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# Heat blanketing envelopes and neutron stars cooling

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# Outline



**Heat blanketing envelopes and neutron stars cooling**

**Introduction**

**Main part**

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- Part II: Diffusively-equilibrated heat blanketing envelopes of neutron stars
- Part III: Cooling of neutron stars with diffusively-equilibrated envelopes

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# Introduction



Interpretation of observations of isolated neutron stars is a difficult task for a lot of reasons. One of the problems is our poor knowledge of the of chemical composition of the outer neutron star envelopes.

The uncertainties in the chemical composition lead to the uncertainties in our theoretical inference of the internal temperature and, thus, the internal structure of a neutron star.

Studies of the impact of different chemical compositions of heat blanketing envelopes on thermal states and thermal evolution of isolated neutron stars were conducted in the past.

However, such studies often relied on “onion”-like models of the envelopes.

Our investigation is focused on proper treatment of the diffusion in the heat blanketing envelopes of neutron stars.

Heat blanketing envelopes contain non-ideal Coulomb plasmas. Diffusion in such plasmas requires special consideration due to specific “Coulomb” effects.

The case of isothermal Coulomb plasma have been studied previously. Now we focus on the non-isothermal case which is required to investigate heat blanketing envelopes.

Then we construct models of diffusively-equilibrated and non-equilibrated heat blanketing envelopes composed of binary ionic mixtures and investigate the effects of chemical composition on thermal states and thermal evolution of isolated neutron stars.

# Part I: Diffusion in non-isothermal dense stellar plasmas

## I.1 Multicomponent Coulomb plasma

Consider multicomponent plasma, which consist of several fully ionized ion species ($\alpha = j$, $j = 1, 2, \ldots$) and neutralizing electron ($\alpha$ = e) background.

Generalized thermodynamic forces in the presence of temperature gradient can be written as

$$\widetilde{\boldsymbol{f}}_\alpha = \boldsymbol{f}_\alpha - \left( \boldsymbol{\nabla} \mu_\alpha - \left. \frac{\partial \mu_\alpha}{\partial T} \right|_P \boldsymbol{\nabla} T \right)$$

In application to heat blanketing envelopes of neutron stars we have ($Z_e = -1$):

$$\boldsymbol{f}_\alpha = Z_\alpha e \boldsymbol{E} + m_\alpha \boldsymbol{g}$$

Deviations from diffusion equilibrium are characterized by

$$\boldsymbol{d}_\alpha = \frac{\rho_\alpha}{\rho}\sum_\beta n_\beta \widetilde{\boldsymbol{f}}_\beta - n_\alpha \widetilde{\boldsymbol{f}}_\alpha, \quad \sum_\alpha \boldsymbol{d}_\alpha = 0$$

Using Gibbs-Duhem relation $\sum_\alpha n_\alpha \nabla \mu_\alpha = \nabla P - S \nabla T$ one obtains

$$\sum_\alpha n_\alpha \widetilde{\boldsymbol{f}}_\alpha = \rho \boldsymbol{g} - \boldsymbol{\nabla} P$$

Taking into account that the envelope as whole is in hydrostatic equilibrium we can simplify $\boldsymbol{d}_\alpha$:

$$\boldsymbol{d}_\alpha = -n_\alpha \widetilde{\boldsymbol{f}}_\alpha$$

Thus, for the envelopes of neutron stars we have:

$$\boldsymbol{d}_\alpha = -\frac{\rho_\alpha}{\rho}\boldsymbol{\nabla} P - Z_\alpha n_\alpha e\boldsymbol{E} + n_\alpha \left( \boldsymbol{\nabla}\mu_\alpha - \left.\frac{\partial \mu_\alpha}{\partial T}\right|_P \boldsymbol{\nabla} T \right)$$

Using adiabatic (Born-Oppenheimer) approximation we can factor electrons out of the problem of ion transport: $\boldsymbol{d}_\mathrm{e} = 0$ (mechanical quasi-equilibrium of the electrons in response to the motion of the ion subsystem). Assuming also $m_\mathrm{e} \to 0$, we obtain $\tilde{\boldsymbol{f}}_\mathrm{e} = 0$ and

$$e\boldsymbol{E} = -\left( \boldsymbol{\nabla}\mu_\mathrm{e} - \left.\frac{\partial \mu_\mathrm{e}}{\partial T}\right|_P \boldsymbol{\nabla} T \right)$$

The final phenomenological expression for the diffusive fluxes is:

$$\boxed{\boldsymbol{J}_\alpha = \frac{n m_\alpha}{\rho k_{\mathrm{B}} T} \sum_{\beta \neq \alpha} m_\beta D_{\alpha\beta} \boldsymbol{d}_\beta - D_\alpha^T \frac{\boldsymbol{\nabla} T}{T}}$$

# Part II: Diffusively-equilibrated heat blanketing envelopes of neutron stars

## II.1 Building diffusively-equilibrated envelopes

Additional assumptions

- Electrons have little impact on the transport of ions → $\boldsymbol{J}_{\mathrm{e}} = 0$ and $\boldsymbol{J}_1 = -\boldsymbol{J}_2$;
- Thermal diffusion is small compared to ordinary diffusion.

Then we can simplify the general expression for the diffusive flux:

$$\boldsymbol{J}_2 = -\boldsymbol{J}_1 = \frac{n m_1 m_2}{\rho k_{\mathrm{B}} T} D_{12} \boldsymbol{d}_1$$

Diffusion equilibrium means $\boldsymbol{J}_2 = -\boldsymbol{J}_1 = 0$. Thus, $\boldsymbol{d}_1 = 0$ and, taking into account $\boldsymbol{d}_\mathrm{e} = 0$, one obtains $\boldsymbol{d}_2 = 0$.

So, diffusion equilibrium conditions are $\boxed{\boldsymbol{d}_1 = \boldsymbol{d}_2 = \boldsymbol{d}_\mathrm{e} = 0}$

Or, writing ***d*** explicitly:

$$\widetilde{\boldsymbol{\nabla}}\mu_e = -e\boldsymbol{E}, \quad \widetilde{\boldsymbol{\nabla}}\mu_j = m_j\boldsymbol{g} + Z_j e\boldsymbol{E}$$

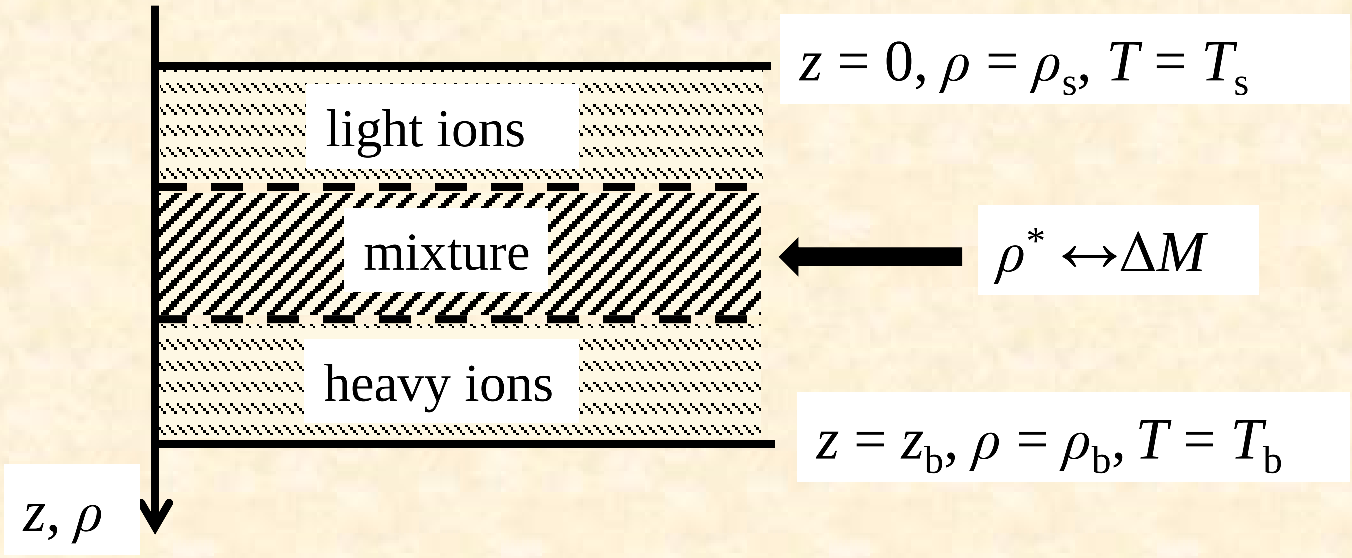
where

$$\widetilde{\boldsymbol{\nabla}}\mu_\alpha \equiv \sum_j \frac{\partial\mu_\alpha}{\partial n_j}\boldsymbol{\nabla}n_j + \frac{\partial P}{\partial T}\left(\sum_j n_j\frac{\partial\mu_\alpha}{\partial n_j}\right)\left(\sum_k n_k\frac{\partial P}{\partial n_k}\right)^{-1}\boldsymbol{\nabla}T$$

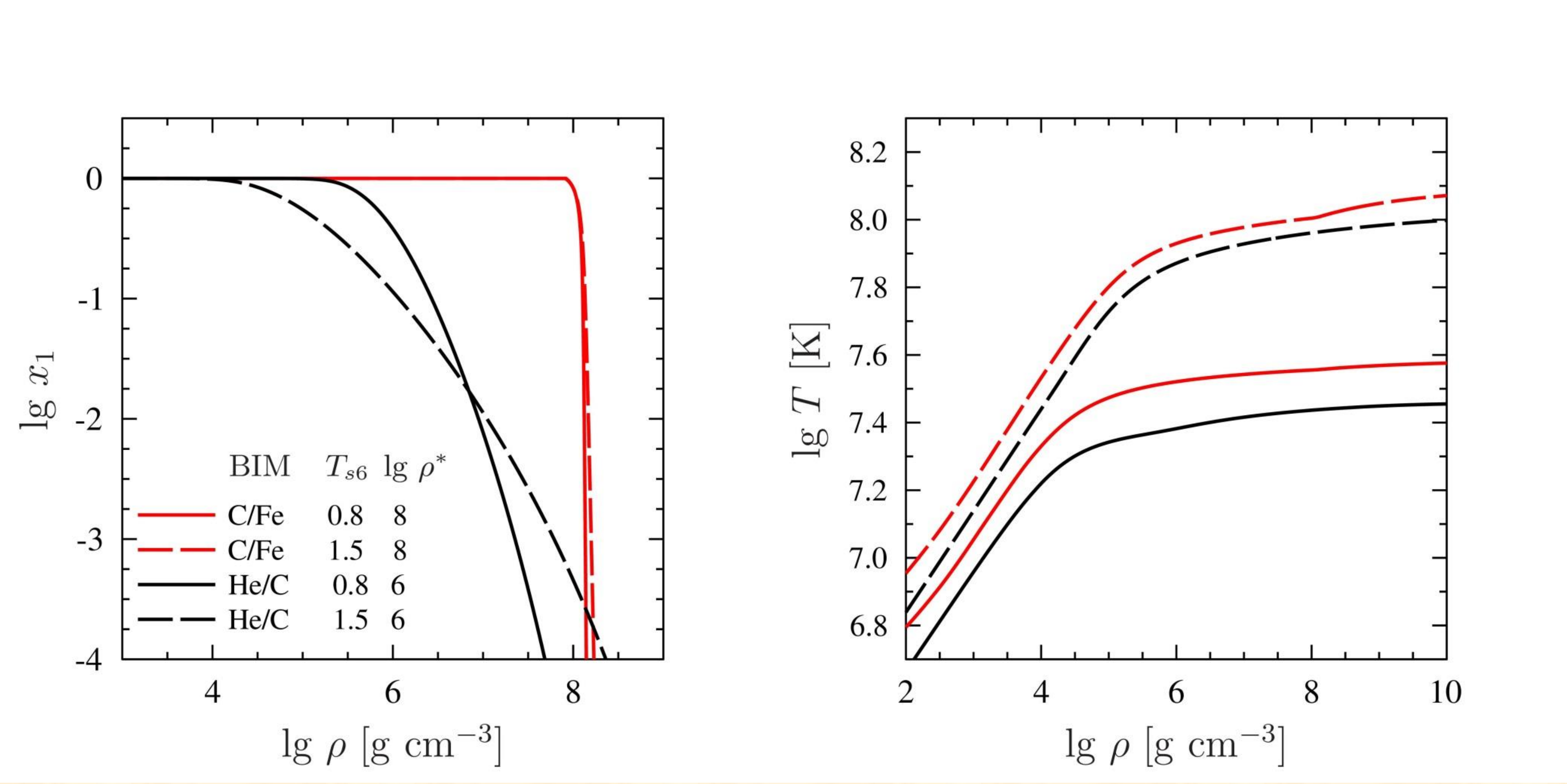
# II.2 Models of the envelopes and their parameters

We have considered heat blanketing envelopes composed of the following binary ionic mixture: $^{1}$H – $^{4}$He, $^{4}$He – $^{12}$C or $^{12}$C – $^{56}$Fe. In all calculations we have used a model of a "canonical" neutron star ($M = 1.4\ \mathrm{M}_{\odot}$ and $R = 10$ km).

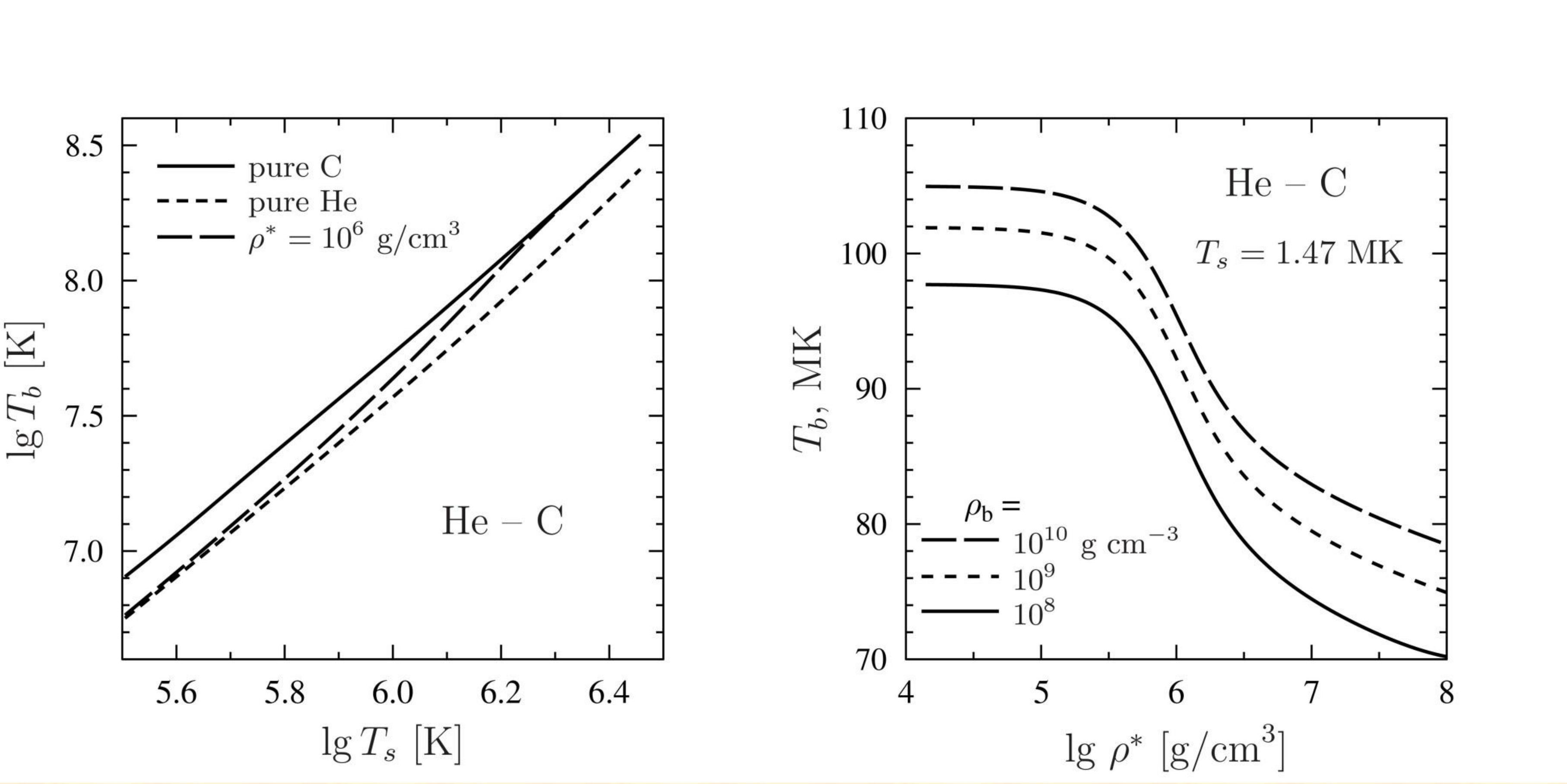
One of the expected results: the stratification of the elements



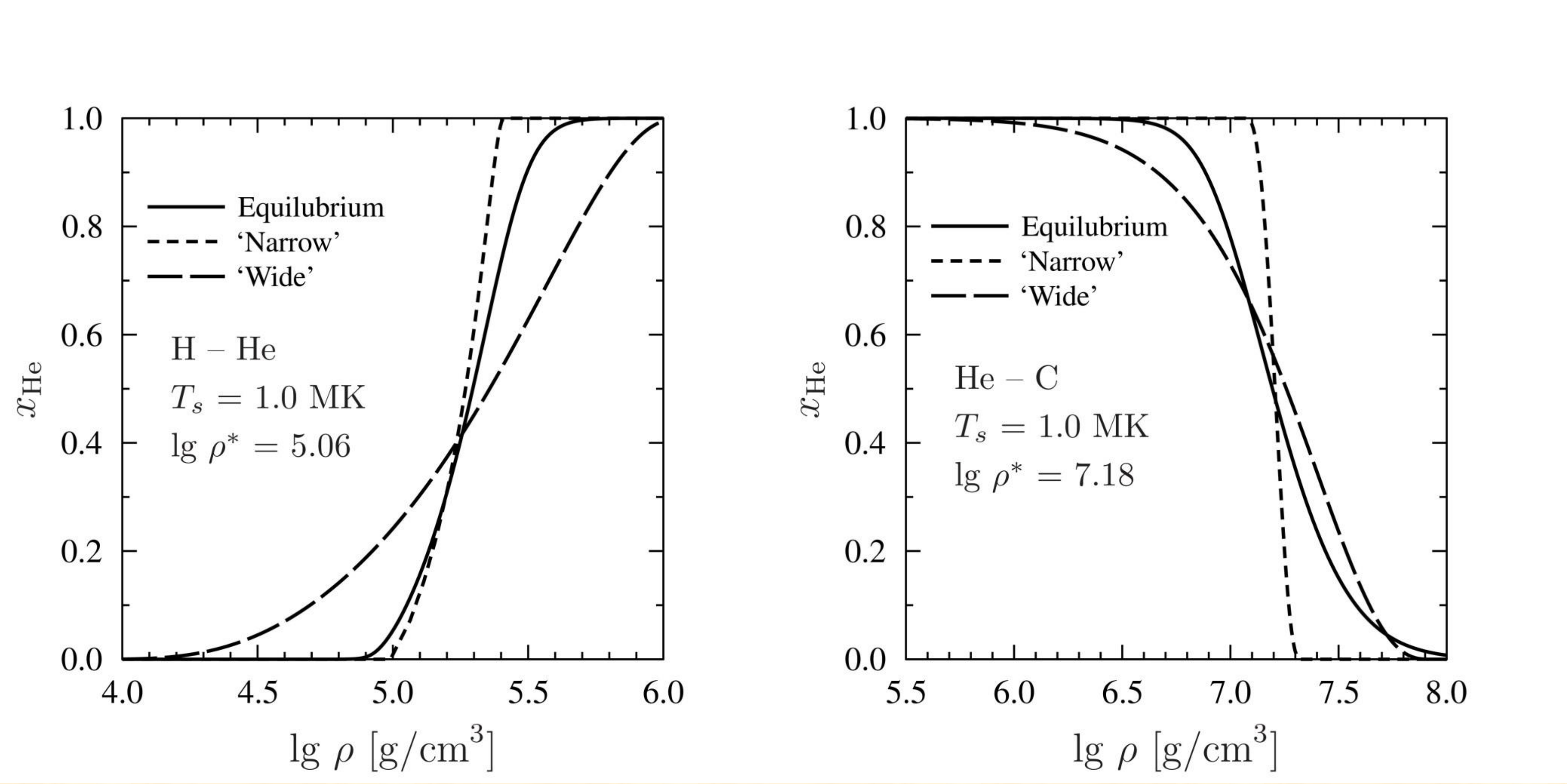
# II.3 Examples of calculations



# Example of $T_s - T_b$ relation (left) and the dependence of $T_b$ on $\rho^*$ (right)



# Diffusively non-equilibrated envelopes



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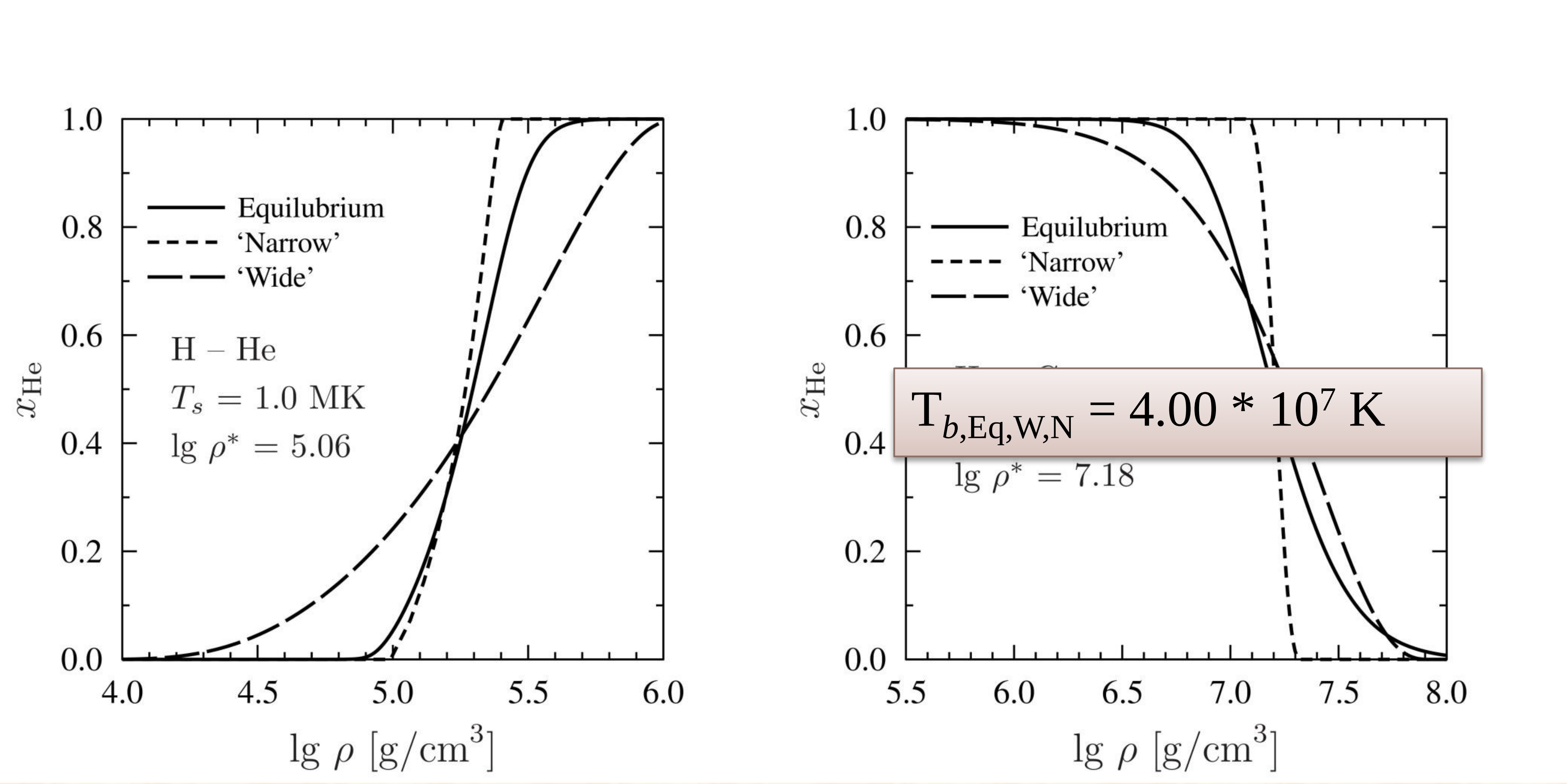
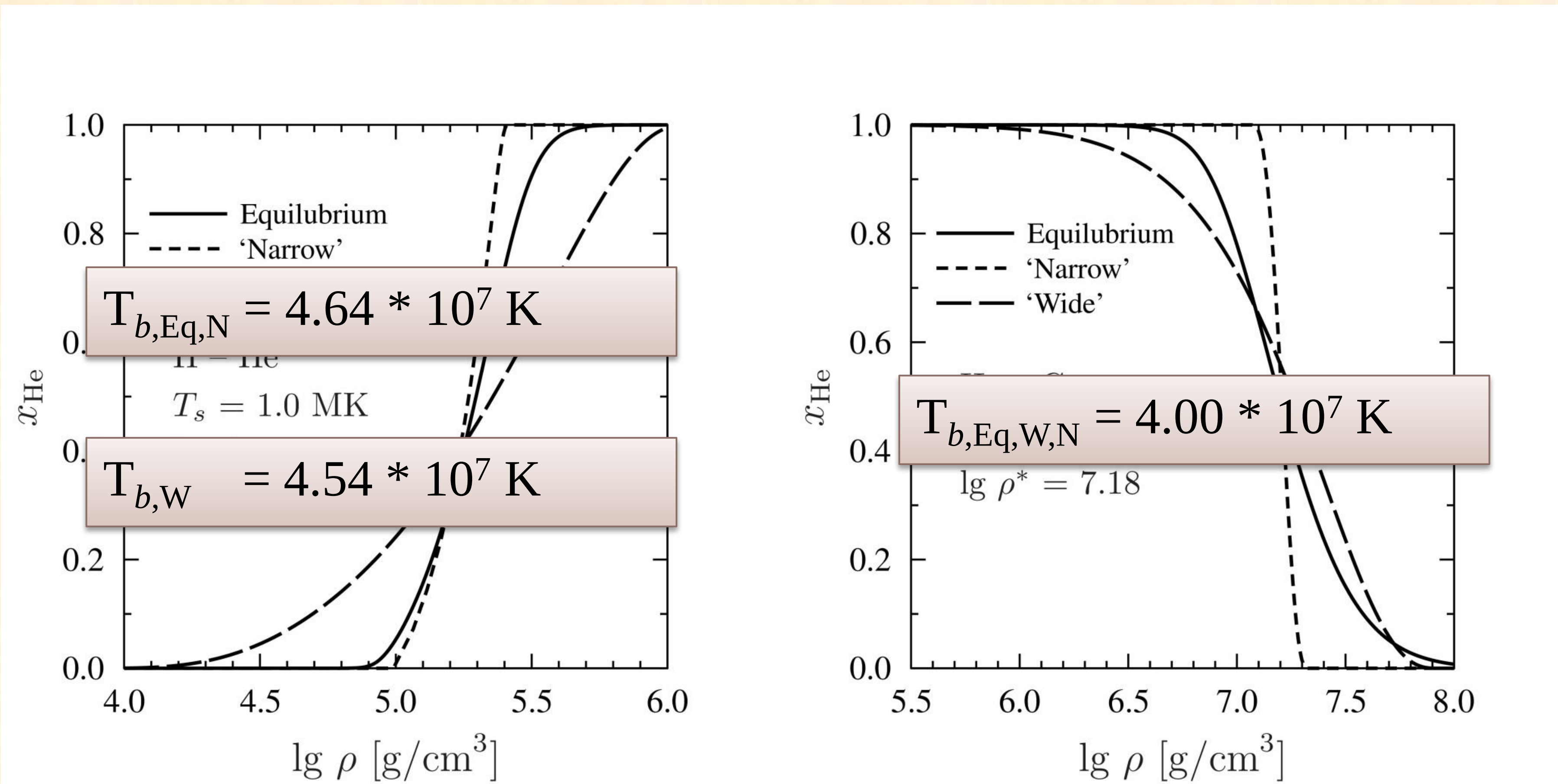



$T_{b,\mathrm{Eq,W,N}} = 4.00 * 10^7$ K

# Diffusively non-equilibrated envelopes




$T_{b,\mathrm{Eq,N}} = 4.64 * 10^7$ K

$T_{b,\mathrm{W}} = 4.54 * 10^7$ K

$T_{b,\mathrm{Eq,W,N}} = 4.00 * 10^7$ K

## Timescale of diffusive equilibration of the envelope

$t_{eq} \sim \Delta z / V_{diff}$

| Mixture | $\Delta z$ | $V_{diff}$, cm/s | $t_{eq}$ |
|---|---|---|---|
| H – He | ~ meters | $\sim 10^{-4} - 10^{-3}$ | ~ days |
| He – C | ~ meters | $\sim 10^{-7} - 10^{-6}$ | ~ years |

## Thermal diffusion contribution to the diffusion velocity

Assume a conservative upper bound on thermal diffusion ratio $k_T = 0.1$.

- H – He, $x_H = x_{He} = 0.5$, thermal diffusion contribution is < 3%;
- He – C, $x_{He} = x_C = 0.5$, thermal diffusion contribution is < 6%.

# Part III: Cooling of neutron stars with diffusively-equilibrated envelopes

## III.1 Formation scenarios

Initially, it was thought that the envelopes (as well as neutron star atmospheres) contain heavy elements (like iron), as a result of the formation of the envelope in a hot and very young star where light elements burnt-out into heavier ones.

However, a more detailed analysis of the observed spectra of the thermal radiation has shown that some spectra are better described by hydrogen atmosphere models and some other – by carbon atmosphere models.

This gives observational confirmations that neutron stars' atmospheres and envelopes can contain different light elements.

## III.2 Thermal states of the Vela pulsar

We know from observations that the Vela pulsar has:

- $t \approx 11$ kyr (characteristical age)
- $T_s^\infty = 0.68 \pm 0.03\,\mathrm{MK}\ (1\sigma) \to T_s = T_s^\infty \big/ \sqrt{1-x_g} = 0.89\,\mathrm{MK}$

(assuming $M = 1.4\ \mathrm{M}_\odot$, $R = 10$ km)

Using the approximations for $T_s - T_b$ relations from part II one can infer the internal temperature $T_b$ and $\tilde{T} = T_b\sqrt{1-x_g}$ as a function of the chemical composition of the Vela's envelope

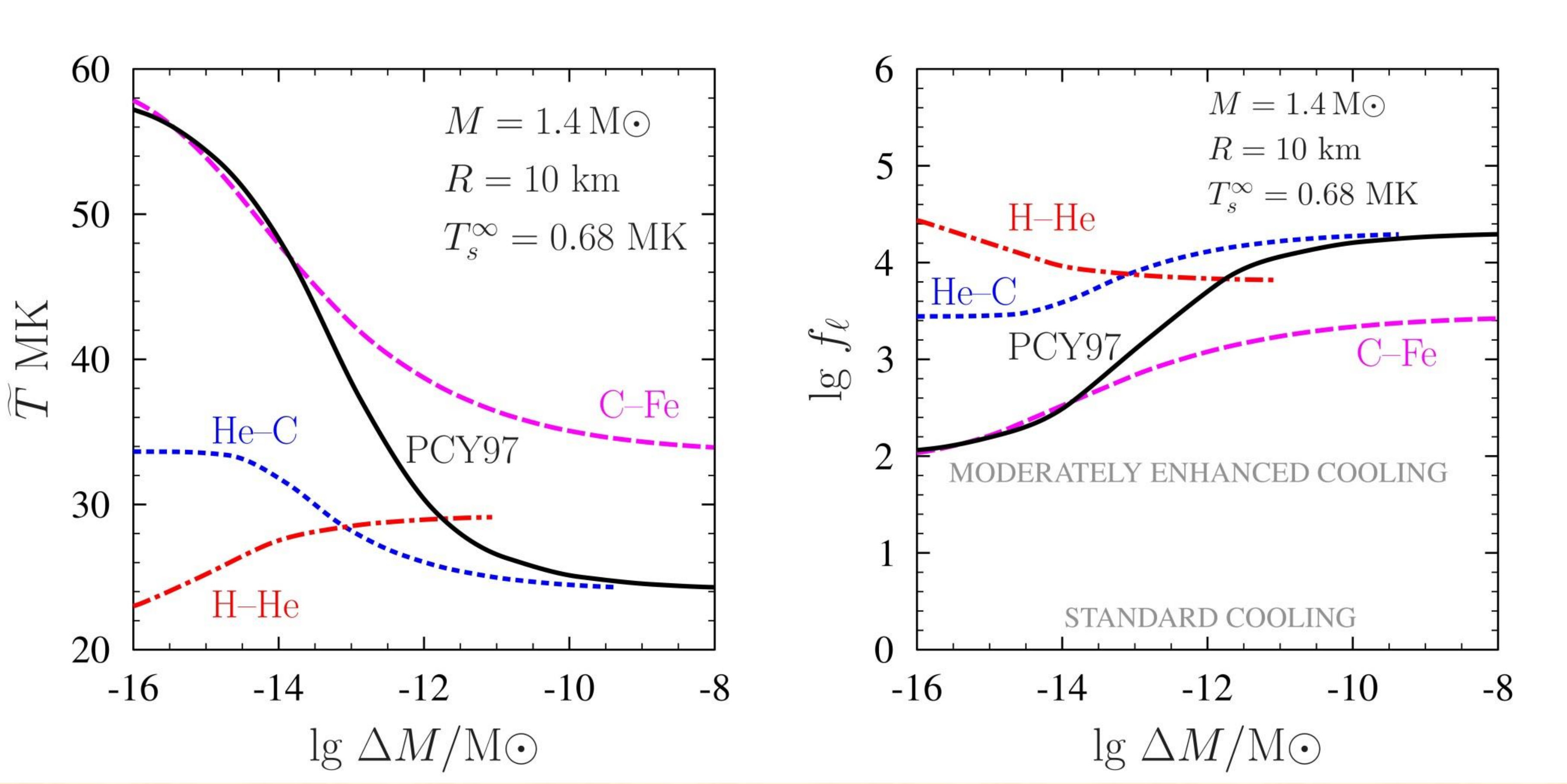
We also define neutrino cooling function

$$\ell(\widetilde{T}) = L_\nu^\infty(\widetilde{T})/C(\widetilde{T})$$

and express it in the units of "standard candles":

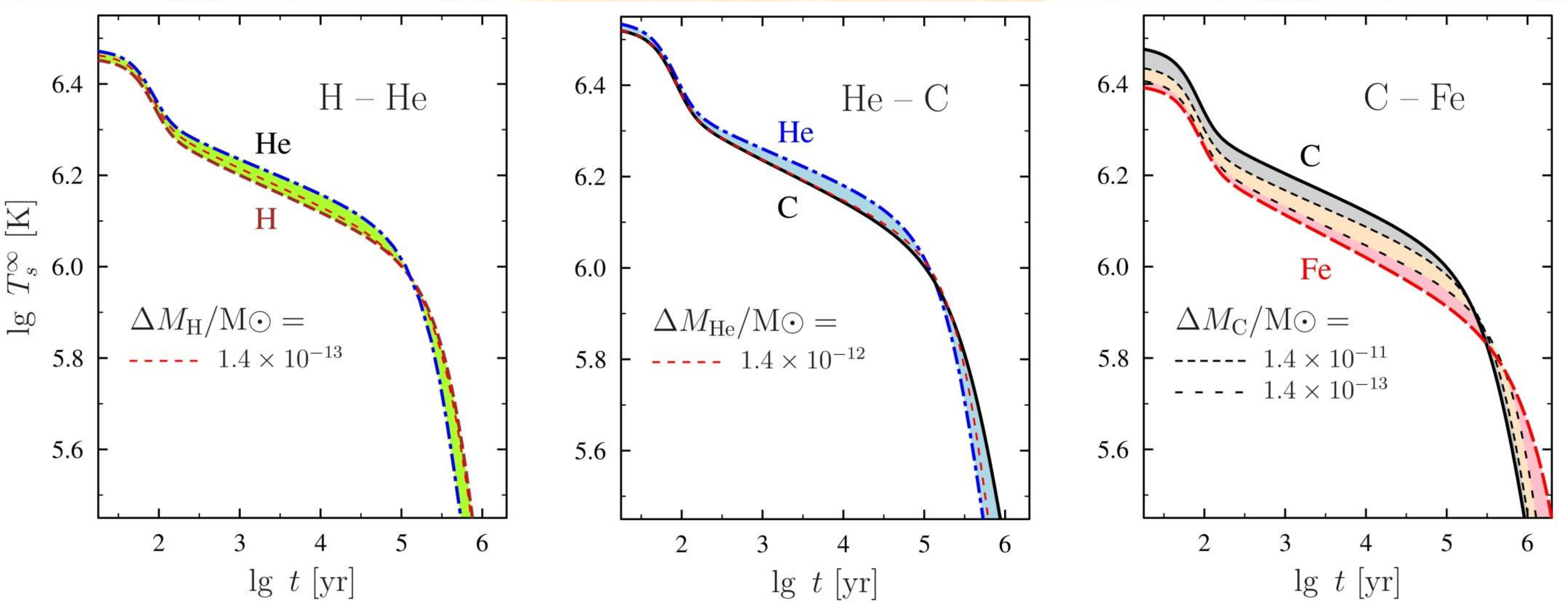
$$f_\ell = \ell(\widetilde{T})/\ell(\widetilde{T})_{\mathrm{SC}}$$

The dependence of the internal temperature (left) and neutrino cooling function (right) on the chemical composition of the envelope of the Vela pulsar.



# III.3 Examples of the cooling of isolated neutron stars

Consider the cooling of an isolated neutron star with a mass $M = 1.4\ \mathrm{M}_{\odot}$ and BSk21 EOS. We assume that the star is cooling as a "standard candle" ($f_{\ell} = 1$).



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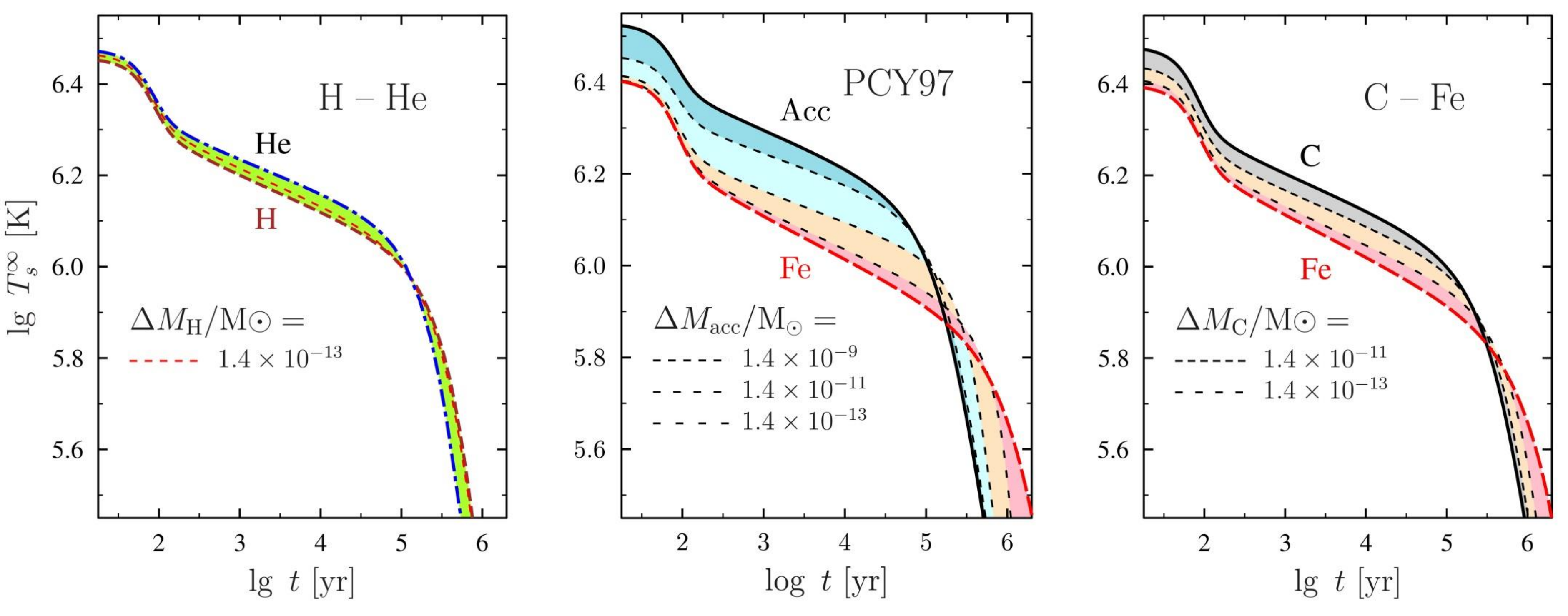

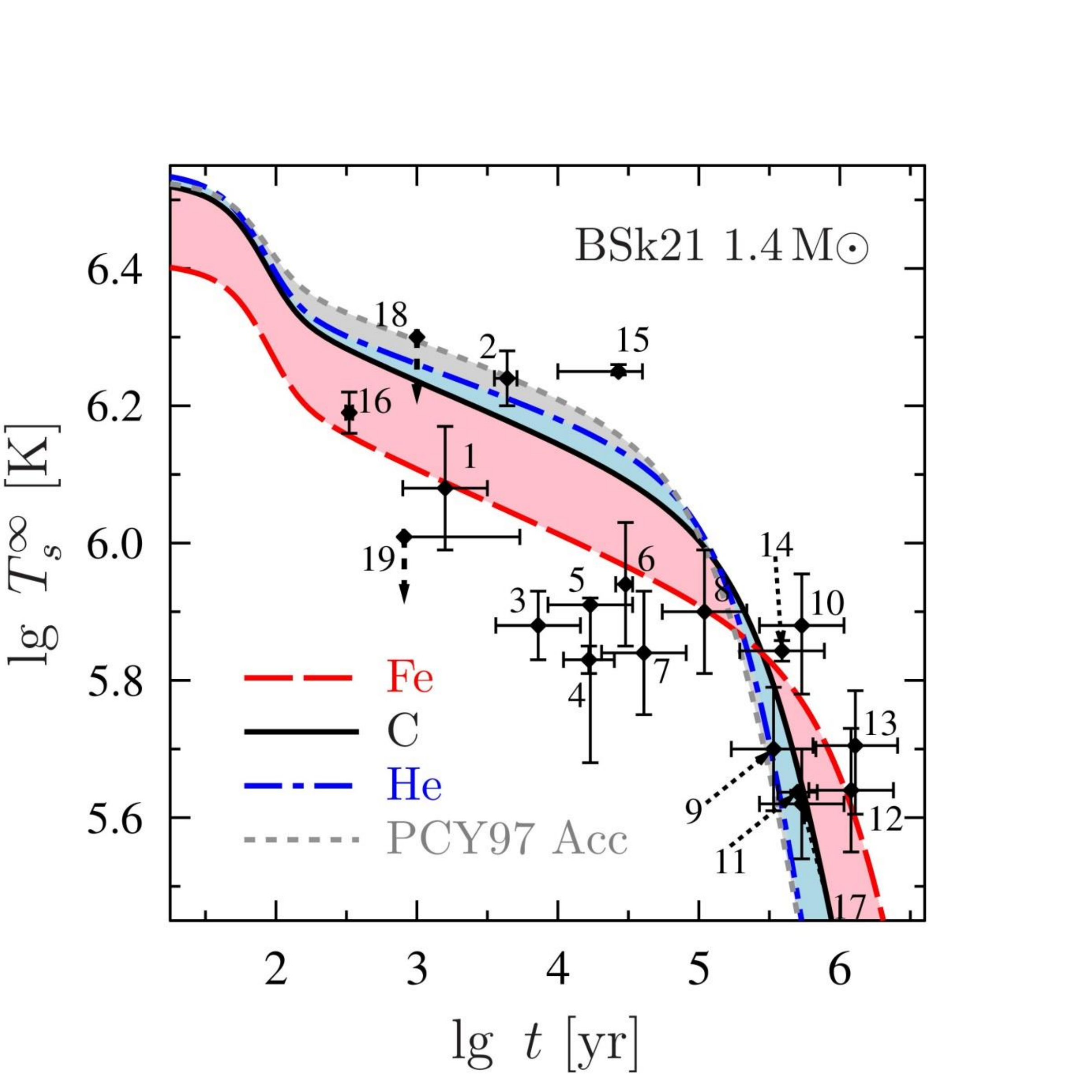
BSk21 1.4 M⊙
lg $T_s^\infty$ [K]
lg $t$ [yr]
Fe
C
He
PCY97 Acc
6.4
6.2
6.0
5.8
5.6
2
3
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# Conclusions

- We have derived general expressions for the diffusive fluxes in non-isothermal multicomponent gaseous or liquid Coulomb systems of ions with arbitrary Coulomb coupling. These expressions are also valid for any gaseous or liquid systems.
- Using these expressions we have constructed models of diffusively-equilibrated heat blanketing envelopes of neutron stars and calculated $T_\mathrm{s} - T_\mathrm{b}$ relations.
- We have confirmed previous results about stratification of elements in the envelopes and about the differences between gravitational and Coulomb separation mechanisms.
- Most significant result of this part is that $T_\mathrm{s} - T_\mathrm{b}$ relations are remarkably insensitive to the structure of the transition layer (its width, distribution of ions and of whether it is diffusively equilibrated or not). These relations depend only on $\Delta M$ ($\leftrightarrow \rho^*$).

## Conclusion

- This insensitivity to the structure of the transition layer justifies previous studies based on “onion”-like models of the envelopes.
- For practical use we have fitted all calculated $T_b(T_s, \rho^*)$ relations with simple analytical expressions.
- Real envelopes can contain more than two ion species but stratification of elements will prevent the formation of essentially multicomponent regions.
- Using general expressions for the diffusive fluxes (along with diffusion and thermal diffusion coefficients) one can not only calculate the diffusively-equilibrated configurations of the envelopes, but also the equilibration of these configurations with time.

## Conclusion



We have demonstrated that uncertainties in the chemical composition of the heat blanketing envelopes of neutron stars can have a considerable impact on the inference of neutron stars' properties from observational data:

- For a fixed surface temperature internal temperature can vary up to ~2.5 times.
- Neutrino cooling function can vary up to ~200 times.
- Cooling curves are also noticeably affected by the changes in the chemical composition.

Other results

- We have shown that in the "minimal cooling" paradigm the Vela pulsar should have an envelope predominantly made of iron.
- Observed neutron stars which have recently entered photon cooling stage are compatible with $f_\ell \lesssim 1$ (*however, other explanations possible*).


Thank you for your attention!
Altitude = 61 m

Ratio of light ions to heavy ions in the envelope is determined by a parameter $\Delta M$ – accumulated (gravitational) mass of light ions. But it is more convenient to determine the location of the transition layer between light and heavy ions by the effective transition density $\rho^*$. This density is determined from the following relation:

$$\frac{\Delta M}{M} = \frac{0.838}{g_{s14}^2} \frac{P^*}{10^{34}\ \mathrm{dyn\ cm}^{-2}} =$$
$$= \frac{1.510 \times 10^{-11}}{g_{s14}^2} \left\{ \xi(\rho^*)\sqrt{1+\xi(\rho^*)^2}\left[\frac{2}{3}\,\xi(\rho^*)^2 - 1\right] + \ln\left[\xi(\rho^*) + \sqrt{1+\xi(\rho^*)^2}\right] \right\}$$

where

$$\xi(\rho) = 0.01009\,(\rho Z/A)^{1/3}$$

## Restrictions on the envelope parameters

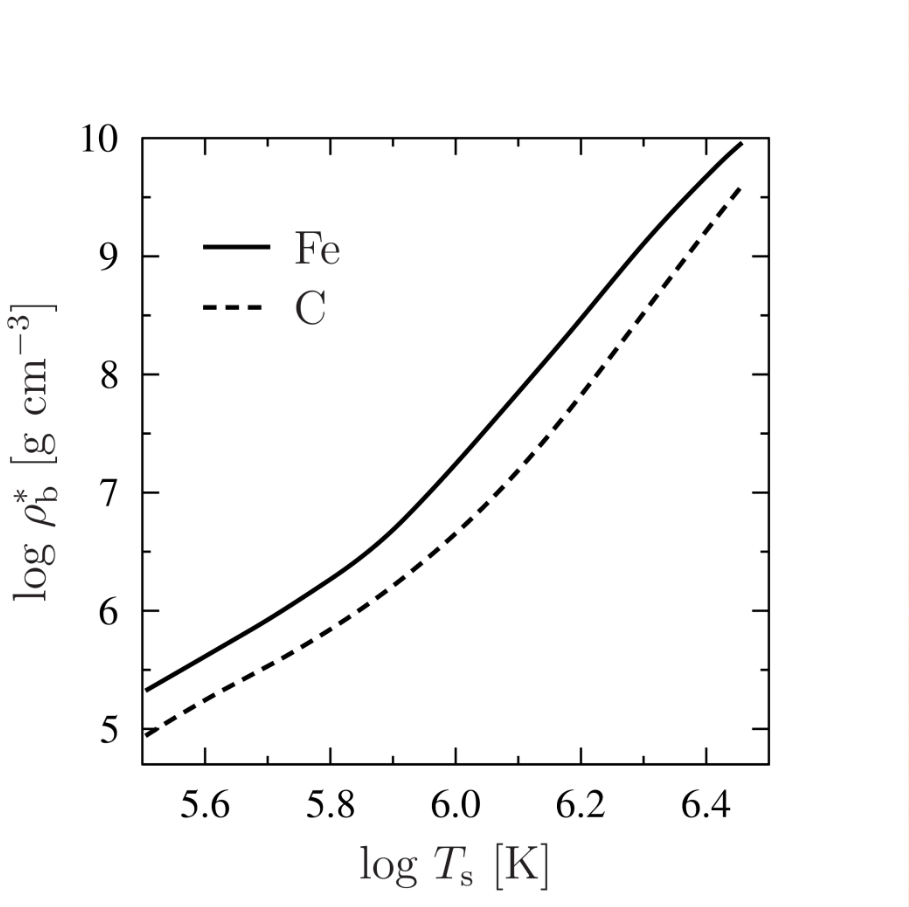
Range of possible values of $\Delta M$ ($\leftrightarrow \rho^*$) is limited from below by the mass of the atmosphere and from above – by the nuclear reactions.

Different elements have different temperatures and densities of nuclear burning. Thus, different mixtures have different restrictions on $\Delta M$. A very rough estimate gives:

- $\lg(\Delta M/M) > -19$ – $-17$ – the mass of the atmosphere
- $\lg(\Delta M/M) < -12$ – for H – He mixture $\leftrightarrow \rho^* < 10^6$ g cm$^{-3}$; $T_{\mathrm{H}} < 4*10^7$ K
- $\lg(\Delta M/M) < -10$ – for He – C mixture $\leftrightarrow \rho^* < 10^8$ g cm$^{-3}$; $T_{\mathrm{He}} < 10^8$ K
- $\lg(\Delta M/M) < -8$ – for C – Fe mixture $\leftrightarrow \rho^* < 10^9$ g cm$^{-3}$; $T_{\mathrm{C}} < 10^9$ K

Another obvious condition to satisfy is $\rho^* < \rho_{\mathrm{b}}$.

## Choice of $\rho_b$ – the density of the bottom of the envelope

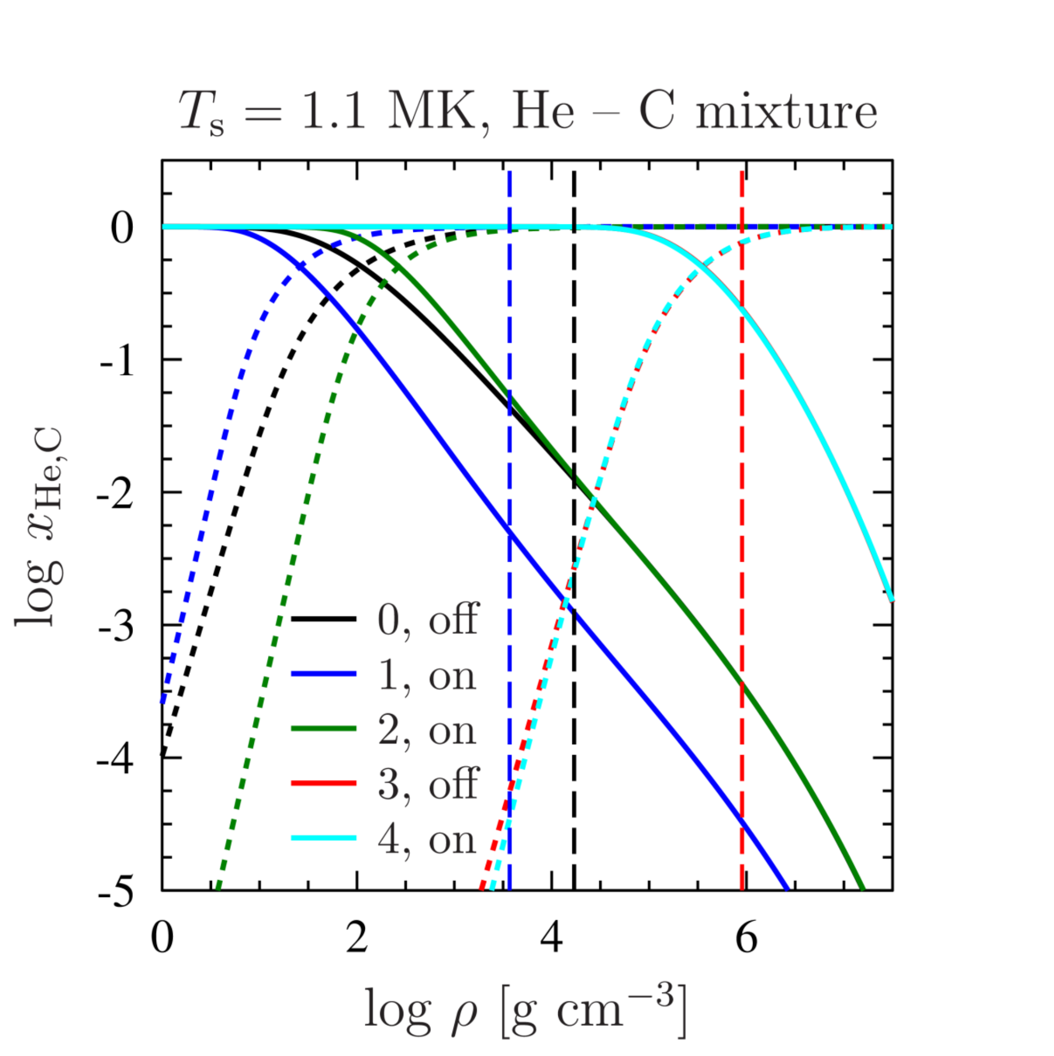



Usually $\rho_b$ is chosen in such a way that at $\rho > \rho_b$, temperature gradient is negligible

Timescale for the heat to diffuse through the envelope (pure iron, $T_s$ = 1MK):

| $\boldsymbol{\rho_b}$ | $10^8$ | $10^{10}$ | $4{\cdot}10^{11}$ |
|---|---|---|---|
| $\boldsymbol{t_d}$ | ~ day | ~ yr. | ~ 10 yrs. |

About the importance of the term with the temperature gradient.



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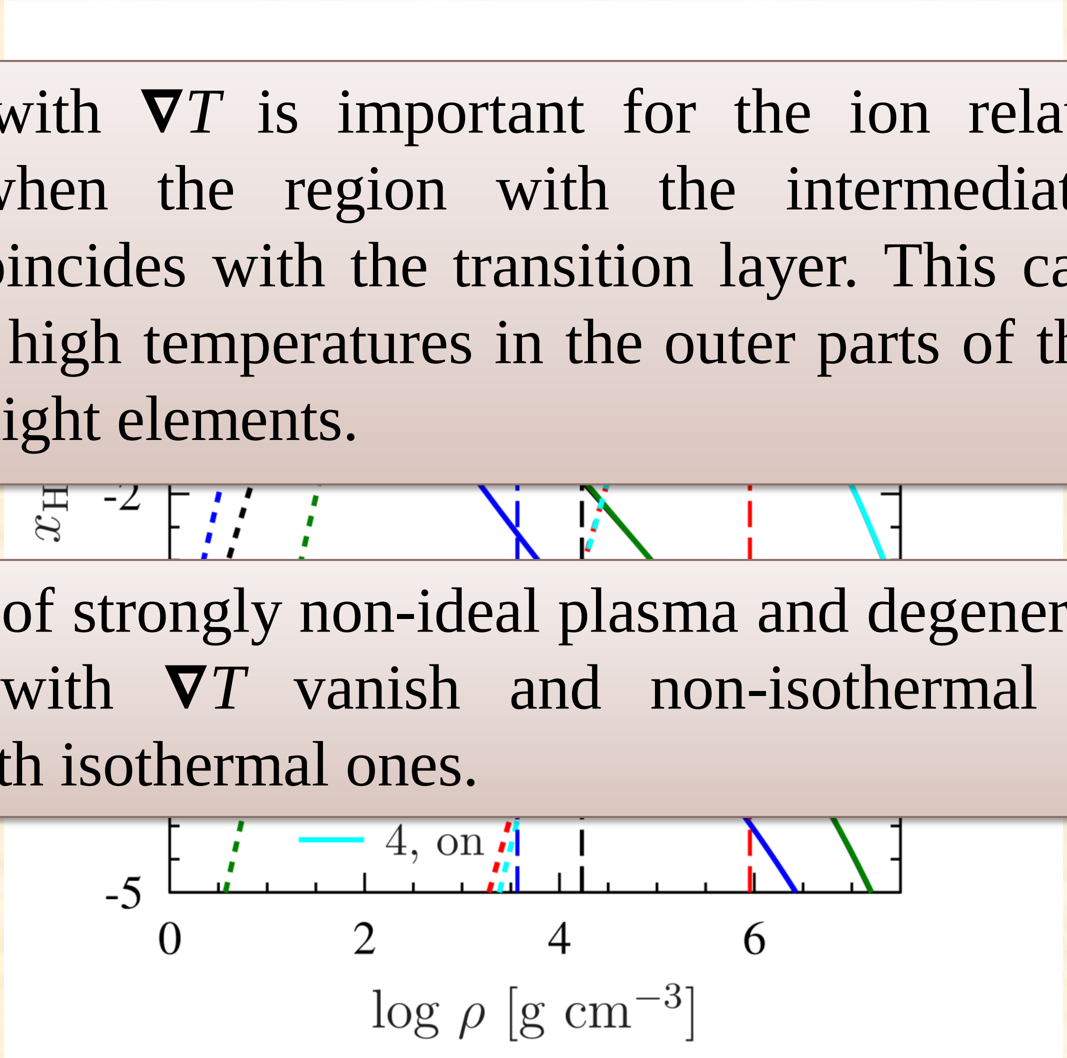


The term with $\boldsymbol{\nabla}T$ is important for the ion relative number densities when the region with the intermediate Coulomb coupling coincides with the transition layer. This can happen at sufficiently high temperatures in the outer parts of the envelopes containing light elements.

In the limit of strongly non-ideal plasma and degenerate electrons all terms with $\boldsymbol{\nabla}T$ vanish and non-isothermal calculations coincide with isothermal ones.

# "Evolutionary tracks" for the Vela pulsar