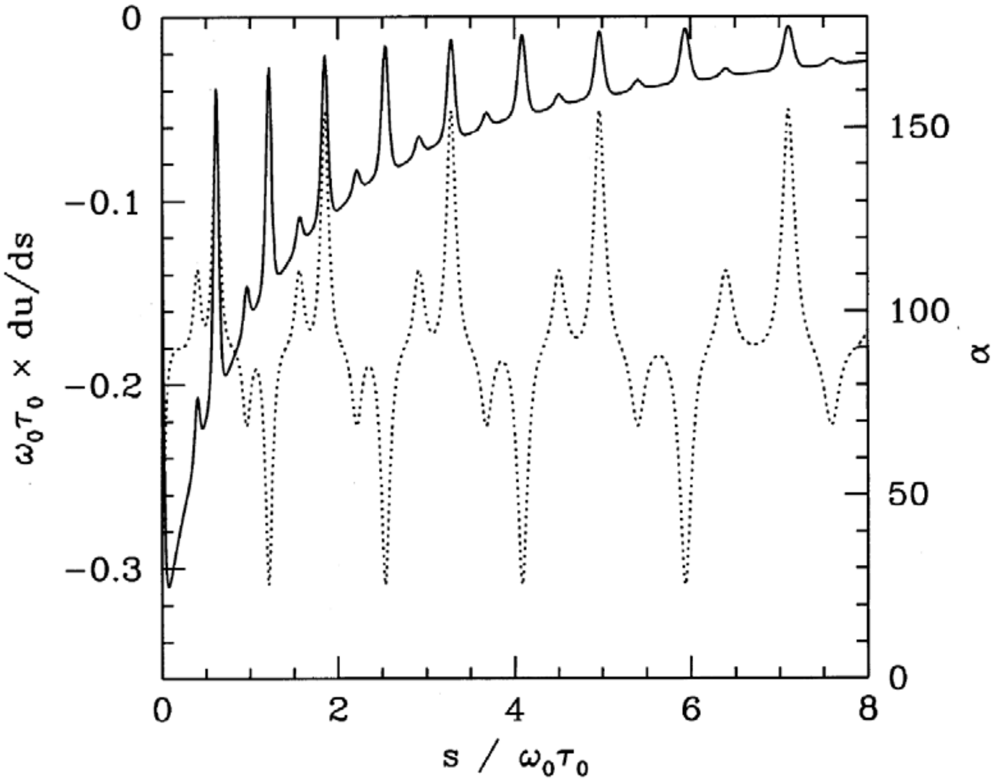
$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\boxed{K_{y'} = \xi\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi}$$

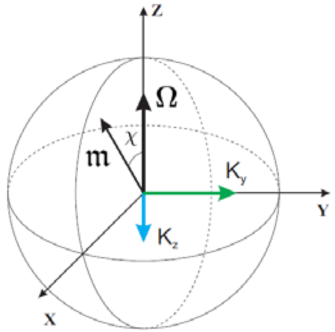
$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)
$\xi = 1/5$ (Good&Ng 1985)
$\xi = 0$ (Michel 1991, Istomin 2005)
$\xi = 3/5$ (Melatos 2000)

Evolution



A.Melatos, MNRAS, **313**, 217 (2000),
D.Barsukov & A.Tsygan, MNRAS, **409**, 1077 (2010),
G.Beskin, A.Biryukov, S.Karpov, MNRAS,**420**, 103 (2012)

Anomalous torque



Basic equation

$$\frac{\mathrm{d}\,\mathbf{L}_{\text{field}}}{\mathrm{d}t} + \mathbf{K}^M + \int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V = 0$$

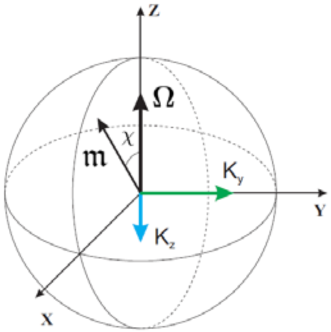
Electromagnetic angular momentum

$$\mathbf{L}_{\text{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c}\mathrm{d}V$$

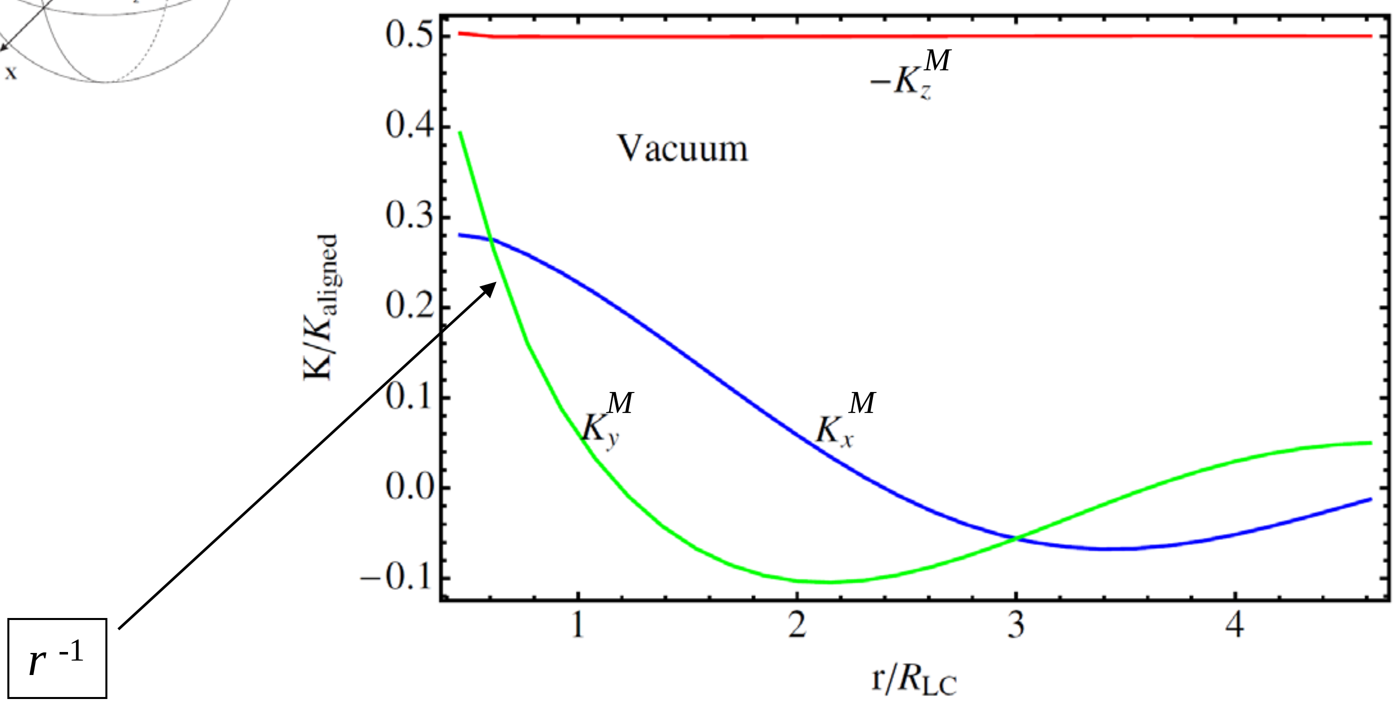
Flux $K_i^M = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$ (spherical surface)

$$\mathbf{K}^M_{y'} = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \mathbf{B}]_{y'} (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}]_{y'} (\mathbf{E}\mathbf{n}) \Big) \mathrm{d}o$$

Anomalous torque

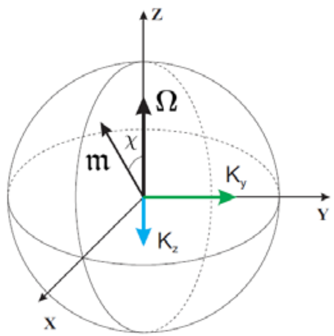


A.Philippov, A.Tchekhovskoy, J.Li,
http://arxiv.org/abs/1311.1513



$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{field}}}{\mathrm{d}t} + \mathbf{K}^M + \cancel{\int [\mathbf{r} \times \mathbf{F}]\mathrm{d}V} = 0$$

Anomalous torque



Torque, acting on the body

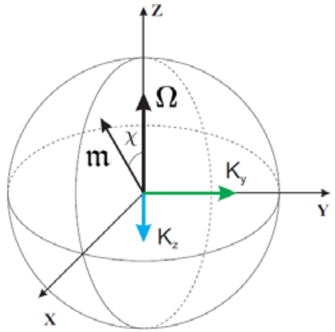
$$\frac{\mathrm{d}\,\mathbf{L}_{\mathrm{mat}}}{\mathrm{d}t} = \int [\mathbf{r}\times\mathbf{F}]\mathrm{d}V$$

$$\mathrm{d}\mathbf{F} = \sigma_{\mathrm{e}}\mathbf{E}\,\mathrm{d}S + [\mathbf{I}_{\mathrm{S}}\times\mathbf{B}]/c\,\mathrm{d}S$$

$$\boxed{\mathbf{K} = \int \mathbf{r}\times\mathrm{d}\mathbf{F} = \frac{R^3}{4\pi}\int\Big([\mathbf{n}\times\{\mathbf{B}\}]\,(\mathbf{B}\mathbf{n}) + [\mathbf{n}\times\mathbf{E}]\,(\{\mathbf{E}\}\,\mathbf{n})\Big)\mathrm{d}o}$$

- for spherical body
- for highly conducting interior
- for corotation currents only

Anomalous torque



Torque, acting on the body

$$\mathbf{K} = \int \mathbf{r} \times \mathrm{d}\mathbf{F} = \frac{R^3}{4\pi} \int \Big([\mathbf{n} \times \{\mathbf{B}\}] \, (\mathbf{B}\mathbf{n}) + [\mathbf{n} \times \mathbf{E}] \, (\{\mathbf{E}\} \, \mathbf{n}) \Big) \mathrm{d}o$$

$$\boxed{K = K^M(R+0) - K^M(R-0)}$$

$$\xi = \frac{3}{5}$$

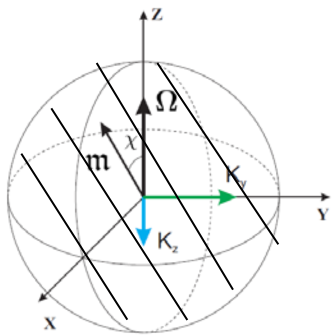
$$\mathbf{K}^M = -\frac{\mathrm{d}\,\mathbf{L}_{\text{field}}}{\mathrm{d}t}$$

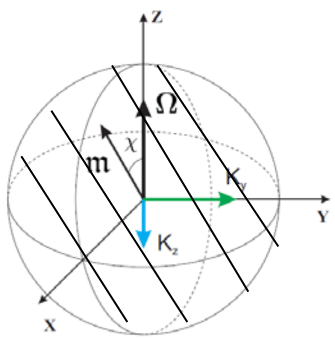
$$\mathbf{L}_{\text{field}} = \int \frac{[\mathbf{r} \times [\mathbf{E} \times \mathbf{B}]]}{4\pi c} \mathrm{d}V$$

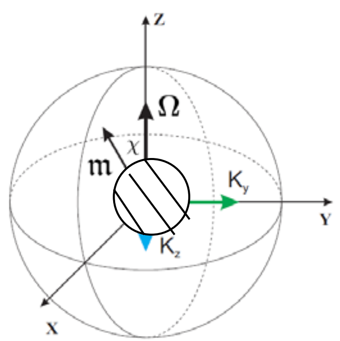
- Does not depend on the internal structure and freedom in $h^{(2)}$
- Obtained by A.Melatos (2000) for Deutsch solution

Depends on the internal structure

Anomalous torque







Three cases

- Homogeneously magnetized sphere

$$K_{y'} = \frac{1}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

- Homogeneously magnetized hollow envelope

$$K_{y'} = \frac{31}{45}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

- Homogeneously magnetized core $R_{\rm in} < R$

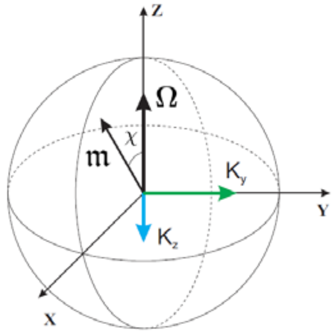
$$K_{y'} = \left(\frac{8}{15} - \frac{1}{5}\frac{R}{R_{\rm in}}\right)\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^2 \sin\chi\cos\chi$$

Comparison with previous results

$\xi = 1$ (Davis & Goldstein 1970, Goldreich 1970)
$\xi = 1/5$ (Good&Ng 1985)
$\xi = 0$ (Michel 1991, Istomin 2005)
$\xi = 3/5$ (Melatos 2000)

$\xi = 1$ – electric current only
$\xi = 0$ – outer sphere only
$\xi = 3/5$ – stress for $r = R+0$

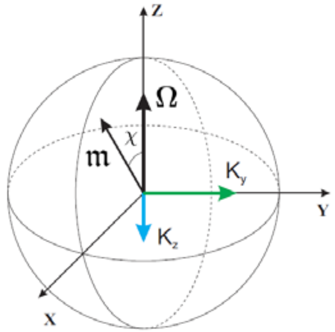
Anomalous torque



Main conclusion

Anomalous torque acting on the body depends on the internal structure of the electromagnetic field inside the body:

Anomalous torque



More conclusions

- Nonzero angular momentum L_{field} of the electromagnetic field exist only in Ω^2 order.
- In $\boldsymbol{\Omega}^3$ order it vanishes. It implies no problem in calculating the braking torque.
- Hence, energy losses can be determined via the flux $K_i = -\int \varepsilon_{ijk} r_j T_{kl} \mathrm{d}S_l$