Neutron Star Dynamics under Time Dependent External Torques

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The Two Component Model

Baym, G., Pethick, C. J., Pines, D., & Ruderman, M., 1969.

$$I_c \dot{\Omega}_c + I_s \dot{\Omega}_s = N_{ext}$$

$$\dot{\Omega}_s = -\frac{\Omega_s - \Omega_c}{\tau_0} = -\frac{\omega}{\tau_0}$$

•Ω_c is the “crust” (normal matter) rotation rate. It obeys the Navier Stokes Equation, but is already in rigid body rotation on superfluid timescales.

•Ω_s is the superfluid rotation rate. It does *not* obey the Navier Stokes Equation.

•Physically, the superfluid spins down by a flow of quantized vortices away from the rotation axis. Under many interactions between the vortices and normal matter, the vortex current, and the superfluid rate are *linear* in the lag ω.

•Under a constant external torque N_{ext} the system has a steady state,

$\dot{\Omega}_s = \dot{\Omega}_c = \frac{N_{ext}}{I}$. ($I = I_s + I_c$), achieved at a lag $\omega_\infty = -\frac{N_{ext}}{I}\tau_0$.

The Two Component Model with Constant External Torques (as applied to radio pulsars)

The *linear response* to any offset $\delta\omega(0)$ from steady state is exponential relaxation:

$$\dot{\Omega}_c = \dot{\Omega}_c(0) - \frac{I_s}{I}\frac{\delta\omega(0)}{\tau}e^{-t/\tau},$$

Several components of exponential relaxation are indeed observed following pulsar glitches.

- I_s / I is typically $\sim 10^{-3} <\sim 10^{-2}$ for each component of exponential relaxation. This points at the *crust superfluid* . With effective masses (entrainment) taken into account (Chamel 2012), $I_s / I <\sim 10^{-1}$, still pointing to the *crust + outer core* (Gügercinoğlu & Alpar 2014).
- The core superfluid is already coupled tightly to the crust; the core is effectively a part of the crust, dynamically, on glitch and postgltich relaxation timescales (Alpar, Langer & Sauls 1984).
- *The two component model is enough*: since $I_{s,i} / I << 1$, the different superfluid components with moments of inertia $I_{s,i}$ can be handled with the crust in separate two component models and then the response of the crust to each can be superposed.

Vortex Creep and the Nonlinear Two Component Model I

In the vortex creep model a superfluid component with vortex pinning spins down by the thermally activated flow (creep) of vortices against pinning potentials. The spindown rate is

$$\dot{\Omega}_s = -\frac{4\Omega_s v_0}{r} \exp\left(-\frac{E_p}{kT}\right) \sinh\left(\frac{\omega}{\varpi}\right), \quad (11)$$

The two component model is now *nonlinear* in the lag ω:

$$\dot{\Omega}_s = -f(\omega)$$

The two component system has a steady state defined by

$$\dot{\omega} = 0$$

$$\dot{\Omega}_s = \dot{\Omega}_c = \frac{N_{ext}}{I}.$$

The steady state value ω_∞ of the lag is determined by

$$f(\omega_\infty) = -\frac{N_{ext}}{I}.$$

Vortex Creep and the Nonlinear Two Component Model II Applications to Radio Pulsars – Constant External Torque

For a constant external torque $N_{\rm ext} = I\dot{\Omega}_\infty$ the solution for the observed crust spindown rate is

$$\dot{\Omega}_{\rm c}(t) = \frac{I}{I_{\rm c}}\dot{\Omega}_\infty - \frac{I_{\rm nl}}{I_{\rm c}}\dot{\Omega}_\infty\left[1 - \frac{1}{1+\left[\exp\left(\frac{t_0}{\tau_{\rm nl}}\right)-1\right]\exp\left(-\frac{t}{\tau_{\rm nl}}\frac{I}{I_{\rm c}}\right)}\right], \quad (16)$$

with a nonlinear creep relaxation time

$$\tau_{\rm nl} \equiv \frac{kT}{E_{\rm p}}\frac{\omega_{\rm cr}}{|\dot{\Omega}|}, \quad (17)$$

and recoupling (waiting) timescale

$$t_0 \equiv \frac{\delta\omega}{|\dot{\Omega}|}. \quad (18)$$

•This is *very nonlinear* behaviour. Depending on the offset from steady state, vortex creep can *stop* for a while.

•The external torque is acting on less moment of inertia, so the crust spindown rate increases by

$$\frac{\Delta\dot{\Omega}_{\rm c}}{\dot{\Omega}_{\rm c}} \cong \frac{I_{\rm s}}{I_{\rm c}}.$$

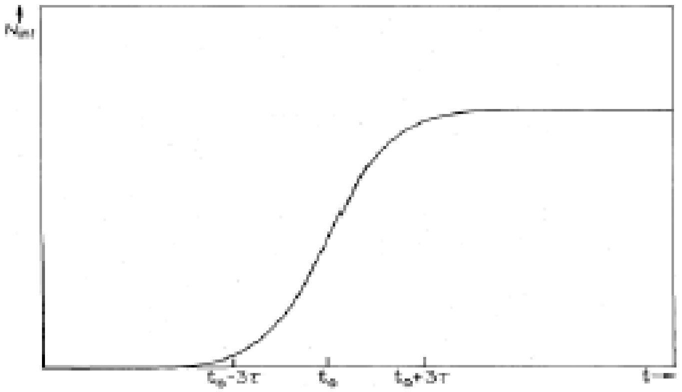
• This does not relax exponentially. The most common relaxation is a recovery at constant $\ddot{\Omega}_c$. *Power law behaviour is characteristic of nonlinear dynamics.*

• Such behaviour is very common in Vela and other pulsars (Akbal et al 2017, etc) .

• Two component models are still enough.

Nonlinear interglitch dynamics, the braking index of the Vela pulsar and the time to the next glitch

O. Akbal[1*], M. A. Alpar[1], S. Buchner[2,3,4] & D. Pines[5]



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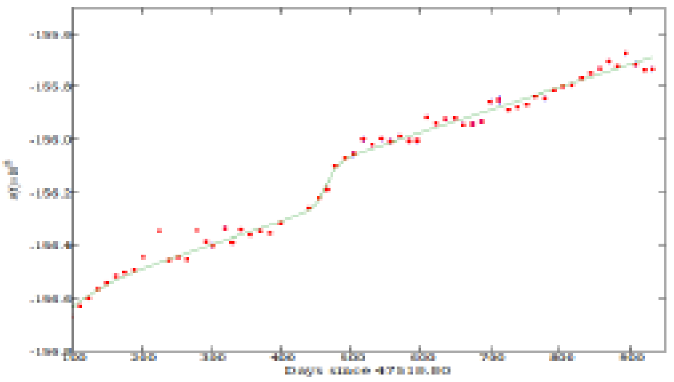

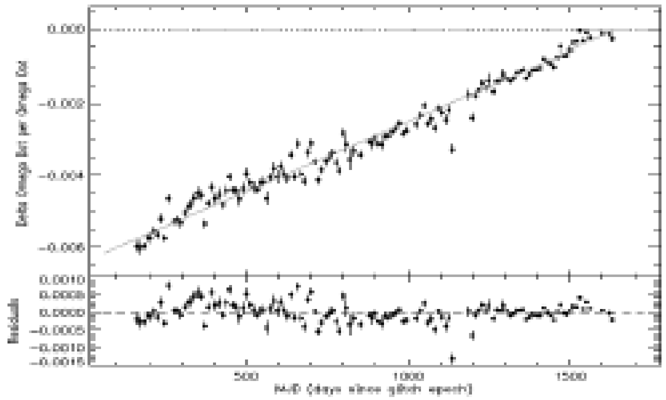

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The Linear Two Component Model under Time Dependent External Torques

•Unlike the case for radio pulsars, the external torque is not constant in neutron stars in transients, HMXBs and magnetars; it is very variable on observational timescales.

• The solution of the linear two component model, with time dependent external torques is:

$$\dot{\Omega}_c(t) = \frac{N_{ext}(t)}{I_c} + \frac{I_s}{I}\left(\frac{e^{(-t/\tau)}}{\tau}\left[\omega(0) - \frac{1}{I_c}\int_0^t e^{(t'/\tau)} N_{ext}(t')dt'\right]\right).$$

• Again, we are lucky, that $I_s/I << 1$ in neutron stars:

•To lowest order, the observed $\dot{\Omega}_c(t) = \frac{N_{ext}(t)}{I_c}$ reflects the external torque.

• Strategy & check: Residuals are related to $\int_0^t e^{(t'/\tau)} N_{ext}(t')dt'$.

The Linear Two Component Model under Time Dependent External Torques : Exponentially Decaying and Power Law Torques

$$N_{\rm ext}(t) = N_0 + \delta N e^{-t/\tau_{\rm d}} = I\dot{\Omega}_\infty + \delta N e^{-t/\tau_{\rm d}}.$$

$$\dot{\Omega}_{\rm c}(t) = \dot{\Omega}_\infty + \frac{\delta N}{I_{\rm c}} e^{-t/\tau_{\rm d}} \left[1 - \frac{I_{\rm s}}{I}\frac{\tau_{\rm d}}{\tau_{\rm d}-\tau}\right] + \frac{I_{\rm s}}{I}\left[e^{-t/\tau}\left(\frac{\omega(0)}{\tau} + \frac{I}{I_{\rm c}}\dot{\Omega}_\infty + \frac{\delta N}{I_{\rm c}}\frac{\tau_{\rm d}}{\tau_{\rm d}-\tau}\right)\right].$$

$$N_{\rm ext}(t) = N_0 + \frac{\delta N t_0^\alpha}{(t+t_0)^\alpha} = I\dot{\Omega}_\infty + \frac{\delta N t_0^\alpha}{(t+t_0)^\alpha}.$$

$$\dot{\Omega}_{\rm c}(t) = \dot{\Omega}_\infty + \frac{\delta N}{I_{\rm c}}\frac{t_0^\alpha}{(t+t_0)^\alpha}\left[1 - \frac{I_{\rm s}}{I}\frac{(t+t_0)}{(1-\alpha)\tau}\right] + \frac{I_{\rm s}}{I}\left[e^{-t/\tau}\left(\frac{\omega(0)}{\tau} + \frac{I}{I_{\rm c}}\dot{\Omega}_\infty + \frac{\delta N}{I_{\rm c}}\frac{t_0}{(1-\alpha)\tau} + \frac{I_{\rm s}}{I}\frac{\delta N}{I_{\rm c}}\frac{t_0^\alpha}{(1-\alpha)\tau^2}\int_0^t \frac{e^{t'/\tau}dt'}{(t+t_0)^{\alpha-1}}\right)\right]$$

Linear Two Component Model with Time Dependent External Torques: Noise

$$N_{ext} = \sum_{i} \alpha_i \delta(t - t_i),$$

$$P(f) = \frac{1}{\sqrt{2\pi}}\left[\frac{2<\alpha^2>}{I_c^2} + \left(\frac{I_s/I}{1+(2\pi\tau f)^2}\right)\left(\frac{<\alpha><\omega>}{I_c} - \frac{<\alpha^2>}{I_c^2}\right)\right] + \frac{1}{\sqrt{2\pi}}\left[\left(\frac{(I_s/I)^2}{1+(2\pi\tau f)^2}\right)\left(\frac{2<\alpha^2>}{I_c^2} + 2<\omega^2> - \frac{<\alpha><\omega>}{I_c}\right)\right],$$

- To lowest order, the power spectrum of $\dot{\Omega}_c(t)$ reflects the power spectrum of the external torque.
- The residual power spectrum has a turnover at $f = (2\pi\tau)^{-1}$ (Lamb, Pines & Shaham 1978; Baykal,Alpar & Kiziloglu 1991). If the residuals can be fitted, I_s / I and the coupling time τ, as well as further moments of the noise process could be measured.
- The example here is white torque noise. The response can be obtained similarly for other torque noise models.

The Non-Linear Two Component Model under Time Dependent External Torques

•The Boltzmann factors in the vortex creep process lead to

$$\dot{\omega} = -\frac{I\varpi}{2I_c\tau_1}e^{\omega/\varpi} - \frac{N_{\rm ext}(t)}{I_c},$$

so the nonlinear two component model is integrable for arbitrary time dependent external torques.

$$\dot{\Omega}_c(t) = \frac{N_{\rm ext}(t)}{I_c} \quad + \frac{I_s}{I_c}\frac{\varpi}{2\tau_1}\left[\frac{1}{\exp\left(\frac{\frac{X(t)}{I_c}-\omega(0)}{\varpi}\right)+\exp\left(\frac{X(t)}{I_c\varpi}\right)\int_0^t \frac{I}{2I_c\tau_1}\exp\left(-\frac{X(t')}{I_c\varpi}\right)dt'}\right].$$

The nonlinear creep timescale and the waiting time are

$$\tau_{\rm nl}(t) \equiv \frac{kT}{E_p}\frac{I\omega_{\rm cr}t}{X(t)}, \qquad t_0(t) \equiv \frac{I\delta\omega(0)t}{X(t)} \qquad X(t) = \int_0^t N_{\rm ext}(t')dt'.$$

• The strategy is the same, with lowest order $\dot{\Omega}_c(t) = \frac{N_{\rm ext}(t)}{I_c}$, and the residuals related to the external torque.

• The model response has to be calculated numerically even for simple time dependent torques, but there is a well defined alogorithm which can be tested.

Applications