An analytic approximation for electron-nucleus bremsstrahlung neutrino emissivity in a neutron star crust of any composition

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Annotation

We present an analytic approximation for neutrino-pair bremsstrahlung emissivity in the inner and outer neutron star envelope (crust). The results are expressed though an effective potential of electron-nucleus scattering. This potential is equally valid in liquid and solid states of neutron star matter. The neutrino emissivity is determined by the generalized Coulomb logarithm which is calculated analytically with the obtained effective potential. The results can be applied for modeling of many phenomena in neutron stars, such as thermal relaxation in young isolated cooling neutron stars and in accreting neutron stars with overheated crust in soft X-ray transients after accretion stops and the star evolves in the quiescent state.

eZ-bremsstrahlung in the neutron star crust

For a long time a neutron star (NS) cools mostly via neutrino emission from its interior.

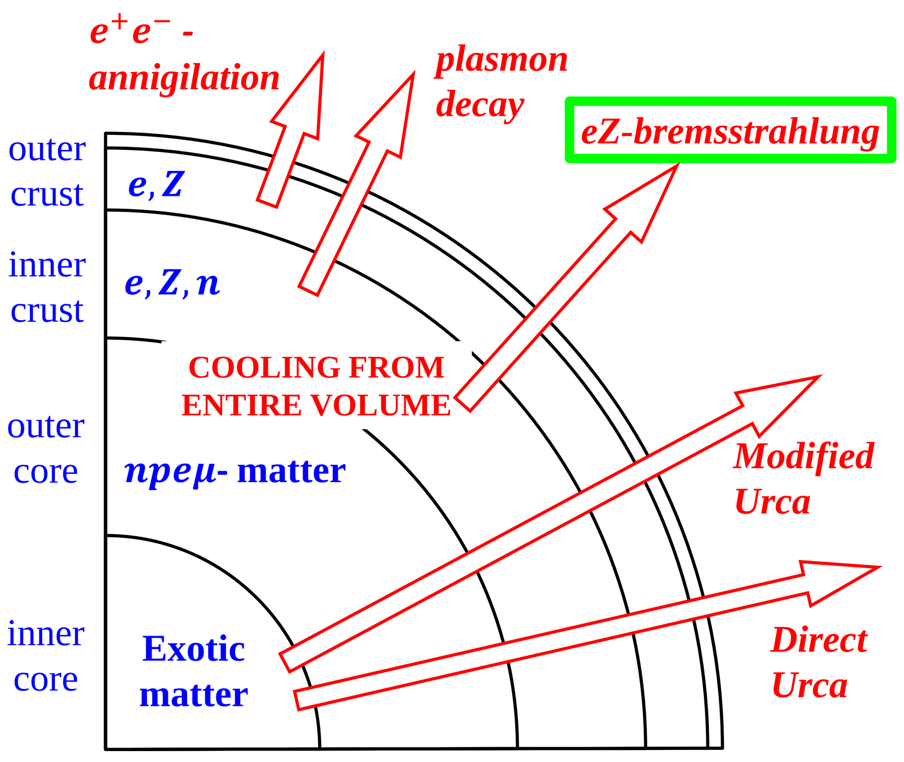


Fig.1. Neutrino emission from neutron star interior

- Basic quantity to be determined: **neutrino emissivity Q [erg cm^{-3} s^{-1}]**
- **Different layers of NS interior ⇒ different Q** (Fig.1.)
- **In NS crust** at $T < 10^9$ K the dominant neutrino emission is due to **electron-nucleus (eZ) bremsstrahlung** (Fig.2.)
- **Important for thermal relaxation of young NSs and for cooling of accreting NSs with overheated crust in SXRTs**

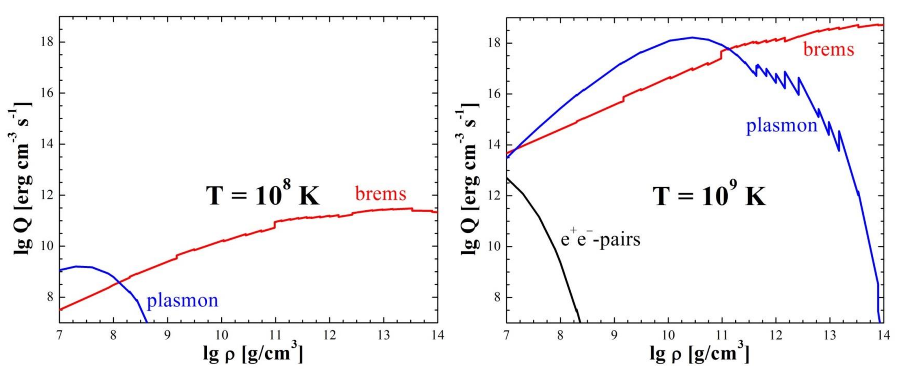


Fig.2. Neutrino emissivity by different processes in the NS crust. "**Plasmon**" – plasmon decay, "e^+e^-**-pairs**" – electron-positron annihilation, "**brems**" – eZ-bremsstrahlung. One can see that *at high densities or $T < 10^9$ K the latter process dominates.*

Approximations in NS cooling code (OYG)

- **One needs fast computation** of coefficients (κ, σ) and Q ⇒ uses **simple analytic approximations** of these quantities
- **Until now** one used the **approximation of eZ-bremsstrahlung by Kaminker et al. (1999)**
  - valid for ground-state crust only
- **One needs to consider non-equilibrium crust ⇒ new approximation are required, valid for all possible EOS's and compostions.**

General formalism of eZ-bremsstrahlung

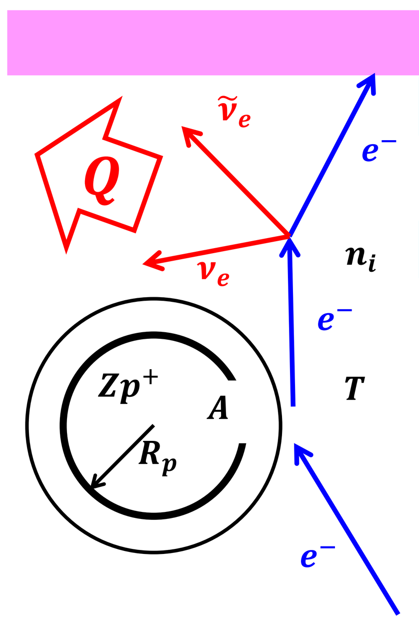


Fig.3. Schematic diagram of eZ-bremsstrahlung

The reaction

$$e^- + (A,Z) \to e^- + (A,Z) + \nu + \bar{\nu} \qquad (1)$$

The fluid model

- **Coulomb liquid or Coulomb crystal of fully ionized atoms** (atomic nuclei), their **number density** n_i lies in range **10^{29}…10^{34} cm^{-3}**;
- **ultrarelativistic and strongly degenerate electrons**,
  - free in liquid
  - Bloch-states in crystal
- There is only **one type of ions** with
  - **charge number Z = 6…50**;
  - **mass number A = (2…3)×Z**;
  - **the proton core radius R_p**; R_p = (0.0…0.2)×R_{ws}, R_{ws} = radius of Wigner-Seitz cell.
- The **temperature T** ranges **from 10^7 to 10^9 K**.

Suitable dimentionless parameters

$$x = \frac{\hbar}{m_e c}\left(3\pi^2 Z n_i\right)^{\frac{1}{3}}, \qquad \Gamma = \frac{Z^2 e^2}{kT}\left(\frac{4\pi n_i}{3}\right)^{\frac{1}{3}}, \qquad \tau = \frac{kT}{\hbar Z e}\sqrt{\frac{A m_u}{4\pi n_i}}, \qquad \xi = R_p\left(3\pi^2 Z n_i\right)^{\frac{1}{3}} \qquad (2)$$

Here m_e = electron mass, m_u = atomic mass unit. Other parameters:

- **x = ratio of electron Fermi-momentum p_F to $m_e c$**;
- **Γ = Coulomb coupling parameter**, **melting point refers to Γ=175**;
- **τ = ratio of temperature T to ion plasma temperature T_p**, determines importance of quantum effects in ion-ion interactions;
- **ξ = ratio of proton core radius to the electron de Broglie wavelength**; determines importance of form factor effects.

Emissivity by Kaminker et al. (1999)

$$Q = \frac{8\pi G_f^2 e^4 C_+^2}{567\hbar^9 c^8} Z^2 (kT)^6 n_i L(Z, A, R_p, n_i, T) R_{\rm NB} \qquad (3)$$

Here G_f = **1.436×10^{-49} erg cm^3** = Fermi weak interaction constant, e = absolute value of electron charge, **C_+^2=1.675** takes into account three neutrino flavours, $\hbar$ = Planck constant, c = speed of light, **$R_{\rm NB}$ = 1 + 0.00554Z + 0.0000737Z^2** = non-Born corrections, **L = generalized Coulomb logarithm**, which can be expressed as

$$L = \begin{cases} L_{liq}, & \Gamma < 175 \\ L_{ph} + L_{sl}, & \Gamma > 175 \end{cases} \qquad (4)$$

liquid state: L_{liq}; solid state: L_{ph} = electron-phonon scattering, L_{sl} = Bragg diffraction of electrons on the lattice cites. The terms takes form **(Kaminker et al., 1999)**:

$$L_{sl} = \sum_{y\neq 0} \frac{|F(2\xi y)|^2 e^{-wy^2}}{\left(y^2 + y_0^2\right)^2} I(y)\, y^2\left(1 - y^2\right) \qquad (5)$$

$$L_{ph} = \int_{(4Z)^{-1/3}}^{1} \frac{S_{ph}(y)|F(2\xi y)|^2}{\left(y^2 + y_0^2\right)^2} R_T(y) y^3 \,\mathrm{d}y, \qquad L_{liq} = \int_0^1 \frac{S_{liq}(y)|F(2\xi y)|^2}{\left(y^2 + y_0^2\right)^2} R_T(y) y^3 \,\mathrm{d}y, \qquad (6)$$

- Summation is over reciprocal lattice vectors
- **y_0** corresponds to **electron screening**; e.g. Kaminker et al. (1999);
- **$I(y)$** given in Kaminker et al. (1999). In high-temperature limit it gives **$R_T(y)$**;
- **$R_T(y)$ = thermal function**, describes neutrino energy losses in the limit of strong electron degeneracy,

$$R_T = 1 + \frac{2y^2 \ln y}{1 - y^2} \qquad (7)$$

- **$F(2\xi y)$ = nuclear formfactor**. Different expressions are given in Haensel, Potekhin & Yakovlev (2007). In this work we use approach of spherical nuclei:

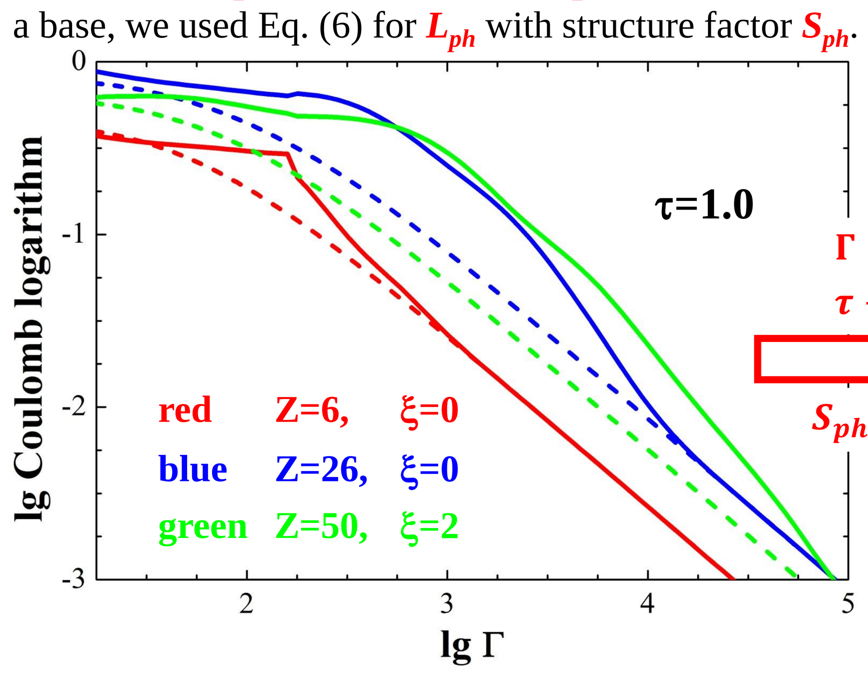
$$F(u) = 3\frac{\sin u - u\cos u}{u^3}, \qquad u = 2\xi y \qquad (8)$$

- **exp($-w\,y^2$) = Debye-Waller factor**;
- **$S_{liq}(y)$ = liquid structure factor**, corresponds to ion screening; approximated by Young et al. (1991);
- **$S_{ph}(y)$ = effective phonon structure factor**, takes into account multiphonon processes.

The analytic approximation of the Coulomb logarithm

In Gnedin et al. (2001) present **approximations** of thermal and electric conductivities **by A.Y. Potekhin, in terms of effective potential of electron-nucleus scattering**. The potential is the same for liquid and solid states and allows analytical integration of Coulomb logarithm. It **does not include Bragg diffraction** which is not important for these kinetic coefficients. Therefore, it **cannot be applied for eZ-bremsstrahlung**.

This work presents an effective potential that includes the static lattice input and valid for eZ brems. As a base, we used Eq. (6) for L_{ph} with structure factor S_{ph}.



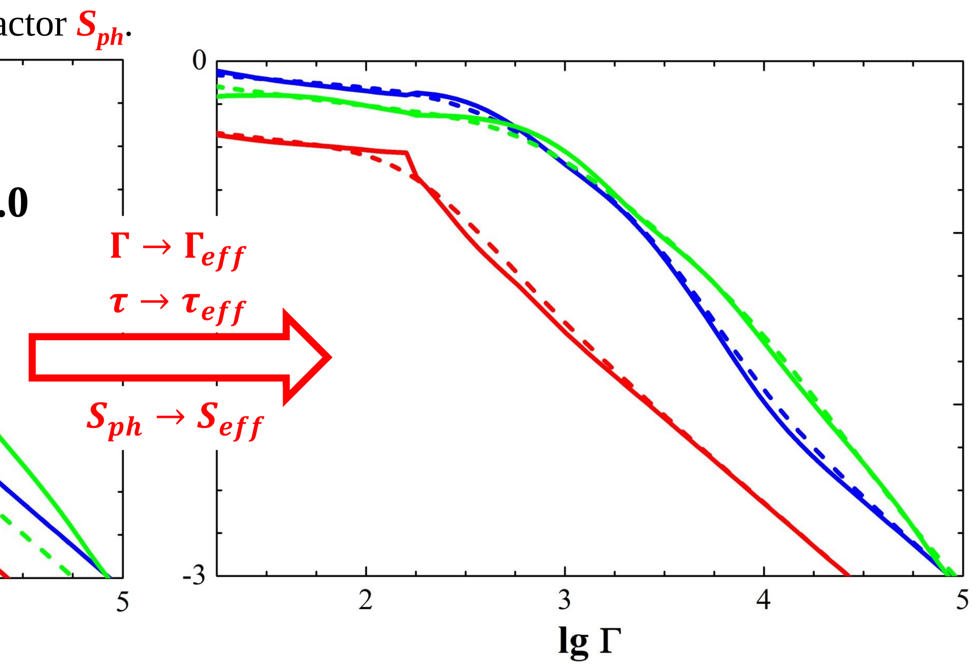


Fig.4. Relation between $L(\Gamma)$ and $L_{ph}(\Gamma)$ in log-log scale. Solid = full L, dashes = L_{ph}.

Fig.5. Calculated $L(\Gamma)$ versus its approximation in log-log scale. Solid = full L, dash = approximated L. Parameters are the same as on the left panel.

The idea: nonlinear scaling of parameters

One can see that **graphs of full $L(\Gamma)$** given in (4) **look similar to graphs of $L_{ph}(\Gamma)$** in (6). This similarity allows us to obtain an approximation using slightly **modified expressions for L_{ph} with non-linearly scaled Γ and τ.**

$$\tau \longrightarrow \tau_{eff}(Z, \Gamma, \tau)$$

$$\Gamma \longrightarrow \Gamma_{eff}(Z, \Gamma, \tau_{eff}, \xi)$$

$$S_{ph}(Z, \Gamma, \tau, y) \longrightarrow S_{eff}(Z, \Gamma_{eff}, \tau_{eff}, \xi, y)$$

The effective potential and an analytic view of Coulomb logarithm

The approximated Coulomb logarithm:

$$L = \int_0^1 \left|V_{eff}(y)\right|^2 R_T(y) y^3 \,\mathrm{d}y, \qquad \boxed{\left|V_{eff}(y)\right|^2 = \frac{S_{eff}(y)|F(2\xi y)|^2}{y^4}} \qquad (9)$$

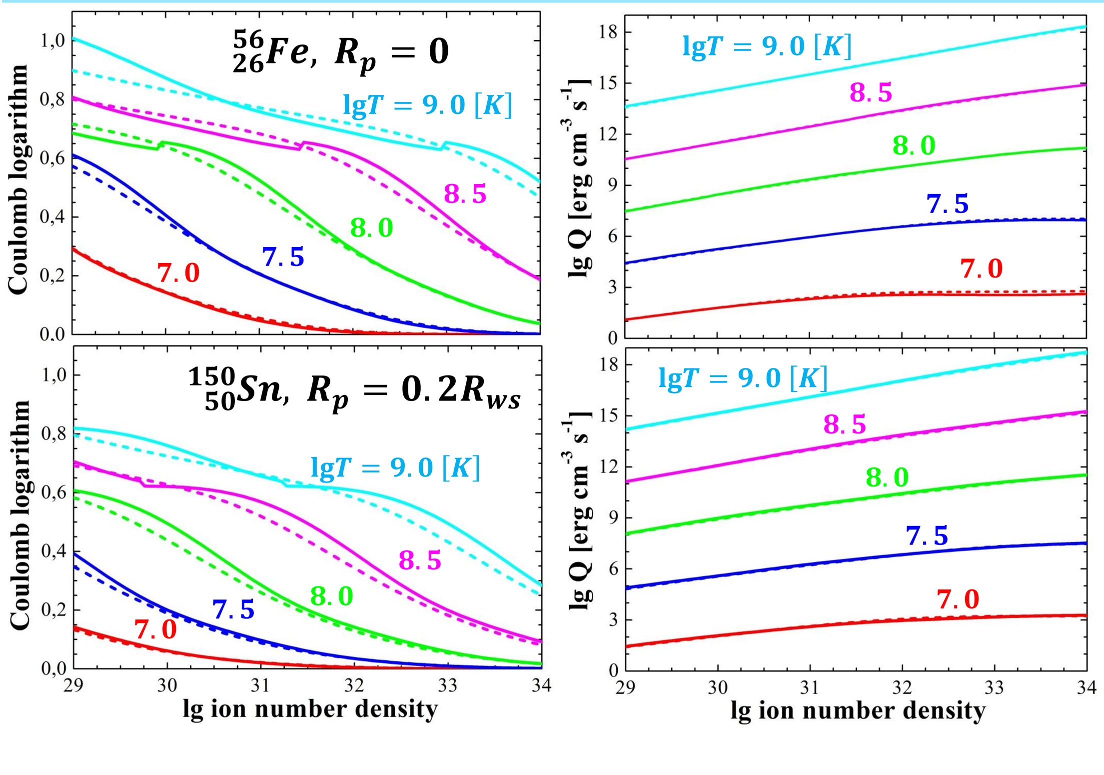
- **V_{eff} = effective Fourier-transformed potential of electron-nucleus scattering in the eZ-bremsstrahlung**.
- the electron screening is neglected as soon as S_{eff} provides a convergence of integral in L.
- For an analytic integration one can use simple approximations
  - of thermal function: **$R_T(y) \approx 1 - y$** with **max absolute error 0.066**
  - of nuclear form factor: **$|F(u)|^2 \approx \exp\{-\alpha u^2\}$**, **$\alpha$ = 0.23** with **max absolute error 0.032**.

The result is

$$\boxed{\begin{gathered} L = \mathrm{Ei}(-p) - \mathrm{Ei}(-q) + \ln\frac{q}{p} - \sqrt{\frac{\pi}{p}}\,\mathrm{erf}(\sqrt{p}) + \sqrt{\frac{\pi}{q}}\,\mathrm{erf}(\sqrt{q}), \\ p = w - w_1 + 4\alpha\xi^2, \qquad q = w + 4\alpha\xi^2, \qquad S_{eff} = \left(e^{w_1 y^2} - 1\right) e^{-wy^2} \end{gathered}} \qquad (10)$$

One can obtain **Q** with (3). *The details are given in the text attached to this poster or can be requested via e-mail.*

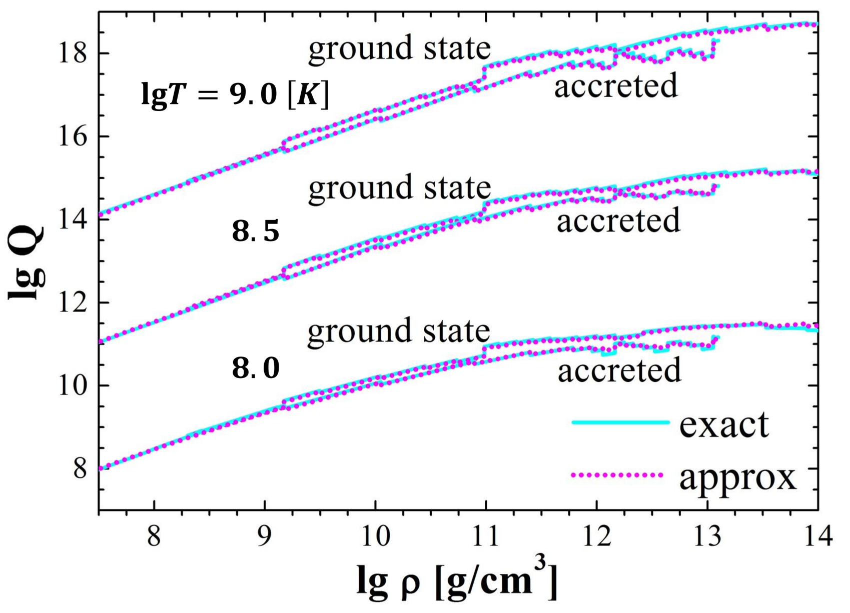
Relative deviation from exact values



- **Optimization of $L(Z, A, R_p/R_{ws}, n_p, T)$**
- **Ranges of arguments:** see *Fluid Model*
- For **Coulomb logarithm L**: **rms = 14%, max error = 140%**;
- For **emissivity log Q**: **rms = 0.6%, max error = 8%**

Fig.6: *left*: relation $L(n_i)$ in semilog scale; *right*: relation $Q(n_i)$ in log-log scale; *upper*: $^{56}_{26}Fe$, R_p=0; *lower*: $^{150}_{50}Sn$, R_p=0.2R_{ws}. Different curves corresponds to different temperatures shown on the panels. Solid = exact calculations, dash = the approximation.

Applications of the approximation



- Calculation of **internal thermal relaxation of young isolated NS**
- modelling of **cooling of accreting neutron stars** with overheated crust in **soft X-ray transients after accretion stops** and the star evolves in the quiescent state.

Fig.7 shows that **the approximation allows us to reproduce all differences** between bremsstrahlung emissivity **in ground state and accreted crust**.

Fig.7. Relation $Q(\rho)$ for ground state and accreted crusts. Ground state composition is given by Haensel, Potekhin & Yakovlev (2007), accreted composition – by Haensel & Zdunik (1990). Exact and approximated curves are close.

Conclusions

We obtain *the universal approximation for eZ-bremsstrahlung emissivity in the NS crust of any composition*. In range of parameters 10^8 ≤ ρ ≤ 10^{14} g/cm^3, 10^7 ≤ T ≤ 10^9 K (ions form Coulomb crystal or liquid, electrons form relativistic degenerate Fermi gas), 6 ≤ Z ≤ 50, 2Z ≤ A ≤ 3Z (all models of NS crust). Deviations from exact values of Q is 0.6%. The approximation is important for modeling of thermal evolution of NSs.

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