So How Do Radio Pulsars Slow-Down?

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with

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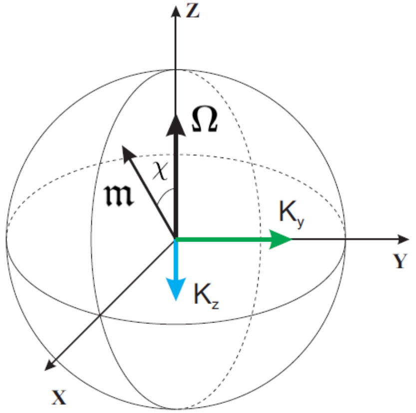
STEP #I

Vacuum: magneto-dipole radiation

Vacuum: magneto-dipole

Energy losses

$$W_{\rm tot} = -\boldsymbol{\Omega}\mathbf{K}$$



$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

$$\varepsilon = \frac{\Omega R}{c} \qquad \beta_{\rm R} = \frac{\boldsymbol{\Omega}\times\mathbf{r}}{c}$$

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

$$W_{\rm tot} = \frac{c}{4\pi}\int (\beta_{\rm R}\mathbf{B})(\mathbf{B}{\rm d}\mathbf{S})$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\mathrm{R}} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\mathrm{tot}} = \frac{c}{4\pi} \int (\beta_{\mathrm{R}} \mathbf{B})(\mathbf{B} \mathrm{d}\mathbf{S})$$

Vacuum (Deusch) $\quad 1 \quad \Omega \quad \Omega^3 \quad 1 \quad 1 \quad = \Omega^4$

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\mathrm{s}} (\mathbf{B}\mathbf{n}) \, \mathrm{d}o = \frac{R^3}{4\pi} \int \{ [\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n}) \} \, \mathrm{d}o$$

Landau-Lifshits, Field Theory

$$\begin{aligned}
B_r^\perp &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\left(2-2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\theta^\perp &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}+\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\varphi^\perp &= \frac{|\mathfrak{m}|}{r^3}\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}+i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_r^\perp &= 0,\\
E_\theta^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_\varphi^\perp &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right).
\end{aligned}$$

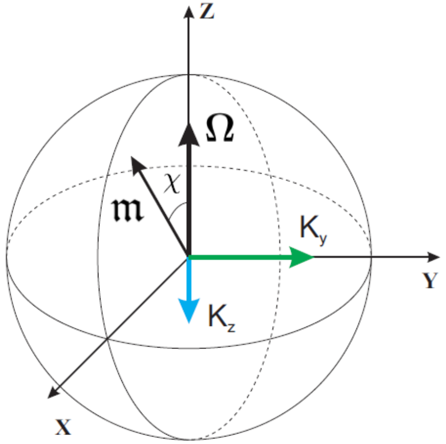
Vacuum: magneto-dipole

Energy losses

$$\beta_{\mathrm{R}} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\mathrm{tot}} = \frac{c}{4\pi} \int (\beta_{\mathrm{R}} \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$



Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\boldsymbol{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi} \int (\beta_{\rm R} \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
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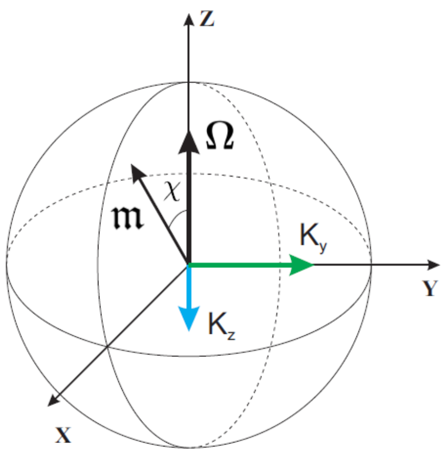
Vacuum (L&L) (1/3)	1	Ω	1	Ω^3	1	$= \Omega^4$



$$\mathbf{B}^{(3)} = -\frac{2}{3} \frac{\mathfrak{m}}{R^3} \left(\frac{\Omega R}{c} \right)^3 \mathbf{e}_{y'}$$

IMPORTANT CONCLUSION

Two terms can play role in energy losses

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\rm s} \left(\mathbf{B}\mathbf{n}\right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \,\mathrm{d}o$$

STEP #II

Pulsar magnetosphere

Force-free approximation

One can neglect energy of particles

$$\frac{1}{c}\mathbf{j} \times \mathbf{B} + \rho_{\mathrm{e}}\mathbf{E} = 0$$

Mestel equation (1973)

$$\nabla \times \tilde{\mathbf{B}} = i\,'\mathbf{B}$$

$$\tilde{\mathbf{B}} = \left\{B_r\left(1 - \frac{\Omega^2 r^2}{c^2}\right),\ B_\theta,\ B_z\left(1 - \frac{\Omega^2 r^2}{c^2}\right)\right\}$$

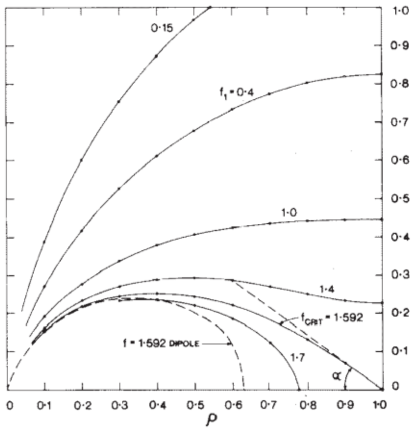
Pulsar equation

$$-\left(1 - \frac{\Omega_{\mathrm{F}}^2 \varpi^2}{c^2}\right)\nabla^2\Psi + \frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi} - \frac{16\pi^2}{c^2} I \frac{\mathrm{d}I}{\mathrm{d}\Psi} + \frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2 \Omega_{\mathrm{F}}\frac{\mathrm{d}\Omega_{\mathrm{F}}}{\mathrm{d}\Psi} = 0$$

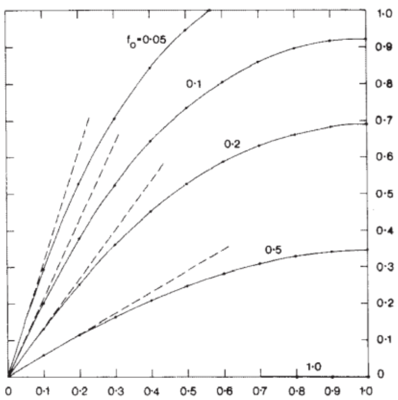
(Michel 1973, Mestel 1993, Scharlemann & Wagoner 1973,
Okamoto 1974, Mestel & Wang 1979)

First solutions

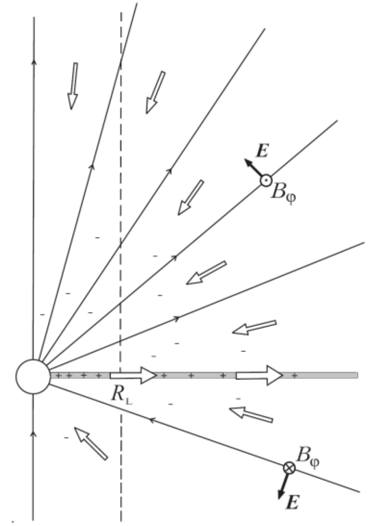
$$-\left(1-\frac{\Omega_{\rm F}^2\varpi^2}{c^2}\right)\nabla^2\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^2}{c^2}I\frac{{\rm d}I}{{\rm d}\Psi}+\frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2\Omega_{\rm F}\frac{{\rm d}\Omega_{\rm F}}{{\rm d}\Psi}=0$$



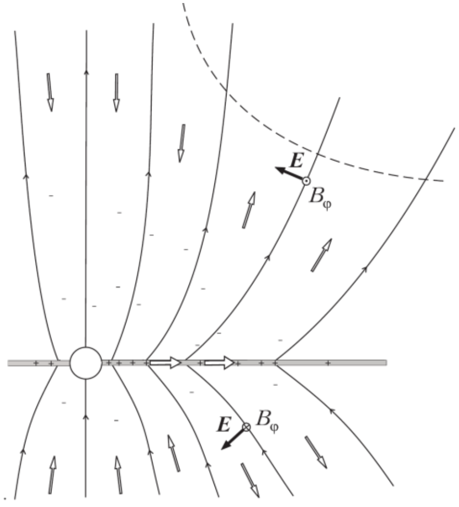
F. Michel (1973)



F. Michel (1973)



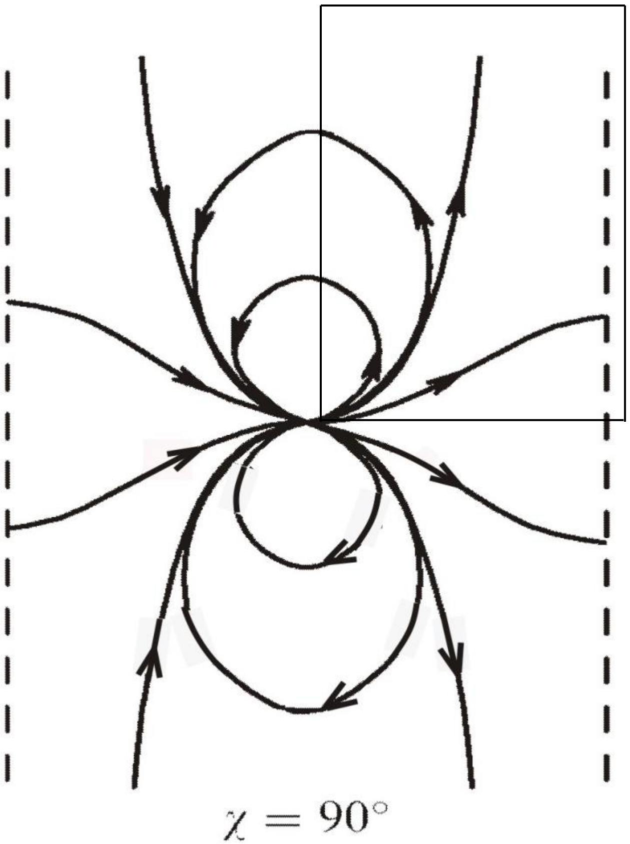
F. Michel (1973)

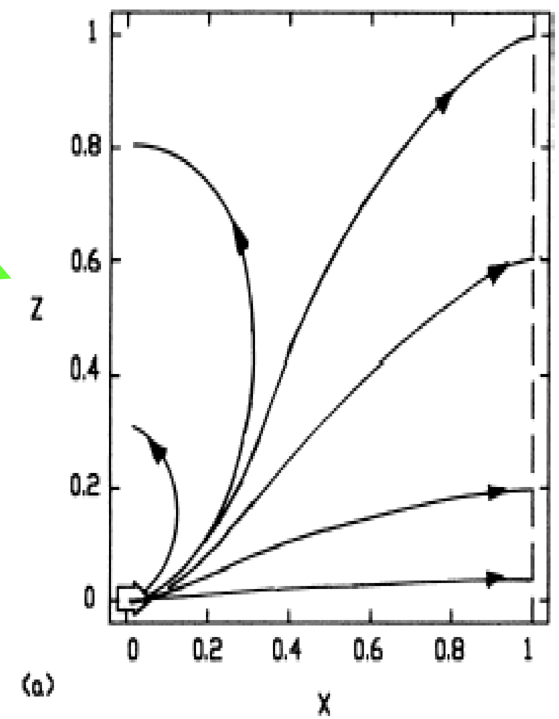


R.Blandford (1976)

Orthogonal Rotator

$$\nabla \times \tilde{\mathbf{B}} = 0$$





VB, A.V.Gurevich, Ya.N.Istomin,
Sov. Phys. JETP, **58**, 235 (1983)

L.Mestel, P.Panagi, S.Shibata,
MNRAS, **309**, 388 (1999)

Orthogonal Rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
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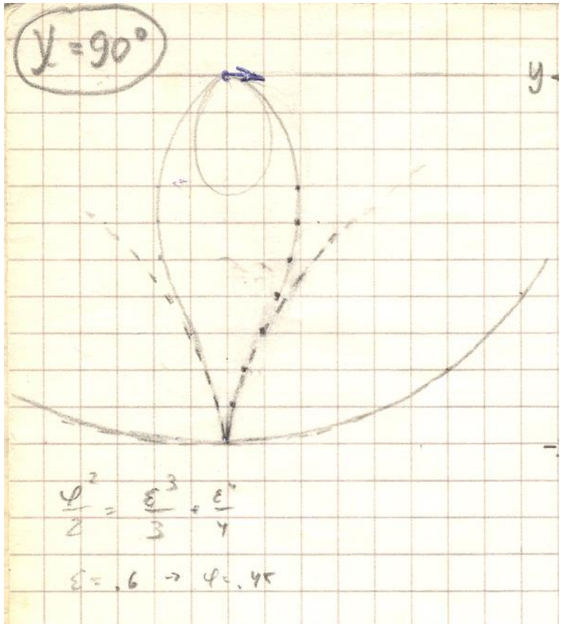
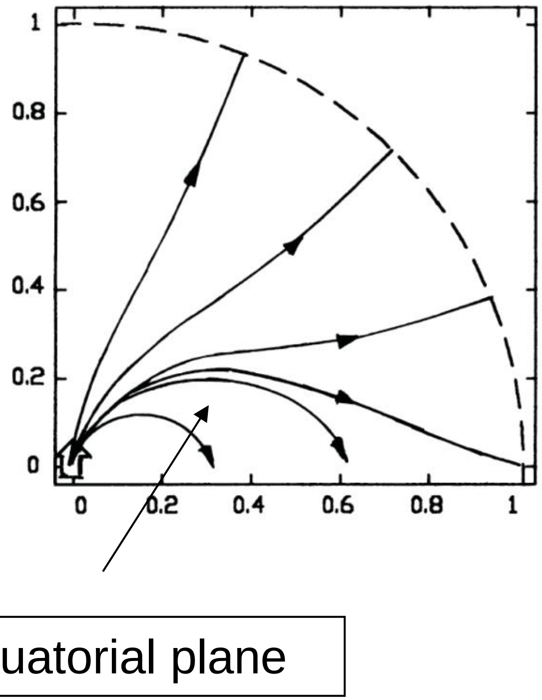


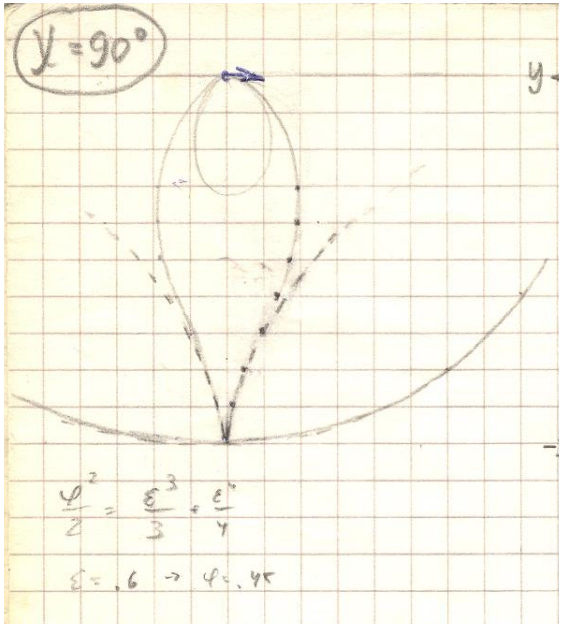


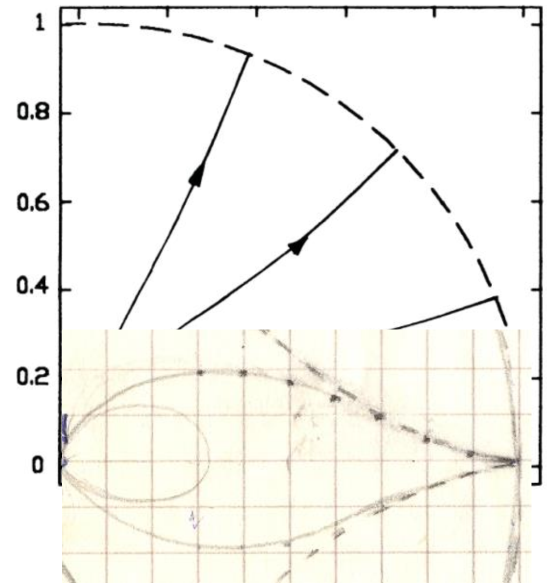
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Orthogonal Rotator

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





Equatorial plane

No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

IMPORTANT CONCLUSION

No energy losses for zero longitudinal current

STEP #III

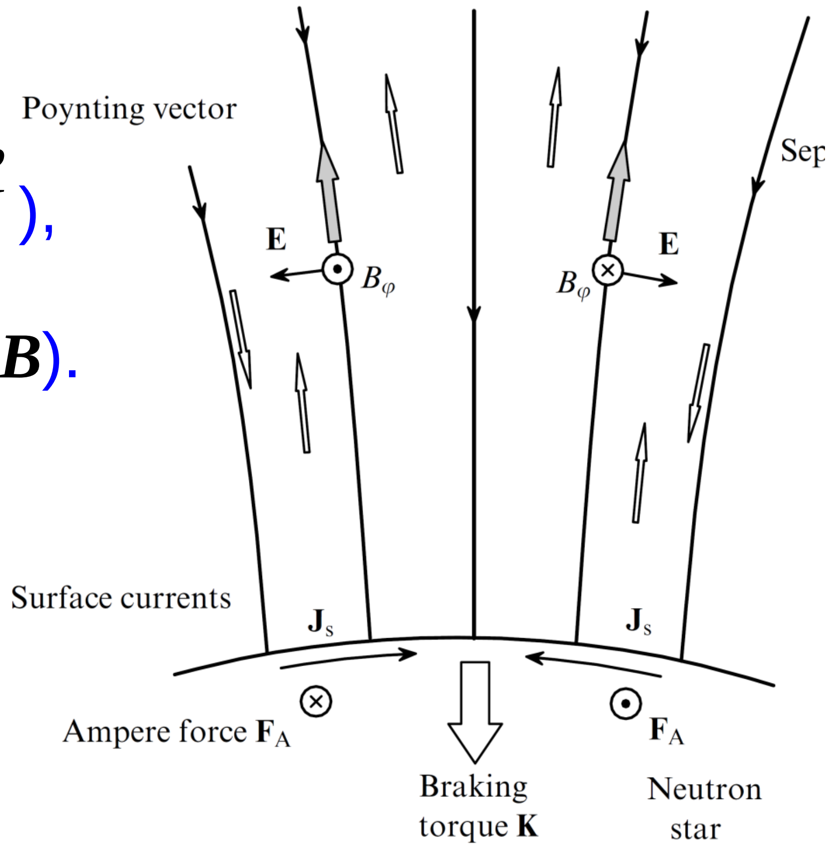
Current losses

Current losses

For current losses mechanism is necessary to have

- Plasma in the magnetosphere,
- regular poloidal magnetic field,
- rotation (inductive electric field $\boldsymbol{E}$, EMF ΔU),
- longitudinal current I (toroidal magnetic field $\boldsymbol{B}$).

$$W_{\rm tot} = I \delta U$$



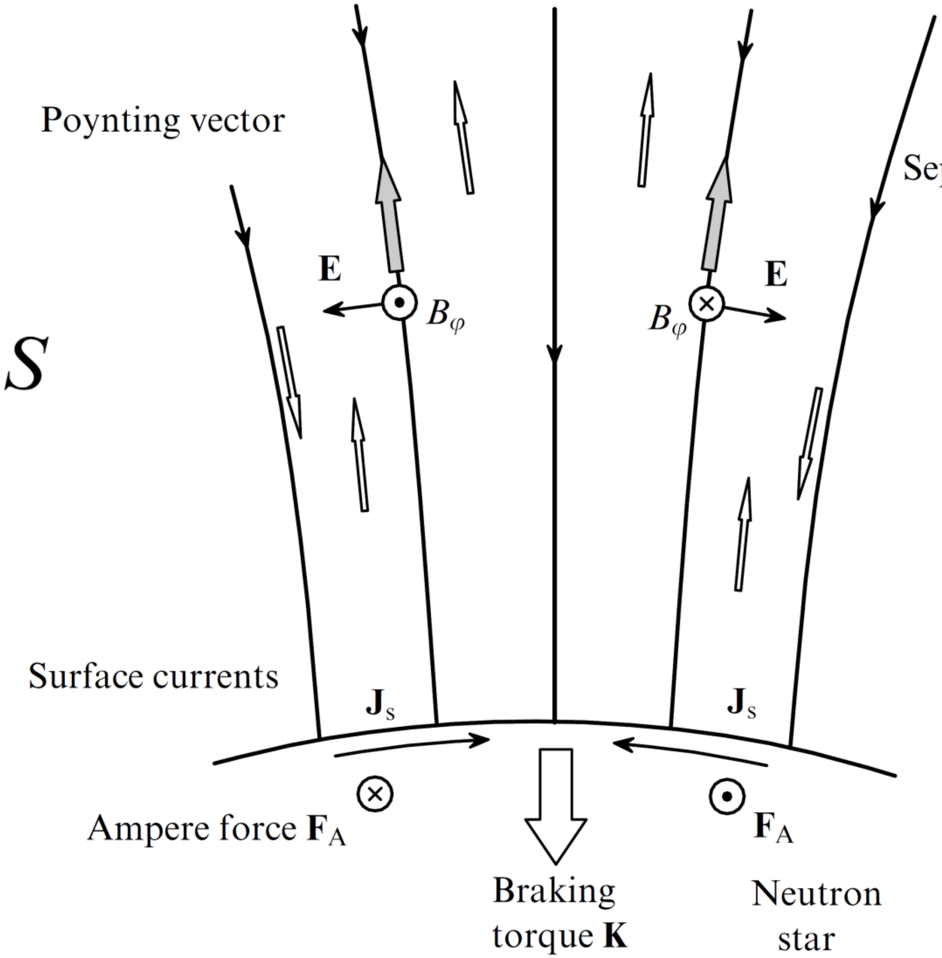
Current losses

$$W_{\rm tot} = -\mathbf{\Omega}\mathbf{K}$$

$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r}\times[\mathbf{J}_{\rm s}\times\mathbf{B}]\right] {\rm d}S$$

$$\nabla_2 \mathbf{J}_{\rm s} = j_{\rm n}$$

$$\mathbf{J}_{\rm s} = \frac{I}{2\pi R\sin\theta}\mathbf{e}_\theta$$



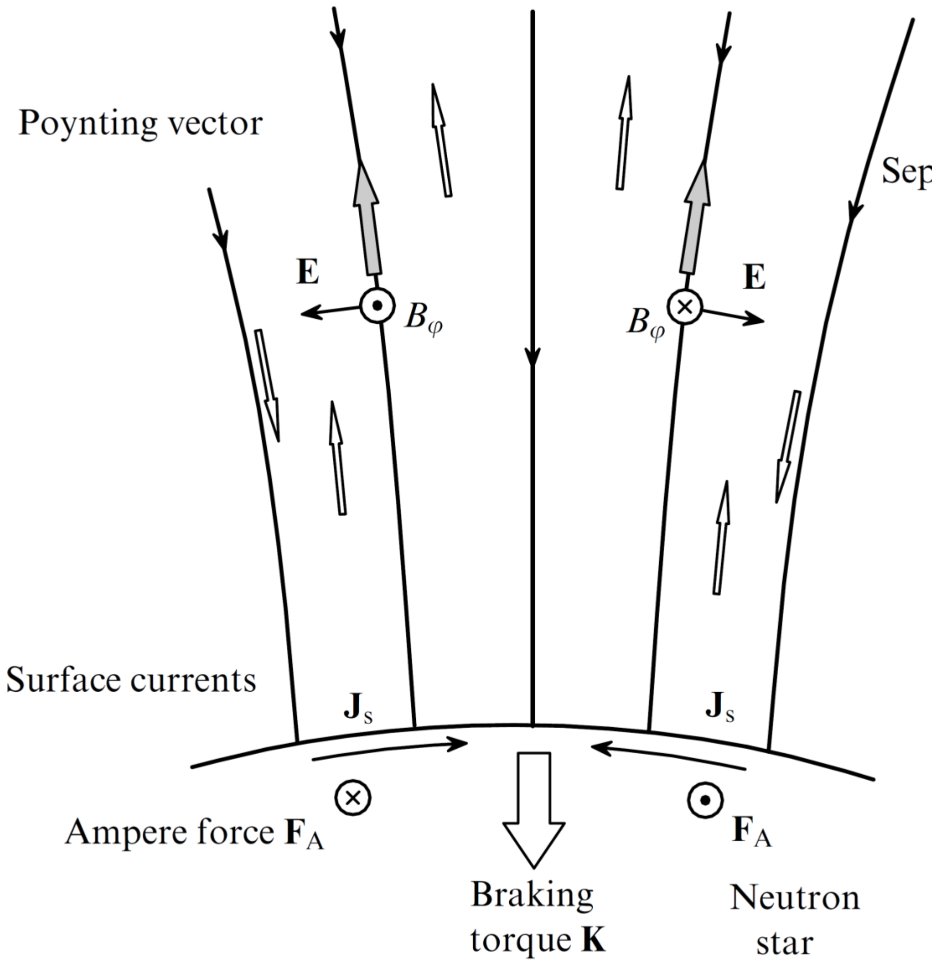
Current losses

$$W_{\rm tot} = c_{\parallel} \frac{B_0^2 \Omega^4 R^6}{c^3} i_0$$

$$i_0 = j_{\parallel}/j_{\rm GJ}$$

$$W_{\rm tot}^{\rm (BGI)} \approx i_{\rm s}^{\rm A} \frac{B_0^2 \Omega^4 R^6}{c^2} \cos^2\chi$$

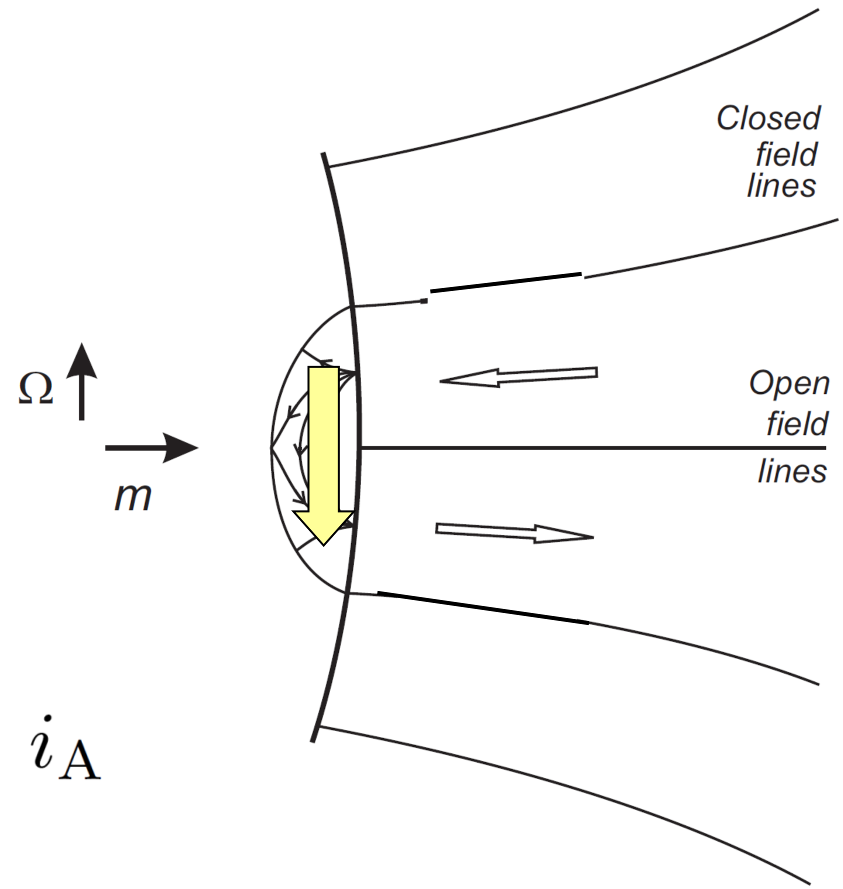
for GJ current



Orthogonal rotator

$$j_{\mathrm{GJ}} \approx \frac{\Omega B}{2\pi} \cos\theta$$

$$\mathbf{K} = \frac{1}{c} \int \left[\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]\right] \mathrm{d}S$$



$$W_{\mathrm{tot}} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}$$

$i_{\mathrm{A}} = j_{||} / \rho_{\mathrm{GJ}} c$

Orthogonal rotator

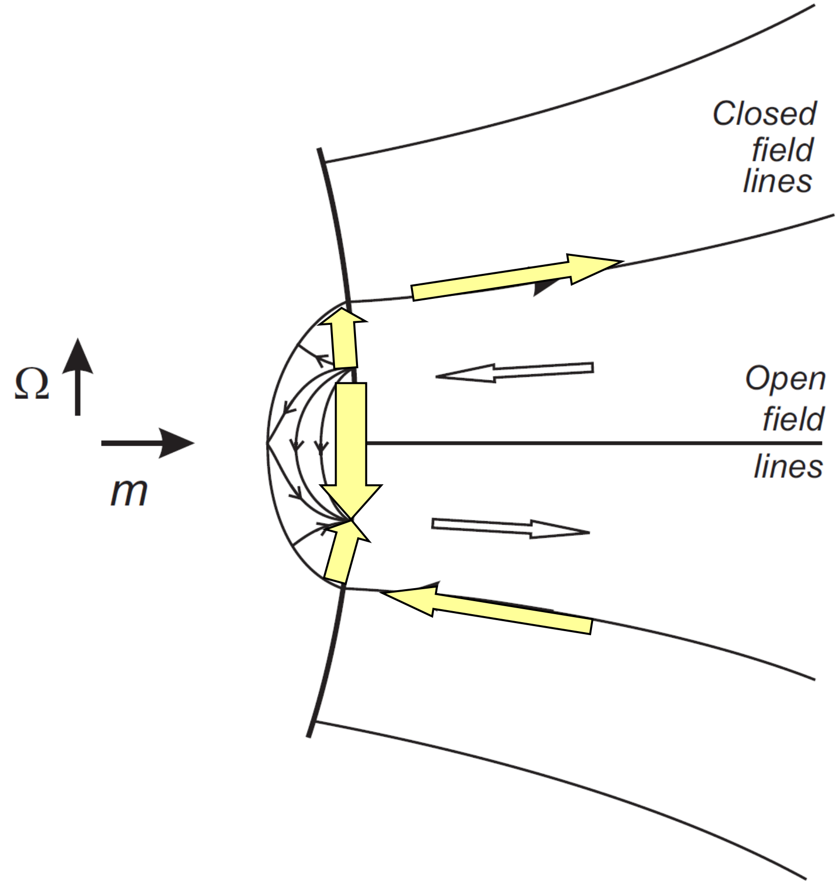
VB, A.V.Gurevich, Ya.N.Istomin JETP **58**, 235 (1983)

$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times [\mathbf{J}_{\rm s} \times \mathbf{B}]\right] \mathrm{d}S$$

$$W_{\rm tot} = c_{\perp}\frac{B_0^2\Omega^4R^6}{c^3}\left(\frac{\Omega R}{c}\right) i_{\rm A}$$

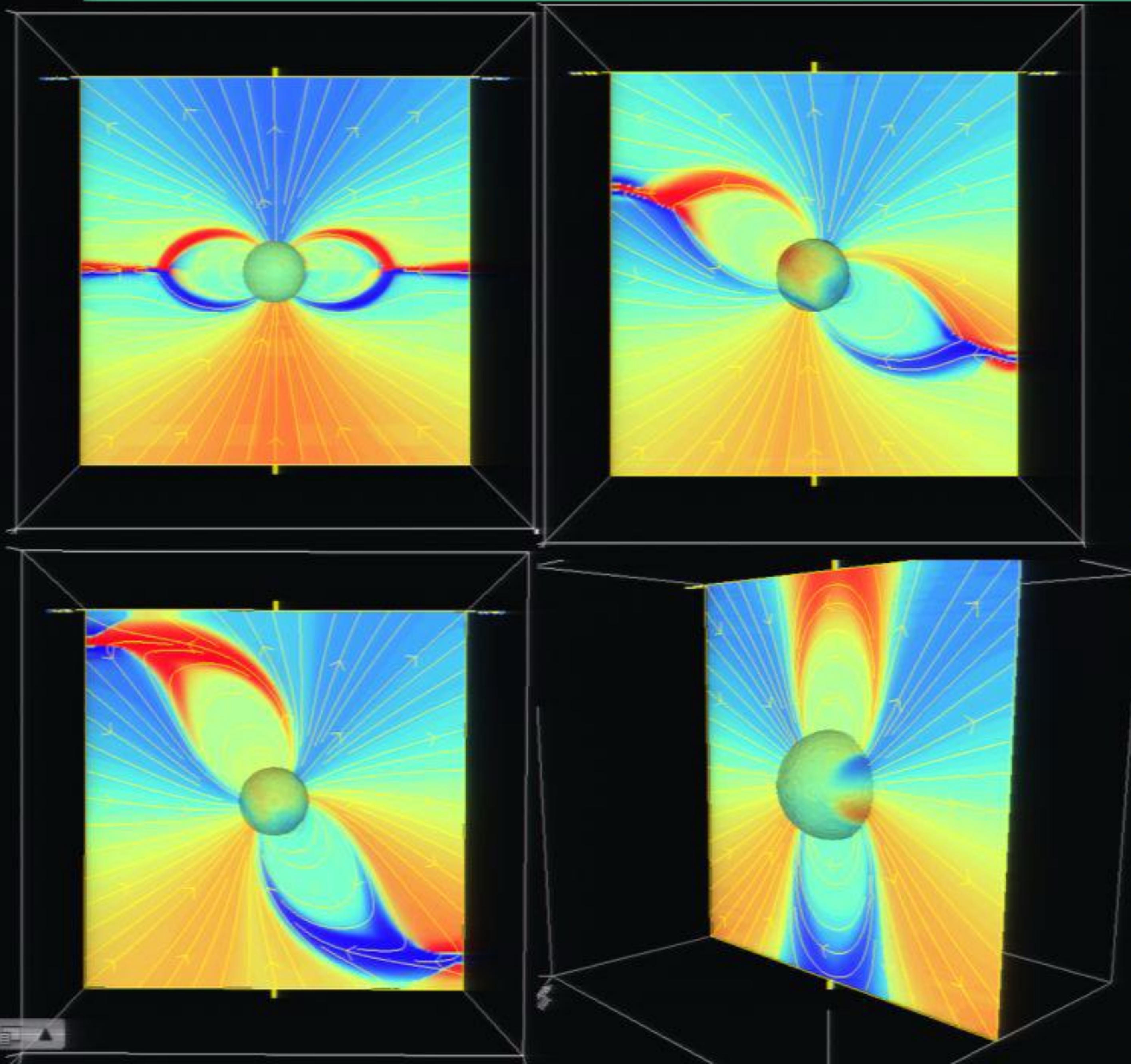
$$< \mathrm{J}_\theta > = 0$$

$$\mathrm{I}_{\rm sep} = 3/4\ \mathrm{I}_{\rm vol}$$



$i_{\rm A} = j_{||}/\rho_{\rm GJ}c$

Magnetospheric currents

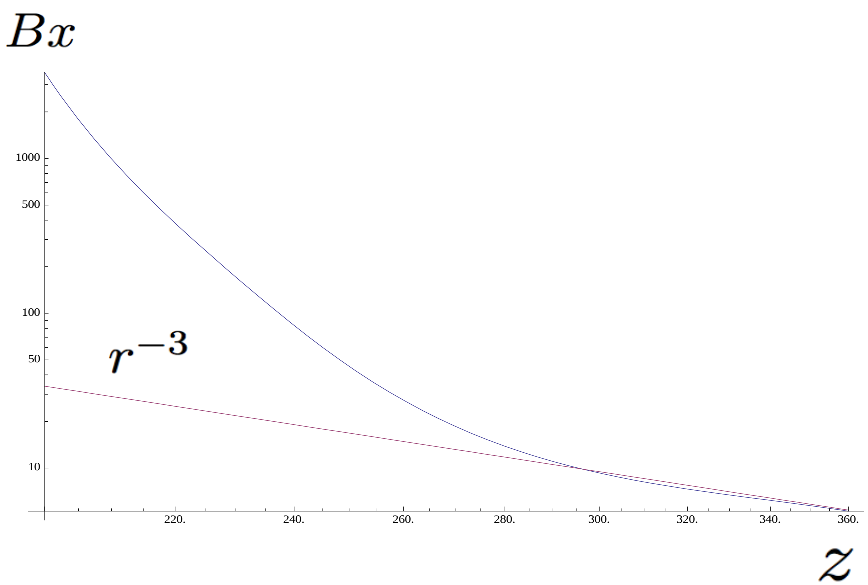


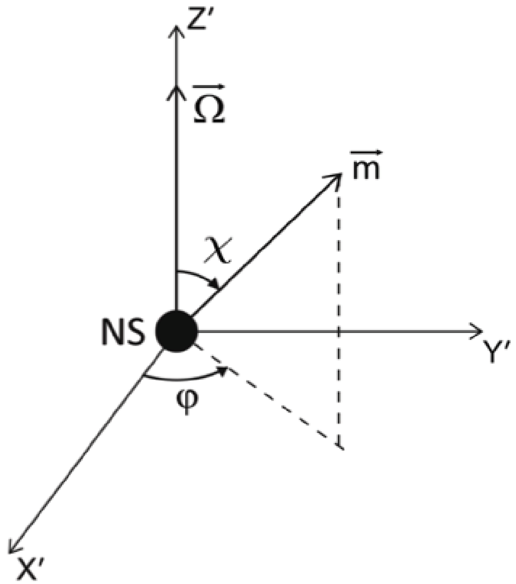
Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

Spitkovsky solution, $\chi = 60^o$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, Ya.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

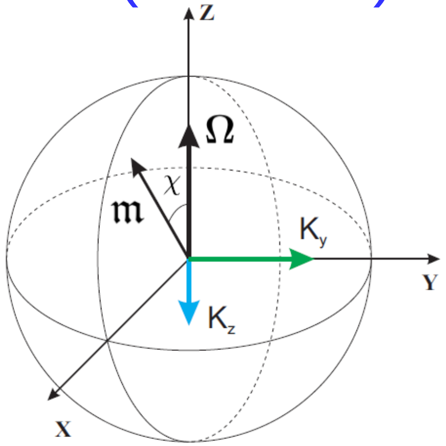
Current losses

$$\beta_R = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

Energy losses

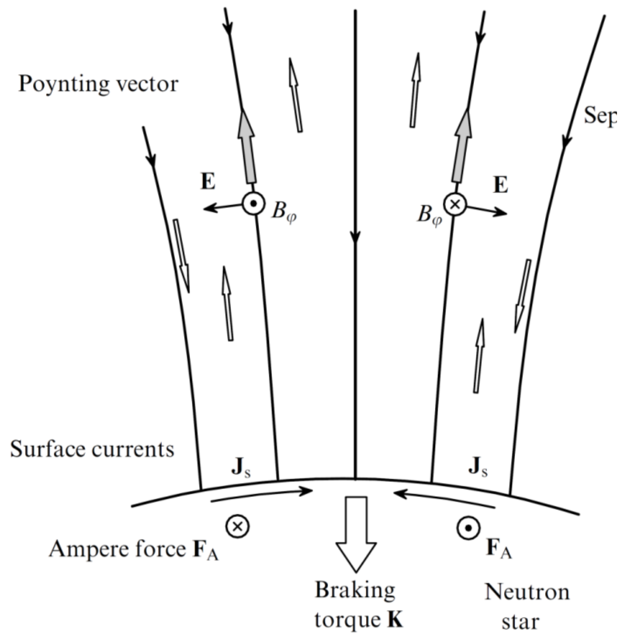
$$W_{tot} = \frac{c}{4\pi} \int (\beta_R \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Current axisymmetric	$i_s \sim 1$	1	$\Omega^{3/2}$	$\Omega^{3/2}$	1	Ω	$= \Omega^4$
Current orthogonal	$i_a \sim 1$	Ω	Ω	Ω^2	1	Ω	$= \Omega^5$
Vacuum (L&L) (2/3)		1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (Duetsch)		1	Ω	Ω^3	1	1	$= \Omega^4$



IMPORTANT CONCLUSION

$$\boxed{W_{\rm tot} = I\delta U}$$



Current losses correspond to first term only

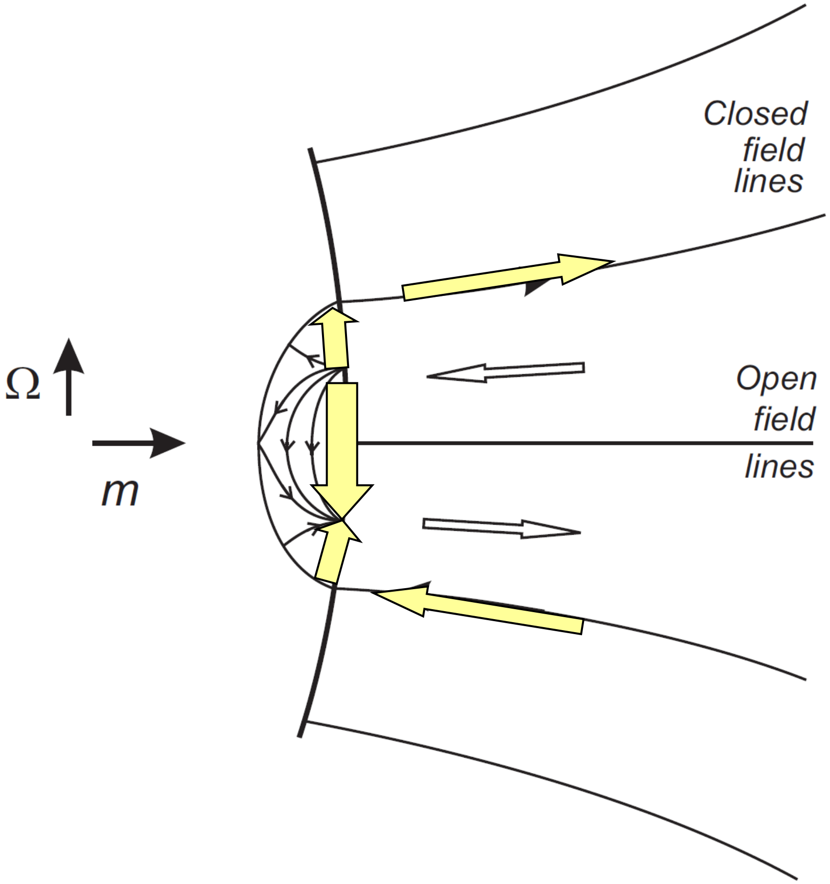
$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{Bn})\,{\rm d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

IMPORTANT REMARK

VB, E.E.Nokhrina. Astron. Lett., **30**, 685 (2004)

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$

No longitudinal currents in close magnetosphere.
No additional currents along the separatrix.



$$I_{\rm sep} = 3/4\ I_{\rm vol}$$

$$<J_\theta> = 0$$

IMPORTANT REMARK

VB, E.E.Nokhrina. Astron. Lett., **30**, 685 (2004)

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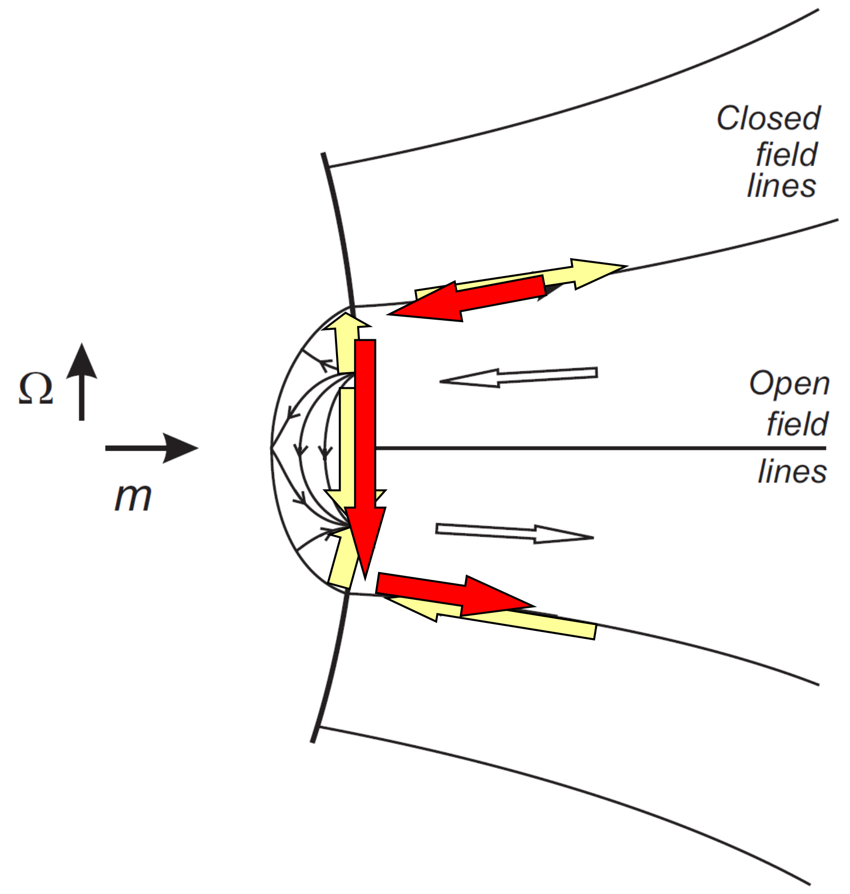


$$I_{sep} < 3/4\ I_{vol}$$

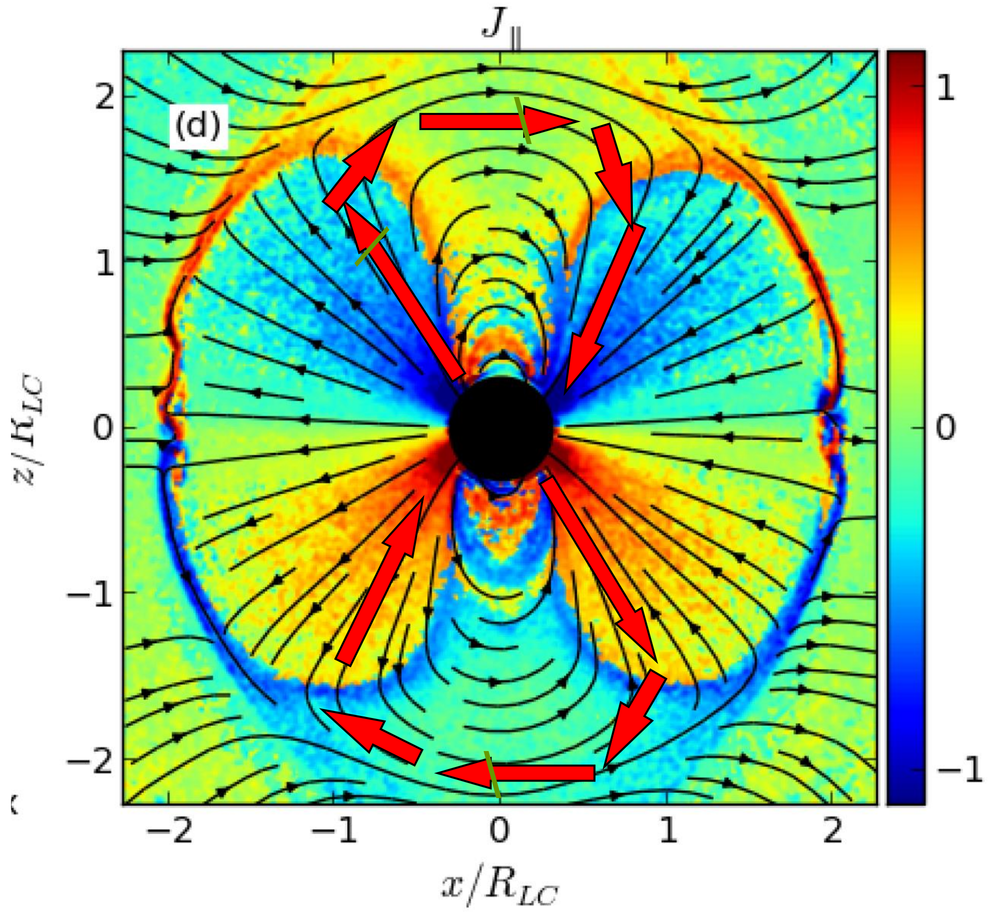
$$\langle J_\theta \rangle \neq 0$$

IMPORTANT REMARK

VB, E.E.Nokhrina. Astron. Lett., **30**, 685 (2004)

$$W_{\mathrm{tot}} = \frac{\Omega R^3}{c}\int J_\theta B_n \mathrm{d}o$$

Current direction corresponds to energy losses.



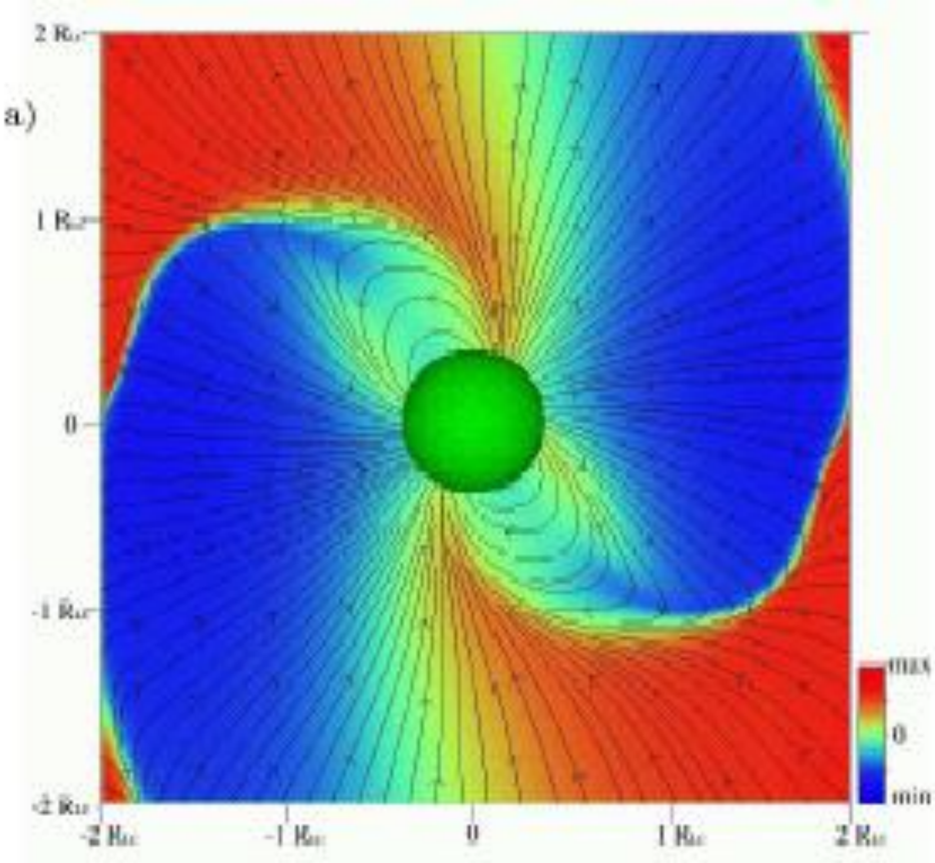
STEP #IV

“Universal solution”

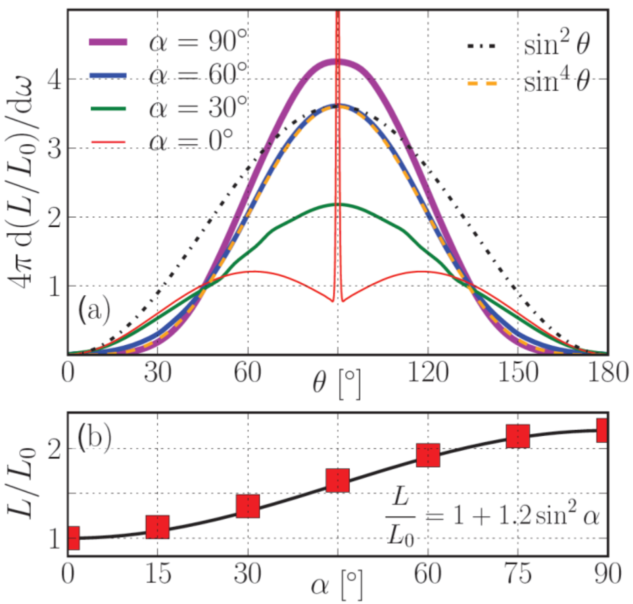
Inclined rotator

A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$



Inclined rotator – numerically



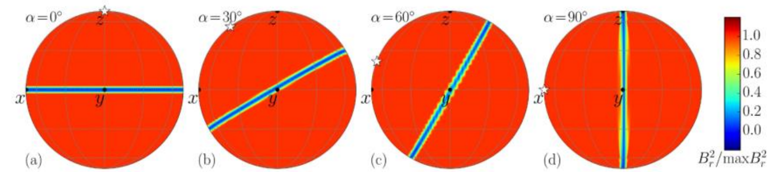
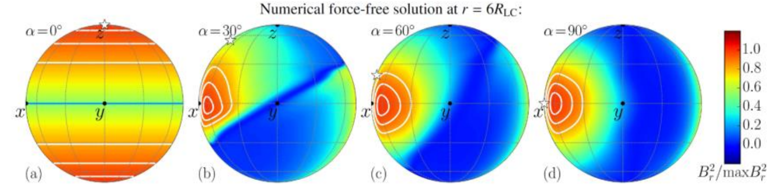
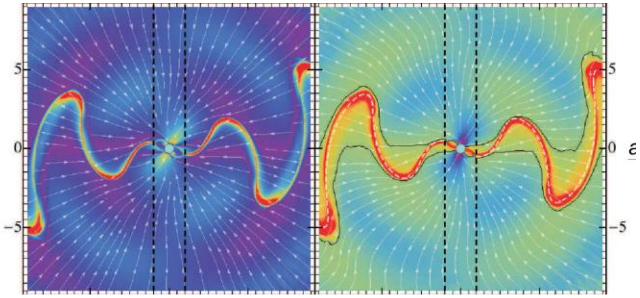


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.



A.Tchekhovskoy, A.Philippov, A.Spitkovsky, MNRAS, **457**, 3384 (2016)



I.Contopoulos et al

$$\langle B_r \rangle \sim \sin\theta$$
$$\langle E \rangle, \langle B_\varphi \rangle \sim \sin^2\theta$$

$$W_{\rm tot}(\theta) = \sin^2\theta\, B_r^2(\theta)$$

Orthogonal rotator – numerically

$$\langle B_r \rangle \sim \sin\theta$$
$$\langle E\rangle\langle B_\varphi \rangle \sim \sin^2\theta$$

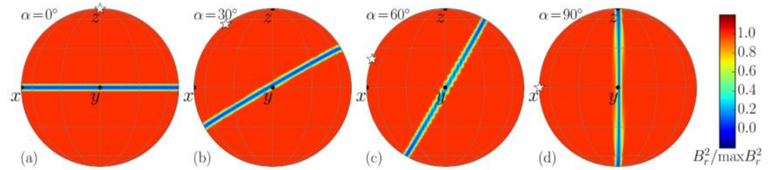
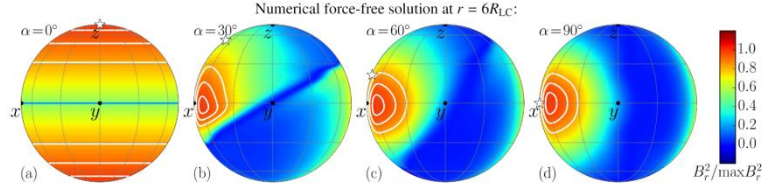


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.



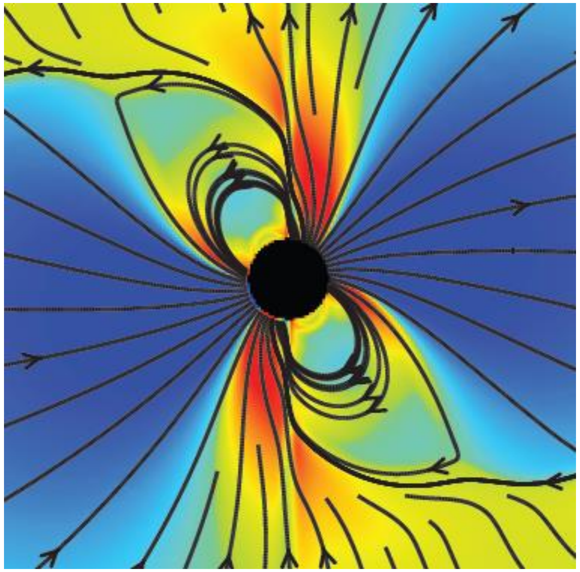
$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

No current sheet for orthogonal rotator!

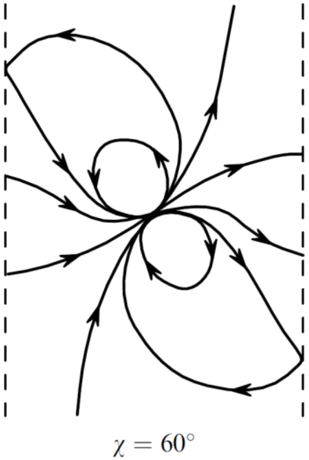
$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

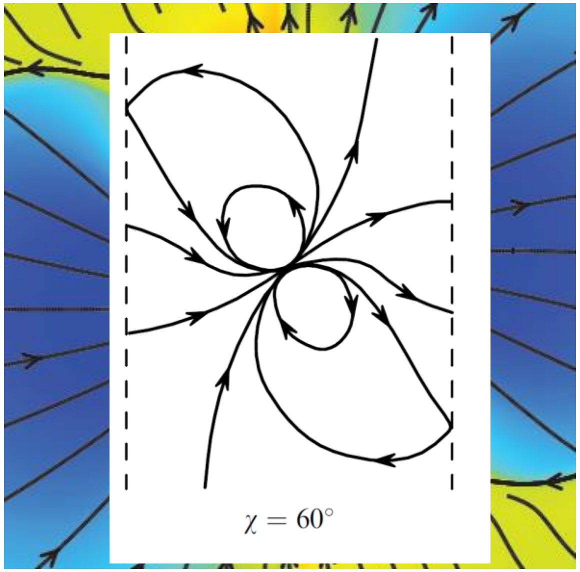
Inclined rotator



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)



Inclined rotator



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)

Polar cap

BGI

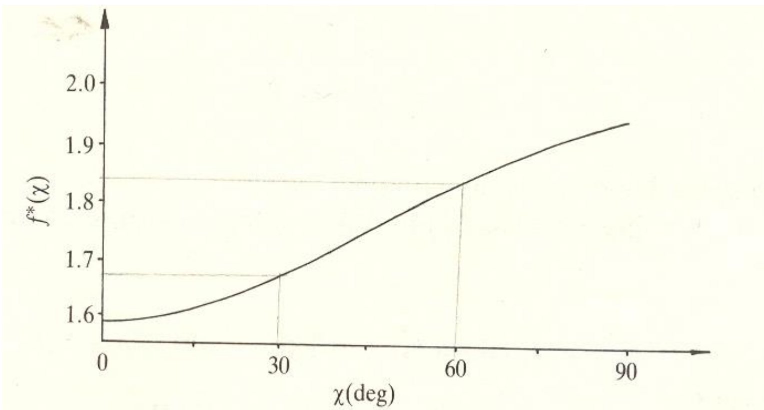
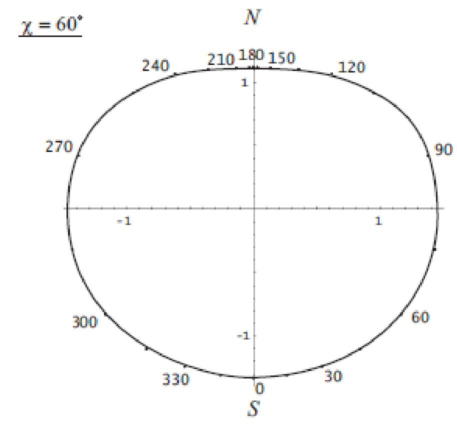


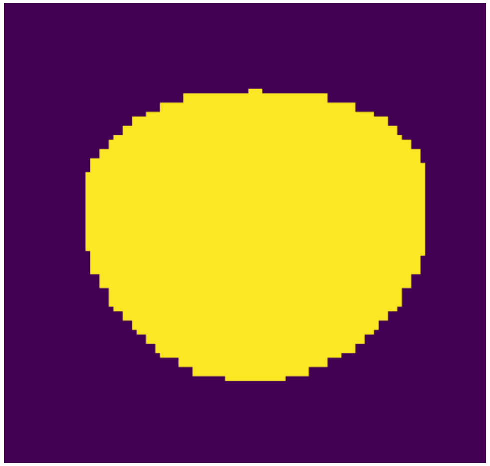
Fig. 4.12. Dependence of the parameter $f^*(\chi)$ on the angle χ.



A.Tchekhovskoy,
A.Spitkovsky, J.Li,
MNRAS, **431**, 1 (2013)

$$s(\chi) = s_0\,(1 + 0.2\,\sin^2\chi)$$

10% precision!



STEP #V

Pulsar braking

Pulsar braking

Current losses

- Direct current losses

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\mathrm{s}},$$
$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\mathrm{a}}.$$

$$i_{\mathrm{A}} \sim (\Omega R/c)^{-1}$$

- 'Second term' (mismatch)

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\mathrm{s}} \, (\mathbf{B}\mathbf{n}) \, \mathrm{d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, \mathrm{d}o$$

- Separatrix currents

How to write down the current

Drift approximation

$$\mathbf{j} = c\,\rho_{\rm e}\frac{[\mathbf{E}\times\mathbf{B}]}{B^2} + a\,\mathbf{B}$$

$$\boxed{\mathbf{j} = \rho_{\rm e}\,[\mathbf{\Omega}\times\mathbf{r}] + i_{||}\,\mathbf{B}}$$

$$\boxed{\mathbf{j} = \frac{(\mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E})\mathbf{B} + (\nabla\cdot\mathbf{E})\mathbf{E}\times\mathbf{B}}{B^2}}$$

$$(\nabla i_{||}\,\mathbf{B}) = 0$$

Mestel, BGI

Gruzinov

Orthogonal rotator – analytically

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$
$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

In the wind

$$i_\parallel = -3\,\frac{\Omega}{c}\cos\theta$$

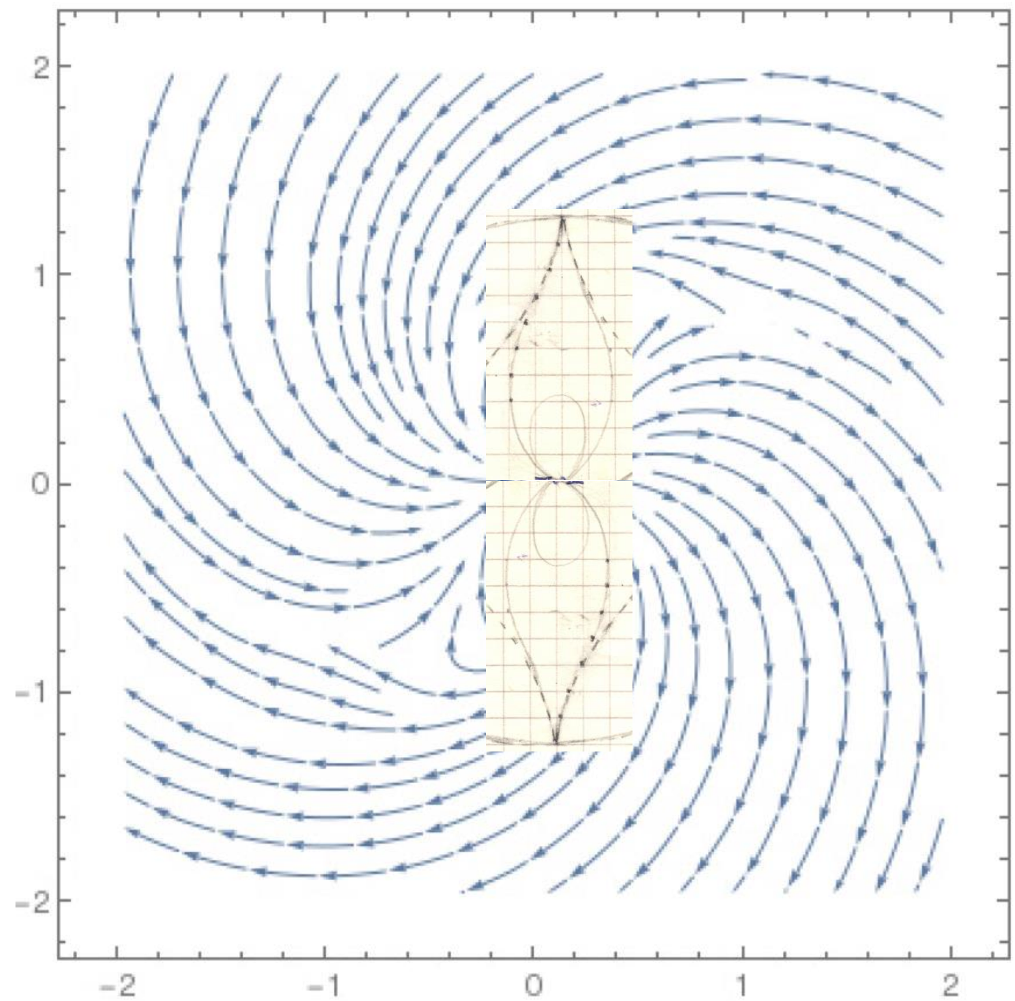
Polar cap

$$i_{\rm a}^{\rm A} \approx f_*^{-1/2}\left(\frac{\Omega R}{c}\right)^{-1/2}$$

TOO LOW!

Mismatch?

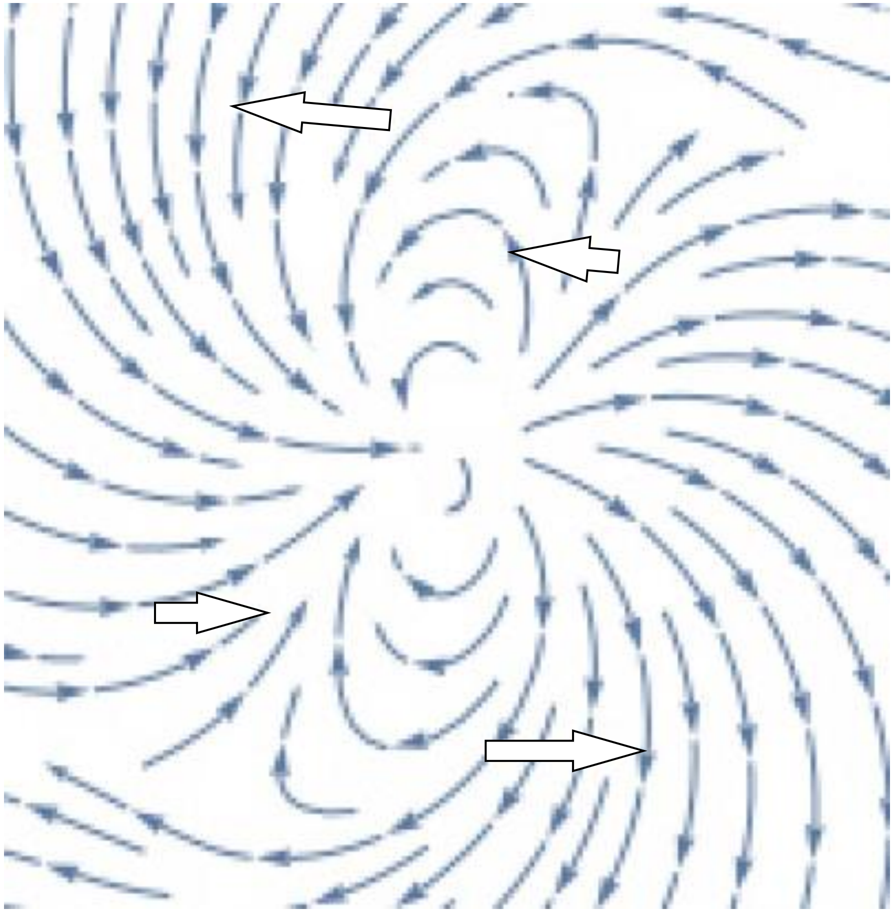
The absence of energy losses for zero longitudinal currents results from exact compensation of central star and magnetospheric losses.
For nonzero currents additional energy losses can appear resulting from mismatch of these two components.



Mismatch?

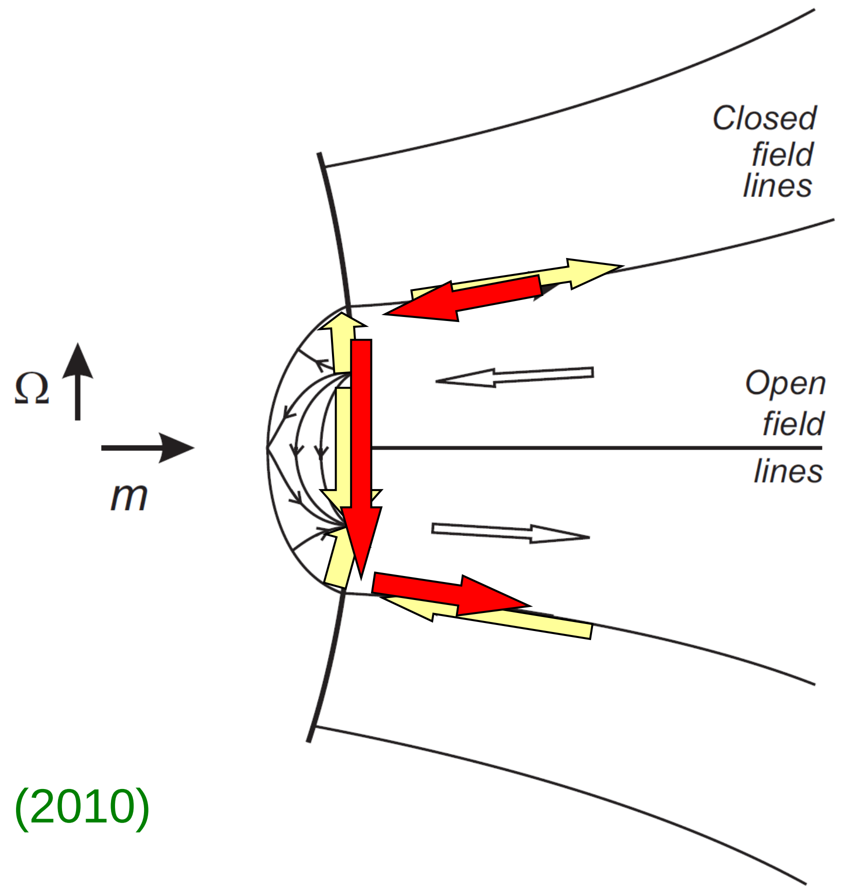
All neutron star surface works

$$\mathbf{S} = \frac{\Omega r}{4\pi}(-B_\varphi \sin\theta\,\mathbf{B} + B^2\mathbf{e}_\varphi)$$



Separatrix current?

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n \mathrm{d}o$$



$I_{sep} = 3/4\ I_{vol}$

X.-N. Bai, A.Spitkovsky ApJ **715**, 1282 (2010)

$I_{sep} = 20\%\ I_{vol}$

STEP #VI

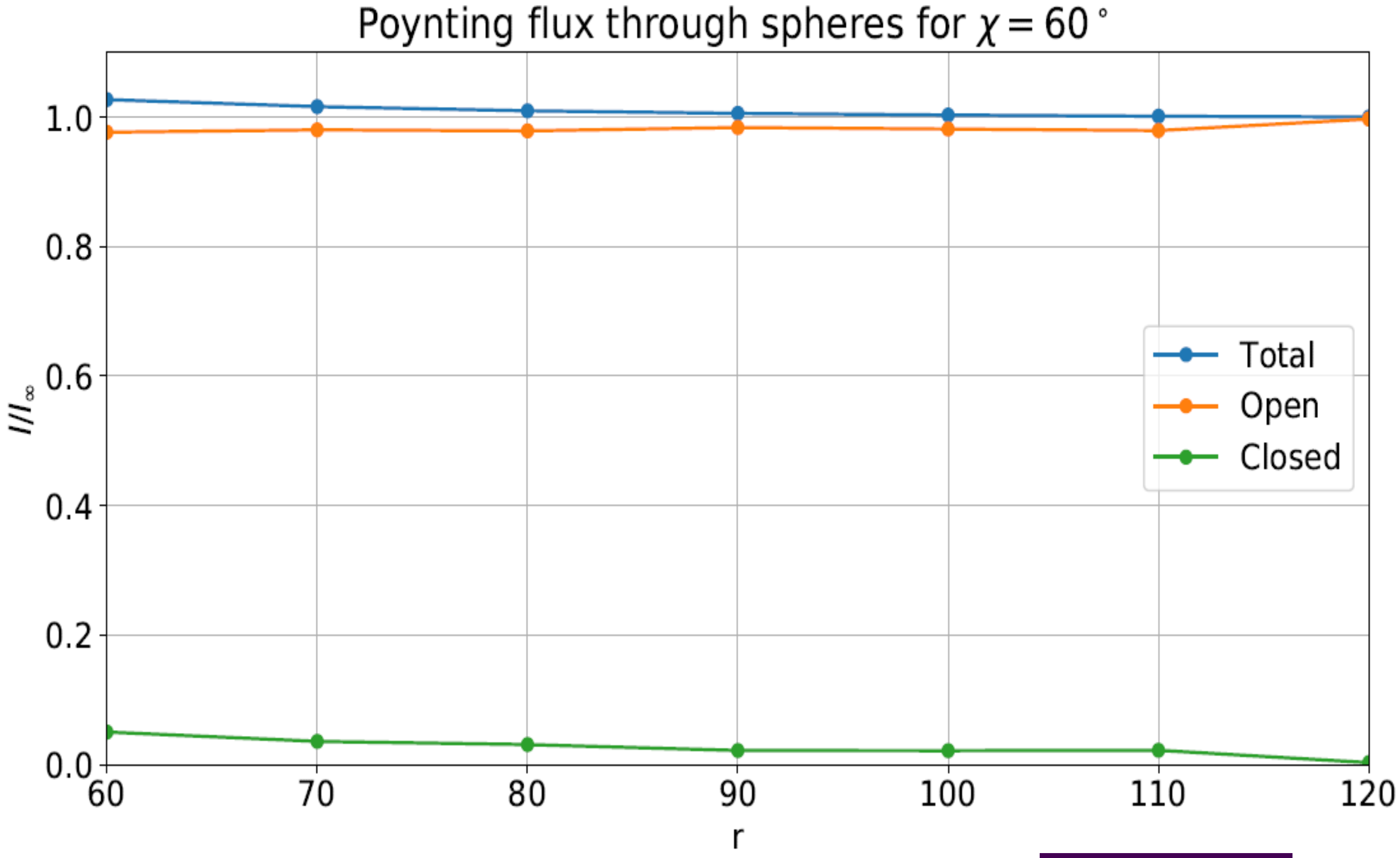
An answer

An answer

What do you prefer?

- Mismatch (all NS surface works).
- Separatrix currents (polar cap only).

An answer


Poynting flux through spheres for $\chi = 60^\circ$
1.0
0.8
0.6
0.4
0.2
0.0
I/I_∞
60
70
80
90
100
110
120
r
Total
Open
Closed

The last remark

Torque

$$
\begin{aligned}
I_{\rm r}\dot{\Omega} &= K_{\parallel}^{A} + [K_{\perp}^{A} - K_{\parallel}^{A}]\sin^2\chi, \\
I_{\rm r}\Omega\dot{\chi} &= [K_{\perp}^{A} - K_{\parallel}^{A}]\sin\chi\cos\chi.
\end{aligned}
$$

$$
K_{\perp}^{\rm mag} = -A\,\frac{B_0^2\Omega^3 R^6}{c^3}\,i_{\rm a}
$$

$$
\boxed{A \approx 2\left(\frac{\Omega R}{c}\right)^{1/2}}
$$

OK for BGI

Conclusion

- To explain energy losses of the 'universal solution', separatrix currents are to be included into consideration.

- In the BGI model ($i_{\mathrm{s}} \approx i_{\mathrm{a}} \approx 1$) the separatrix losses can be neglected, as was proposed.

Hope, it’s all…

THANKS A LOT!