FEATURES OF STEADY STATES OF COLLISIONLESS PLASMA LAYER WITH COUNTERSTREAMING ELECTRON-POSITRON BEAMS

A.Ya. Ender[1], V.I. Kuznetsov[1], A.A. Gruzdev[1,2]

[1] Ioffe Institute, St. Petersburg, Russia

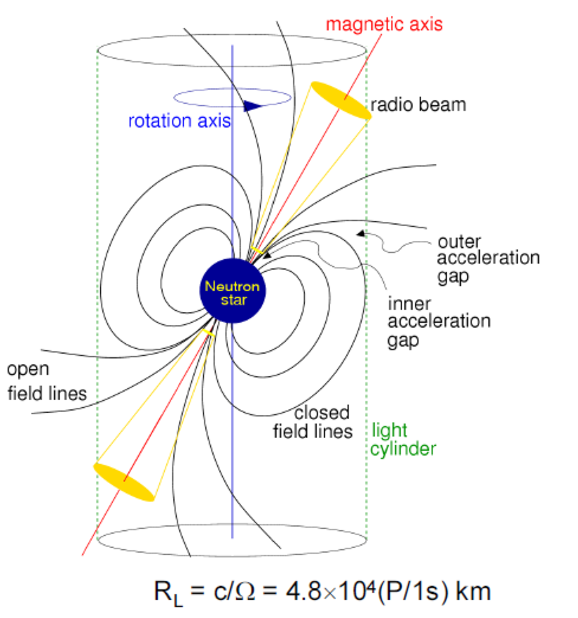
[2] St. Petersburg Polytechnical University, St. Petersburg, Russia

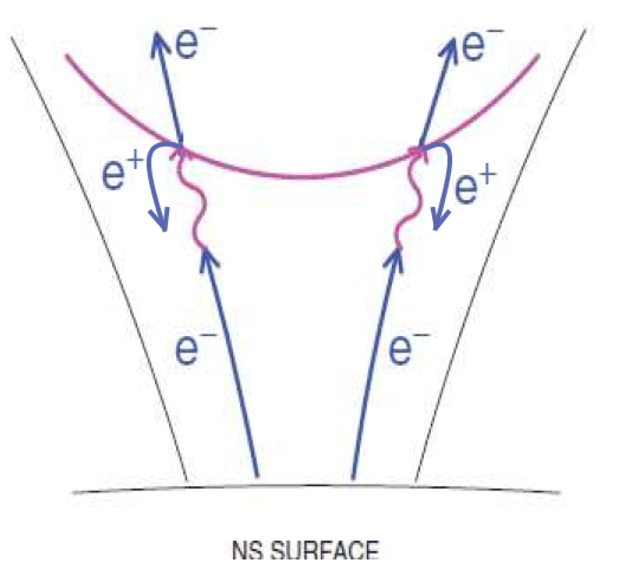
Introduction

Counterstreaming flows of oppositely charged particles are met in various fields of plasma physics. One of them is pulsar astrophysics. In accordance with Goldreich-Julian model, in the vicinity of a pulsar's polar cap there are vast regions containing plasma, formed with counterstreaming flows of relativistic electrons and positrons. It is thought that high-power radiation of different spectral ranges is generated precisely from these regions, named ''gaps'' [1], [2]. Our study follows goal of investigating steady-state solutions of self-consistent electric field which forms in these regions in attempt to understand how radiation is formed.

Statement of the problem

Electrons and protons exit the surface of neutron star permanently. Because of powerful magnetic field they can only move in parallel with magnetic field lines. Those regions, where magnetic field lines do not cross neutron star light cylinder (i.e. closed field lines), are being filled with plasma, which compensates electric field. However, in regions where magnetic field lines cross light cylinder (i.e. open field lines) full compensation of electric field is not possible, for particles leave neutron star forever, otherwise they would have been moving faster than light.

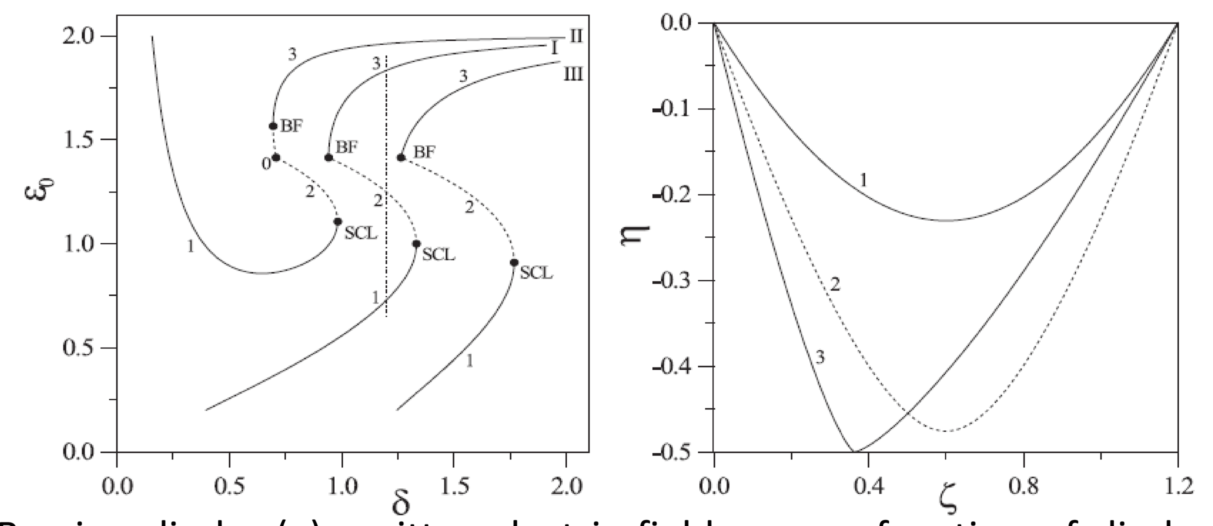




Particles that leave polar cap of neutron star, are being accelerated by uncompensated electric field and gain high energy. They produce gamma-photons, that decay into electron and positron couples in presence of powerful magnetic field. New-born electron is carried away from neutron star, positron turns back and gets accelerated by the same electric field. Thus, counterstreaming flows of electrons and positrons are formed.

Previous Studies

Studies under condition that potential distribution is monotonic: [1], [2].
Study with electron beam only: V.R. Bursian 1923 [3].



Bursian diode: (a) emitter electric field ε_0 as a function of diode length δ, 3 values of voltage: **I** – $V_C=0$, **II** – $V_C=0.2$, **III** – $V_C=-0.4$; (b) potential distribution examples

Initial conditions

We study a planar diode in which nonrelativistic electron and positive ion (with orbitrary charge) flows are monoenergetic, are supplied from opposite electrodes and move in the interelectrode space with no collision under self-consistent electric field. As for this study we consider positrons of the same energy as electron one, and zero potential difference between electrodes.

We introduce dimensionless quantities and use, as length and energy units, the Debye length and the kinetic energy of electrons at the emitter, respectively:

$$\lambda_D=\left[2\epsilon W_{be}/\left(e^2 n_{be}\right)\right]^{1/2}, \qquad W_{be}=m_e v_{be}^2/2$$

Then, coordinate $\zeta=z/\lambda_D$, potential $\eta=e\phi/(2W_{be})$ and electric field $\varepsilon=e E\lambda_D/(2W_{be})$. An interelectrode distance $\delta=d/\lambda_D\sim j_{be}^{1/2}$ and a potential difference between electrodes $V_C=e\varphi_C/(2W_{be})$.

The diode with electron and positron flows

We have obtained the full classification of the potential distributions (PDs). As a rule, PD has wavy form. There are various solutions related with particle reflection. These are solutions equipped with either virtual electron emitter (VEE), positron virtual one (VPE), or both. If PD minimum η_m takes a value -1/2, the VEE exists and a portion of the electron beam is reflected at it and returns to the emitter. If PD maximum is $\eta_M=1/2$, the VPE exists and positron reflection occurs at it.

Note, that in a real device, the possibility of a beam splitting is further supported by a small dispersion in the momentum of the particles. For the quantitative description of partial particle reflection, we introduce the reflection coefficient: r_e for electrons and r_p for positrons, respectively, which represents the density ratio of reflected and injected particles.

To obtain 1D steady-state solutions, it is sufficient to make use of the energy conservation law of electrons and positrons, the continuity equations of both kind of particles, and Poisson's equation.

Then the equation for potential becomes:

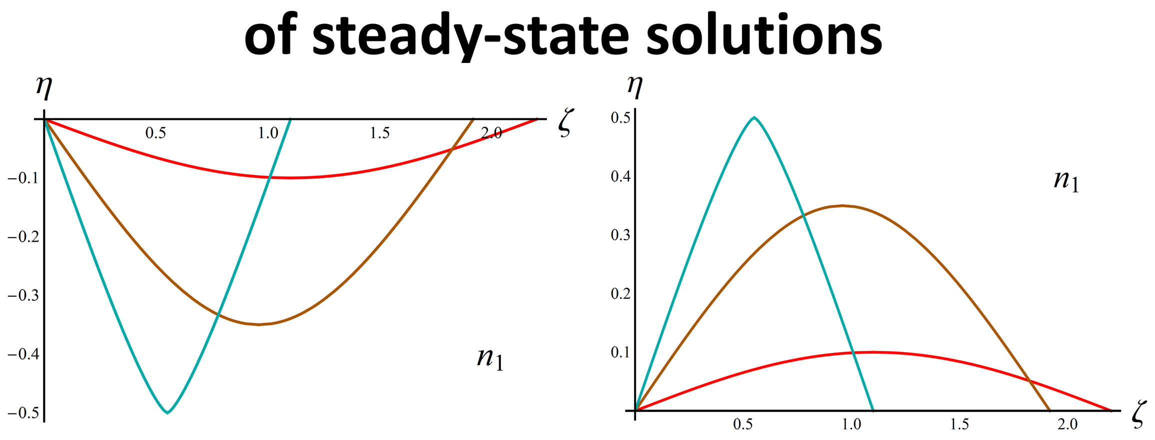
$$\frac{d^2\eta}{d\zeta^2} = \frac{H_e(\zeta;r_e)}{\sqrt{1+2\eta}} - \frac{H_p(\zeta;r_p)}{\sqrt{1-2\eta}}$$

with the boundary conditions

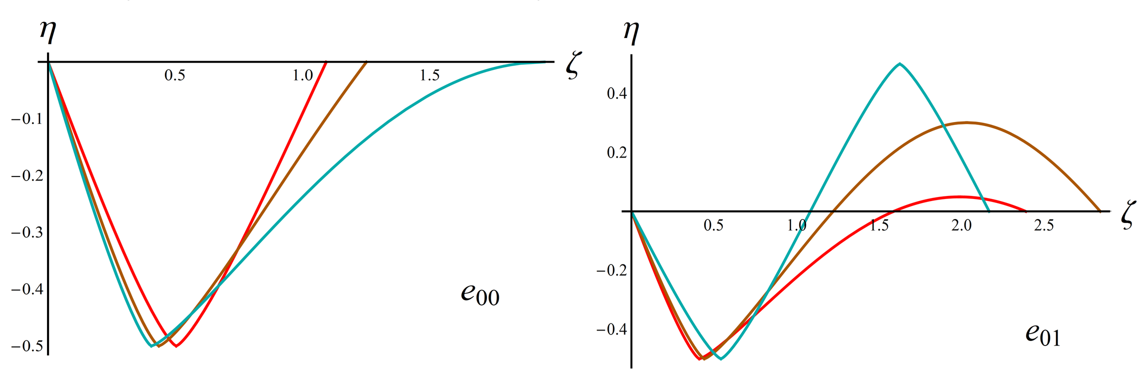
$$\eta(0) = 0, \qquad \eta(\delta) = V_C = 0.$$

By integrating this equation twice, we obtain implicit expression for the potential distribution (PD).

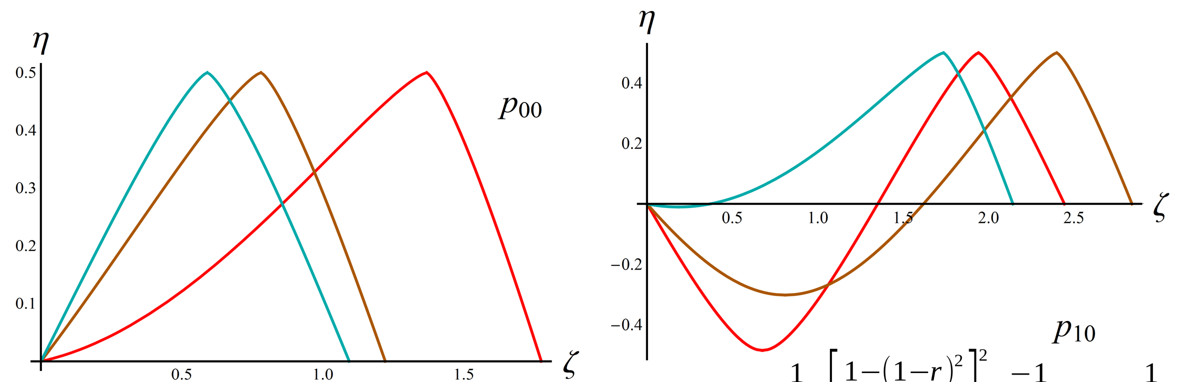
Examples of PDs for different types of steady-state solutions



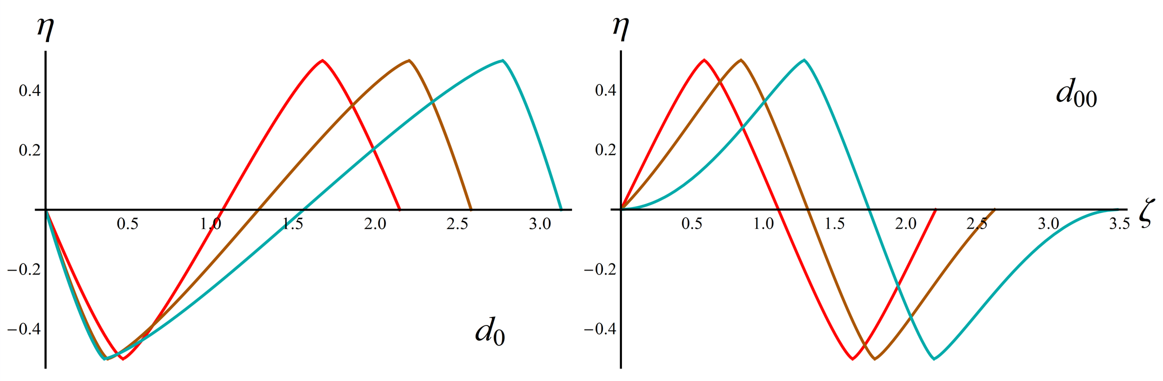
The solutions without any particle reflection; ε_0 = 0.1423 for the red curve, 0.5443 for the brown one, and 1.0732 for the blue curve.



The e_{00} and e_{01} branches involve solutions with partial electron reflection. They merge in the point corresponding to maximal $\varepsilon_0\approx1.5307$, r≈0.5858.
e_{00} branch: red – ε_0=1.1857, r=0.1172; brown – 1.4113, 0.4101; blue – 1.5307, 0.5857.
e_{01} branch: red – ε_0=1.5115, r=0.5565; brown – 1.3904, 0.3808; blue – 1.0825, 0.0001.



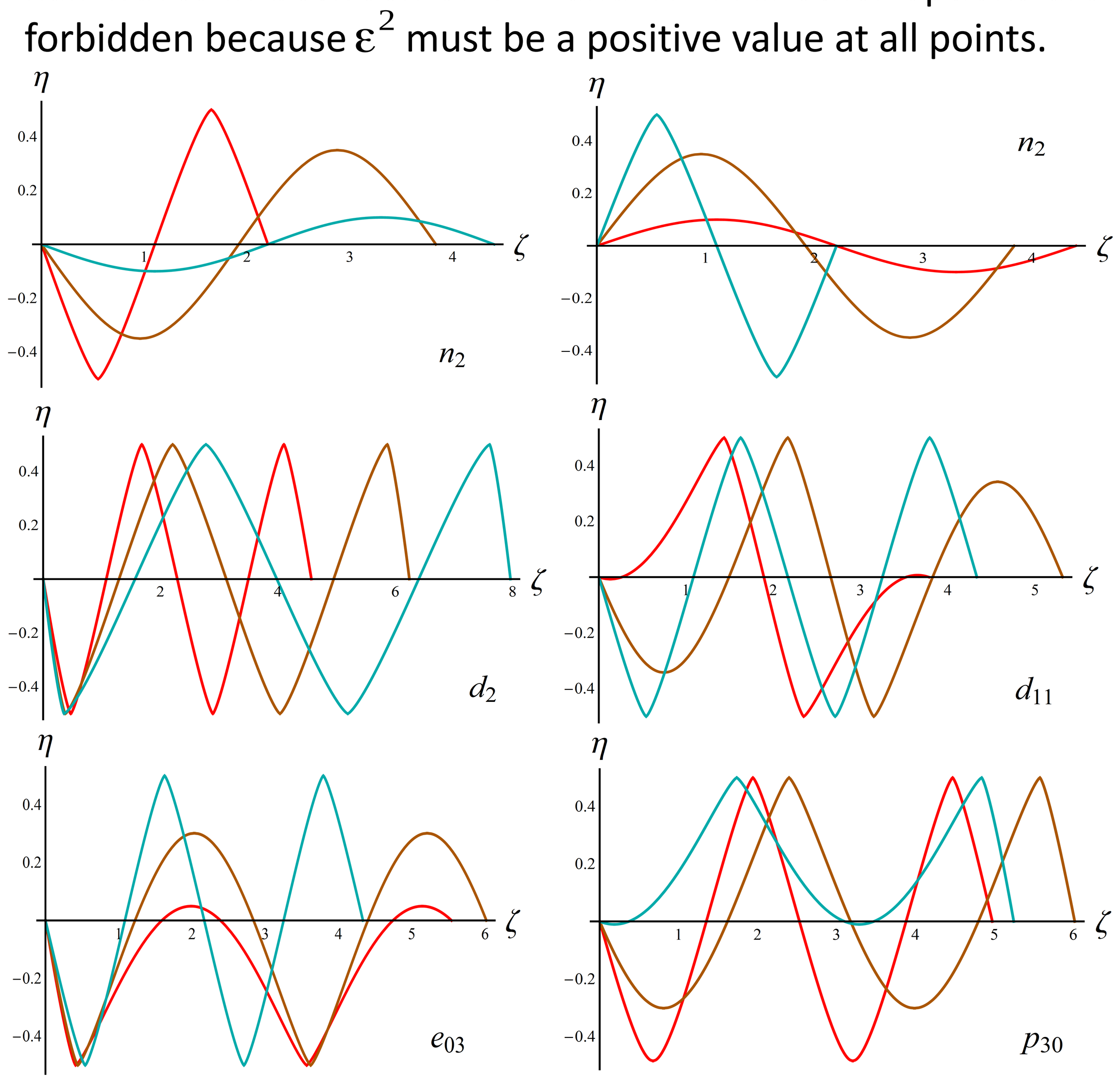
Solutions with partial positron reflection. $\eta_m = -\frac{1}{2}+\left[\frac{1-(1-r)^2}{1+(1-r)^2}\right]^2 > \frac{-1}{2}, \quad \eta_M=\frac{1}{2}$
P_{00} branch: red – ε_0=-0.1082, r=0.5799; brown – -0.6404, 0.3808; blue – -0.9681, 0.1172.
P_{10} branch: red – ε_0=0.9681, r= 0.1172; brown – 0.6474, 0.3808; blue – 0.1082, 0.5799.



Branches that involve partial reflection of both electrons and positrons. We have proven that reflection coefficients are equal in these cases.
branch d_0: red – ε_0=1.3178, r= 0.1998; brown – 1.7344, 0.6494; blue – 1.8526, 0.7992.
branch d_{00}: red – ε_0==-0.9681, r= 0.0828; brown -0.4841, 0.3314; blue -0.0108, 0.4142.
Notice a position swap between VEE and VPE in branch d_{00} versus branch d_0.

Infinite number of steady-state solutions

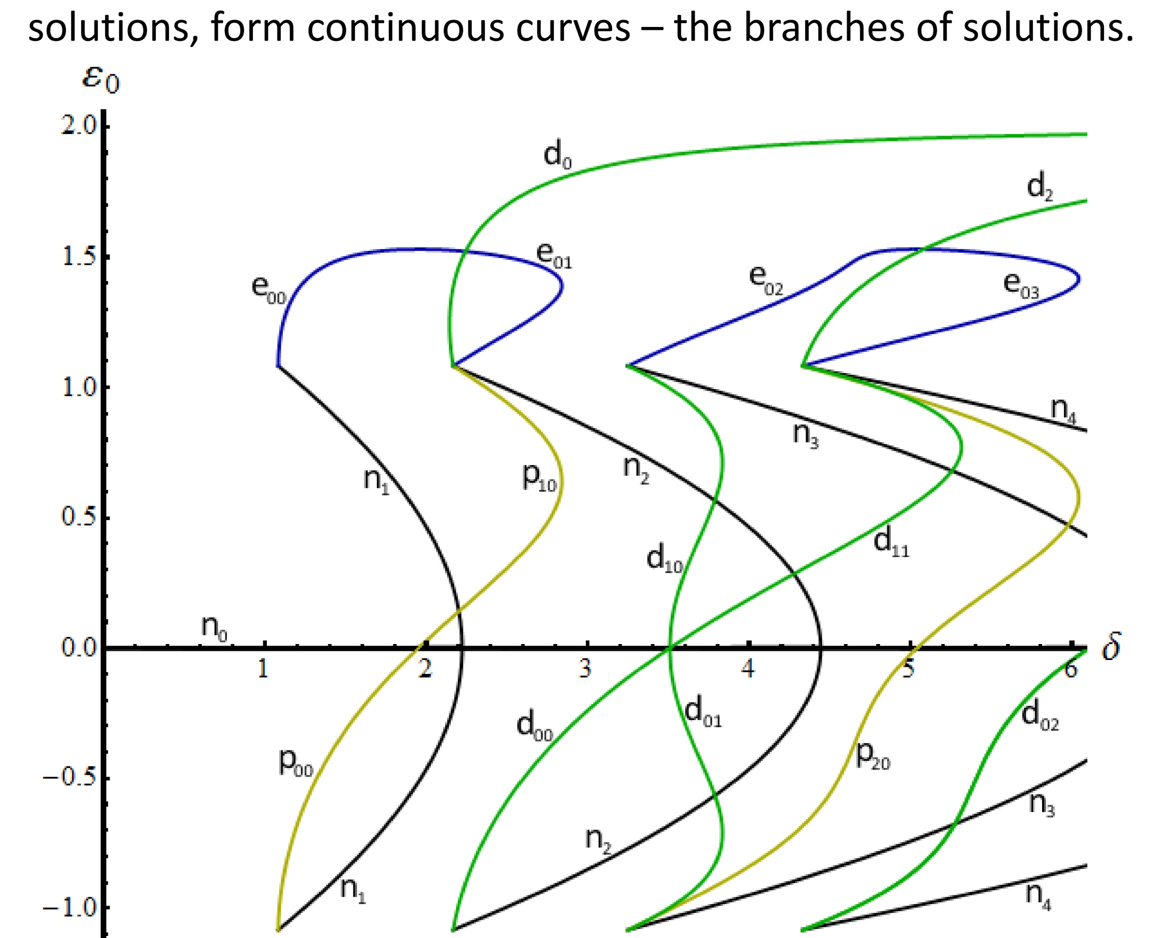
Due to a wavy form of PD, it appears that new solutions can be built out of existing ones. If we have ε_0 fixed, it is an option to add to corresponding potential distribution any integer number of half-wave pieces and thus obtain a new solution. However, for each type of steady-state solutions this process is individual. In case of electron reflection it is only possible to add half-vawe pieces to the right side from VEE. In case of positron reflection – only to the left side of VPE. For branch d_{00} it is possible to add half-waves either to the right of VEE, to the left of VPE or to the both sides at the same time. In case of d_0 branch, you can only add even number of half-waves in between of VEE and VPE. All other options are forbidden because ε^2 must be a positive value at all points.



Examples of branches that can be obtained from primal ones by adding half-waves to them in certain positions.

Systematization

Aggregation of all solutions of the problem with a certain fixed potential difference between electrodes V_C may be effectively represented on a {ε_0,δ}-plane, i.e., emitter electric field—electrode distance plane [4]. Dots that correspond to solutions, form continuous curves – the branches of solutions.



An example of {ε_0,δ}—diagram for V=0. n_s denotes a PD without particle reflection, **s** is number of potential extrema; d_s denotes a PD both with electron and positron reflection, **s** is number of potential extrema located between VEE and VPE; e_{0j} denotes a PD with electron reflection; p_{i0} – PD with positron reflection; d_{ij} denotes a PD both with electron and positron reflection, **i** and **j** is number of potential extrema located to the left of VPE or to the right of VEE, respectively.

Conclusion & the perspectives for study

Thus, we have found the full family of steady-state solutions for the diode with two counterstreaming flows.
Now, the question arises: how one can examine these solutions?
(1) It is necessary to study stability of obtained steady-state solutions to relatively small perturbations as well as variable time-dependent transition processes. This gives possibilities (a) to create switches based on transitions between different states; (b) to study nonlinear oscillations in the diode which may develop in the region corresponding to unstable solutions.
(2) To study steady-state solutions with variation in magnitude of the potential difference V_C between electrodes.
(3) To study steady-state solutions for the relativistic beams.
Perhaps, this shall shed light on the nature of the high-power radiation which is generated from pulsars.

References

[1] *S. Shibata.* The field eligned accelerator in the pulsar magnetosphere. Non. Mot. R. Astron., 1997. V. 287. p. 262—270.
[2] *S.C. Litwin and R.Rosner.* Relativistic space-charge-limited bipolar flow. Phys. Rev. E. 1998. V.58. No. 1, p. 1163—1164.
[3] *V.R.Bursian, V.I.Pavlov.* Zh. Russ. Fiz.-Khim. O-va. 1923. V.55. P.71 (in Russian).
[4] *V.I. Kuznetsov and A.Ya. Ender.* On the self-consistent states of a planar vacuum diode with an electron beam. Tech.Phys. 2013. V. 58, p. 1705.