LFN, QPO and fractal dimension of X-ray light curves from X-ray binaries

A.V. Prosvetov[1] S.A. Grebenev[1]
1) IKI RAN
E-mail: prosvetov@gmail.com

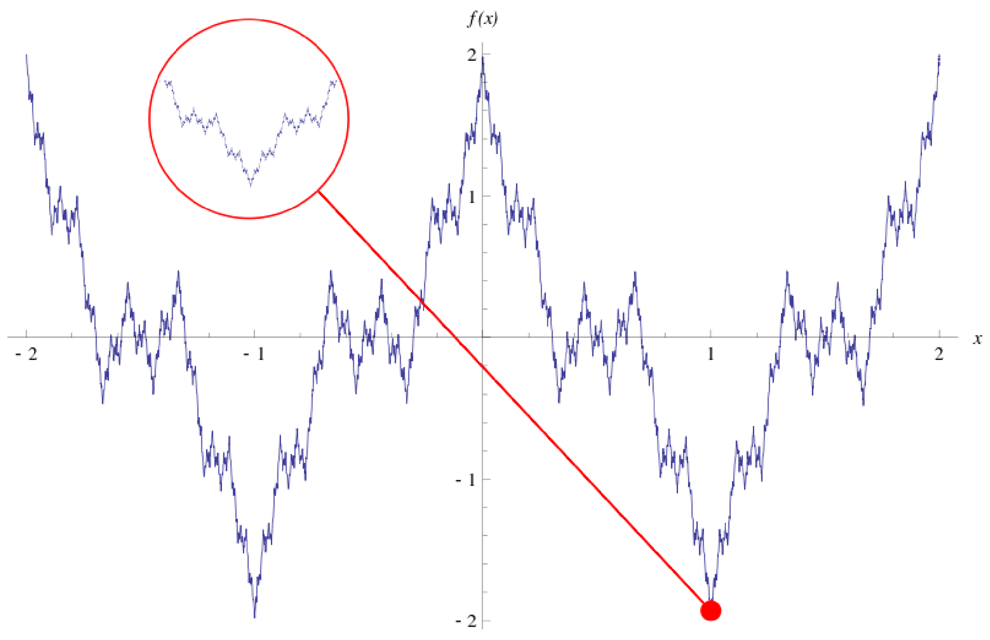

Fig 1. Weierstrass function exhibits self-similarity. Fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a pattern changes with the scale at which it is measured.

Fractal dimension

The term "fractal dimension" was brought to the fore by Benoit Mandelbrot based on his 1967 paper on self-similarity in which he discussed fractional dimensions of coastline. A fractal dimension is an index for characterizing fractal patterns or sets by quantifying their complexity as a ratio of the change in detail to the change in scale (fig.1) [*Benoit B. Mandelbrot (1983). The fractal geometry of nature*]. Fractal dimensions were first applied as an index characterizing complicated geometric forms for which the details seemed more important than the gross picture [*Albers; Alexanderson (2008)*] and fractal dimension does not have to be an integer.

Fractal analysis is now widely used in many areas of science; the methods have solid scientific validation so we decide to apply some elements of fractal analysis on the area of accretion in black hole binaries.

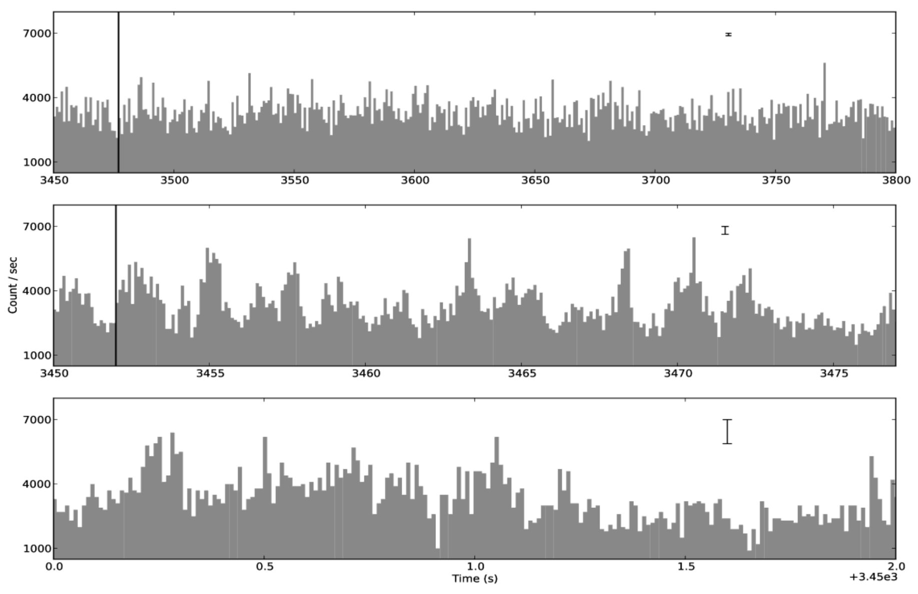

Fig 2. GX 339-4 light curve from PCA/RXTE in 2.9-36 keV. The light curve shows strong irregularity on the scale of 1 s (top), 0.1 s (middle), and 0.01 s (bottom). There is a self-similarity of the light curve at different scales and therefore it can have the fractal properties.

R/S analysis

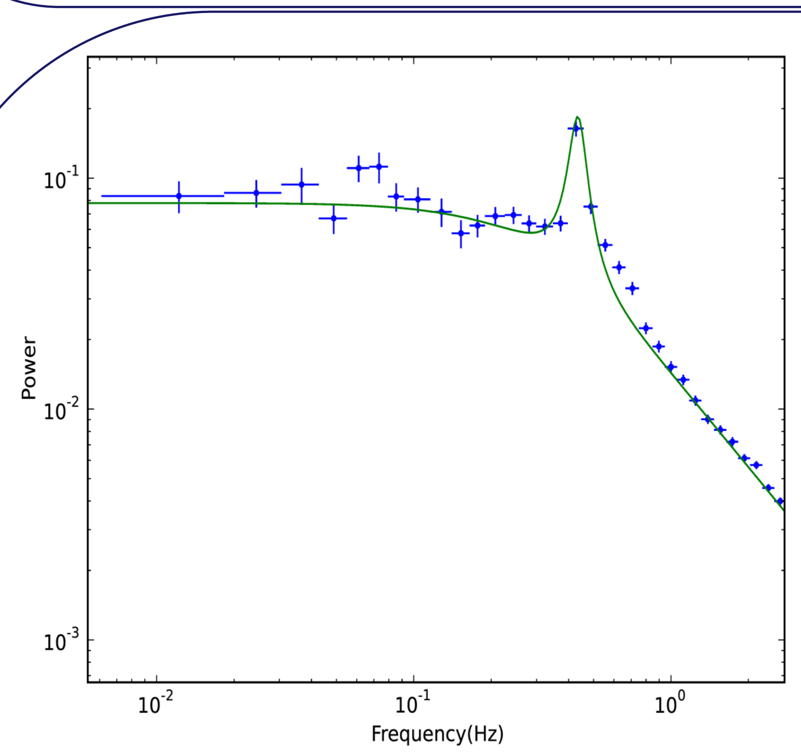

Fig 3. The typical QPO in power spectrum of GX 339-4 X-ray light curve.

We used PCA RXTE observations of the black hole GX 339-4 during outbursts in 2007 and 2010 contained QPO (fig. 3). Applying R/S analysis to the light curve of GX 339-4 (2.9-36 keV) we got the diagram log(R/S)-log(N/2), where S is standard deviation, R is a range (max — min), N is a bin size. The slope of the line on diagram gives the Hurst exponent H [Hurst, H. E. (1951)], and fractal dimension D is equals to $2 — H$ (fig. 4).

The results of R/S analysis are:

- It is possible to divide observations into two main groups by the fractal dimension of the light curves (fig. 5).
- There is a strong dependence of the fractal dimension of the light curves on the QPO presents and their frequency (fig. 6).
- The fractal dimension of the light curves correlates with the count rate of the inner zone of accretion disk (fig. 7).
- Changes in the fractal dimension resulting from short-range autocorrelation in the time series. The main changes are in the low energy range (fig.8)

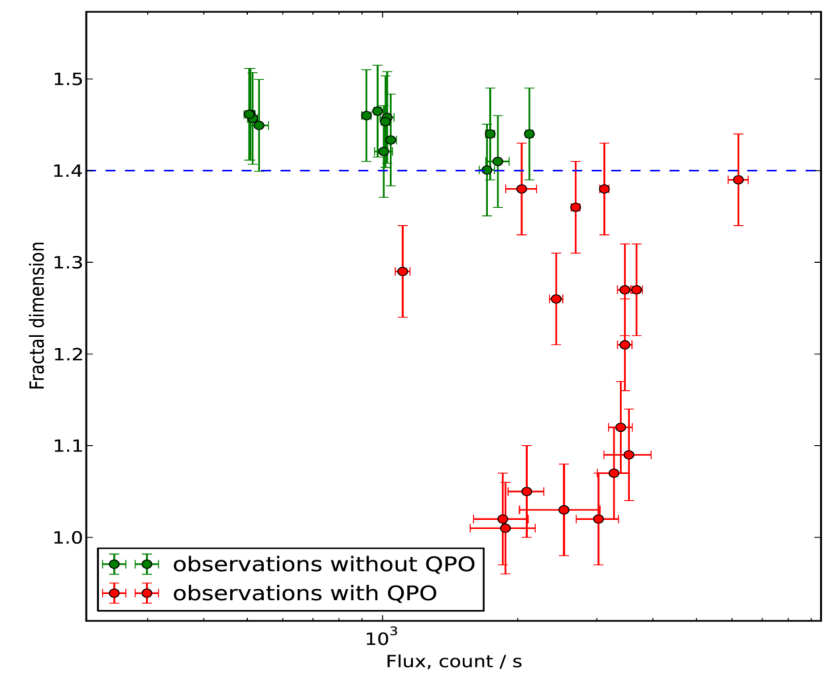

Fig 5. The fractal dimension of X-ray light curve as a function of X-ray flux.

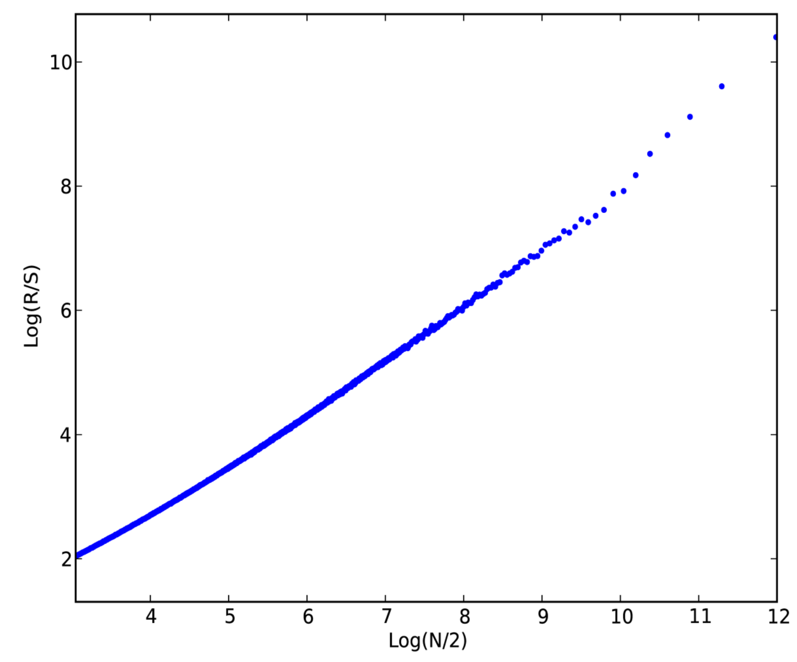

Fig 4. The typical log(R/S) — log(N/2) diagram of the GX 339-4 light curve.

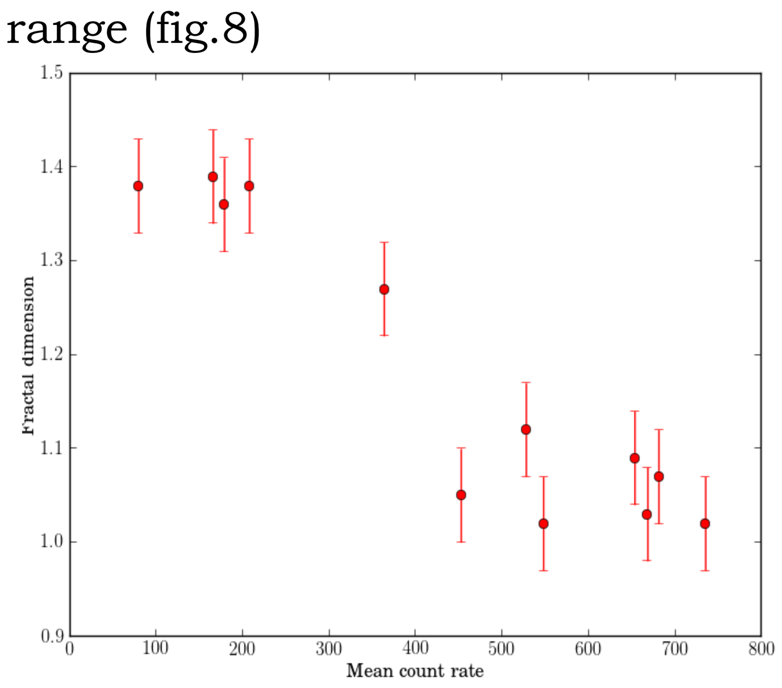

Fig 7. The relationship between fractal dimension of light curve and mean count rate from frequencies higher than the QPO peak.

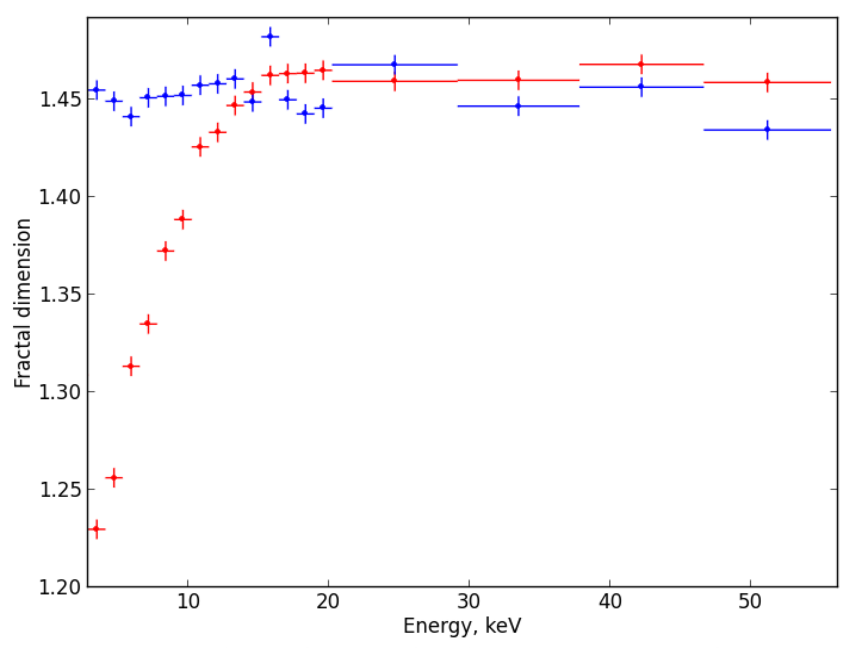

Fig 8. The fractal dimension of X-ray light curve in different energy channels with and without QPO.

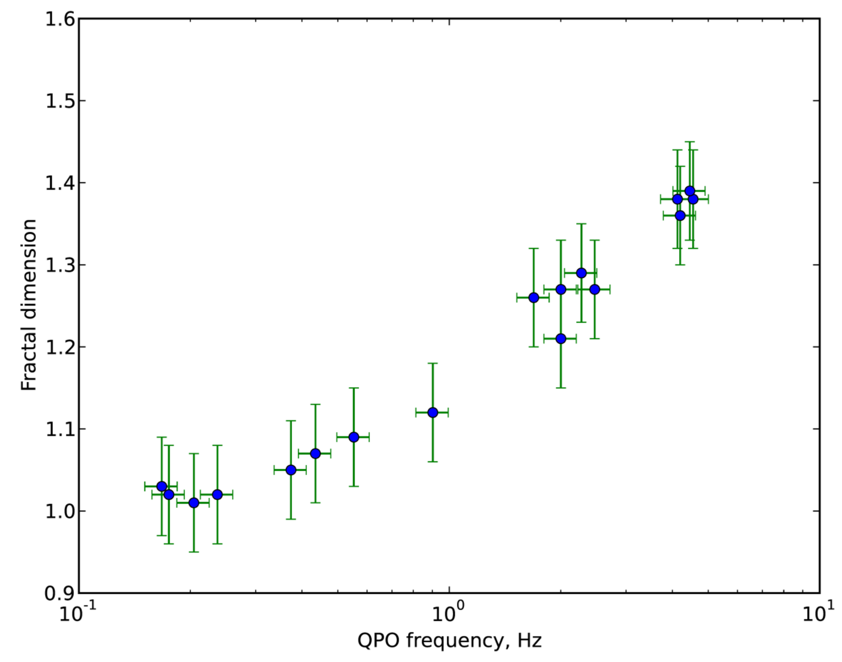

Fig 6. The fractal dimension of X-ray light curve as a function of QPO frequency.

Max-Spectrum analysis

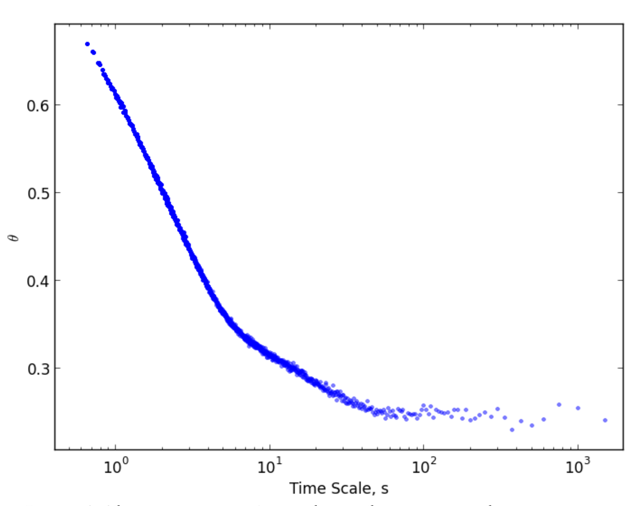

Fig 9. The typical dependency of the extremal index θ on a time scale from Max-Spectrum analysis of the GX 339-4 light curve contained QPO

Using one of the new methods based on the use of scaling properties of the data maxima [Stoev et al (2006)] to log(R/S)-log(N/2) diagram, we gain the extremal index θ introduced by Leadbetter et al [1983]. The extremal index quantifies the temporal clustering of extreme events and allows exploring the distribution of cluster size on time scales by the analogy with power spectrum (fig. 9). The distribution of extreme cluster size on time scales reflects the QPO existence with the break in power law: up to and down to QPO frequency the accretion disk is characterized by different power laws (fig. 10).

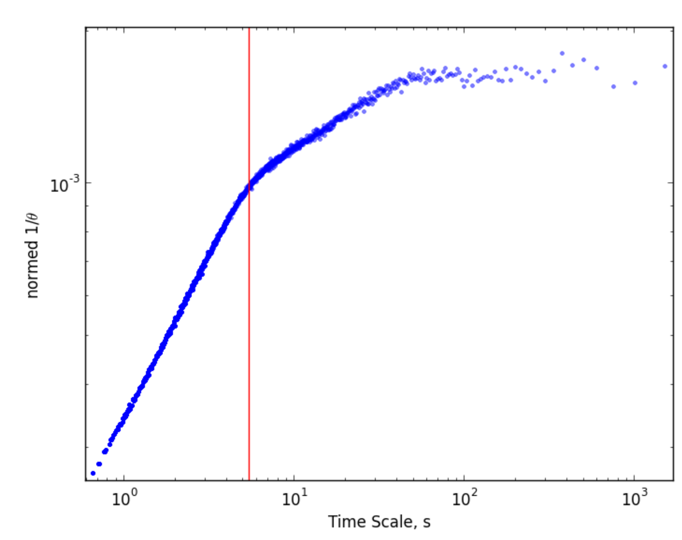

Fig 10. The typical dependency of mean extreme cluster size on time scale from GX 339-4 light curve contained QPO.

Neutron star AQL X-1

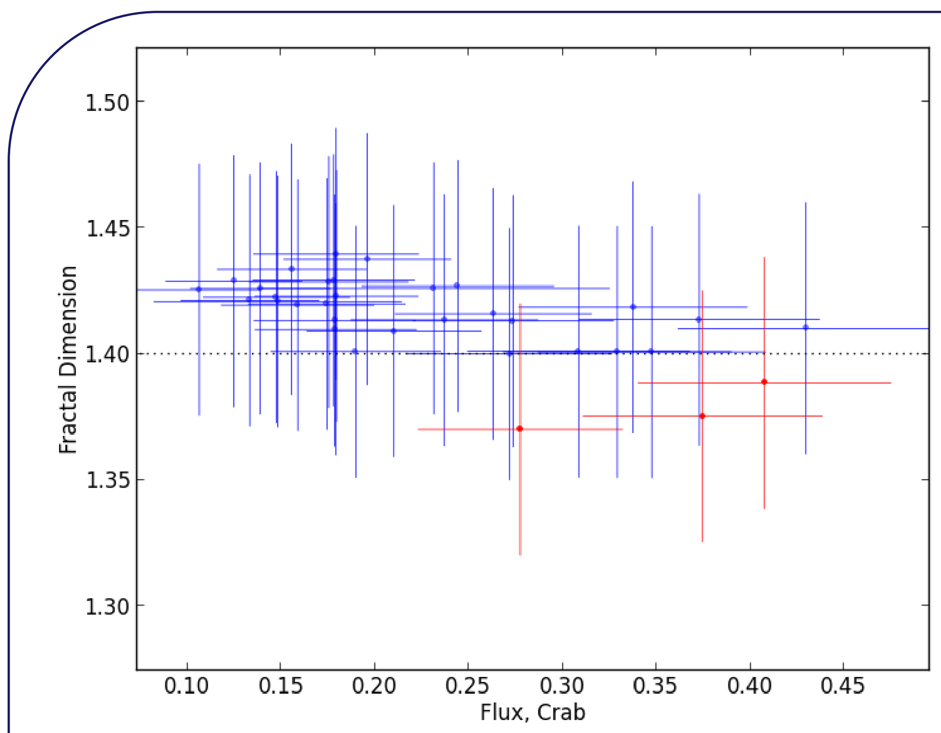

Fig 11. The fractal dimension of AQL X-1 light curve as a function of X-ray flux.

Applying R/S analysis to the light curves of neutron star AQL X-1 during outburst of 2011 we gain the relationship between fractal dimension and flux that is very similar to the case of accretion to a black hole (fig.11). The usual observations have fractal dimension of light curves $D > 1.4$, however there are several unusual observations with fractal dimension $D < 1.4$. The usual and unusual observations have different relationship between extremal index and time scale. After subtracting the Poisson noise and normalization to the mean count rate we get the relationship between mean cluster size of extremes and time scale (fig. 12). Also unusual observations differ from usual by features in the power spectra that correspond to activity near the boundary layer of accreting neutron star. All mentioned above features allow boundary layer of a neutron star to be studied and accreting neutron stars to be distinguished from black holes.

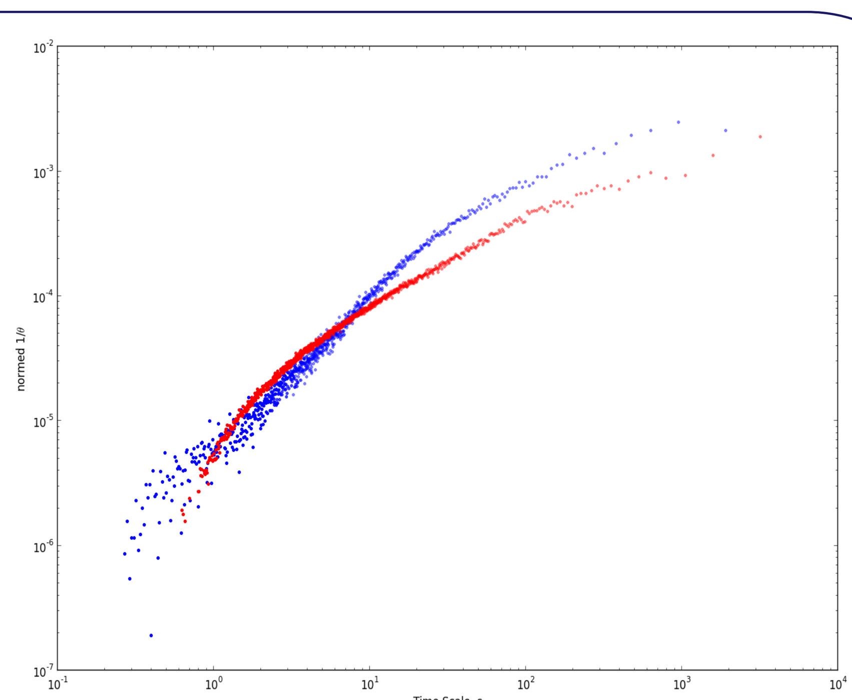

Fig 12. The relationship between mean extreme cluster size and time scale for usual (blue) and unusual (red) observations of AQL X-1 during outburst in 2011.

Conclusion

- Fractal dimension and extremal index may provide clues to the physics of the QPO in black hole binaries and boundary layer of accreting neutron star since they allow looking at the phenomenon from new unexpected side.
- The accretion disk of black holes may be divided into inner and outer regions by the keplerian orbit of QPO. These regions are characterized by different relationship between mean cluster size and time scale, as well as different fractal dimension of X-ray light curves.
- The continued work in this area could help to propose new methods of the compact object's study and identification.