On the Asymptotic Structure of the Pulsar Wind

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Abstract

We investigate the asymptotic structure of the pulsar wind under force-free and two-fluid MHD approximation. Within the force-free approximation we obtain simple asymptotic solutions of the Grad-Shafranov equation for quasi-spherical pulsar wind. In the case of oblique rotator we have a good agreement with the results of recent numerical simulations. In particular, we show that the shape of current sheet does not depend on the radial structure of magnetic field.

For the internal region of the current sheet in the pulsar wind where the force-free approximation is not valid we use two-fluid approximation. Passing into the comoving reference frame we determine electric and magnetic field structure as well as the velocity component perpendicular to the sheet. It allows us to estimate the efficiency of particle acceleration. Finally, investigating the motion of individual particles in the time-dependent current sheet we find self-consistently the width of the sheet and its time evolution.

1. Introduction

Key element of the pulsar wind is believed to be the current sheet separating regions with oppositely directed magnetic fluxes [1, 2, 3]. According to numerical simulation [4], angular distribution of pulsar wind parameters differs from the simplest "split-monopole" model. In the case of large inclination angles between magnetic and rotation axes the energy flux concentrates near the equatorial plane as well as radial magnetic field. It is necessary to construct a more realistic model which will allow us to obtain main parameters of pulsar wind for arbitrary geometry.

Another important issue is the internal structure of current sheet. Existing models do not describe it neither analytically [3] nor numerically [4]. We consider pulsar wind in a comoving frame in order to eliminate dominant components of the electromagnetic fields and distinguish the field of the current sheet itself.

2. Force-free approximation.

Using the force-free approximation one can find stationary configurations of the electromagnetic field by solving one second order differential equations for magnetic flux function $\Psi(r, \theta)$ [6]:

$$-\left(1-\frac{\Omega_F^2\varpi^2}{c^2}\right)\nabla^2\Psi+2\frac{1}{\varpi}\frac{\partial\Psi}{\partial\varpi}+\frac{\varpi^2\Omega_F}{c^2}(\nabla\Psi)^2\frac{\mathrm{d}\Omega_F}{\mathrm{d}\Psi}-\frac{16\pi^2}{c^2}I\frac{\mathrm{d}I}{\mathrm{d}\Psi}=0$$

It is called Grad-Shafranov equation. We search the solution of this equations as asymptotic series:

$$\Psi(r,\theta)=\sum_{n=0}^{\infty}\Psi_n(\theta)\left(\frac{R_L}{r}\right)^{2n} \qquad \frac{\mathrm{d}\Psi_0}{\mathrm{d}\theta}=\Psi^0\sin^m\theta$$

If the Pointing flux at large distances depends on θ according to the results of paper [6], the first term of these series satisfies the equation

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Psi_1}{\mathrm{d}\theta}\right)-\Psi^0(m-1)\cot\theta=\Psi_1(m+3-(m+1)^2\cot\theta)$$

According to numerical simulation [4], $m = m(\alpha)$, where α is the inclination angle. For large inclinations $m = 2$ and thus $B_p(\theta)$ is proportional to $\sin\theta$.

In oblique case we verified that configuration:

$$B_\phi=E_\theta=-\frac{\Omega\Psi}{2\pi rc}\mathrm{sgn}(\Phi) \qquad B_r=\frac{1}{2\pi r^2}\frac{\partial\Psi}{\partial\theta}\mathrm{sgn}(\Phi)$$

satisfied Maxwell equation and force-free assumption. The shape of the current sheet is similar to the famous Bogovalov solution [3]:

$$\Phi=\sin\alpha\sin\theta\sin(\varphi-\Omega t+\Omega r/c)+\cos\theta\cos\alpha$$

In more realistic solution with smoother than sign(Φ) function Maxwell's equations remain satisfied though force-free assumption is broken. It means that inside the current sheet we have to use more precise model.

3. Fields in the comoving frame.

To study the mechanism of particle acceleration inside the current sheet we consider the following configuration

$$B_\varphi=E_\theta=-\frac{B_LR_L}{cr}\sin\theta f(r-ct)$$

which within the force-free approximation satisfies Maxwell's equations for arbitrary function $f(r-ct)$ [5].

We can use such a solution in orthogonal case because the shape of the current sheet in this case is similar to a spherical wave. We modify this solution in order to work with a current sheet moving with a speed βc. We also neglect B_r as it is negligible beyond the fast magnetosonic surface. Then the configuration changes to

$$B_\varphi=\frac{1}{\beta}\frac{B_LR_L}{r}\sin\theta\tanh\left(\frac{r-\beta ct}{\Delta}\right) \qquad E_\theta=\frac{B_LR_L}{r}\sin\theta\tanh\left(\frac{r-\beta ct}{\Delta}\right)$$

The next step is to write the field configuration in the reference frame moving with a velocity βc at the angle θ to the equatorial plane. If we put $f(\xi) = \tanh(\xi/\Delta)$, we obtain the following leading components of electromagnetic field for arbitrary m:

$$B_y=\frac{B_LR_L\sin^m\theta}{ct\beta^2\gamma^2}\tanh\left(\frac{x}{\gamma\Delta}\right) \qquad E_x=\frac{zB_LR_L\sin^m\theta}{c^2t^2\beta^2\gamma^2}\tanh\left(\frac{x}{\gamma\Delta}\right)$$

Moreover, two of the Maxwell's equations (without currents and charges) are satisfied both inside and outside the current sheet.

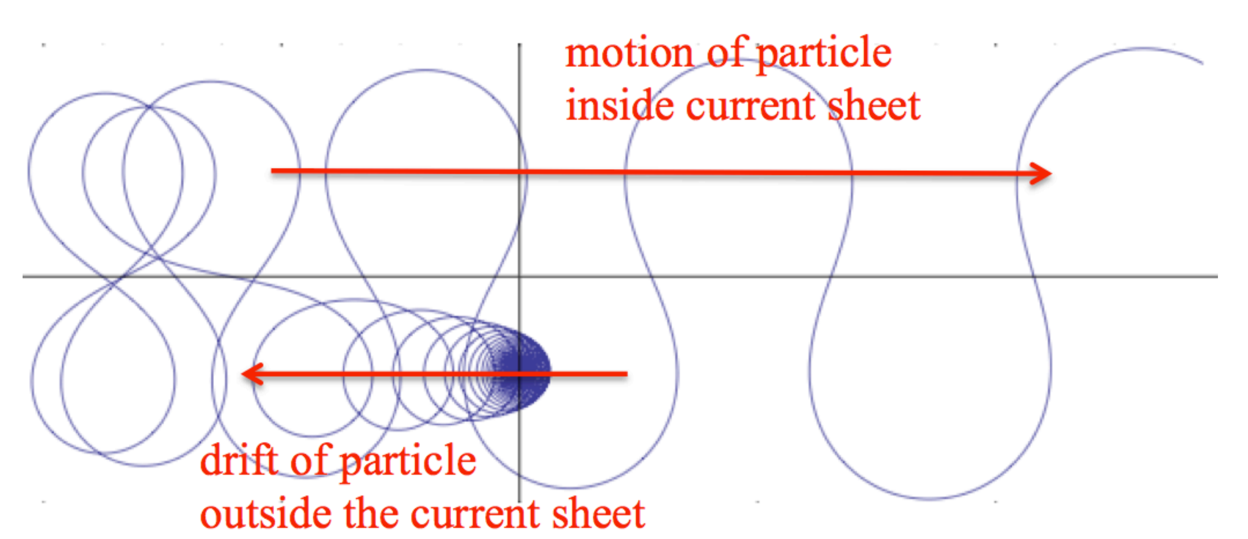

Fig.1: Particle trajectory inside the time-dependent current sheet. Initially particles do not cross the zero surface and drift in slowly changing fields. As sheet expands, particles become trapped and begin to move in opposite direction.

4. Estimating the particles acceleration

In the comoving reference frame

$$B_y=\frac{A}{ct}f(x,t) \qquad E_x=\frac{A}{c^2t^2}zf(x,t)$$

According to [1,2], the density is proportional to $1/t^2$ and $f(x,t)$ is proportional to $1/t$. We set $f(x,t)=x/kct$.

Now, to satisfy Maxwell's equations we need to add z-component of the electric field. As a results, electric field in the center of sheet equals $E_z=kA/2ct$.

In ultrarelativistic case we found the parameter $k=1/4\lambda\gamma$ as well as thickness of the current sheet $\Delta = R_L\,\sigma^{1/3}/4\gamma\lambda$.

To estimate particle acceleration we obtain expression for derivative of particle gamma factor at the initial moment of time

$$\left.\frac{\mathrm{d}\gamma}{\mathrm{d}t/t_0}\right|_{t=t_0}=\frac{\sigma}{2\gamma}$$

Here $\sigma=\gamma^3$ outside the fast magnetosonic surface [6]. This estimation shows that the particle acceleration is effective enough so our assumption of ultrarelativistic motion is correct.

After formation of the current sheet its internal pressure is small so it begins to collapse and the density starts to increase. It is natural to assume that the final thickness of the sheet is comparable to the skin depth c/ω_p. In this case particle concentration inside the sheet is $\sigma^{1/3}$ times larger than outside the sheet.

One should also notice that described solution is not self-consistent as the number of particles inside the sheet is not constant: $nr^2\Delta \sim r$. We discuss self-consistent solutions in the next section.

5. Simple self-consistent solution

Consider now an axisymmetric pulsar magnetosphere. In this case the fields in the comoving frame could be found in the same way as in section 3

$$B_y=\frac{B_\mathrm{L}R_\mathrm{L}}{\gamma^2\beta^2}\frac{1}{ct}f\left(\frac{z}{\beta ct}\right) \qquad E_x=\frac{B_\mathrm{L}R_\mathrm{L}}{\gamma^2\beta^2}\frac{z}{c^2t^2}f\left(\frac{z}{\beta ct}\right)$$

As the size of particle orbit inside the current sheet is larger than a characteristic scale of magnetic field variation we can use an approach introduced in [8]. According to this prescription the quasi-adiabatic invariant

$$I_z=\int p_z\mathrm{d}z$$

is approximately constant at timescales larger than characteristic period of particle motion. Setting the thickness L of the current sheet to be proportional to t^β we found $I_z\propto t^{1/2}p^{3/2}L^{1/2}\psi(s)$, where $s = p_x/p$ and

$$\psi(s)=\int_{-\sqrt{s+1}}^{\sqrt{s+1}}\sqrt{1-(s-\xi^2)^2}\mathrm{d}\xi.$$

Now we have a system of equations including the pressure balance, Maxwell's equations and conservation of quasi-adiabatic invariant. To uniquely determine the power β we set the size of particle orbit to be proportional to the width of current sheet.

Finally, we find $\beta = 1/2$ and $L\propto t^{1/2}$ $p\propto t^{-1/2}$ $n\propto t^{-1}$

Example of particle trajectory inside the current sheet is presented in Fig. 1.

6. Conclusion

Within two fluid MHD approximation we investigate particle motion inside the current sheet. In particular, we

- obtain the fields structure in the comoving frame,
- show the existence of accelerating electric component inside the sheet,
- show that electric field accelerates particles to ultrarelativistic velocities,
- give the estimations of the width of current sheet and density inside it,
- find the simple self-consistent solution for current sheet evolution.

Under force-free approximation we build the model consistent with recent numerical simulations:

- we obtain the equation for second term of the asymptotic series,
- we show that for oblique case the shape of the current sheet is similar to the shape obtained in [3] and is universal.

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