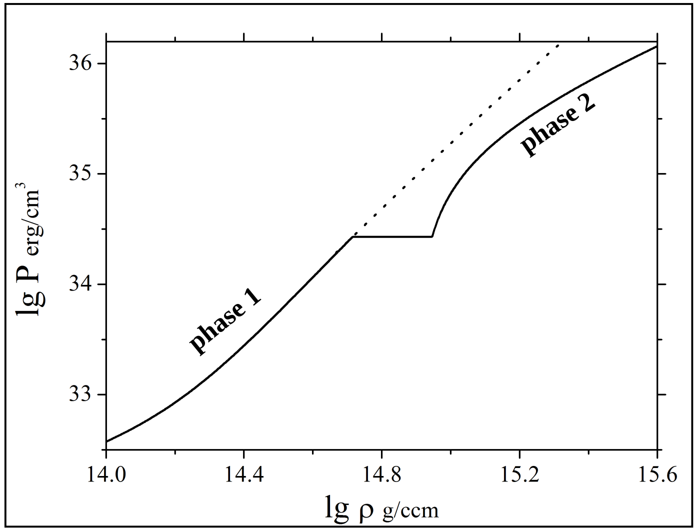
On some properties of hybrid stars

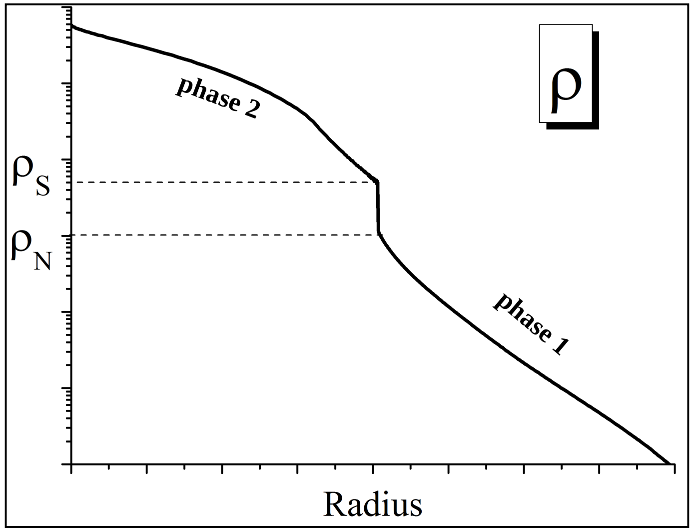
A.V. Yudin[1], T.L. Razinkova, D.K. Nadyozhin, A.D. Dolgov

Physics of neutron stars - 2014

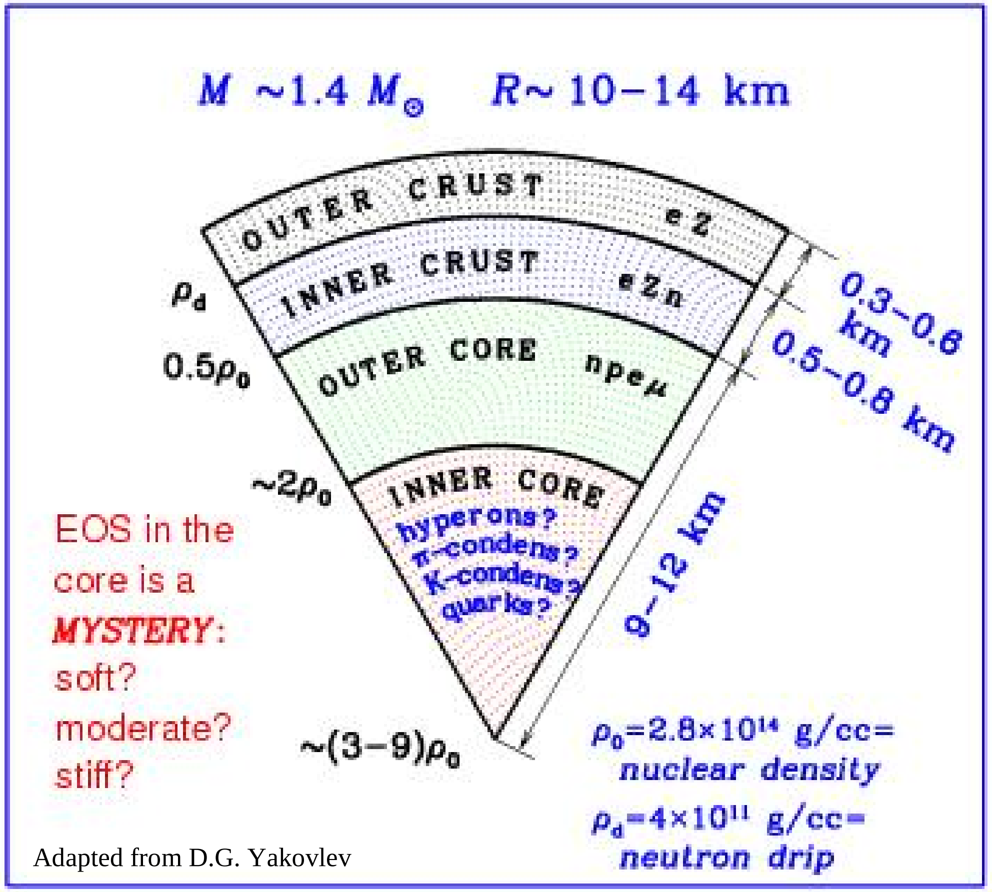
E-mail: yudin@itep.ru (ITEP)

Possible phase transitions in the matter at high density





Maxwellian type phase transition causes a density jump inside the star



Mass-radius diagram for hybrid stars

Phase transition:

Nuclear Matter

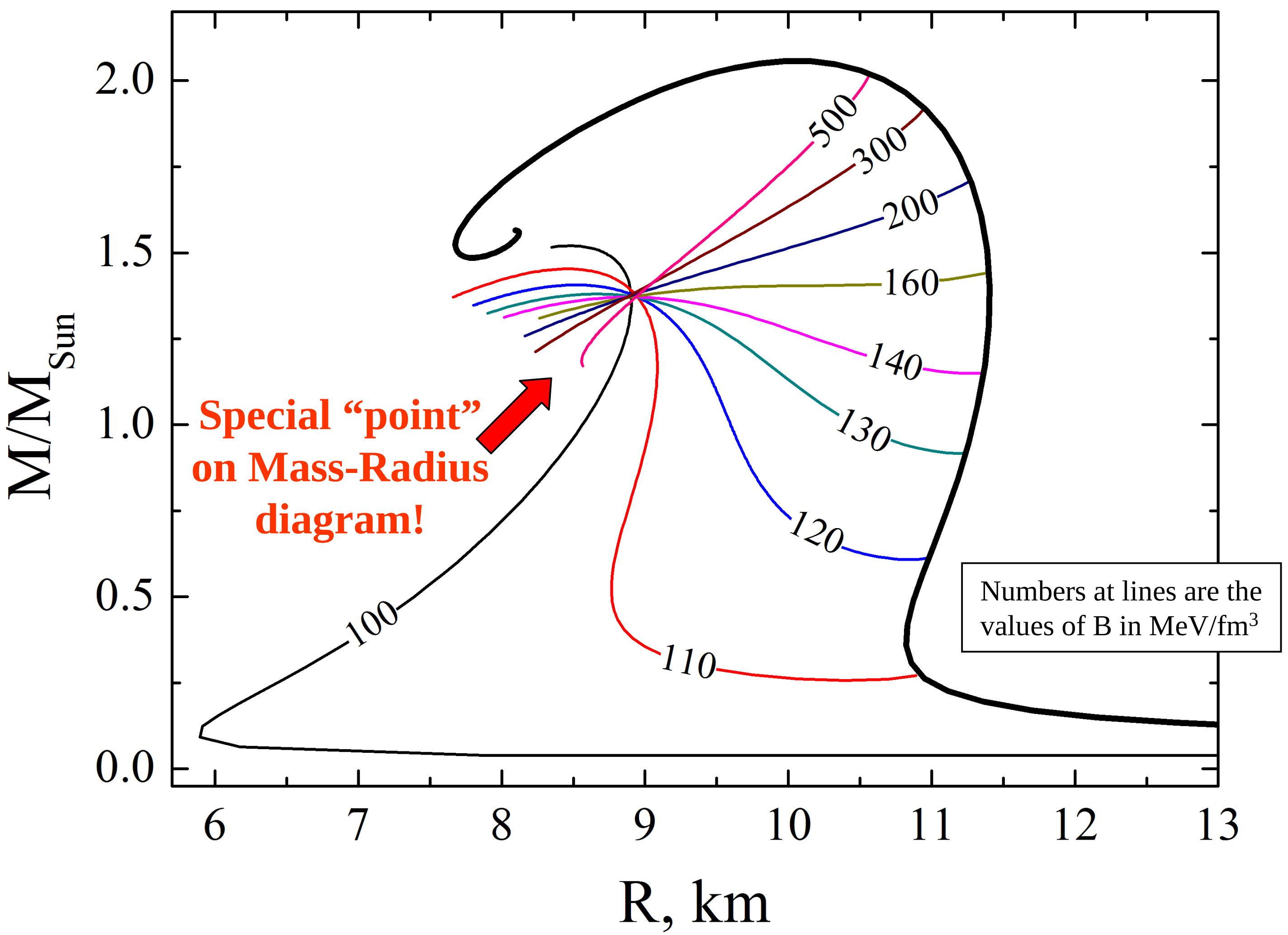
Quark Matter

EOS for pure nuclear matter is from Douchin & Haensel Astron. Astrophys., 380 151-167 (2001)

EOS for quark matter according to BAG model is linear: pressure P_q is proportional to internal energy E_q. B is a vacuum constant (model parameter)

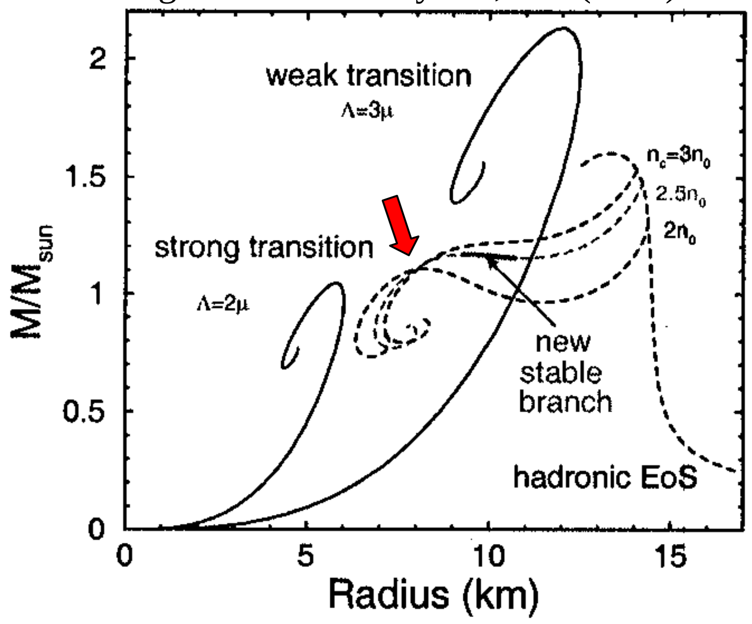
$$P_q = \frac{1}{3}\left(E_q - 4B\right)$$

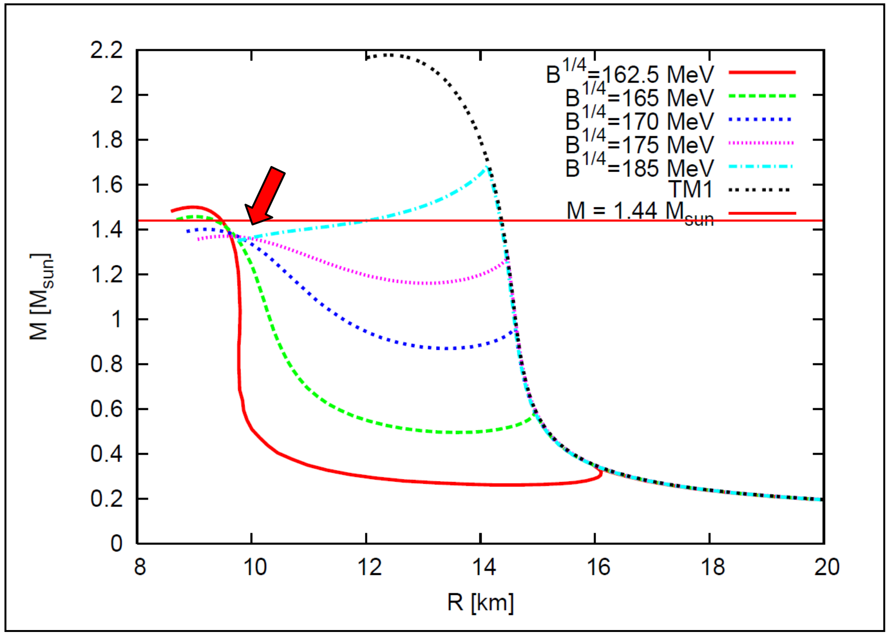
Thick black line – Mass-Radius relation for pure nuclear matter EOS



Density of phase transition ρ_{pt} is connected with B: $\rho_{pt} / \rho_n \approx -3 + \ln\left[B - 91\right]$ $\rho_n = 2.6\text{x}10^{14}$ g/ccm

Fraga et al. Nucl. Phys. A, 702 (2002)

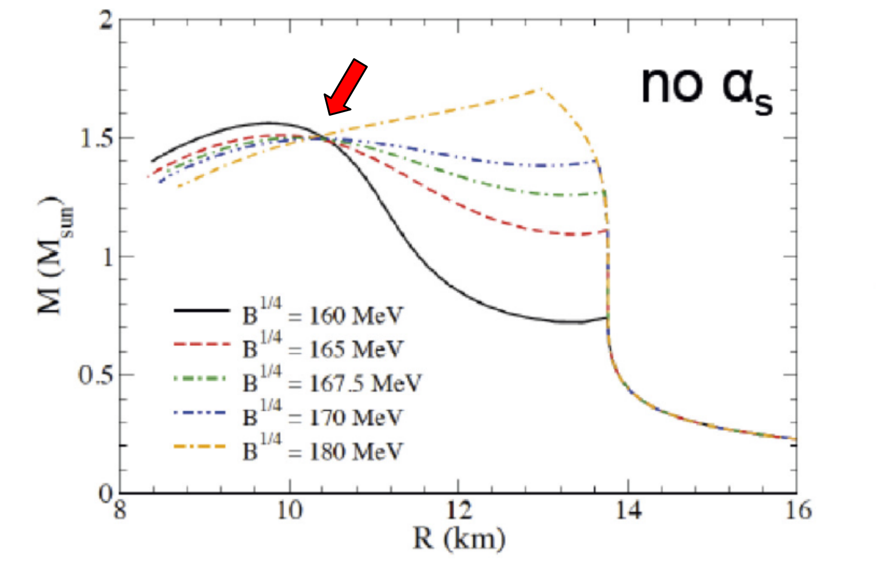


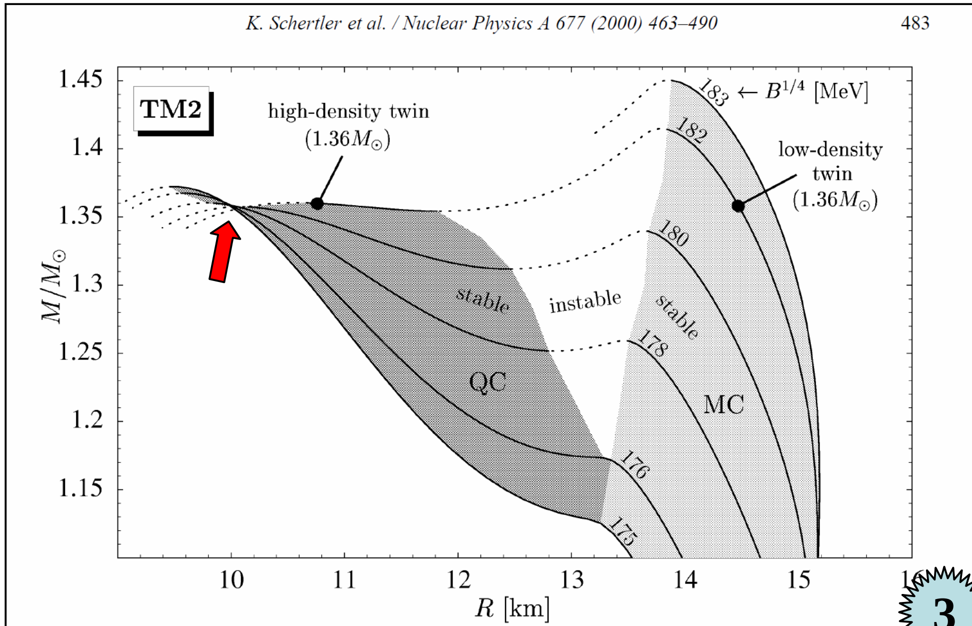


Sagert et al. J Phys G. 36, 6 (2009)

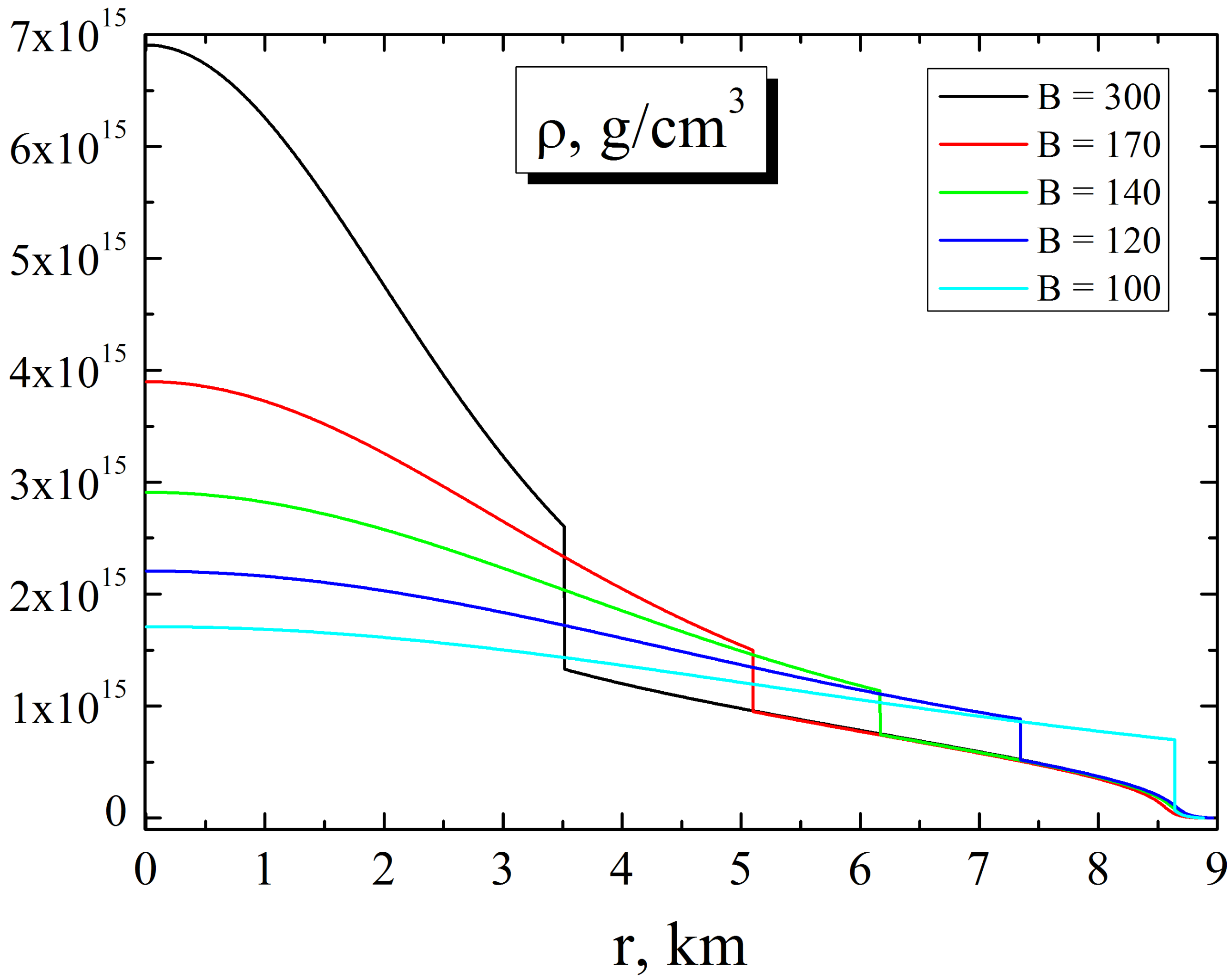
The are other examples of special point on M-R diagram!

Schramm et al arXiv:1310.5804





What is the structure of the stars at special point?



The stars have different quark cores but very similar envelope

The conditions for the envelope to remain unchanged at varying B:

OVT-equations of equilibrium

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{G(P+E)\left(m + \dfrac{4\pi r^3}{c^2}P\right)}{c^2 r\left(r - \dfrac{2Gm}{c^2}\right)}, \\ \dfrac{dm}{dr} = 4\pi r^2 \dfrac{E}{c^2}. \end{cases}$$

$$\Delta P = \left(\frac{\partial P}{\partial r}\right)_1 \delta r = \frac{1}{\lambda}\left(\frac{\partial P}{\partial r}\right)_2 \delta r.$$

$$\Delta m = 4\pi r_*^2 \frac{E_1}{c^2}\delta r = 4\pi r_*^2 \frac{E_2}{c^2}\left[1-(\lambda-1)\frac{P_2}{E_2}\right]\frac{\delta r}{\lambda}.$$

Phase equilibrium

$$\begin{cases} P_{pt} = P_1(n_1) = P_2(n_2) \\ \mu_{pt} = \dfrac{P_1+E_1}{n_1} = \dfrac{P_2+E_2}{n_2} \end{cases}$$

$$E_2 = E_2(n_2, B)$$

$$\Delta P = \frac{\delta B}{\lambda - 1}\left(\frac{\partial E_2}{\partial B}\right), \quad \lambda \equiv \frac{n_2}{n_1}.$$

$$\begin{cases} \Delta P = \left(\dfrac{dP}{dr}\right)_2 \delta r + \left(\dfrac{dP}{dP_c}\right)_{r,B} \delta P_c + \left(\dfrac{dP}{dB}\right)_{r,P_c} \delta B, \\ \Delta m = \left(\dfrac{dm}{dr}\right)_2 \delta r + \left(\dfrac{dm}{dP_c}\right)_{r,B} \delta P_c + \left(\dfrac{dm}{dB}\right)_{r,P_c} \delta B. \end{cases}$$

The whole set of the equations:

$$\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial E_2}{\partial B}\right)\partial B,$$

$$-\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial P}{\partial P_c}\right)_{r,B}\delta P_c+\left(\frac{\partial P}{\partial B}\right)_{r,P_c}\delta B,$$

$$-\frac{dm}{dr}\left[\frac{P_2+E_2}{E_2}\right]\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial m}{\partial P_c}\right)_{r,B}\delta P_c+\left(\frac{\partial m}{\partial B}\right)_{r,P_c}\delta B.$$

All the quantities in main equation refer to quark phase only!

Non-trivial solution
Determinant = 0!

The main equation

$$\left(\frac{\partial P}{\partial P_c}\right)_{r,B}\left[\frac{dm}{dr}\frac{(P_2+E_2)}{E_2\frac{dP}{dr}}\left(\frac{\partial E_2}{\partial B}\right)+\left(\frac{\partial m}{\partial B}\right)_{r,P_c}\right]=\left(\frac{\partial m}{\partial P_c}\right)_{r,B}\left[\left(\frac{\partial E_2}{\partial B}\right)+\left(\frac{\partial P}{\partial B}\right)_{r,P_c}\right].$$

This is the main equation. If it fulfilled at some point, the whole mass M and radius R of the star will remain constant with small changes in B. Pay attention that λ dropped out from the main equation! Thus this property does not depend on the parameters of star’s envelope.

Dimensionless form of the Main Equation

For quark phase ⇨ $$\boxed{P=\alpha(E-E_0),\quad E_0\equiv 4B,\quad \alpha_q=\frac{1}{3}}$$

$$\boxed{\rho=\frac{E}{E_0},\quad x=\frac{r}{r_{\dim}},\quad \mu=\frac{m}{m_{\dim}}}$$

$$\boxed{r_{\dim}=\frac{c^2}{\sqrt{4\pi G E_0}},\quad m_{\dim}=\frac{c^4}{G\sqrt{4\pi G E_0}}.}$$

Equilibrium equations:

$$\begin{cases}\alpha\dfrac{d\rho}{dx}=-[\rho+\alpha(\rho-1)]\dfrac{\mu+\alpha x^3(\rho-1)}{x(x-2\mu)},\\[2ex] \dfrac{d\mu}{dx}=x^2\rho.\end{cases}$$

This is the main equation in dimensionless variables:

$$\boxed{\left(\frac{\partial\mu}{\partial\rho_c}\right)_x\left[\rho-\frac{\alpha}{1+\alpha}+\frac{x}{2}\frac{d\rho}{dx}\right]\frac{d\rho}{dx}=x^2\left(\frac{\partial\rho}{\partial\rho_c}\right)_x\left[\rho-\frac{\alpha}{1+\alpha}+\frac{x}{2}\frac{d\rho}{dx}\left(\rho-\frac{\mu}{x^3}\right)\right]}$$

The way to solve main equation:

Homology-invariant variables

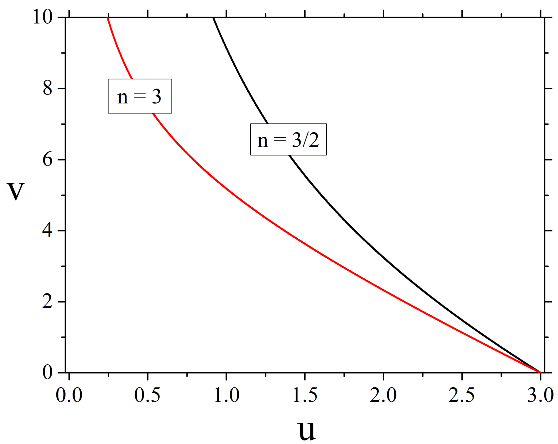
For some specific EOSes TWO TOV-equations of equilibrium can be combined to ONE differential equation with homological variables.

$$\begin{cases} x\dfrac{du}{dx} = f_u(u,v), \\ x\dfrac{dv}{dx} = f_v(u,v). \end{cases} \Rightarrow \quad \frac{dv}{du} = \frac{f_v(u,v)}{f_u(u,v)} = f(u,v).$$

Examples:

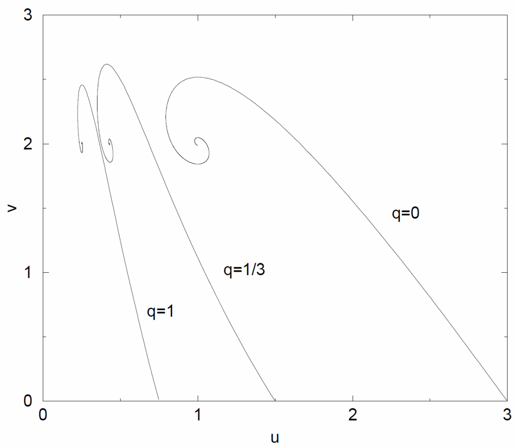
Newtonian limit, polytropes:

$$\begin{cases} u = \dfrac{d\ln m}{d\ln r} = \dfrac{4\pi r^3\rho}{m}, \\ v = -\dfrac{d\ln P}{d\ln r} = \dfrac{Gm\rho}{rP}. \end{cases} \Rightarrow \quad \frac{u}{v}\frac{dv}{du} = -\frac{u + \dfrac{v}{1+n} - 1}{u + \dfrac{nv}{1+n} - 3}$$



General Relativity, P=qE

$$\frac{u}{v}\frac{dv}{du} = \frac{-1 - \frac{2q}{1+q}v + (1+3q)u + \frac{q(3+5q)}{1+q}uv + \frac{2q^2(1-q)}{(1+q)^2}uv^2}{3 - \frac{1-q}{1+q}v - (1+3q)u - \frac{q(3+q)}{1+q}uv - \frac{4q^2}{(1+q)^2}uv^2}.$$

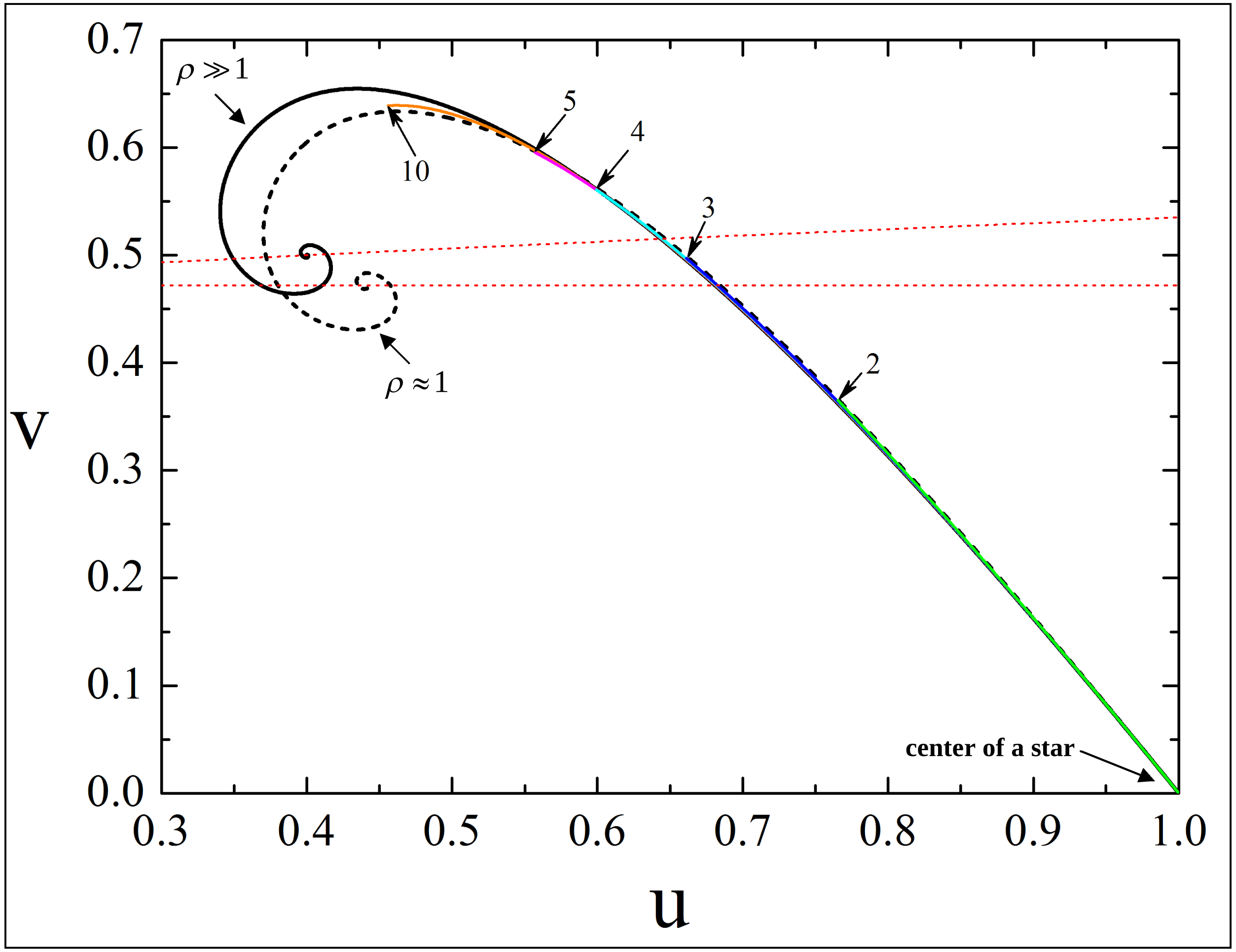


From P.-H. Chavanis, Astron. Astrophys.,381, (2002)

"Almost" homological variables u and v

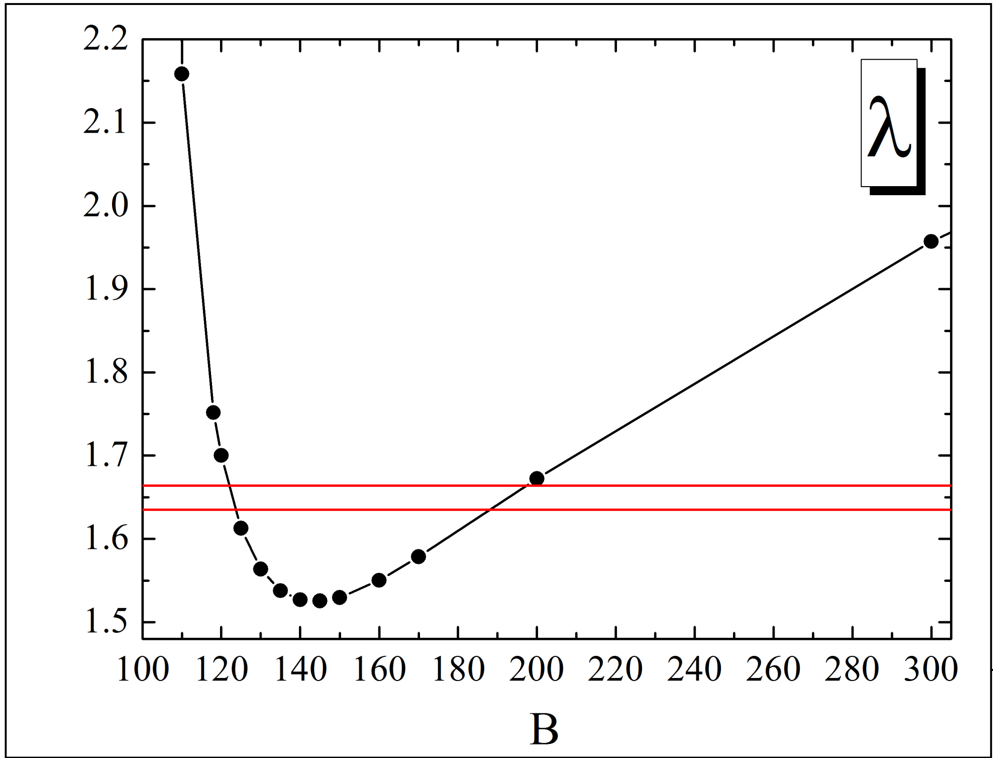
$$\begin{cases} v = \dfrac{\mu + \alpha x^3(\rho - 1)}{x - 2\mu}, \\[2ex] u = \dfrac{x^3\left[\rho + \alpha(\rho - 1)\right]}{3\mu + \alpha x^3(\rho - 1)}. \end{cases}$$

The structure of real quark stars in homologous variables is indicated by the lines of various colors, almost coincident with the spiral ones. The arrows indicate the points corresponding to the surface (ρ =1) of these stars. The number at the arrow indicates the corresponding dimensionless central density ρ_c



This is the solution of main equation in variables (u,v)

$$u_* = \left[v_*^2(3+\alpha) + v_*(3-\alpha) - 6\alpha\right]\frac{1 + (1+\alpha)(\rho_* - 1)}{4\alpha^2(\rho_* - 1)(1 - v_*)(3 - v_*)}$$



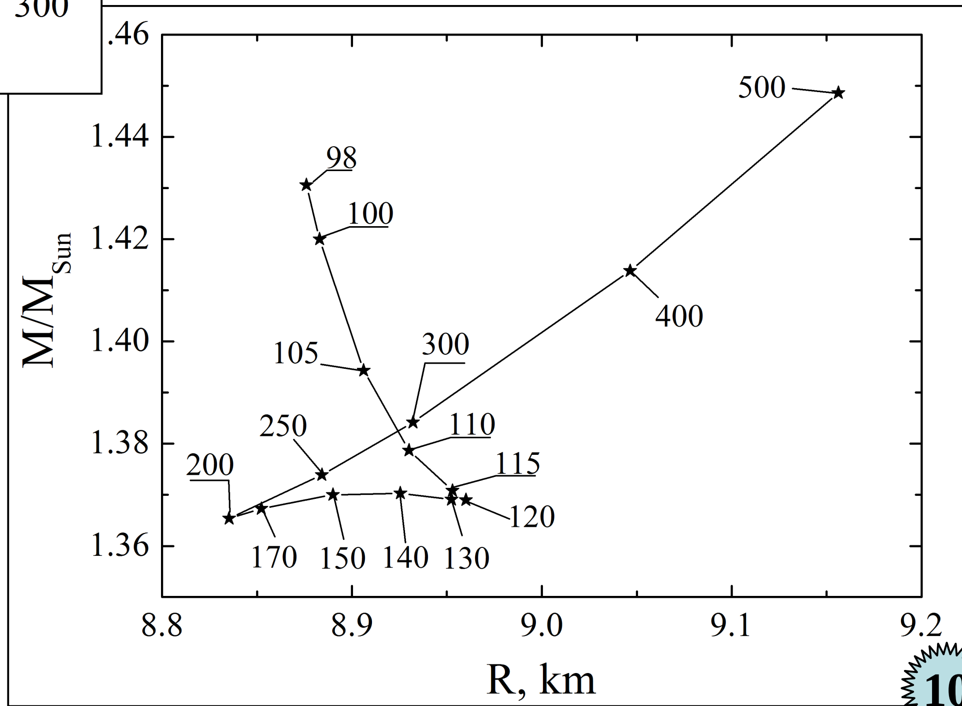
The reasons for the existence of a special point on the M-R diagram of hybrid stars:

1. **The linearity of quark EOS**

$$\boxed{P = \alpha(E - E_0)}$$

2. **The “phase diagram” of quark matter**

Global structure of fixed points



The condition that a small shift from a fixed point bring us to another fixed point

$$\boxed{\lambda = \frac{u(3-v)\left[(1+v)^2(3-v)+8(1-v^2)\alpha-(3-v)^2\alpha^2\right]}{(1+v)(7v^2-6v+3)+8(1-v^2)(3-v)\alpha-(3-v)^3\alpha^2}}$$

$$\boxed{\lambda \equiv \frac{n_2}{n_1}}$$

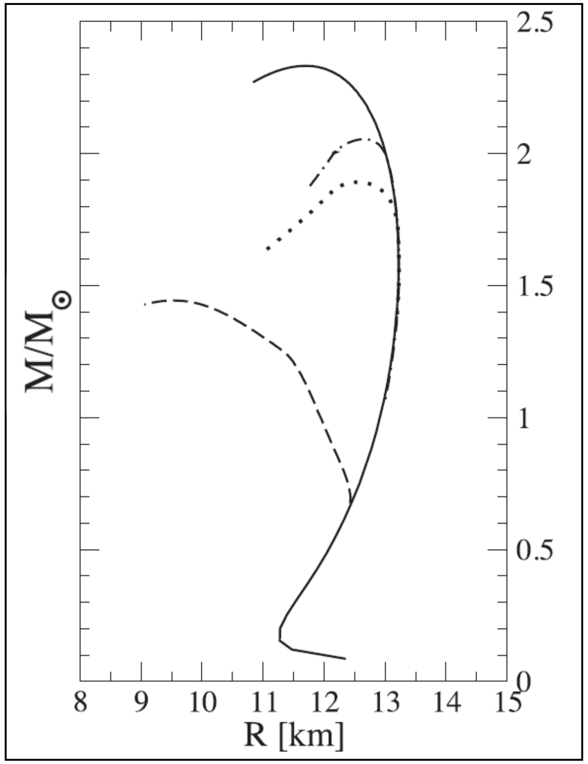
Large scale

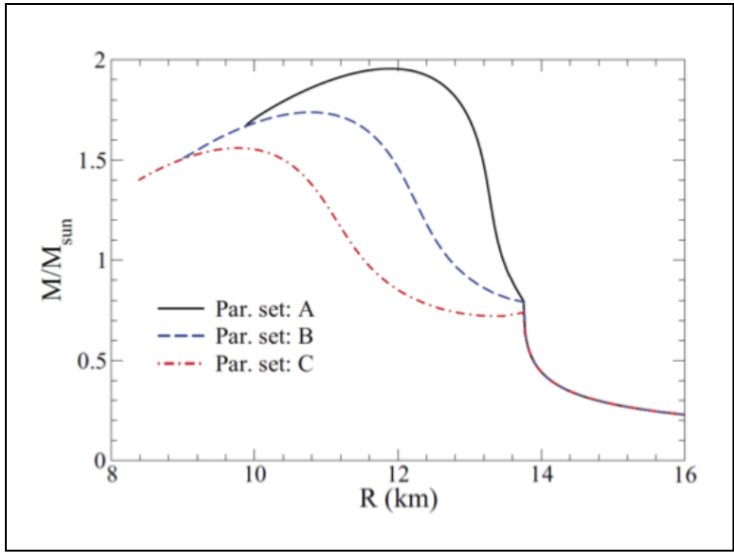
For a given EOS the point on M-R diagram where the main equation holds we call fixed point. This is a local property.

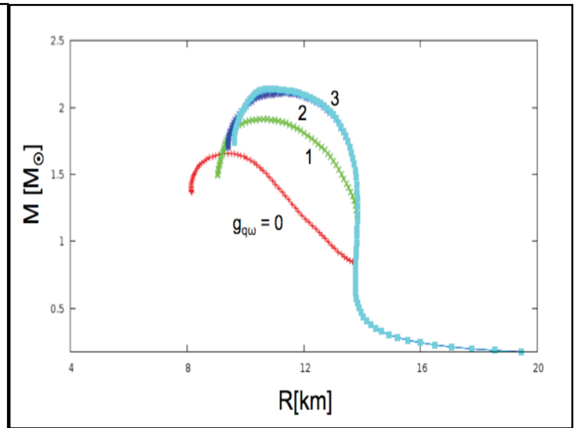
Open questions:

Other (non-linear) EOS as the solution of the main equation?
Other topology of fixed-points because different envelope?

Calculations with non-linear quark EOS:







Schramm et al. arXiv:1306.0989v2

Bombachi and Lagoteta, MNRASL 433, L79-L82 (2013)

Linear EOS (P being a linear function of ρ) is characteristic of a simplest bag model of quark matter that assumes massless quarks, but it also holds with very high accuracy for more realistic bag model with massive s-quark (Zdunik, 2000). **Zdunik and Haensel A&A 551, A61 (2013)**

The existence of special point means that the hybrid stars with the same Mass and Radius can have nevertheless absolutely different inner structure depending on EOS