Physics of Magnetars and Its Astrophysical Significance

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1) Origin of Ultra strong B:

Ferromagnetism of Anisotropic ($^3P_2$) neutron superfluid

2) Fermi energy of electrons

under ultra strong magnetic field

3) Activity: Physical origin of the abnormal high X-ray luminosity For AXP :

Instability due to high Fermi energy of electrons under the ultra strong B

Observation

Ultra strong magnetic field

For AXPs (Anormalous X-ray Pulsars) and SGRs (Soft Gamma Repeaters)

Long spin period and spin-down rate

$$P \sim 5-12 \quad \mathrm{s}$$

$$\dot{P} \approx (10^{-11} - 10^{-12}) \quad ss^{-1}$$

$$\Rightarrow \quad B_{p,14} = 0.32\sqrt{P\dot{P}_{-12}} \quad Gauss$$

$$B \sim 10^{14} - 10^{15} \quad Gauss$$

Absorption line at 10 keV

Abnormal high X-ray luminosity For AXP

$$L_x \sim (10^{34} - 10^{36}) ergs/\sec$$

Decaying pulsating tails of giant flares with energy 10^{44} ergs/sec

It can not be explained by loss of the rotational energy.
It is guessed by transformed from the energy of the ultra strong magnetic field of the magnetars.

I. Origin of Ultra strong Magnetic Field

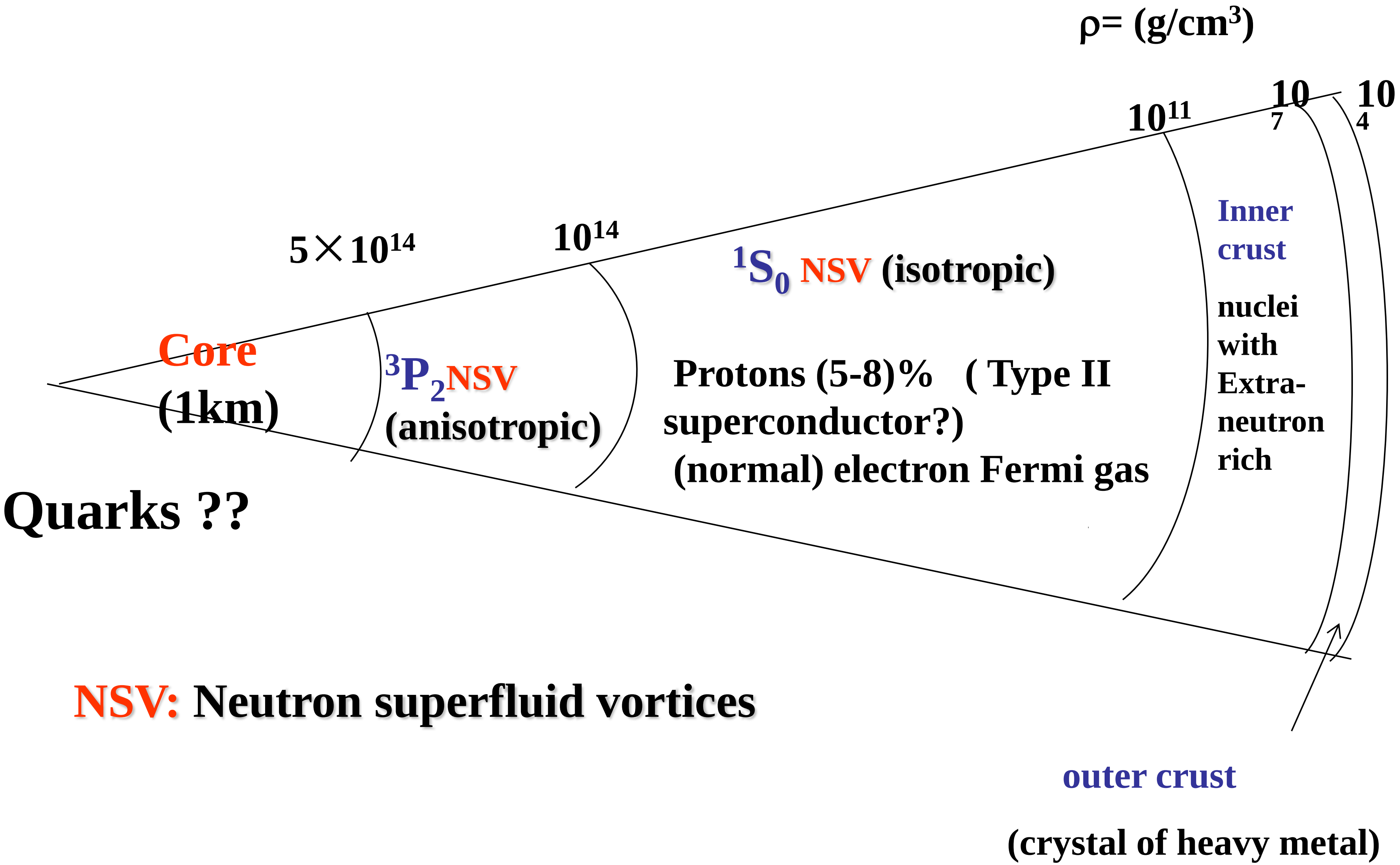
Proposed Models for the ultra strong B

- Duncan & Thompson (1992, 1993): α-Ωdynamo with initial spin period less than 3ms
- Ferrario & Wickrammasinghe(2005)suggest that the extra-strong magnetic field of the magnetars descends from their stellar progenitor with high magnetic field core.
- Vink & Kuiper (2006) suggest that the magnetars originate from rapid rotating proto-neutron stars.
- Iwazaki(2005)proposed the huge magnetic field of the magnetars is some color ferromagnetism of quark matter.

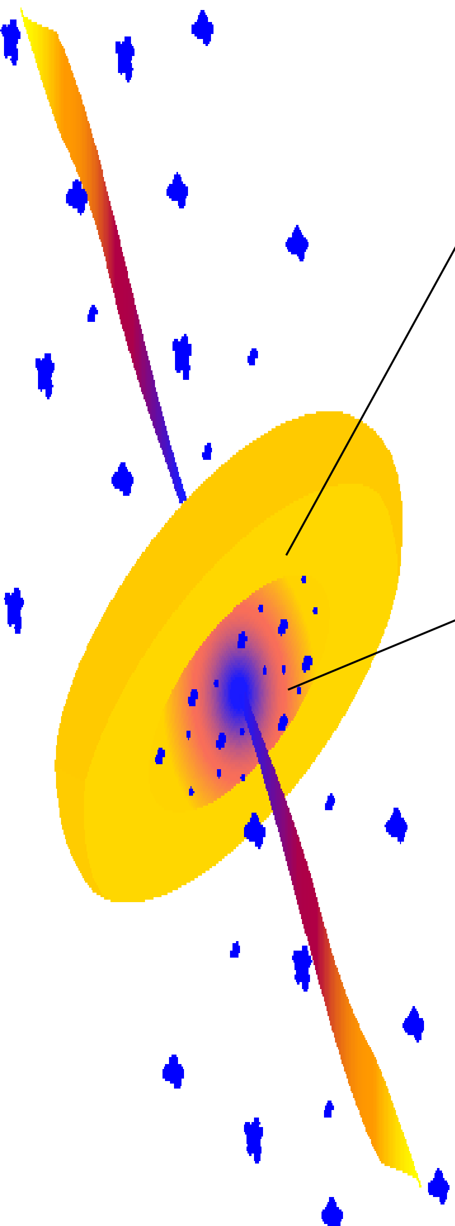
The question is still open!

Our idea: **It Is Origin from Ferromagnetism of Anisotropic ($^3P_2$) neutron superfluid**

Structure of a neutron star



$^1S_0$ & $^3P_2$ Neutron superfluid



$^1S_0$ neutron Cooper pair: S=0, isotropic

$\vec{S} = 0 \quad \uparrow\downarrow$

Energy gap :

$\triangle(^1S_0) \geq 0$, $\quad 10^{11} < \rho(g/cm^3) < 1.4\times10^{14}$

$\triangle(^1S_0) \geq 2MeV \quad 7\times10^{12} < \rho(g/cm^3) < 5\times10^{13}$

$^3P_2$ neutron Cooper pair: S =1, anisotropic,

$\vec{S} = 1 \cdot \hbar\vec{\sigma}$

$\uparrow\uparrow \quad \downarrow\downarrow \quad \Rightarrow (Or \Leftarrow)$

Abnormal magnetic moment ~10^{-23} c.g.s.

Energy gap:

$\triangle_n(^3P_2) \sim 0.05MeV$

$(3.3\times10^{14} < \rho\ (g/cm^3) < 5.2\times10^{14})$

$$\rho_{nuc} = 2.8\times10^{14} \quad g/cm^3$$

Anisotropic $^3P_2$ neutron superfluid

Critical temperature:

$$T_c(^3P_2(n)) = \Delta_{\max}(^3P_2(n))/2k \approx 2.78\times10^8\,K$$

Magnetic moment: $2\mu_n$

$$\mu_n \sim -0.966\times10^{-23} \quad erg/gauss$$

A magnetic moment tends to point at the direction of applied magnetic field with lower energy due to the interaction of the magnetic field with the magnetic moment of the $^3P_2$ neutron Cooper pair.

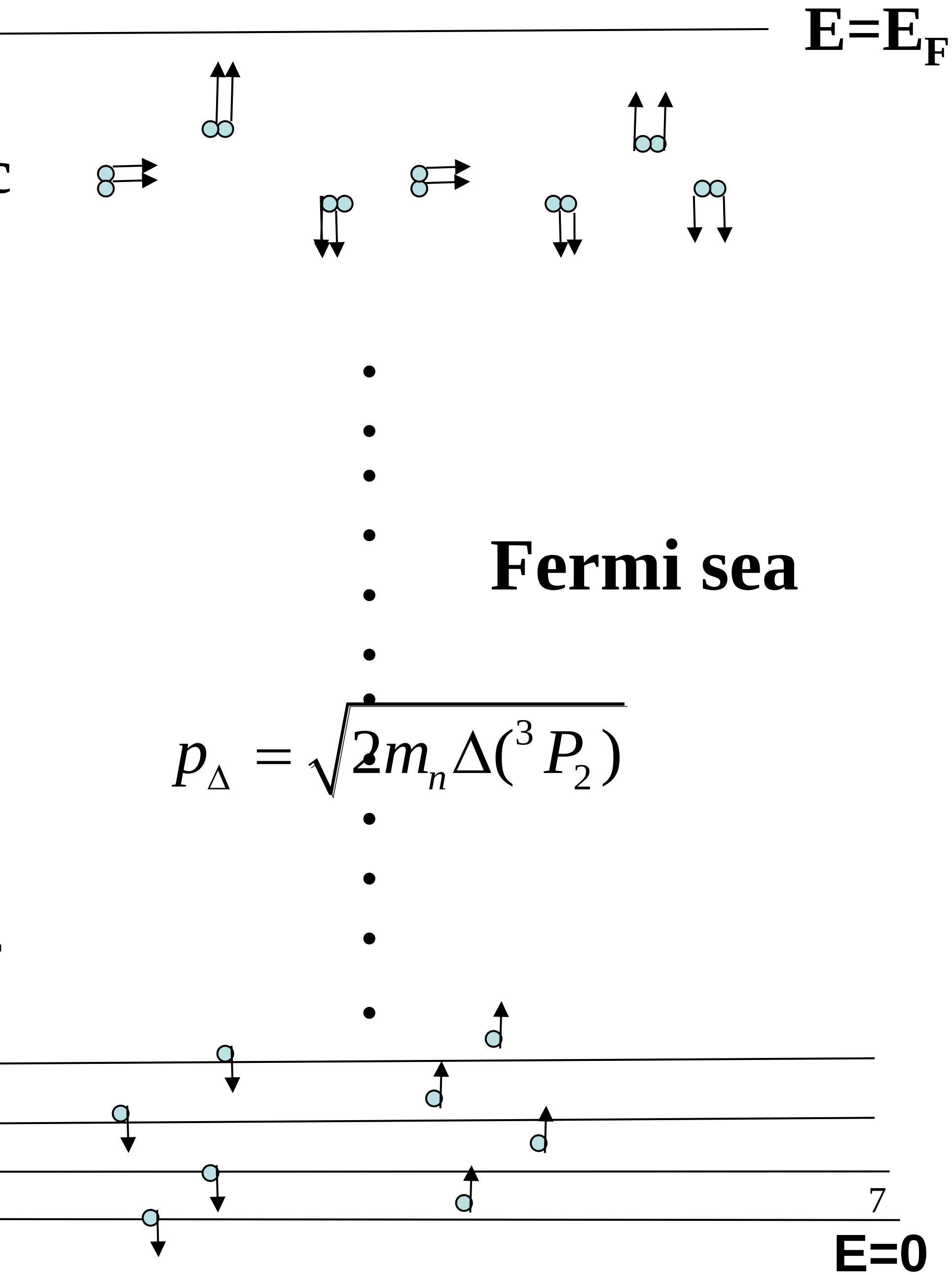
orientation of the $^3P_2$ Cooper pairs

The magnetic moment tends to the direction of applied magnetic field

Thermal motion will cause the magnetic moments in a chaos

The paramagnetism of the $^3P_2$ neutron pairs is decided by competition between the two opposite factors above.

The neutrons in the deep level of the Fermi sea do not contribute to paramagnetism.

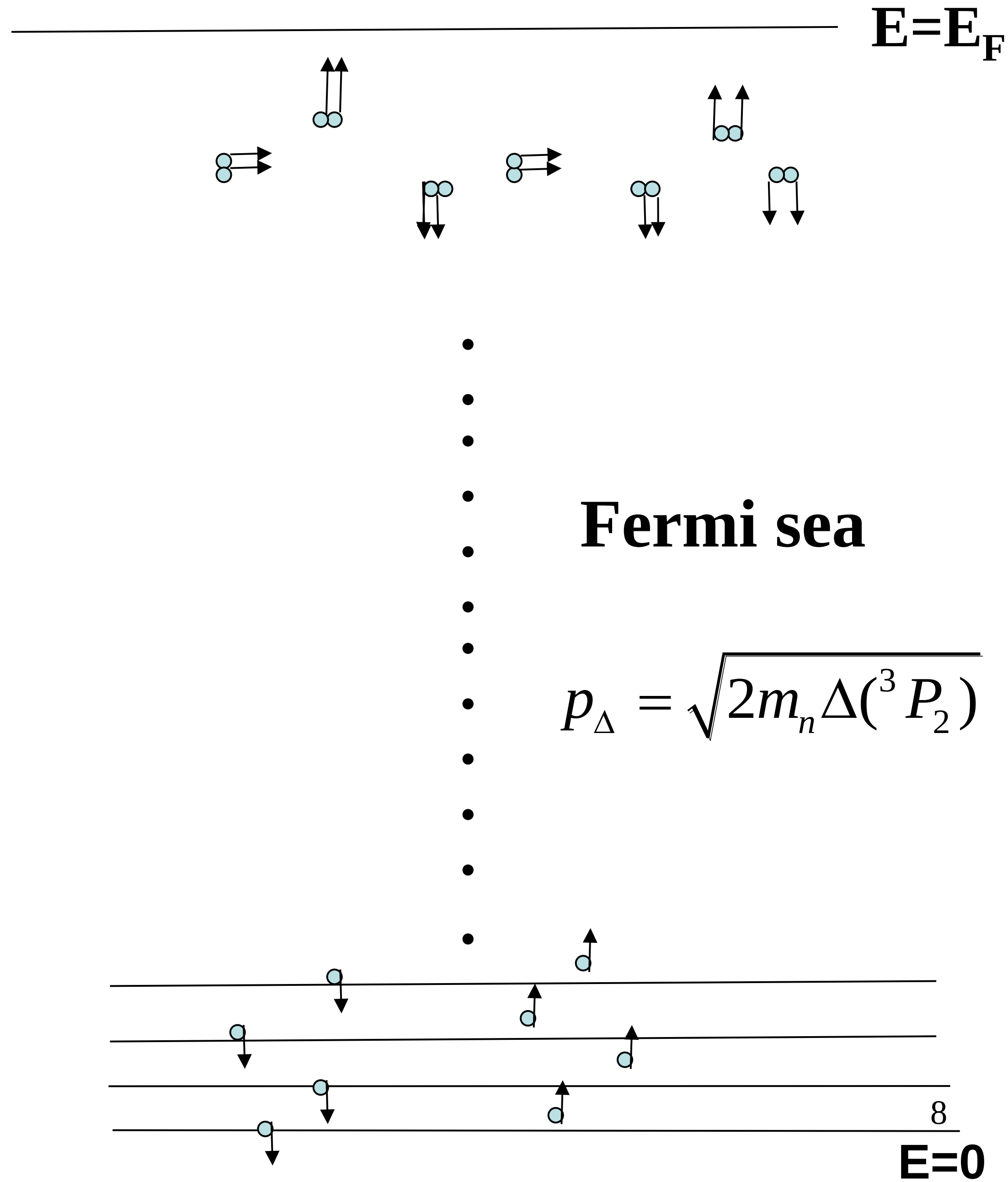


$$p_\Delta = \sqrt{2m_n\Delta(^3P_2)}$$

orientation of the $^3P_2$ Cooper pairs

The paramagnetism is caused only by the Cooper pairs. All Cooper pairs almost congregate in a thin layer with thickness p_Δ near the Fermi surface in the momentum space.

The paramagnetic Cooper pairs are slightly more than one of diamagnetic ones at temperature T>2×10^{7}K.



$$p_\Delta = \sqrt{2m_n\Delta(^3P_2)}$$

Statistics

(For the magnetic moment of $^3P_2$ neutron Cooper pair)

- **Hamiltonian:** $$H = -2\vec{\mu}_n \cdot \vec{B} = -2\mu_{n,z} B$$

•**Ensemble average:** $$< 2\mu_n >= 2\mu_n f(\frac{\mu_n B}{kT})$$

· **The Brillouin function** $$f(x) = \frac{2\sin h(2x)}{1 + 2\cos h(2x)}$$

$$f(x) \approx 4x/3 \qquad x << 1 \qquad (i.e. \frac{\mu_n B}{kT} << 1)$$

$$f(x) \to 1 \qquad x >> 1 \qquad (i.e. \frac{\mu_n B}{kT} >> 1)$$

Energy gap --- Combing energy of a Cooper pair

A key idea: The energy gap, Δ, is a combining energy of couple of of neutrons (the Cooper pair). It is a real energy, rather than the variation of the Fermi energy due to the variation of neutron number density.

$$\Delta \neq \delta E_F$$

Corresponding momentum of the combing energy of the neutron Cooper pair is (in non-relativity)

$$p_\Delta = \sqrt{2m_n\Delta}$$

q Value

How many neutrons have been combined into the $^3P_2$ Cooper pairs?

Since only particles in the vicinity of the Fermi surface contribute (Lifshitz et al. 1999), there is a finite probability q for two neutrons being combined into a Cooper pair.

$$q = \frac{4\pi p_F{}^2 \times p_\Delta}{\frac{4\pi}{3} p_F{}^3} = 3\sqrt{\frac{\Delta}{E_F}} = 0.087$$

$$N(^3P_2 - pair) = q \times N_A m(^3P_2)/2$$

$$\mu(^3P_2) = N(^3P_2 - pair) \times < 2\mu_n >$$

Total induced magnetic field by the $^3P_2$ superfluid

$$B^{(in)} = \frac{2\mu(^3P_2(n))}{R^3} = \frac{2m(^3P_2(n))N_A\mu_n}{R^3} qf(\mu_n B/kT)$$

or

$$B^{(in)} = B_{\max} f(\mu_n B/kT)$$

$$B_{\max} = \frac{2\mu_n q N_A m(^3P_2)}{R_{NS}^3} \approx 2.02\times 10^{14}\eta \qquad gauss$$

$$\eta = \frac{m(^3P_2)}{0.1 m_{Sun}} R_{NS,6}^{-3}$$

From paramagnetism to ferromagnetism

$$B^{(in)} = B_{\max} f(\mu_n B / kT)$$

$$B = B^{(in)} + B^{(0)}$$

$$b = f(x) \qquad f(x) \approx 4x/3 \qquad x << 1 \qquad \mu_n B << kT$$

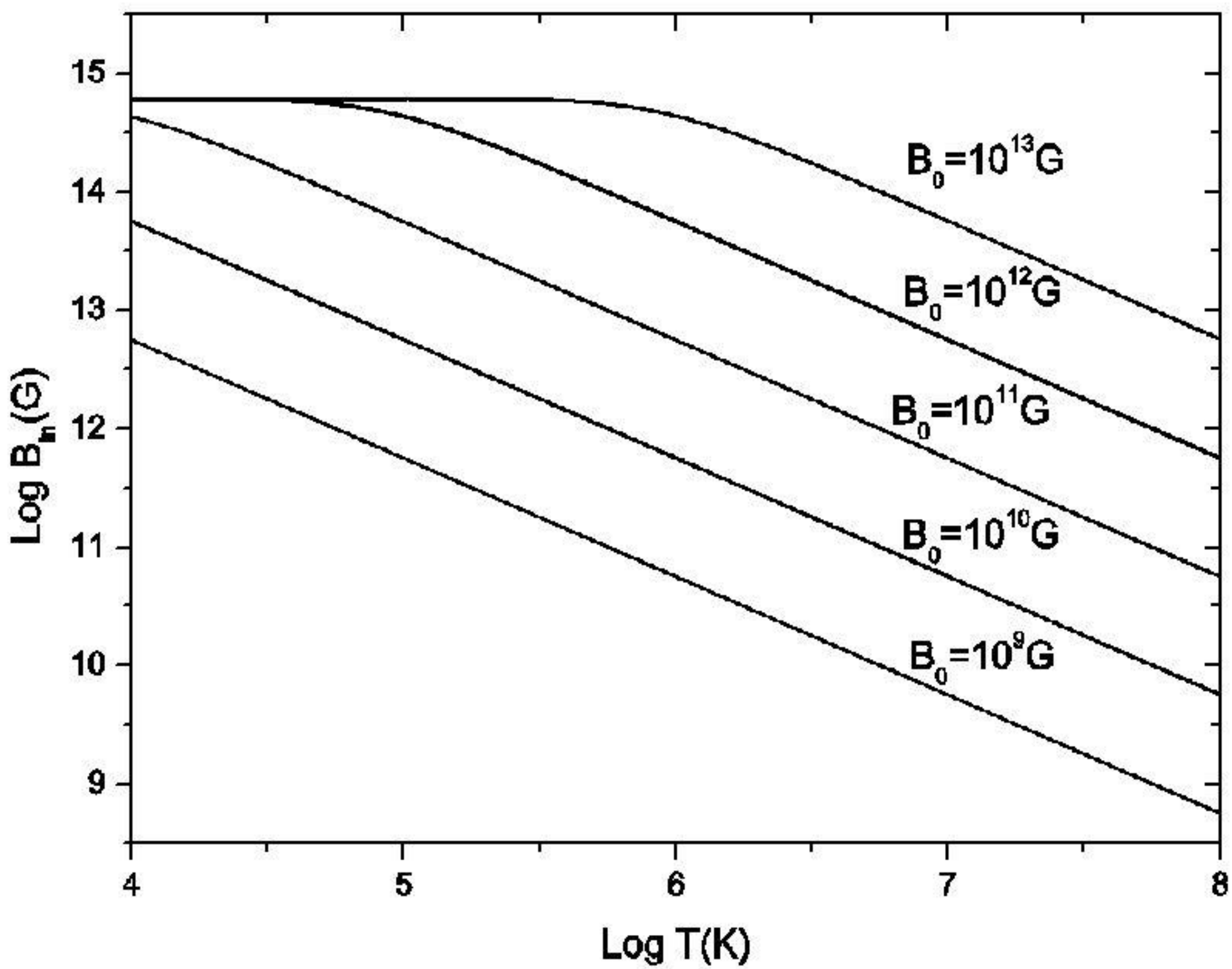
(B < 10^{13} gauss , T > 10^7 K)

$$x = \frac{1.40\ \eta}{T_7}(b + b^{(0)}) \qquad b = \frac{B^{(in)}}{B_{\max}} \qquad b^{(0)} = \frac{B^{(0)}}{B_{\max}}$$

Set $b^{(0)} = 0$ → $T_c \approx 2\eta \times 10^7 K$ **(Curea Temparature)**

Paramagnetism is not important when T>T$_c$

→ **Phase Transition From paramagnetism to ferromagnetism When T↘ down to T→T$_c$ and the induced magnetic is very strong**



$B_0=10^{13}G$
$B_0=10^{12}G$
$B_0=10^{11}G$
$B_0=10^{10}G$
$B_0=10^{9}G$
Log B_{in}(G)
Log T(K)
15
14
13
12
11
10
9
4
5
6
7
8

Increase of magnetic field of NS

a) **The induced magnetic field for the anisotropic neutron superfluid increases with decreasing temperature due to More and more neutron $^{3}P_{2}$ Copper pairs transfer into paramagnetic states.**

b) **The region and then mass of anisotropic neutron superfluid is increasing with decreasing temperature**

Energy gap of the $^3P_2$ neutron pair (Elgagøy et al.1996, PRL, 77, 1428-1431)

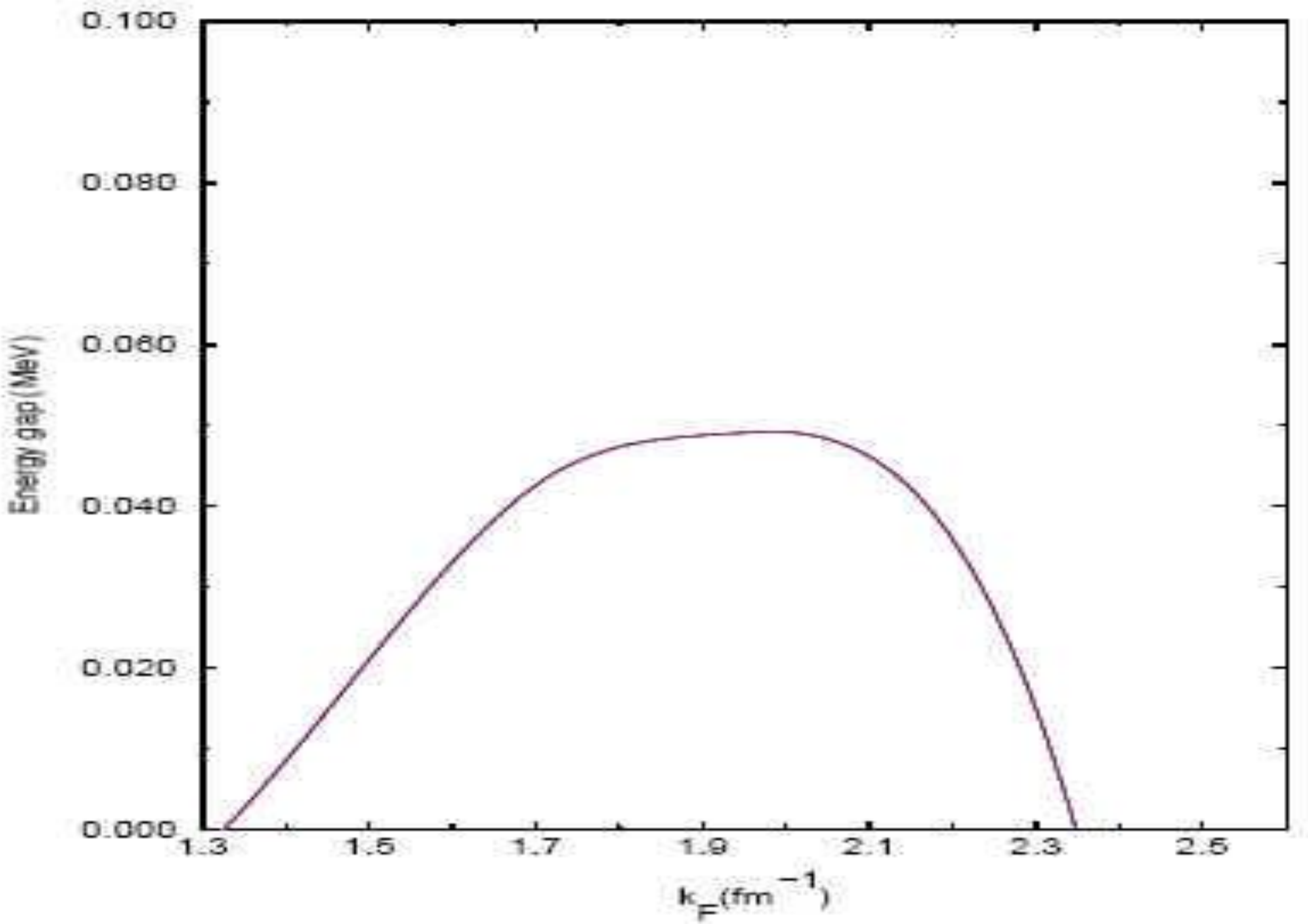


Fig. 8. Neutron energy gap at β equilibrium.

The up limit of the magnetic field for magnetars

$$B_{\max} = \frac{2\mu_n q N_A m(^3P_2)}{R_{NS}^3} \approx 2.02\times10^{14}\eta \qquad gauss$$

$$\eta = \frac{m(^3P_2)}{0.1 m_{Sun}} R_{NS,6}^{-3}$$

$$m_{\max}(NS) \le 2.5 m_{Sun}$$

$$m_{\max}(^3P_2) \le 1.5 m_{Sun}$$

$$B_{\max} < 3\times10^{15} \qquad gauss$$

Conclusion: **All assumptions with $b > 3\times10^{15}$ *gauss* are unphysical.**

II.

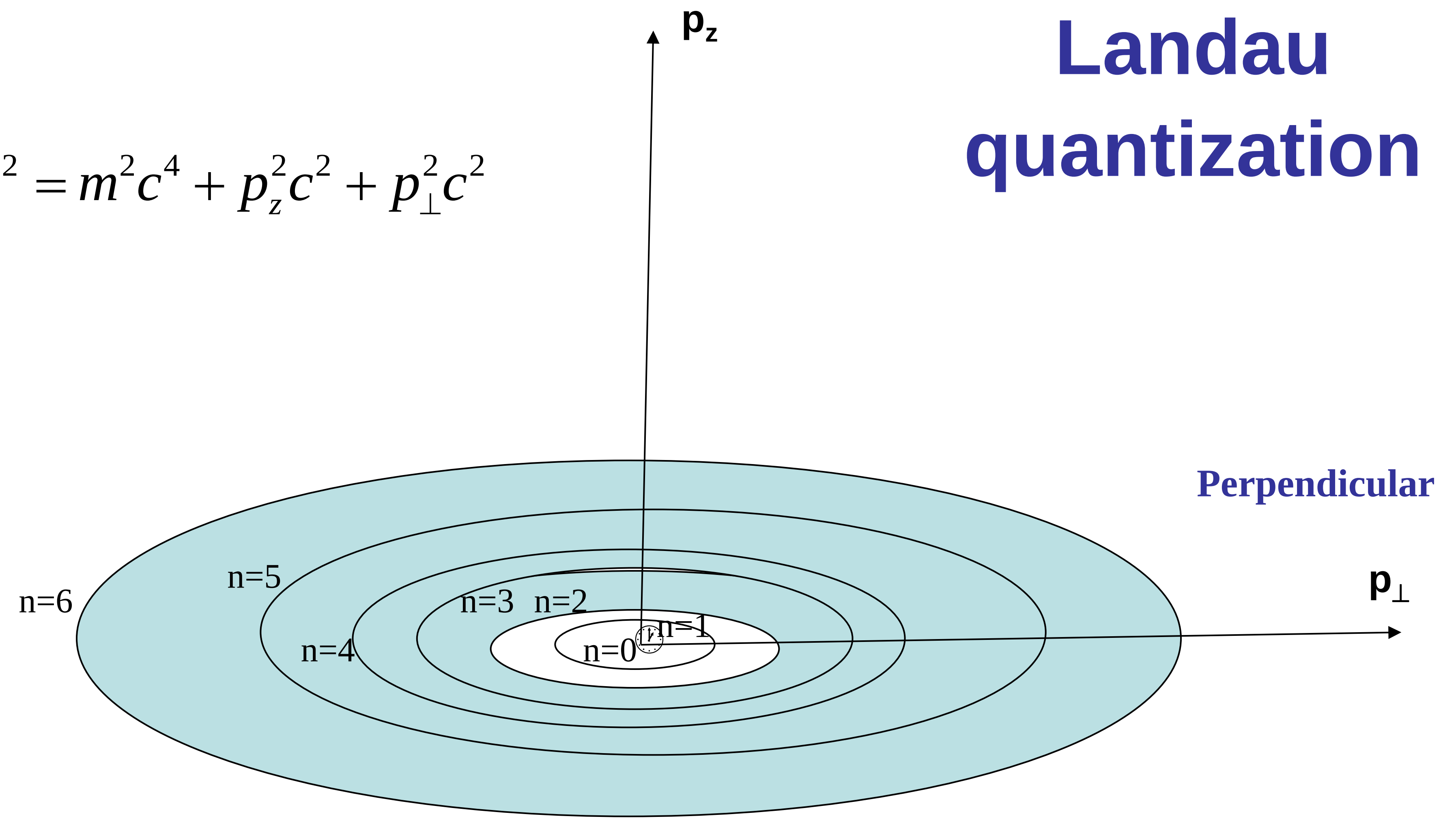
$E_F(e)$

under

strong magnetic field

Landau quantization

$$E^2 = m^2c^4 + p_z^2c^2 + p_\perp^2c^2$$



$$\left(\frac{p_\perp}{m_e c}\right)^2 = (2n+1+\sigma)b$$

$$b = B/B_{cr}$$

$$\boldsymbol{B_{cr} = 4.414 \times 10^{13}\ Gauss}$$

Classical electron gas under strong magnetic field

For Boltzmann' classical electron gas

$$\left(\frac{\rho}{10^4\,(g.cm^{-3})} \ll 2.4\left(\frac{T}{10^8 K}\right)^{3/2}\right)$$

a) $n = 0, 1, 2, \ldots \to \infty$ $(theoretically)$

b) For Boltzmann' classical electron gas, probability distribution:

$$P(E(n)) \propto \exp\{-\frac{E(n)}{kT}\}$$

$$\left(\frac{E}{m_e c^2}\right)^2 = 1 + \left(\frac{p_z}{m_e c}\right)^2 + (2n + 1 + \sigma)\left(\frac{B}{B_{cr}}\right)$$

In a super strong magnetic field ($B \gg B_{cr}$),

The ratio $P(E(n))/ P(E(n{=}0)) <<<<1$ *when* $n>>1$

Acturally, the probability for the state n may be neglected.

Intuitively the electrons populate on the quantum state $n = 0,1,2,3$ only when $B \gg B_{cr}$

Fermi sphere in strong magnetic field:

Fermi sphere without magnetic field :

Both dp_z and $dp_\perp$ change continuously.

the microscopic state number in a volume element of phase space $d^3x\, d^3p$ is $d^3x\, d^3p/h^3$.

Fermi sphere in strong magnetic field:

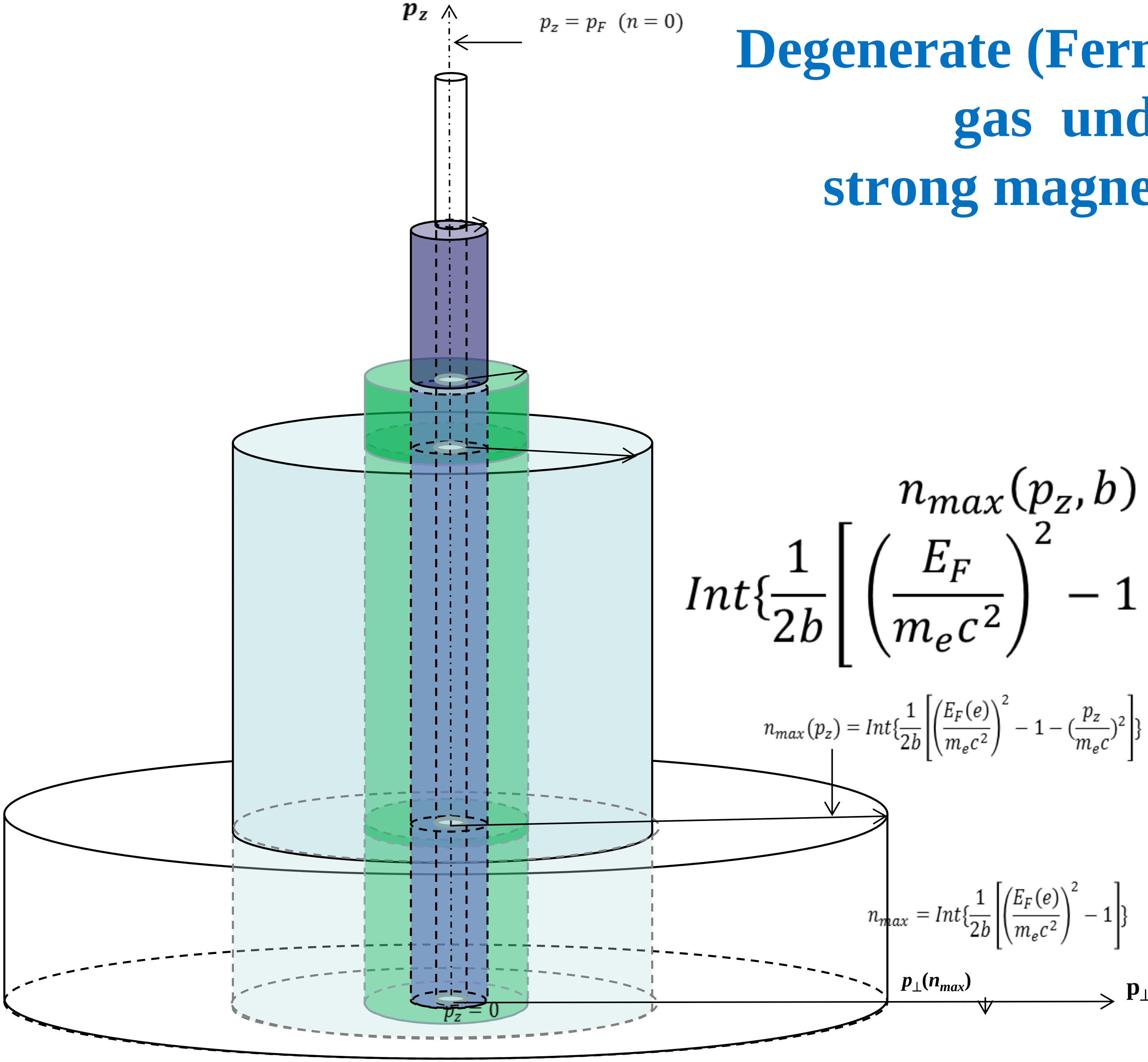
along the z-direction dp_z changes continuously.

In the x-y plane, electrons are populated on discrete Landau levels with n=0,1,2,3…

For a given p_z (p_z is still continuous)，there is a maximum orbital quantum number $n_{max}(p_z,b,\sigma)\approx n_{max}(p_z,b)$.

In strong magnetic fields, an envelope of these Landau cycles with maximum orbital quantum number $n_{max}(p_z,b,\sigma)$ ($0 \le p_z \le p_F$) will approximately form a spherical sphere, i.e. Fermi sphere.

Degenerate (Fermi) electron gas under strong magnetic field



$$n_{max}(p_z, b) \approx$$

$$Int\{\frac{1}{2b}\left[\left(\frac{E_F}{m_e c^2}\right)^2 - 1 - \left(\frac{p_z}{m_e c}\right)^2\right]\}$$

Degenerate (Fermi) electron gas under ultra strong magnetic field

In complete degenerate electron gas , given p_z, only limited quantum states, $n = 0, 1, 2 \ldots, n_{max}(p_z, b)$, along the direction perpendicular to the magnetic field are occupied by electrons, in which each microscopic state is only occupied by one electron according to the Pauli exclusion principle .

$\Rightarrow$ deduction:

$B \uparrow \;\Rightarrow\; n_{max} \downarrow \;\Rightarrow$ The number of electrons occupied on the microscopic states perpendicular to the magnetic field decreases

$\Rightarrow (p_z)_{max}$ increases due to $n_e = N_A Y_e \rho = \mathrm{Const.}$

$\Rightarrow E_F(e) \uparrow$.

The stronger the magnetic field, the more the Fermi energy of electrons.

Discrepancy

My idea:

The stronger the magnetic field, the higher the Fermi energy of the electron gas.

However, my idea is opposite with the popular theory.

A popular theory:

Electron Fermi energy decreases with increasing magnetic field. And the effect of magnetic is insignificant in the high density.

[1] V. Canuto and H.Y. Chiu, 1968, Phys. Rev. 173:1210

[2]V. Canuto and H.Y. Chiu, 1971, Space Science Reviews 12:3-74

[3]D. Lai, S.L. Shapiro,1991, ApJ., 383: 745-761

[4] D. Lai, 2001, "Matter in Strong Magnetic Fields" .
Reviews of Modern Physics, 73:629-661

[5] Harding & Lai , 2006, Rep. Prog. Phys. 69 : 2631-2708)

The popular idea and its result is totally country with the picture of Landau column above.

Why?

The key reason

The key is that all of those popular papers used an incorrect formulae in some text books of statistic physics , which calculate the number of microscopic states for a degenerate electron gas in strong magnetic field .

In some text-book:

Kubo, R. 1965, *Statistical Mechanics*, Amsterdam:North-Holland Publ.Co. pp.278-280

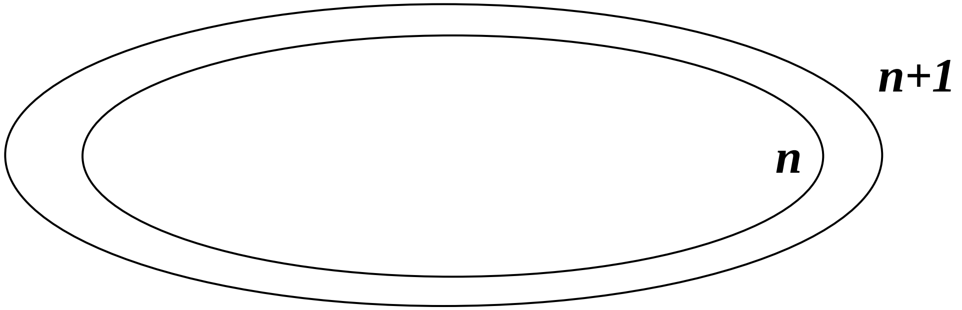
Pathria R.K., 2003, Statistical Mechanics, 2nd ed. Lsevier, Singapore

The state number of electrons in the interval $p_z \rightarrow p_z + dp_z$

In these text-books, The state number of electrons in the interval ***$p_z \rightarrow p_z + dp_z$ along the direction of magnetic field is calculated as***

$$\frac{1}{h^2}\int dp_x dp_y = \frac{1}{h^2}\pi p_\perp^2\Big|_n^{n+1} = \frac{4\pi m\mu_B B}{h^2} \qquad \textbf{(B)}$$

$$\mu_B = \frac{e\hbar}{2m_e c}$$



The result is the same with previous one for the non relativistic case

And it is usually referenced by all popular papers in common .

But, The formula (B) is questionable .

My idea

No any state between $p_{\perp}(n)$ - $p_{\perp}(n+1)$ exists according to the idea of Landau quantization. It is inconsistent with Landau idea. Besides, this result in the next book above has not been checked by any physical experiment up to now. And no any important observation in neutron stars (for example, for magnetars) has been well explained by theory in strong magnetic field.

In my opinion, we should use the Dirac' δ - function to represent Landau quantization of electron energy

Microscopic state number in an unite volume

Microscopic state number in volume $d^3x\ d^3p$ is

$$\delta N_{phase} = \frac{1}{h^3} dxdydzdp_x dp_y dp_z$$

Total microscopic state number in an unite volume is

Landau quantization

$$N_{phase} = 2\pi(\frac{m_e c}{h})^3 \int_0^{p_F/m_e c} d(\frac{p_z}{m_e c})\{\sum_{n=0}^{n_{\max}(p_z,b,\sigma=-1)} g(n)\int \delta(\frac{p_\perp}{m_e c} - \sqrt{2nb})(\frac{p_\perp}{m_e c})d(\frac{p_\perp}{m_e c})$$
$$+\sum_{n=1}^{n_{\max}(p_z,b,\sigma=+1)} g(n)\int \delta(\frac{p_\perp}{m_e c} - \sqrt{2(n+1)b})(\frac{p_\perp}{m_e c})d(\frac{p_\perp}{m_e c})\}$$

Here *g(n)* is the statistic weight of the energy level with quantum *n* . The Dirac's δ- function is induced to express the Landau quantization in the perpendicular to the direction of magnetic field.

Relation between $E_F(e)$ and B

According to the Principle of Pauli's incompatibility

The total number states (per unite volume) occupied by the electrons in the complete degenerate electron gas should be equal to the number density of the electrons

$$N_{phase} = N_A Y_e \rho$$

$$\Rightarrow$$

$$E_F(e) = 42.9\left(\frac{Y_e}{0.05}\right)^{1/4}\left(\frac{\rho}{\rho_{nuc}}\right)^{1/4}\left(\frac{B}{B_{cr}}\right)^{1/4} MeV$$

→ **We may explain the phenomena of very high x-ray luminosity for magnetars.**

III.
Ultra strong X-ray Luminosity
—
Magnetars are unstable due to the ultra high Fermi energy of electrons

Basic idea

Electron capture by protons will happen

$$e^{-} + p \rightarrow n + \nu_e$$

When the magnetic field is more strong than B_{cr} and then

$$E_F(e) > E_F(n) \approx 60 MeV$$

Energy of the outgoing neutrons is high far more than the Binding energy of a $^3P_2$ Cooper. Then the $^3P_2$ Cooper pairs will be broken by nuclear interact with the outgoing neutrons.

$$n + (n\uparrow, n\uparrow) \rightarrow n + n + n$$

It makes the induced magnetic field by the magnetic moment of the $^3P_2$ Cooper pairs disappearing , and then the magnetic energy the magnetic moment of the $^3P_2$ Cooper pairs $2\mu_n B$

Will be released and then will be transferred into x-ray radiation

$$kT \approx \mu_n B \approx 10 B_{15} \quad keV$$

Total thermal energy

Average energy of outgoing neutrons after breakdown of the $^3P_2$ Cooper pairs

$$\bar{\varepsilon}(n) \approx \frac{1}{3}[E_F(e) + E_F(p) - (m_n - m_p)c^2 - \Delta(^3P_2)]$$

It will be transfer into thermal energy (with x-ray emission) .

Total thermal energy will be released after all $^3P_2$ Cooper break up

$$E = qN_A m(^3P_2) \times 2\mu_n B \approx 1 \times 10^{47} \frac{m(^3P_2)}{0.1 m_{Sun}} \quad \text{ergs}$$

Life time of magnetar activity

X ray luminosity of AXPs :

$$L_x = 10^{34} - 10^{36}\, ergs / \sec$$

It may be maintained ~ 10^4 -10^6 yr

The X-ray luminosity by calculating the Electron capture rate

$$L_x = \zeta V(^3P_2)\frac{(2\pi)^4}{\hbar V_1}G_F^2 G_V^2(1+3a^2)\times$$

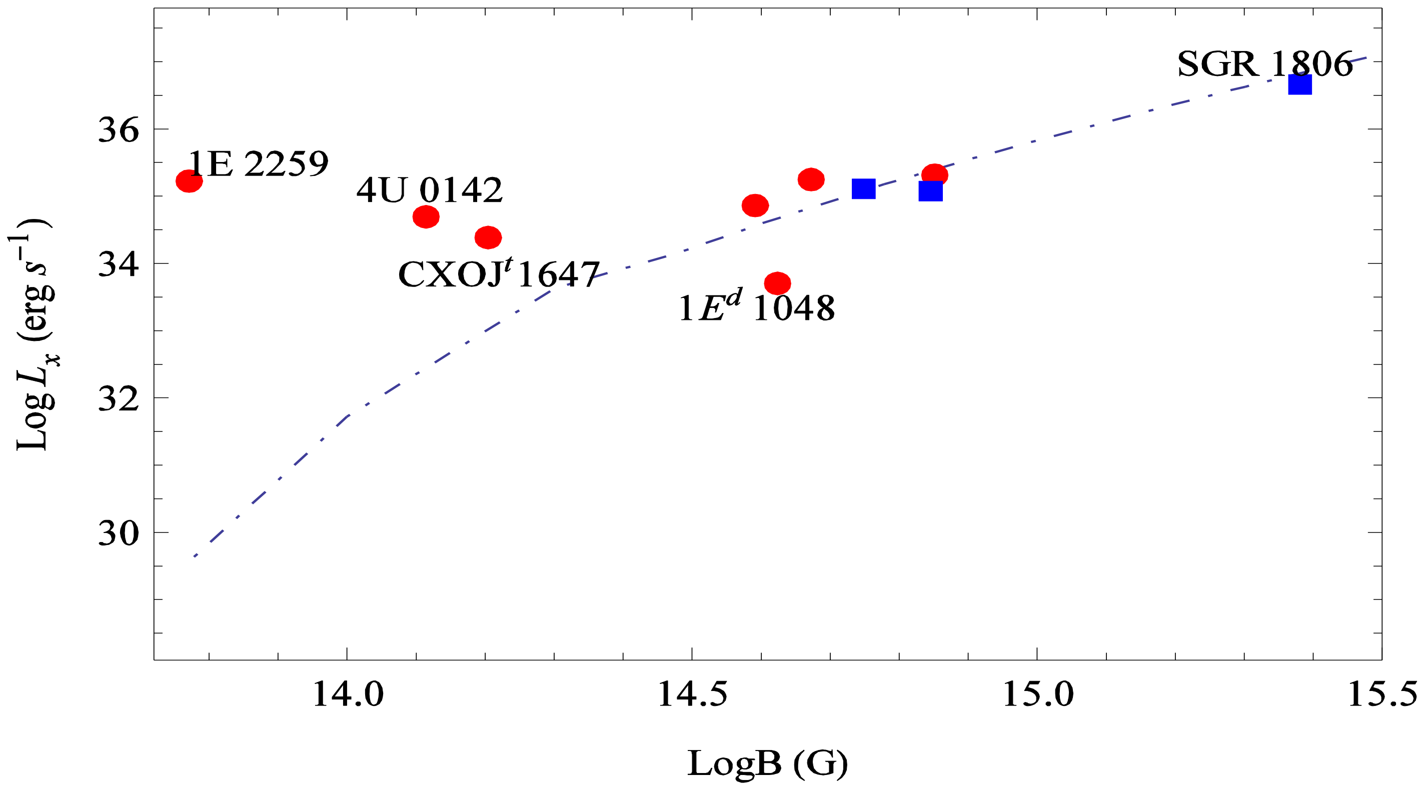
$$\int d^3n_e d^3n_p d^3n_n d^3n_\nu \delta(E_\nu + E_n + 0.61MeV - E_e)\delta^3(\vec{k}_f - \vec{k}_i)S\times 2\mu_n B$$

$$S = f_e(E_e)f_p(E_p)[1-f_n(E_n)][1-f_\nu(E_\nu)]$$

$$f_j(E) = [\exp(E-\mu_j)/kT+1]^{-1}$$

$$\boldsymbol{E_F(e) = 42.9(\frac{Y_e}{0.05})^{1/4}(\frac{\rho}{\rho_{nuc}})^{1/4}(\frac{B}{B_{cr}})^{1/4}\ MeV}$$

Comparing the theoretical calculation with the observation



red circle: SGR blue pane: AXP
Accretion phenomena has been obviously detected for the three AXPs in the left region

Future Works

1. **The Physical reason for the glitches of Pulsars due to the oscillation between A and B phase of anisotropic superfluid”.**

2. X-ray flare and burst for some magnetars

Combining with the Glitch mechanism, the magnetic field line will be twisted interior the magnetar and the energy of magnetic field will be shortly transferred into the high kinetic energy of charged particles which is similar to the case of solar flares.

Thanks