Electromagnetic tornado semiclassical quantization and origin of the bands in the giant pulses frequency spectrum of the Crab pulsar

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Giant radio pulses of pulsars [1] have a circular polarization and characteristic (typical) bands (**strips**) in the inter pulse frequency spectrum (T.H. Hankins and J.A. Eilek, Ap.J., **670**, 693, 2007; [2]), an adequate explanation of which at present there is absent.

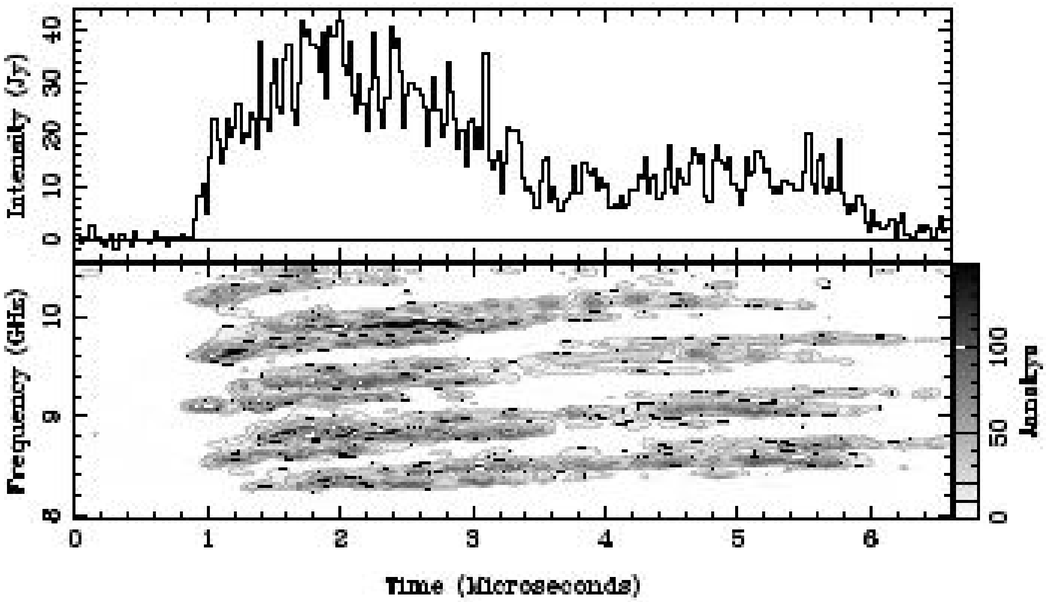
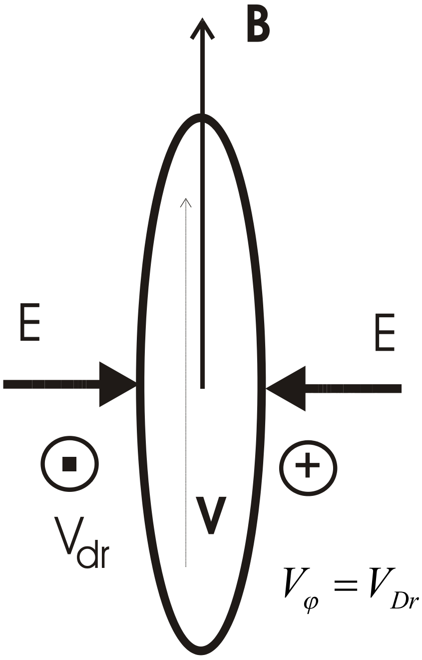


Fig. 1. *Example of an inter pulse radiation spectrum of the Crab pulsar (Hankins and Eylek [2]). Alternating bands of frequency correspond to the "permitted" and "forbidden" zones.*

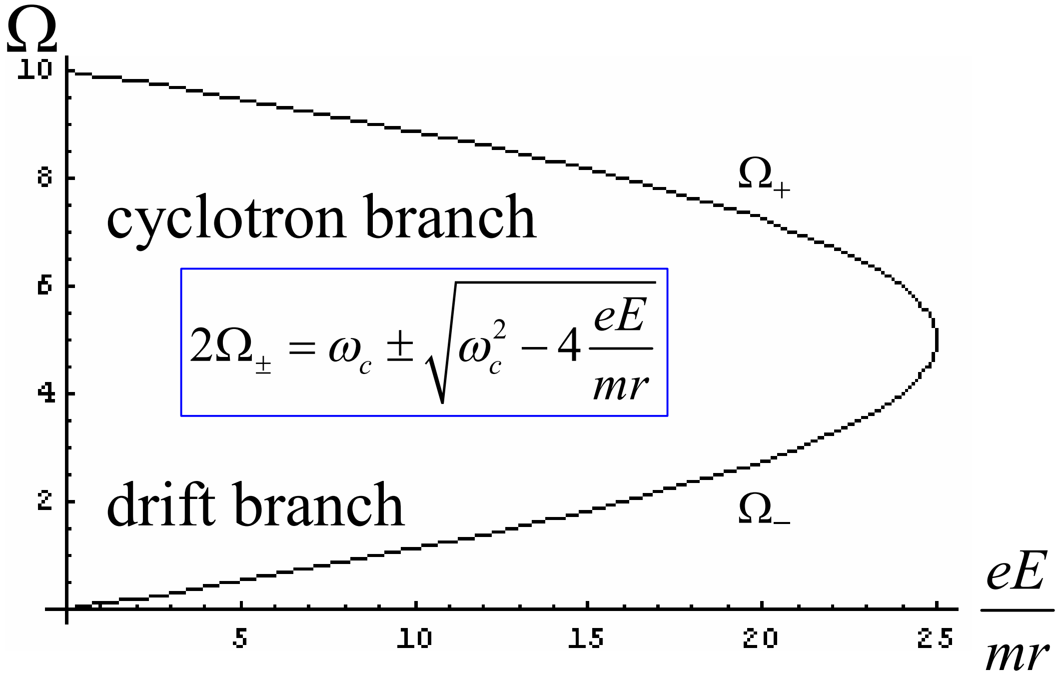
To date, made assumptions about the nature of bands use the analogy with the solar plasma effects, but in this case the magnetic field must be sufficiently weak. By now several assumptions have been suggested about the nature of strips. One of them is to interpret them as fuzzy levels (anomalous) cyclotron resonance [3] or of an analogue of the solar zebra structure (or Bernstein modes) [4]. That is, the formation of bands have to associate with a specially introduced dense region far away from the star (near the light cylinder at the periphery of the magnetosphere). Another idea is to use effects associated with superluminal rotation, and also locates the source of radiation away from the surface of the star at the region of the light cylinder [5].

Explanation, which we discuss below, uses the **quantization of the electromagnetic whirlwinds** (**tornadoes**) introduced to explain the circular polarization of giant pulses from pulsars. The tornadoes can occur at breakdown in the inner gap of pulsars – the particle acceleration region, located near the pulsar magnetic poles. They are rotating around its axis cylindrical flow of electrons (or positrons) in crossed beam space charge field and super strong magnetic field of the pulsar and can be responsible for the generation of giant pulses (V.M. Kontorovich, JETP, **110**, 966, 2010).



$$V_{\varphi} = V_{Dr} = cE_r / B$$

Fig. 2. *There are two branches in the tornado spectrum: the high energetic modified Landau branch and low energetic drift one. The great value of the magnetic field B ~ 10^12 G leads to* ***quantization*** *of electron motion in the tornado. The result is the quantization of current in the gap, and the quantum of current is determined only by the magnetic field.*



In the context of pulsars in such strong electric and magnetic fields at the relativistic motion of particles is necessary to consider the possible influence of the electron spin on the quantization of electron motion as done in this report.

. In the quantization rule the electron spin enters through the topological Berry phase. Just as in the absence of an electric field by virtue of compensation orbital and spin summands there is no gap of zero vibrations in the spectrum. Due to this fact the semiclassical quantization is possible**.**

The expression for the energy of a relativistic electron in an external constant magnetic field (in the absence of an electric field) is [8]:

$$\varepsilon = \sqrt{m^2c^4 + p_z^2c^2 + |e|\hbar Bc(2n_L+1) - e\hbar Bc\sigma}\ , \tag{1}$$

e is the charge of the electron (positron), p_z is its longitudinal momentum directed along the magnetic field, the integer *n* numbers the Landau levels, σ is determined by the spin projection on the axis z. In the approximation linear in Planck's constant this expression takes the form:

$$\varepsilon \simeq \varepsilon_0 + \varepsilon_\perp + \varepsilon_s\ ;\ \varepsilon_0 = \sqrt{m^2c^4 + p_z^2c^2}\ ,\ \varepsilon_\perp = \frac{\hbar|e|cB}{\varepsilon_0}\left(n_L + \frac{1}{2}\right),\ \varepsilon_s = -\frac{\hbar ecB}{2\varepsilon_0}\sigma\ , \tag{2}$$

($\varepsilon_\perp + \varepsilon_s \ll \varepsilon_0$, $B \ll B_{\text{crit}} = m^2c^3/e\hbar = 4.4\ 10^{13}$ Гс).

In the crossed fields of the external magnetic field and an inhomogeneous space-charge field, forming a tornado in the pulsar, there are two classical branches corresponding differential rotation of the particles around the magnetic field: a high-energy cyclotron and low-energy drift:

$$2\Omega_\pm = \omega_c \pm \sqrt{\omega_c^2 - 4\frac{eE}{mr}}\ , \tag{3}$$

Slow rotation in the case of pulsars corresponds to drift velocity orthogonal to the electric field [6]. Rotational energy due to this orthogonality is served. This makes possible the existence of stationary quantum states. To implement such a rotation is needed (like having a place in the absence of an electric field), the exact compensation of zero oscillations of the cyclotron branch by the spin term in the energy.

The quantization allows to offer a natural explanation for the observed bands in the frequency spectrum of inter pulse radiation of pulsar PSR J0534 +22 in the Crab Nebula and to determine the tornado physical parameters.

$$\varepsilon_\perp = \hbar\Omega_+(n_L + \frac{1}{2}) + \hbar\Omega_-(n + \frac{1}{2}) \tag{4}$$

$$\varepsilon_\perp + \varepsilon_s = \hbar\Omega_- \cdot n \qquad (n_L = 0). \tag{5}$$

The resulting compensation allows us to consider the usual rule of semiclassical quantization

$$\oint \mathbf{R} d\mathbf{p} = 2\pi\hbar \cdot n \,, \tag{6}$$

that leads to the condition that coincides with obtained in [6]:

$$mrV_{\varphi} = n \cdot \hbar \,, \tag{7}$$

where orbital electron velocity must be expressed in terms of non-relativistic rotation rate of drift in the crossed fields

$$\Omega(r) = \Omega_{-}(r) \approx \frac{cE}{rH} \,, \quad \left(\Omega(r) \ll \omega_c = \frac{eB}{mc} \right) \tag{8}$$

Circular motion is achieved by virtue of the Lorentz force exact compensation of the Coulomb repulsion between the electrons in the bunch. For the angular rotation drift frequency in the laboratory system that gives

$$\tilde{\Omega}_n(r) = n \cdot \frac{\Omega(r)}{\Gamma} \cdot \,, \tag{9}$$

where $\Omega(r)$ is the quantum of rotational speed in the rest frame:

$$\Omega(r) = \frac{\hbar}{mr^2} \,. \tag{10}$$

By duration Δt we can reconstruct Γ - factor of the radiating electrons using the connection

$$\Delta t \simeq \delta\varphi \cdot \frac{P}{2\pi} \,, \quad \delta\varphi \approx \frac{1}{\Gamma_{\Delta}} \,, \tag{11}$$

where the azimuthal angle $\delta\varphi$ is defined aberration

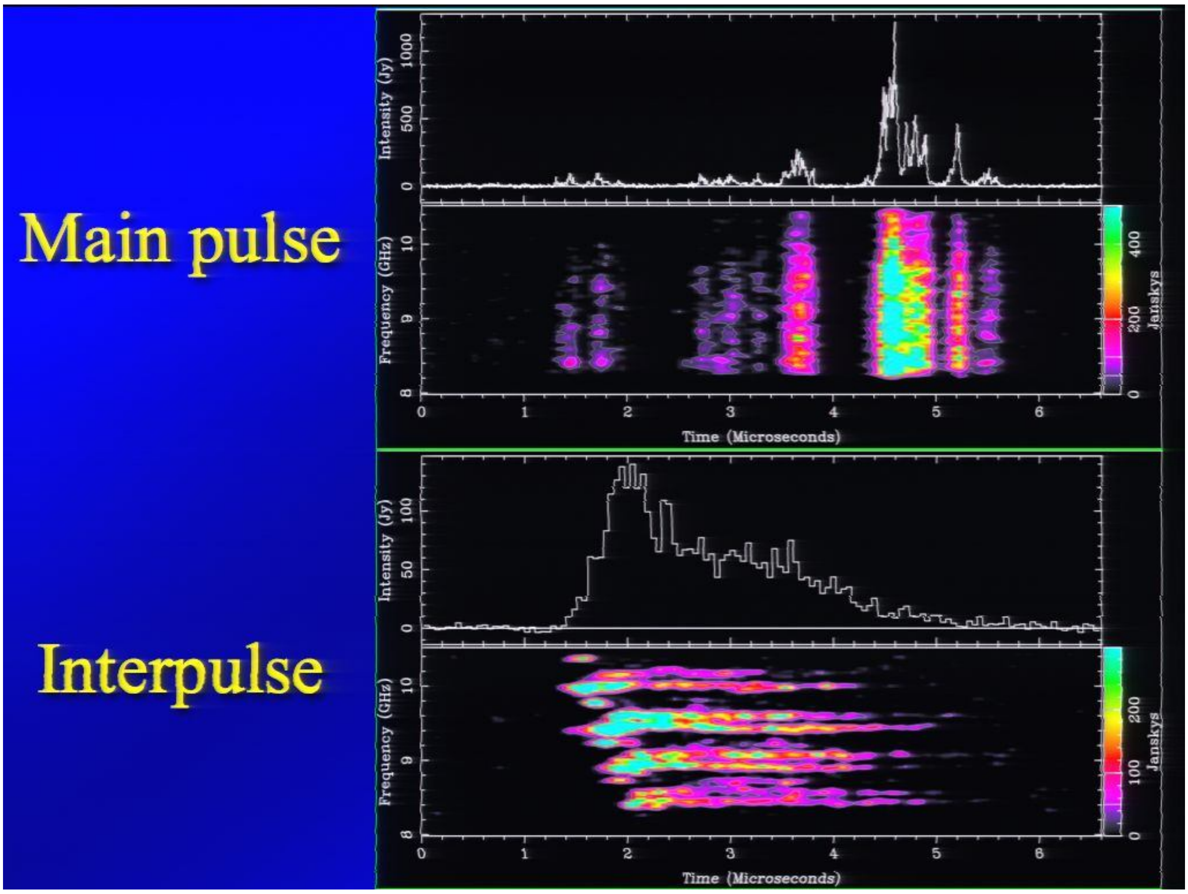


Fig. 3. The difference in the main pulse and inter pulse spectra of GP [2].

That gives

$$\Gamma_{\Delta} \approx \frac{P}{2\pi\Delta t} \approx 10^{3}. \tag{12}$$

$$\omega_n \approx n \cdot \Omega(0) \cdot 2\Gamma_{\Delta}, \tag{13}$$

$$\Delta\omega = n \cdot (\Omega(0) - \Omega(r_0)) \cdot 2\Gamma_{\Delta}. \tag{14}$$

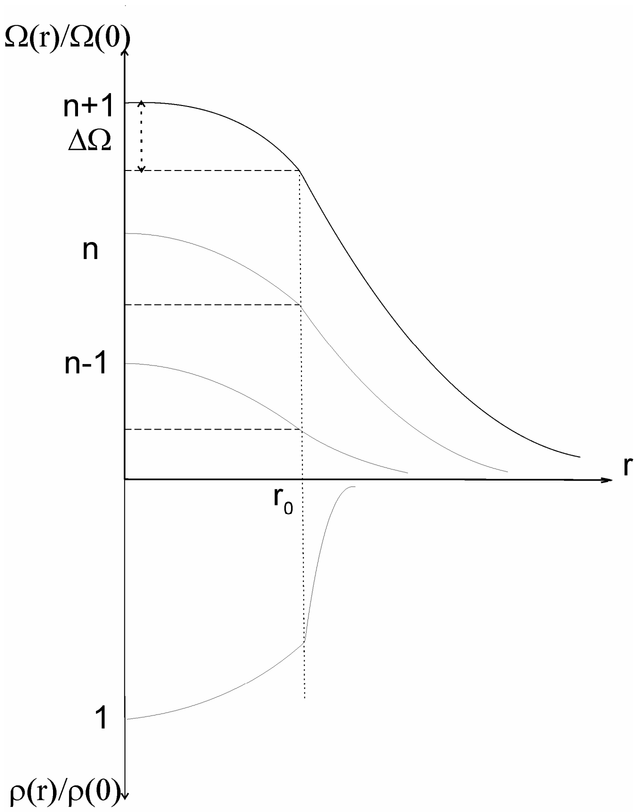


Fig. 4. *The distribution of the charge density on the radius (the bottom) and the corresponding rotational angular velocity of the tornado (above) at the different quantum states (a scheme). Marked the band width of the radiation at the level n+1. Because at a frequency of 10 GHz the allowed band width is 0.4 and the forbidden one is 0.1 [2] we obtain an estimate (16-17). Condition of no overlaping zones: the whirlwind border shall be sufficiently sharp and should be performed (15)*

$$(n+1)\Omega(r_0) > n\Omega(0). \tag{15}$$

To estimate the level height use the obvious relation

$$n = \omega / (\Delta\omega + \delta\omega). \tag{16}$$

that gives

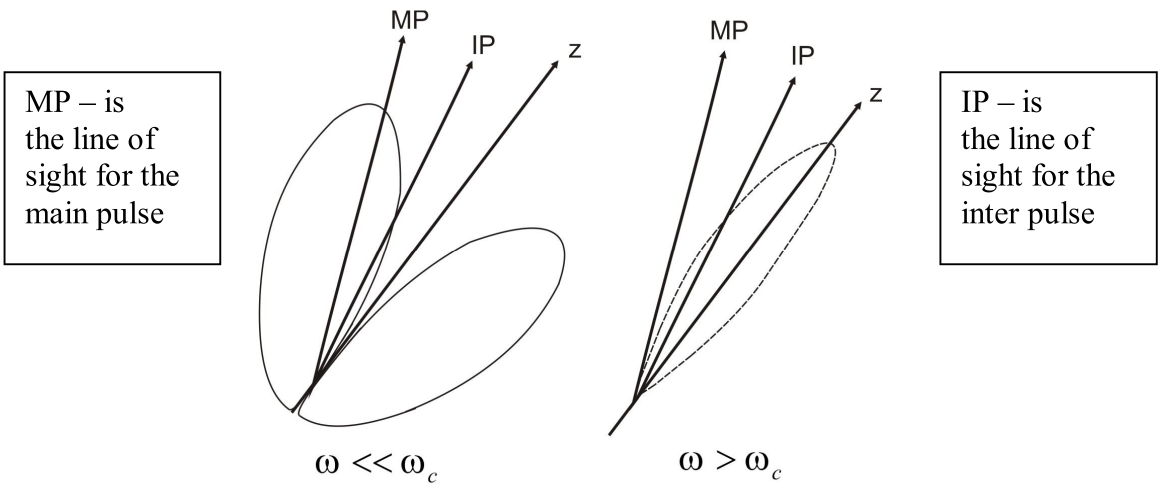
$$n \simeq 20. \tag{17}$$

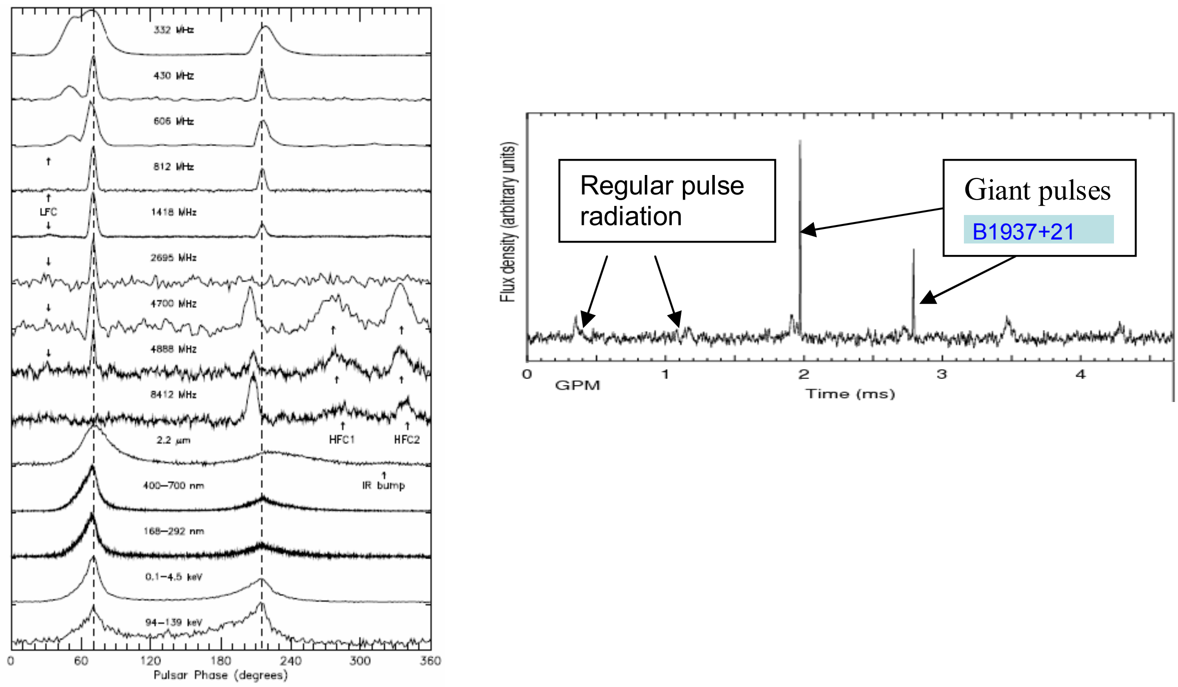
and $\Omega \approx \omega / 2\Gamma_{\Delta}$, $\Omega_n / 2\pi \simeq 10^{7}$ Hz.

Expression for angular speed is true for $r \gg r_0$ where r_0 is the classically rotating tornado core.

$$r_0 \simeq \sqrt{\frac{\hbar}{n \cdot m \Omega_0}} \approx 10^{-4}\,\text{cm}. \tag{18}$$

Relativistic aberration allows us to understand the difference in the spectra of the main pulse and inter pulse at a frequencies of change emission mechanisms near of the high-frequency spectrum break (V.M. Kontorovich and A.B. Flanchik: Radio emission with acceleration of electrons in a polar gap of a pulsar. Physics of Neutron Stars – 2011, Saint-Petersburg, Russia, Abstract book, p.75; http://www.ioffe.ru/astro/NS2011/index.html) of the Crab pulsar, where the bands are observed.





Figs. 5-6. *The difference in the diagrams relative to the line of sight by changing the mechanism of high-frequency radiation in the range of fracture (The Scheme on Fig. 5) [8] and main-pulse and inter-pulse observations in the pulsar PSR B0531 +21 (Fig. 6) Moffett and Hankins [9]. In the Scheme the rays are summarized in one quadrant.*

Some details of the observed spectral bands (two maximum in the band, frequency trend, etc.) is discussed and also received the natural explanation in the frame of the model.

Summary

Thus, the drift velocity of slow rotation is orthogonal to the electric field [6]. Rotational energy by virtue of this orthogonality is conserved. This makes possible the existence of stationary quantum states. It is implemented (similar taking place for Landau levels) the exact compensation of zero oscillations by the spin term.

Quantization of motion in an electromagnetic tornado may lead to appearance of bands in its frequency spectrum. Really, the rotation frequency of the drift branch [6, 10] of electrons in the tornado depends on the distance to the axis and for a given value level n spans some range of values determined by the geometrical dimensions of the tornado. The observed bands in the spectrum of giant pulses [2], coming directly from the internal polar gap [7], are connected with quantization of electron motion in the electromagnetic tornado [6]. Two maxima in the band (Fig. 1, [2]) can correspond radiation in the two aberration beams. One beam corresponds to emission of accelerated electrons in the polar gap (with gamma factor $\approx 10^3$) flying from the surface of the pulsar, the second one corresponds to the emission of accelerated positrons flying to the stellar surface in the same gap (with the same gamma factor in this band). Positron emission is directed to the star surface and we see reflected from the surface radiation. The incidence of positron radiation occurs orthogonally to the surface, whereby the reflected light is directed strongly backword to the incident radiation and accordingly coincides with the direction of electron emission. Therefore, it is observed as a continuation of the emission band (due to delay, which has the correct order of magnitude for a given gamma factor $\Gamma_\Delta \approx 10^3$). Indeed, the delay of the radiation emitted at the elevation z_c of order of the gap altitude is $\delta t \approx z_c / c$. At a height of the gap $z_c = 10^4 \mathrm{cm}$ it gives for δt the estimate $\delta t \approx 10^{-6}\,\mathrm{sec}$, which coincides with the pulse duration (emission band) for (11). Each peak corresponds to the passage of the radiation beam on the diagram of the telescope (the input and output of the diagram). Due to the reflection losses and deviations from exact orthogonality to the star surface of the incident and, respectively, the reflected beam, the intensity of the second peak is smaller than intensity of the first one.

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Appendix

Semiclassical motion of relativistic electron in external fields

The movement in the spin and coordinate spaces for relativistic electron is linked [A1]. For quasiclassical description it is necessary to separate the electron states with positive energies from the positron with negative ones. We will use the approach using the topological (geometric) Berry phase [A3] and the spin shift $\delta\mathbf{r}$ of covariant electron coordinate **R**:

$$\mathbf{R} = \mathbf{r} + \delta\mathbf{r}, \quad \delta\mathbf{r} = \hbar\mathbf{A}, \qquad \mathbf{A} = \frac{c^2}{\varepsilon_0(\varepsilon_0 + mc^2)}[\mathbf{p},\mathbf{S}] \qquad \text{(A.1)}$$

Singling out in the phase ϕ of the electron wave function $\psi \propto \exp\{\mathrm{i}\int \mathbf{R}d\mathbf{p}/\hbar\} \equiv \exp\{\mathrm{i}\phi\}$ the spin-dependent summand $\delta\phi$, we obtain $\delta\phi = \int \delta\mathbf{r}d\mathbf{p}/\hbar = \int \mathbf{A}d\mathbf{p}$ where **A** (Berry connectedness or geometric vector-potential [A4]) is defined by (A.1). When the drift motion of an electron in a circular orbit is formed by the joint action of ortogonal each other electric and magnetic fields, we obtain the phase shift $\theta_B = \oint_C \mathbf{A}d\mathbf{p}$ for orbital period (Berry phase). This phase increment depends only on the geometrical characteristics of the electron trajectories in space and does not depend from the time of its dynamics

$$\dot{\mathbf{S}} = [\mathbf{S}, [\frac{ec}{\varepsilon_0}\mathbf{B} - \frac{ec^2}{\varepsilon_0(\varepsilon_0 + mc^2)}[\mathbf{p},\mathbf{E}]]]. \qquad \text{(A.2)}$$

In our case of cylindrical geometry the condition that leads to solutions. This corresponds to electron spin states in a whirlwind $S_z = \pm 1/2$ and the spin part of energy equal to $\delta\varepsilon_s = -S_z B \cdot ec\hbar \cdot mc^2 / \varepsilon_0^2$, leading to compensation of zero oscillations. The resulting compensation allows us to consider the usual semiclassical quantization rule with the spin contribution. Bohr quantization rule, taking into account the contribution of Berry phase leads to

$$\oint \mathbf{r}d\mathbf{p} = 2\pi\hbar \cdot (n - \frac{\theta_B}{2\pi}), \qquad \theta_B = \oint \mathbf{A}(\mathbf{p})d\mathbf{p} = -2\pi\frac{p_\varphi^2 c^2}{\varepsilon_0(\varepsilon_0 + mc^2)}S_z, \qquad \text{(A.3)}$$

where the velocity of the electron in its orbit must be expressed in terms of non-relativistic electron frequency of revolution due to the drift in crossed fields. In the nonrelativistic limit the Berry phase tends to zero, which leads to condition (9) used in the text.

References to application

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