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Analytic approximation for electron-nucleus bremsstrahlung emissivity in the NS crust of any composition

User's guide

Saint-Petersburg
2014

We approximate the generalized Coulomb logarithm L as a function of five arguments: nuclear charge number Z, mass number of bounded nucleons A, proton core radius R_p, number density of ions (atomic nuclei) n_i, and temperature T.

Let us define suitable dimensionless parameters:

$$\Gamma = \frac{Z^2 e^2}{kT}\left(\frac{4\pi n_i}{3}\right)^{1/3} = 5.798 \frac{Z^2}{T_9} n_{i\,37}^{1/3}, \tag{1}$$

$$\tau = \frac{kT}{\hbar Z e}\sqrt{\frac{A m_u}{4\pi n_i}} = 0.03135 \frac{T_9}{Z}\sqrt{\frac{A}{n_{i\,37}}}, \tag{2}$$

$$\xi = R_p \left(3\pi^2 Z n_i\right)^{1/3} = \left(\frac{9\pi}{4} Z\right)^{1/3} \frac{R_p}{R_{ws}} = 0.6665 R_{p\,\mathrm{fm}} \left(Z n_{i\,37}\right)^{1/3}. \tag{3}$$

Here $T_9 = T/(10^9\text{ K})$, $n_{i\,37} = n_i/(10^{37}\text{ cm}^{-3})$, $R_{p\,\mathrm{fm}} = R_p/(1\text{ fm})$; R_{ws} is a Wigner-Seitz cell radius.

The Coulomb logarithm is expressed as

$$L = \int_0^1 \left|V_{\mathrm{eff}}(y)\right|^2 R_T(y) y^2 \mathrm{d}y, \qquad \left|V_{\mathrm{eff}}(y)\right|^2 = \frac{S_{\mathrm{eff}}(y)|F(2\xi y)|^2}{y^4}, \tag{4}$$

where $y = q/2p_{\mathrm{F}}$ is a dimensionless momentum transfer (p_{F} being the electron Fermi momentum); $R_T(y)$ is the thermal function,

$$R_T(y) = 1 + \frac{2y^2 \ln y}{1 - y^2} \approx 1 - y, \quad \text{max abs error} = 0.066; \tag{5}$$

$V_{\mathrm{eff}}(y)$ is the Fourier image of the effective potential of electron-nucleus scattering for eZ-bremsstrahlung process, the main subject of our study. Its constituents are: $F(2\xi y)$ = the nuclear form factor,

$$F(u) = 3\frac{\sin u - u\cos u}{u^3} \approx e^{-\alpha u^2}, \ \alpha = 0.23, \quad \text{max abs error} = 0.032; \tag{6}$$

$1/y^4$ = squared Fourier transform of the Coulomb potential; $S_{\mathrm{eff}}(y)$ = the effective structure factor,

$$S_{\mathrm{eff}} = \left(e^{w_1 y^2} - 1\right) e^{-w y^2}, \tag{7}$$

where w_1 and w are

$$w_1 = B\, \frac{b\tau_{\mathrm{eff}}}{\sqrt{(b\tau_{\mathrm{eff}})^2 + u_{-2}^2 \exp\left\{-7.6\left(\tau_{\mathrm{eff}} + \frac{18.0}{\Gamma_{\mathrm{eff}}}\right)\right\}}}, \tag{8}$$

$$w = B\,\left(\frac{u_{-1}}{2u_{-2}\tau_{\mathrm{eff}}} \exp\left\{\tau_{\mathrm{eff}} + \frac{216}{\Gamma_{\mathrm{eff}}}\right\} + 1\right), \tag{9}$$

$$B = \frac{3}{5} \frac{\left(12\pi^2\right)^{1/3}}{1 + \frac{\xi}{Z} \frac{\Gamma_{\rm eff}}{200+\Gamma_{\rm eff}}} \frac{u_{-2} Z^{4/5}}{\sqrt{\Gamma_{\rm eff}^2 + \frac{1037}{\left(1+\frac{\Gamma_{\rm eff}}{204}\right)^4} \frac{\Gamma_{\rm eff}^{\frac{1}{2}(1+0.15\xi)}}{Z^{0.10}}}}. \tag{10}$$

Here $b = 231$, $u_{-1} = 2.798$, $u_{-2} = 12.972$, and $\tau_{\rm eff}$ and $\Gamma_{\rm eff}$ are non-linearly scaled τ and Γ, respectively. They are related to original Γ, τ, ξ and Z via the expressions:

$$\tau_{\rm eff} = 0.095 \left(\frac{2\tau}{0.095\sqrt{\tau^2+4}} \right)^{\overline{1+\left(\frac{Z}{35.5}\right)^2\left(1+\frac{\Gamma}{223Z}\right)^{-1}}^{\,1}}, \tag{11}$$

$$\Gamma_{\rm eff} = \Gamma \exp \left\{ - \frac{\tau_{\rm eff}^{0.053} \left(\frac{Z}{11.3}\right)^{4/9}}{1 + \tau_{\rm eff}^{0.50+0.002Z\xi} \left(\frac{\Gamma}{19.3Z^{1.7}}\right)^{\frac{1.52+0.90\left(1-\tau_{\rm eff}\right)}{1+0.50\xi}}} \right\} \tag{12}$$

Now the Coulomb logarithm (4) is integrated analytically:

$$L = f\left(w + 4\alpha\xi^2\right) - f\left(w - w_1 + 4\alpha\xi^2\right), \tag{13}$$

$$f(x) = \sqrt{\frac{\pi}{x}} \operatorname{erf}\left(\sqrt{x}\right) + \ln x - \operatorname{Ei}(-x). \tag{14}$$

Here w and w_1 are given by Eqs. (9) and (8); α is defined by (6). A useful advice is to use an asymptotic value of f at $x \sim 0$:

$$f(x) \xrightarrow[x\to 0]{} 2 - \gamma, \tag{15}$$

where $\gamma = 0.557216...$ is the Euler's constant. It helps to avoid computational errors.

Finally, the neutrino emissivity takes form

$$Q = 5.362 \times 10^{18} Z^2 n_{i\,37} T_9^6 L R_{\rm NB} \, \left[\frac{\rm erg}{\rm cm^3 \ s}\right], \tag{16}$$

where $R_{\rm NB} = 1 + 0.00554Z + 0.0000737Z^2$ is a non-Born correction.

The details of the eZ neutrino bremsstrahlung formalism can be found in Kaminker *et al.* Astron. Astrophys. **343**, 1009 (1999).