

# The apparent decay of pulsar magnetic fields

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# Outline

- 1 Estimating pulsar magnetic fields ( $B$ )
- 2 Estimating pulsar ages ( $t$ )
- 3 Analyzing the apparent  $B(t)$
- 4 Observational selection and the intrinsic  $B(t)$
- 5 Interpretation and conclusions

# Estimating the magnetic fields

- Spin-down of a spherical isolated NS:

$$P \dot{P} = \frac{4\pi^2 R^6}{I c^3} \cdot B^2 \cdot (k_0 + k_1 \sin^2 \alpha)$$

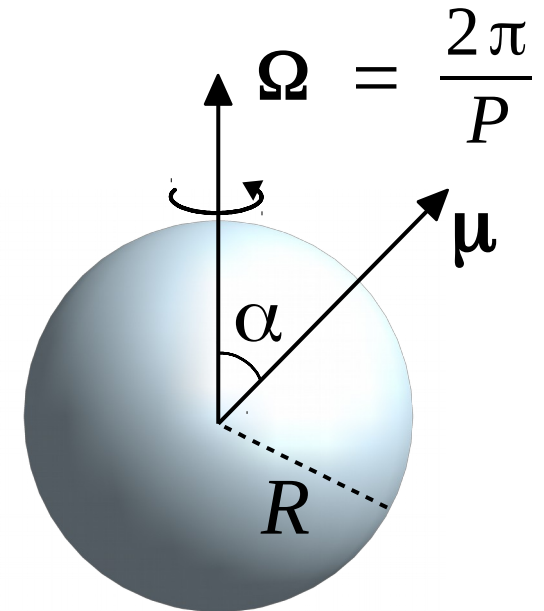
- The magnetic field strength (at the magnetic equator):

$$B = \sqrt{\frac{c^3}{4\pi^2}} \times \sqrt{\frac{I(M)}{R^3(M)}} \times \sqrt{\frac{P \dot{P}}{k_0 + k_1 \sin^2 \alpha}}$$

**equation of state**

**mass**

**magnetic obliquity**



# Estimating the magnetic fields

The logarithm of the magnetic field:

$$\log B^{(\text{eos})}(P, \dot{P}, M, \alpha) = \log B_{\text{md}}(P, \dot{P}) + \Delta_B^{(\text{eos})}(M, \alpha)$$

*additive correction depending on the accepted EOS, NS mass and obliquity*

Standard «magneto-dipolar» formula:

$$\left\{ \begin{array}{l} B_{\text{md}}(P, \dot{P}) = \sqrt{\frac{3 I_0 c^3}{8 \pi^2 R_0^6 \sin^2 \alpha_0}} P \dot{P} = 3.2 \times 10^{19} \sqrt{\overline{P \dot{P}}} \text{ Gs} \\ R_0 = 10 \text{ km}, I_0 = 10^{45} \text{ g} \cdot \text{cm}^2, \alpha_0 = 90^\circ \end{array} \right.$$

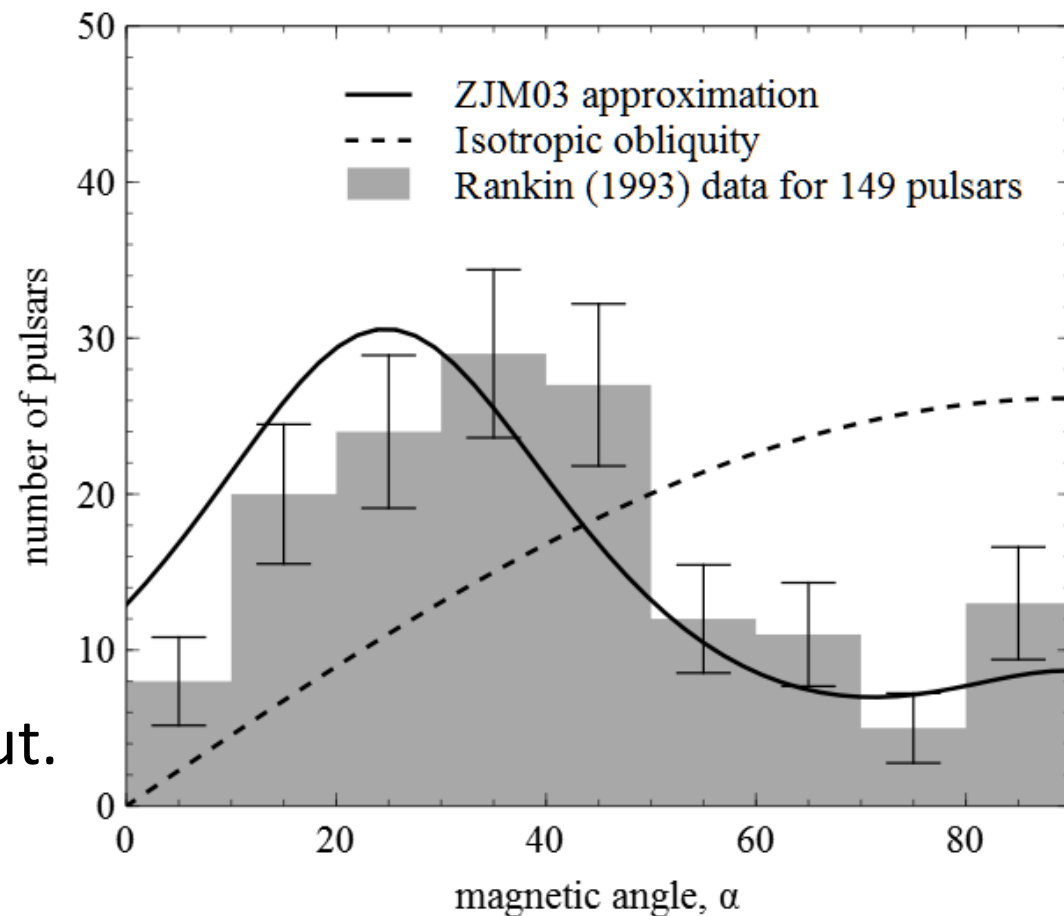
# Correction $\Delta_B$ constituents

- $L_{\text{sd}} \propto B^2 (k_0 + k_1 \sin^2 \alpha)$   
 $k_0 \approx 1, \quad k_1 \approx 1.4$

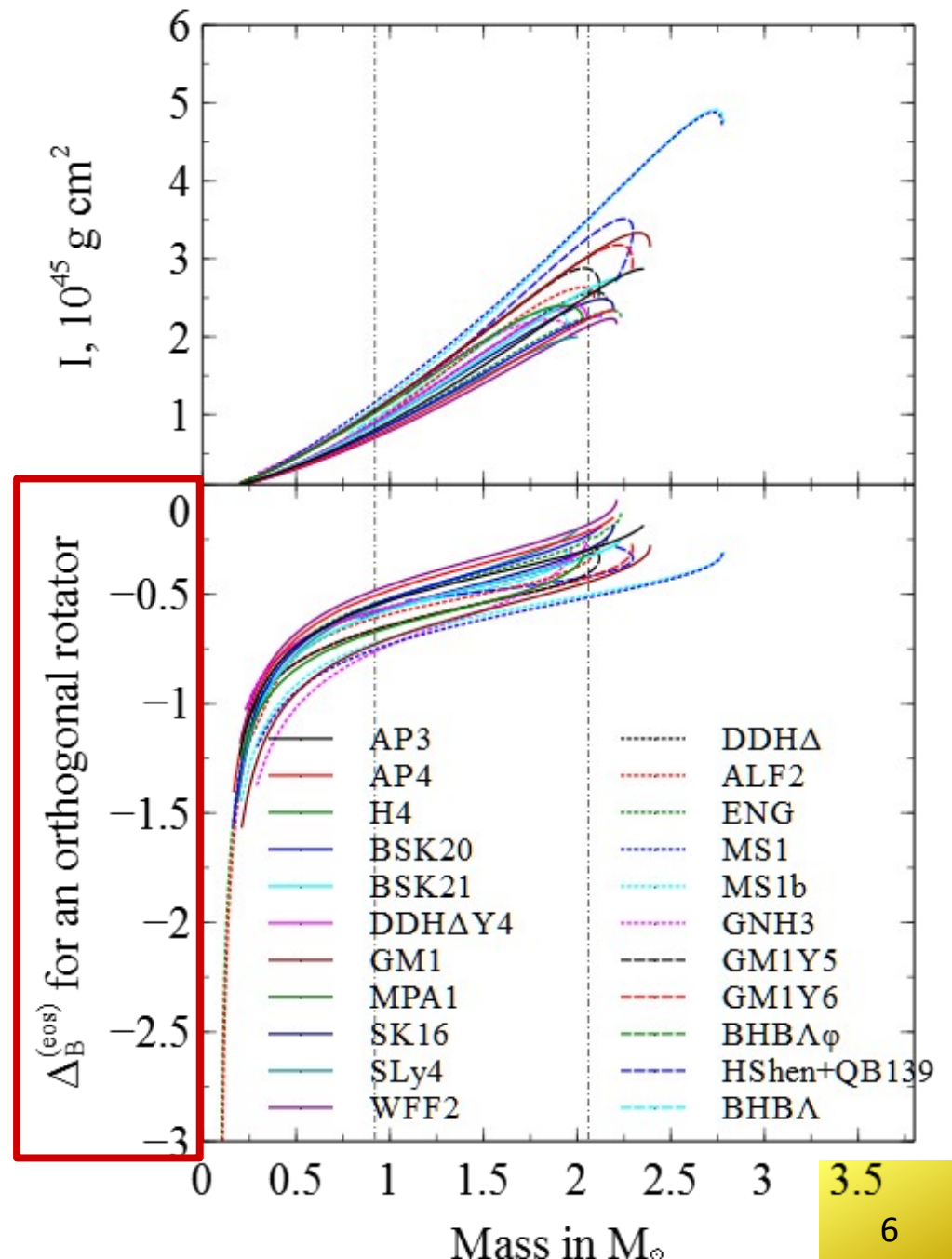
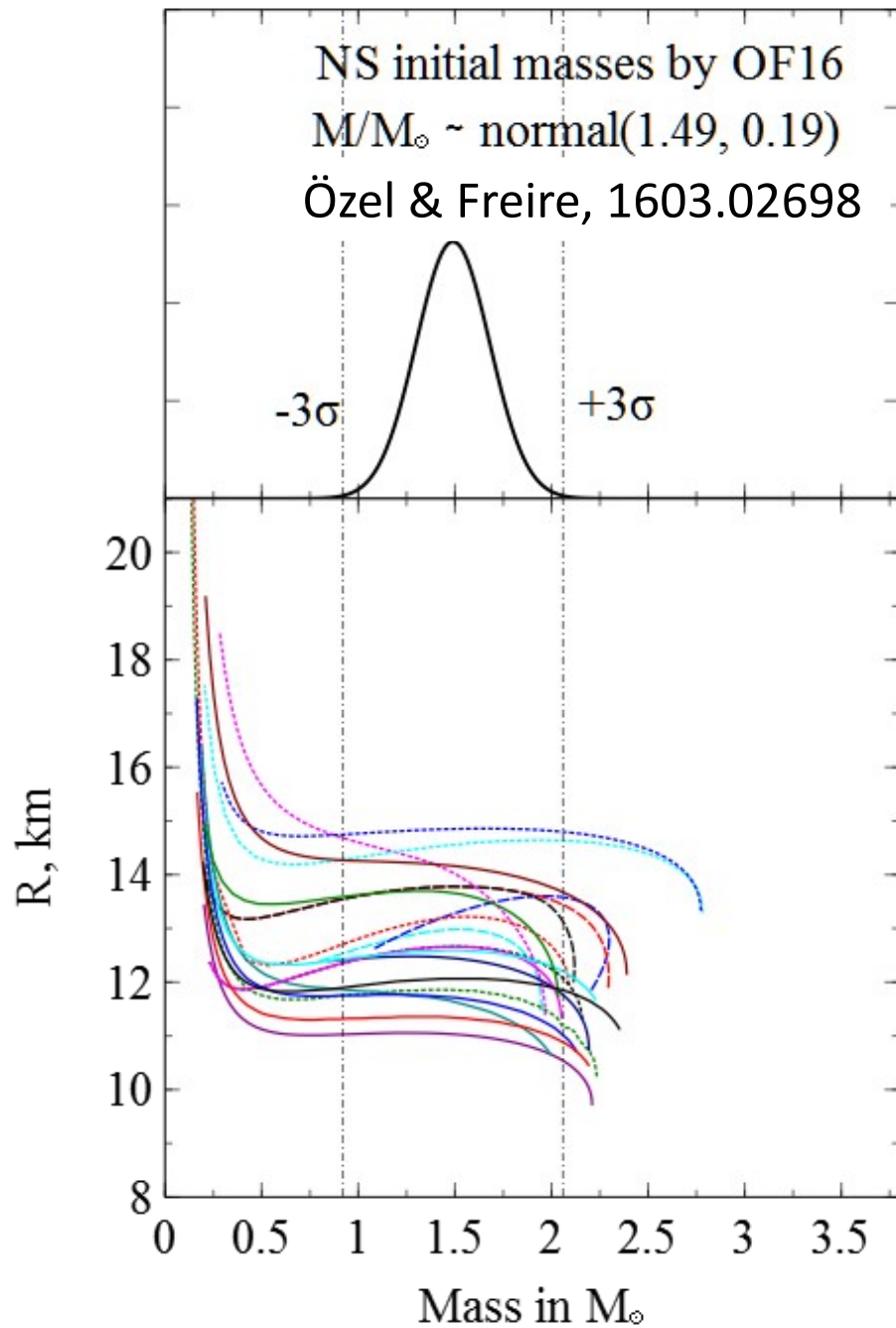
Spitkovsky, astro-ph/0603147

Philippov, Tchekhovskoy & Li, 1311.1513

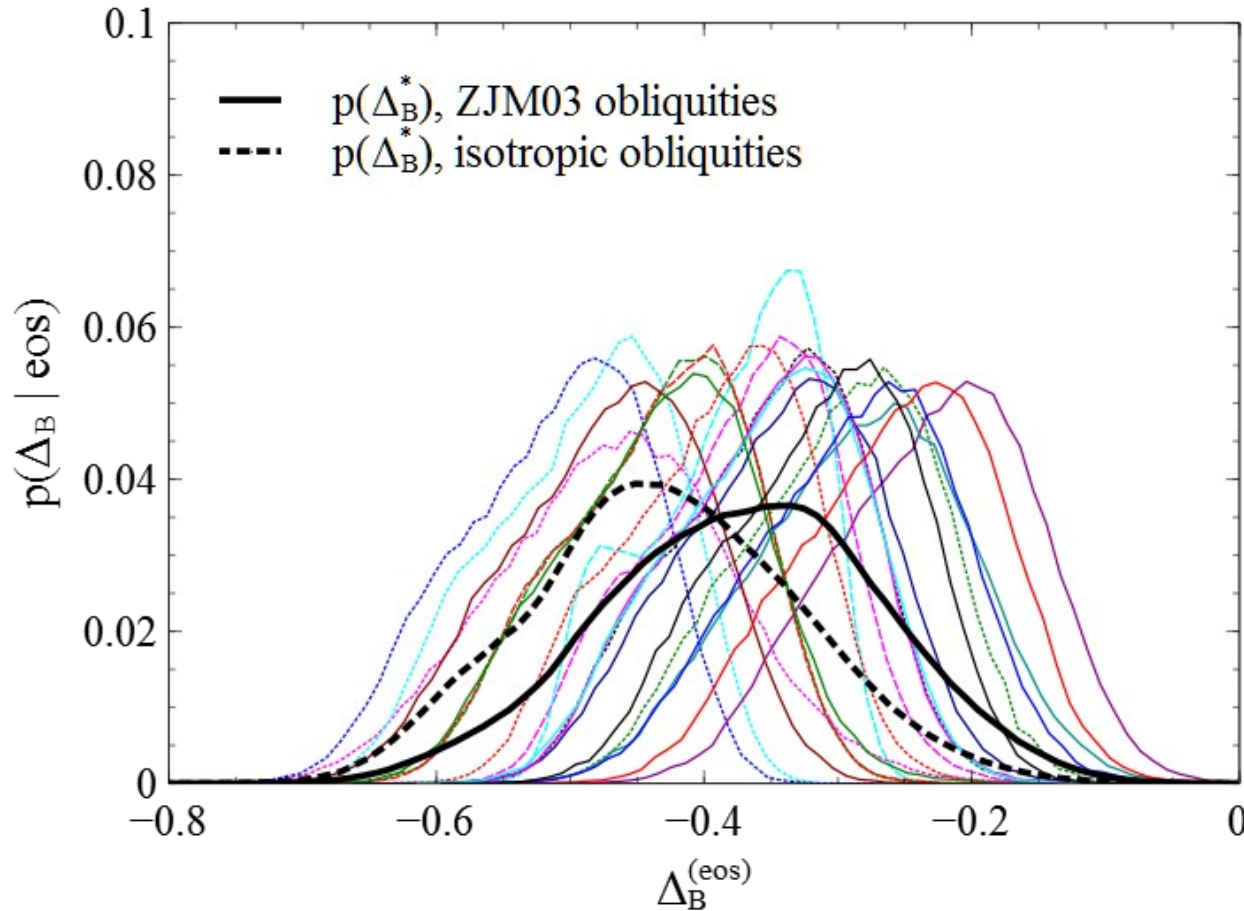
- Distribution of the magnetic angles is based on the data by Rankin (1993) with their parametrization by Zhang et al. (2003)
- Isotropic  $\alpha$  was also checked out.



# NS masses and equation(s) of state



# $\Delta_B$ distribution(s)



Distributions averages:

$$\langle \Delta_B^{(\text{eos})} \rangle \approx -0.55 \dots -0.25 \text{ dex}$$

Distributions widths:

$$\sigma[\Delta_B^{(\text{eos})}] \approx 0.06 \dots 0.09 \text{ dex}$$

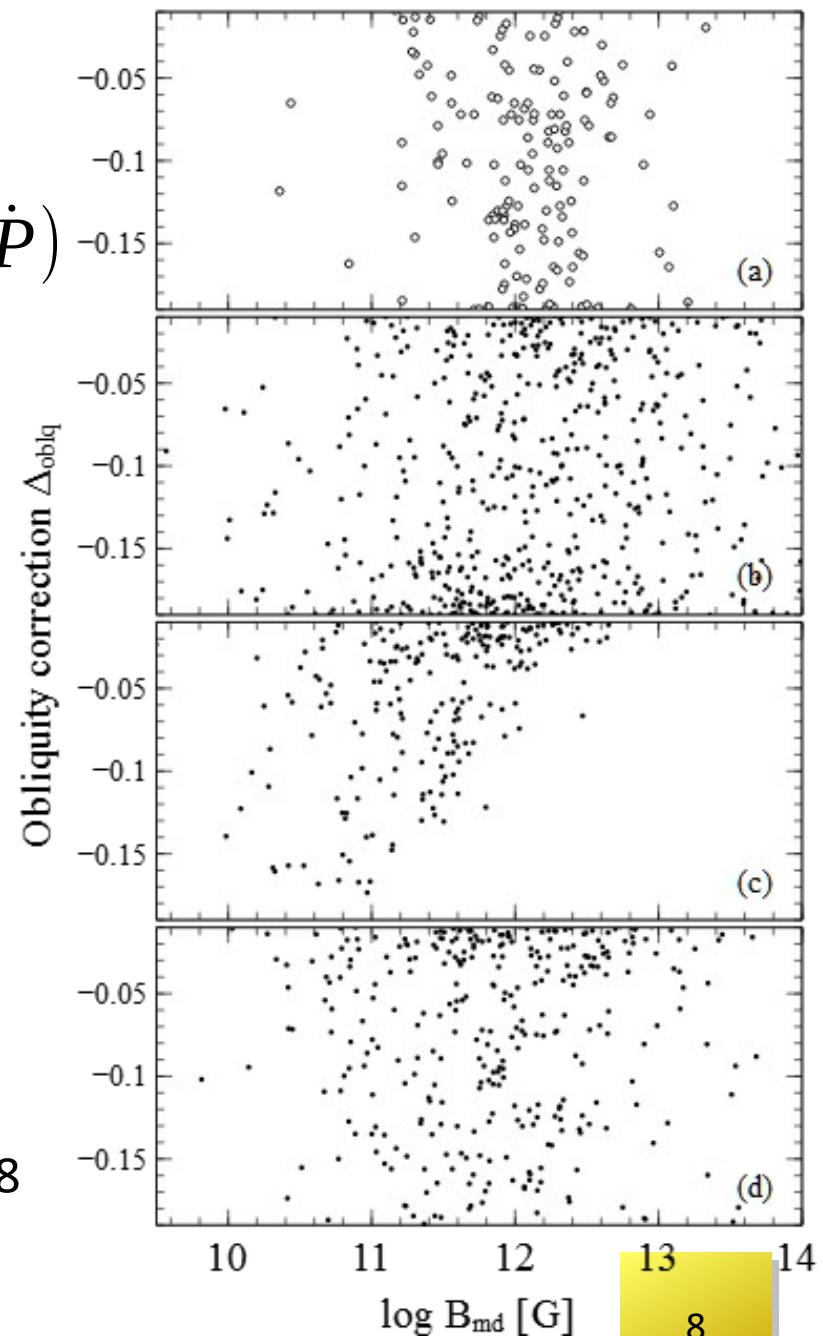
Mixture PDF for all 22 considered EOSs:

$$p(\Delta_B^*) = \sum w_i p(\Delta_B | \text{eos}) \Rightarrow \Delta_B^* \approx -0.37 \pm 0.10 \quad \text{or} \quad B^* \approx \frac{3}{7} B_{\text{md}}$$

# Properties of $\Delta_B$

- $\Delta_B(M, \alpha)$  does not correlate with  $B_{\text{md}}(P, \dot{P})$
- $\langle \Delta_B \rangle$  is the same for all pulsars  
(when a particular EOS is adopted)
- $\sigma[\Delta_B]$  can be considered as a formal  
precision of the magnetic field  
estimation for an individual pulsar

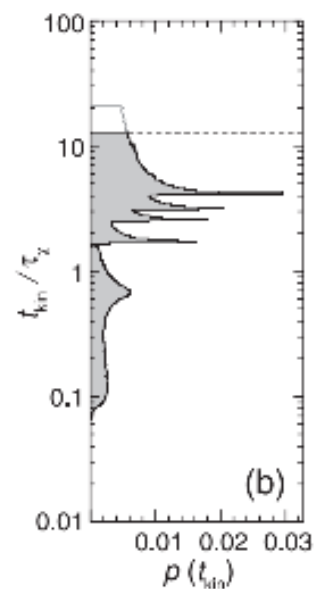
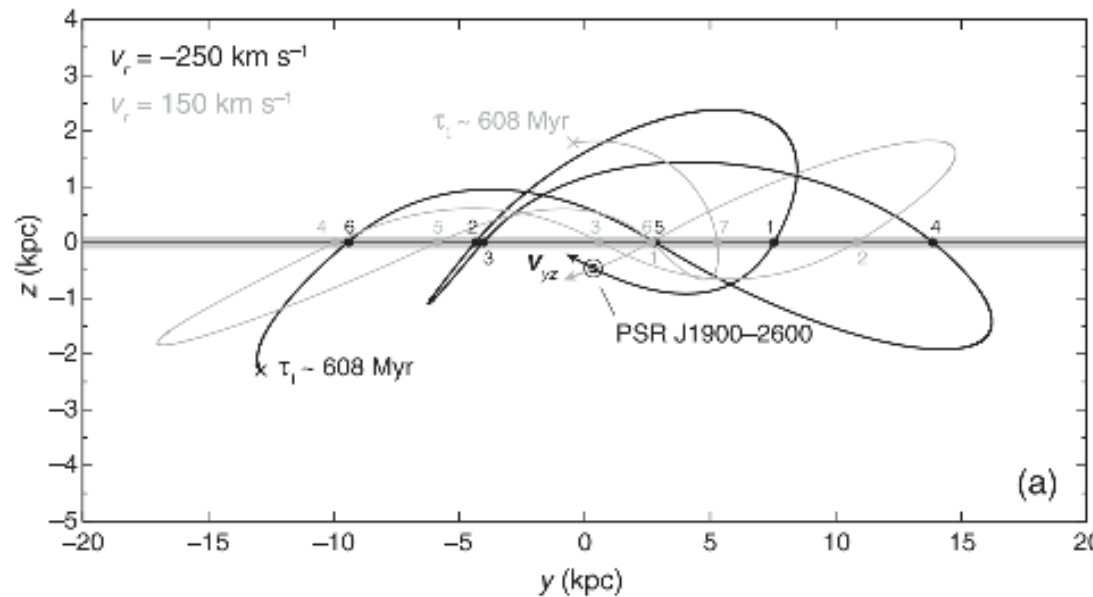
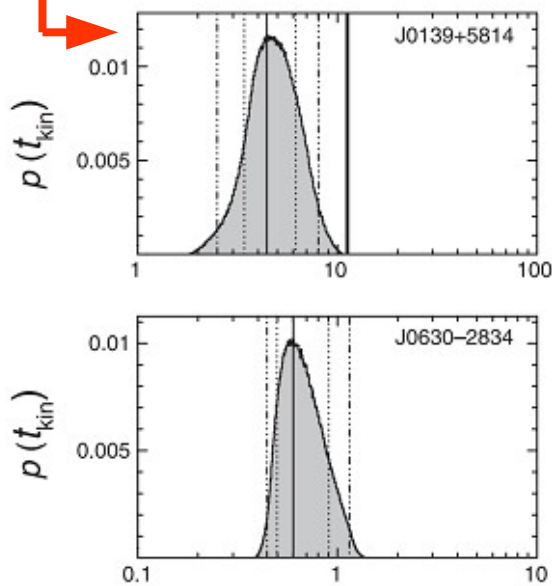
AB, Astashenok, G. Beskin, 1702.00018





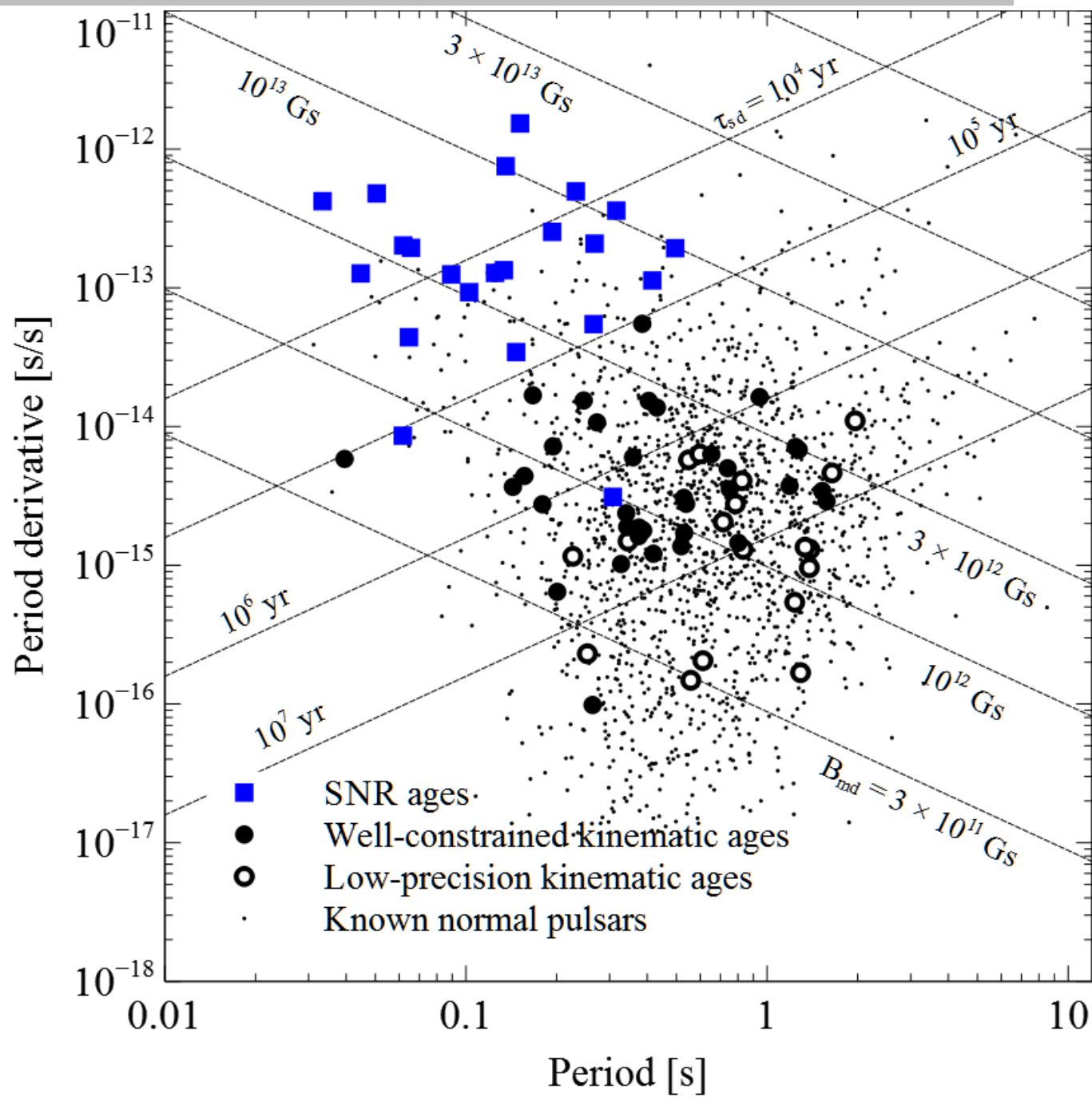
# Pulsar ages

- 22 PSR-SNR associations with ages within  $10^3 \dots 10^5$  years (mostly from Popov & Turolla, 1204.0632 and Gill & Heyl, 1305.0930);
- 54 pulsars with kinematic ages within  $10^5 \dots 10^8$  years (mostly from Noutsos+, 1301.1265; also: Chmyreva, G. Beskin, AB, 1203.2836; Tetzlaff+, 1401.4678)
  - 36 pulsars with «well-constrained»  $t_{kin}$
  - 18 pulsars with «low-precision»  $t_{kin}$

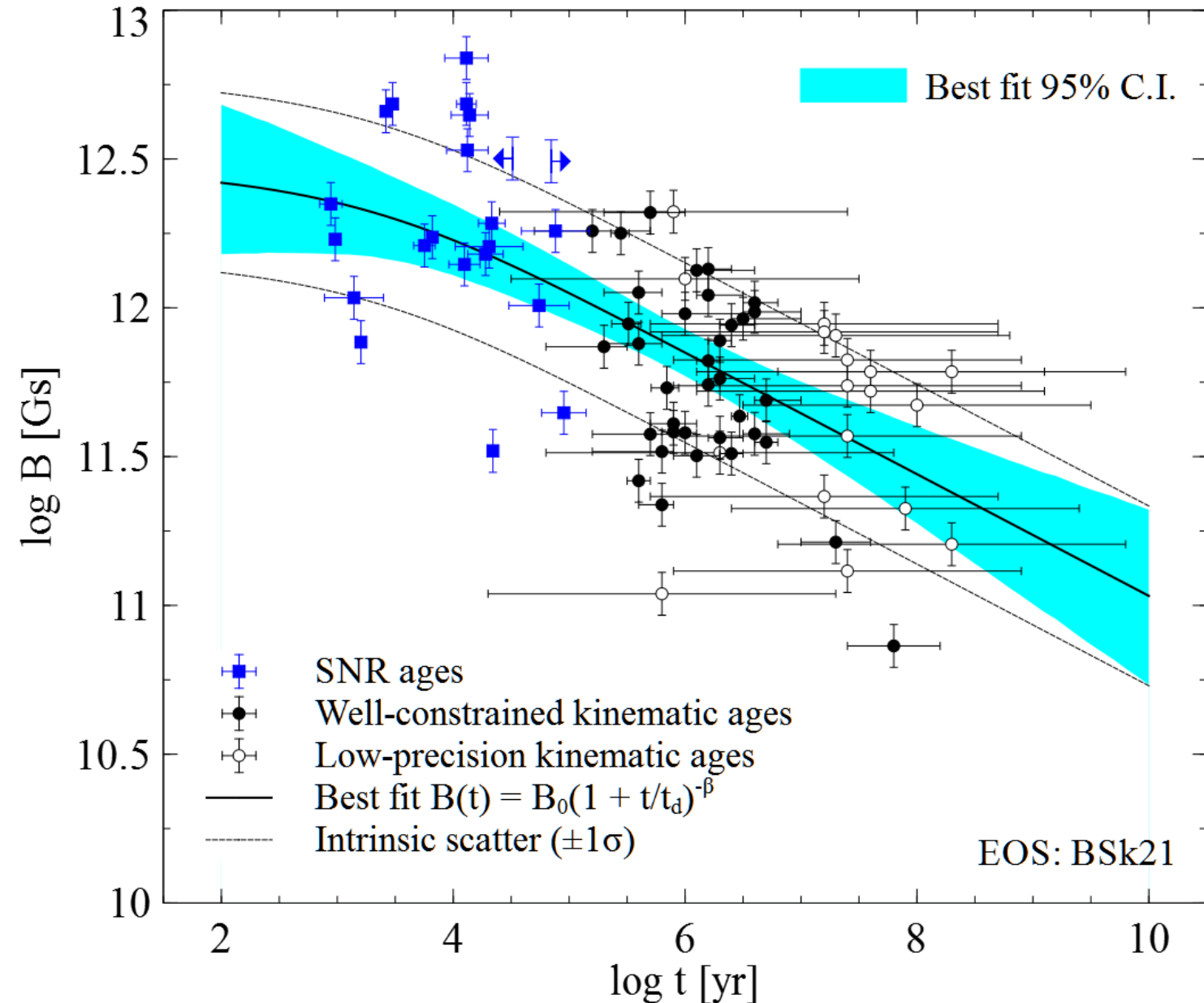


Figures from Noutsos+ 2013

# 76 pulsars



# Apparent field evolution

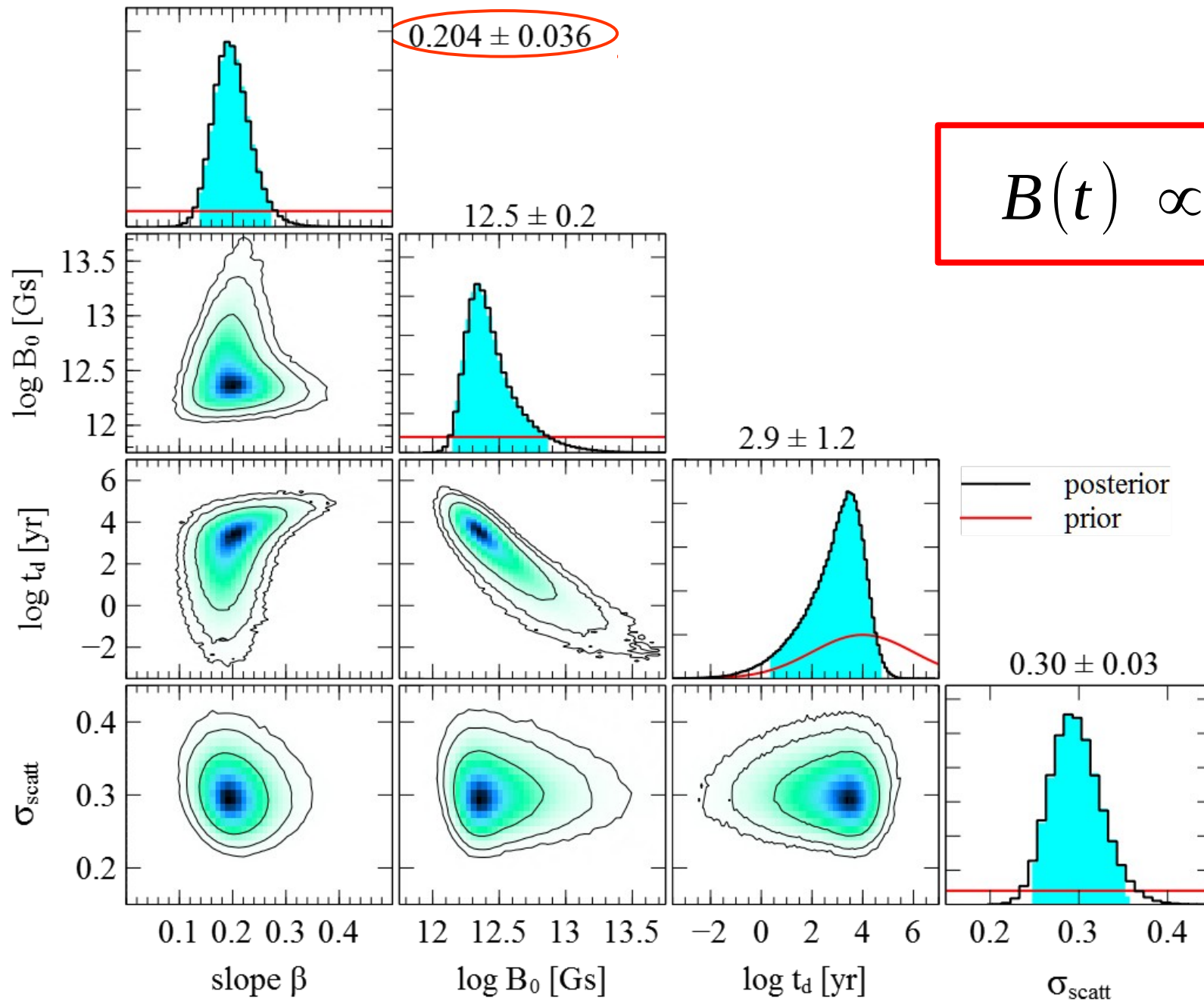


Bayesian fit by the model:

$$B(t) = B_0 \left( 1 + \frac{t}{t_d} \right)^{-\beta}$$

The shape and the slope of the cloud are independent on the choice of EOS

# Apparent field evolution

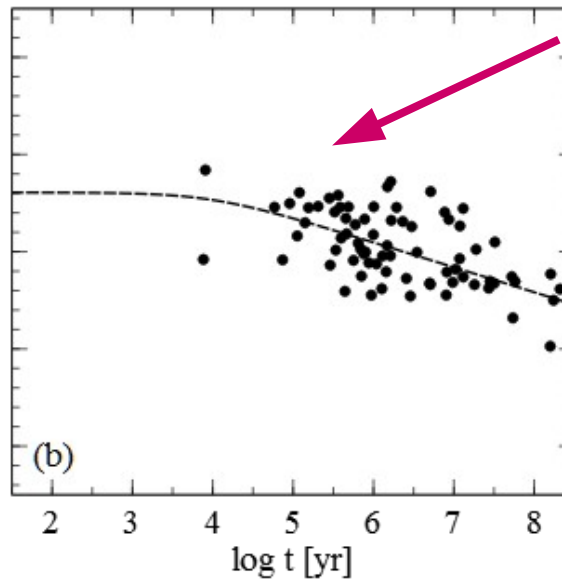
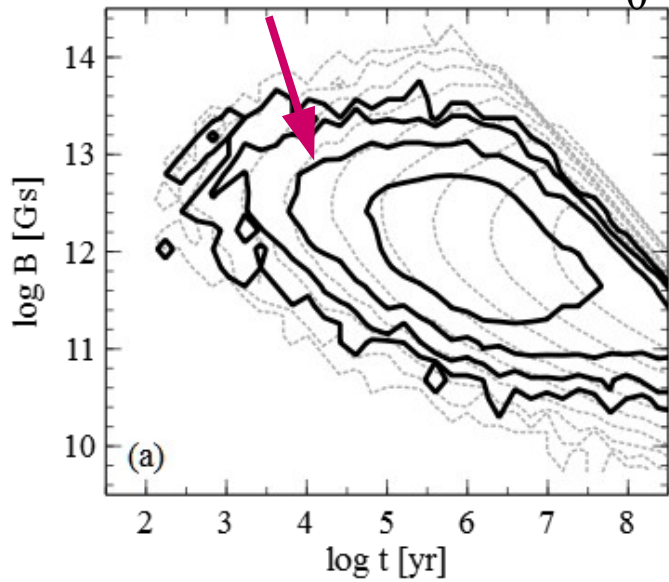


# Population synthesis

- Based on the best model derived by Faucher-Giguere & Kaspi, astro-ph/0512585 (including initial  $P_0$ ,  $B_0$ , luminosities, kinematics etc.), death line equation:  $\dot{P} = (2.82 \times 10^{-17} \text{ sec}^{-3}) \cdot P^3$
- Some extensions of the model:
  - Direct modeling of pulsar beam direction to the observer and pulse width  $W_{10}$ . Assume beam half-width  $\rho = 5.7^\circ P^{-1/2}$
  - Modeling of the magnetic angle evolution (from isotropic  $\alpha_0$ )
  - Spin-down + field decay:  $P \dot{P} \propto B_0^2 \left(1 + \frac{t}{10^3 \text{ yr}}\right)^{-2\beta_0} \cdot [1 + 1.4 \sin^2 \alpha(t)]$
  - $\alpha$ -dependend death line:  $\cos \alpha < (P/\text{sec})^{15/7} (B/10^{12} \text{ Gs})^{-8/7}$   
Beskin, Gurevich & Istomin 1993  
(see also Arzamasskiy, Beskin & Pirov, 1612.04820)

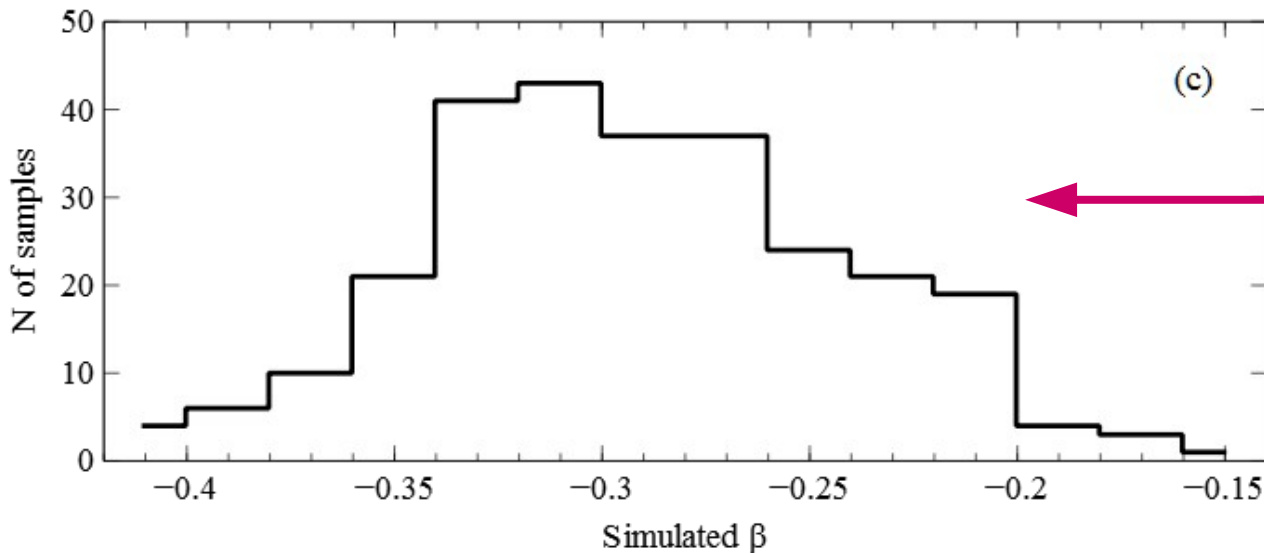
# Modeling the observational selection

Modeling  $\log B$ - $\log t$  for synthetic pulsars  
assuming a particular  $\beta_0$



Extracting and fit the independent samples of 76 pulsars with

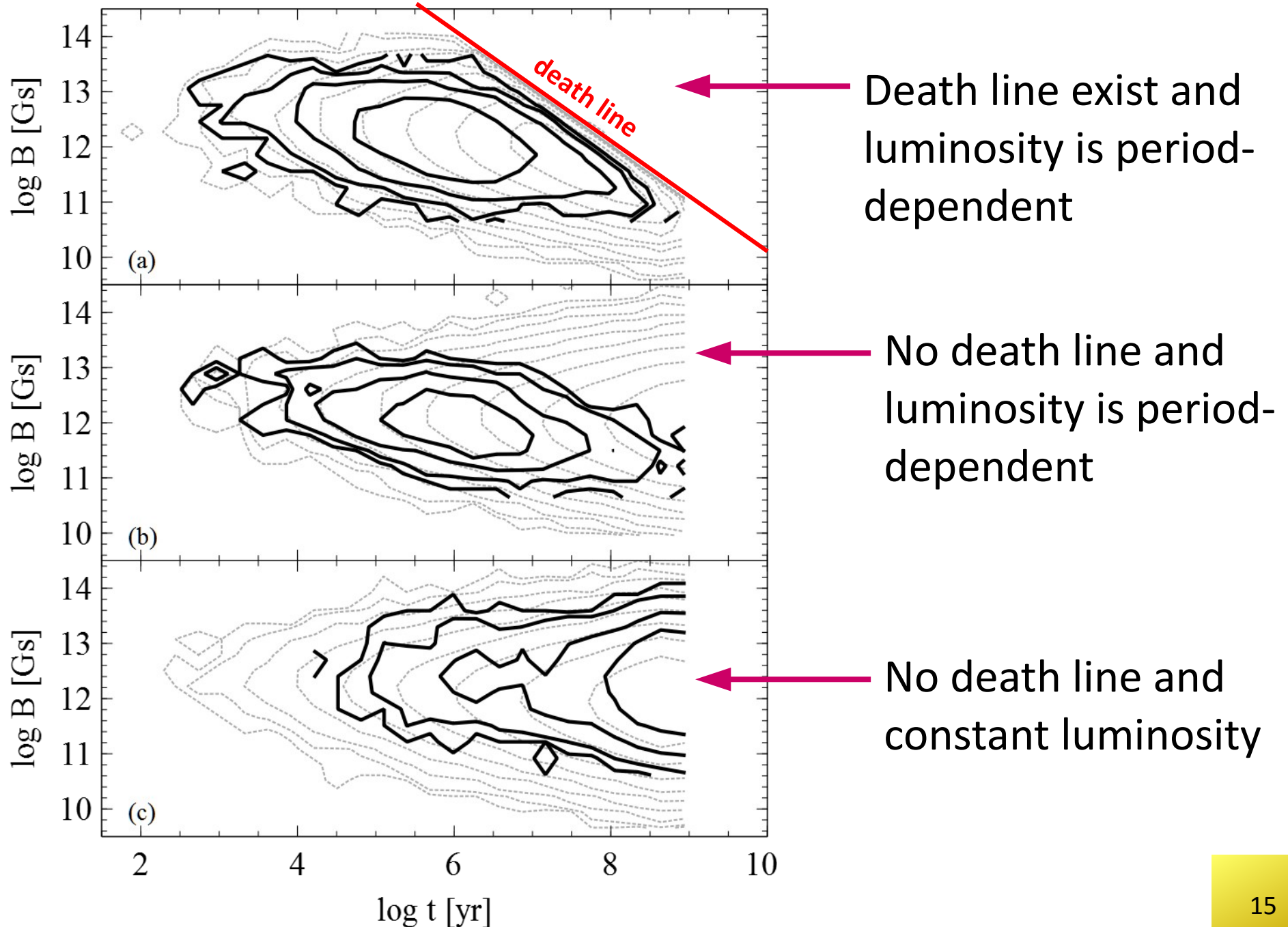
$$B = B_0 \left( 1 + \frac{t}{10^3 \text{ yr}} \right)^{-\beta}$$



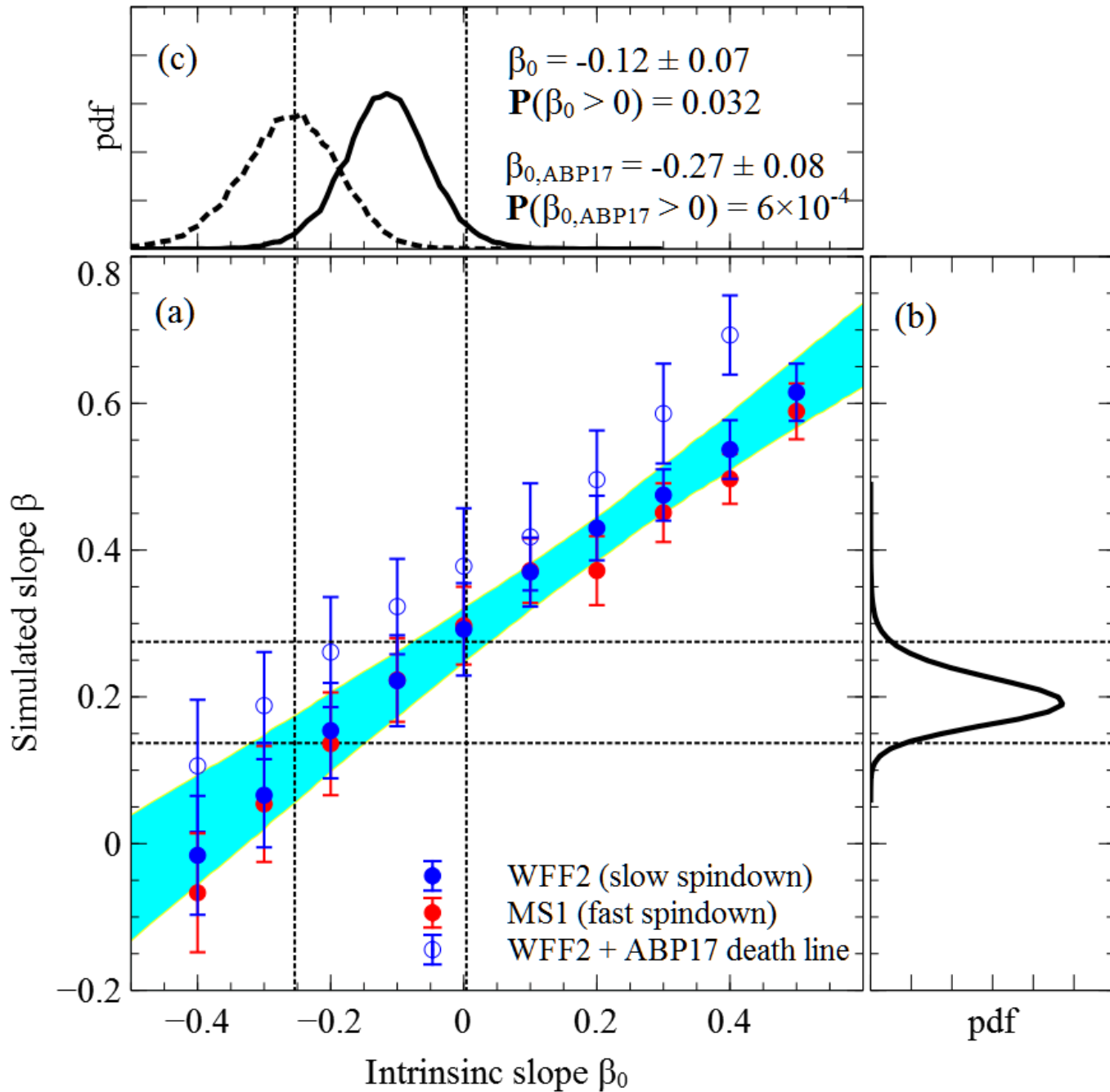
Plot the distribution of simulated  $\beta$



# Selectional effects (for $\beta_0 = 0$ )



# Intrinsic field decay



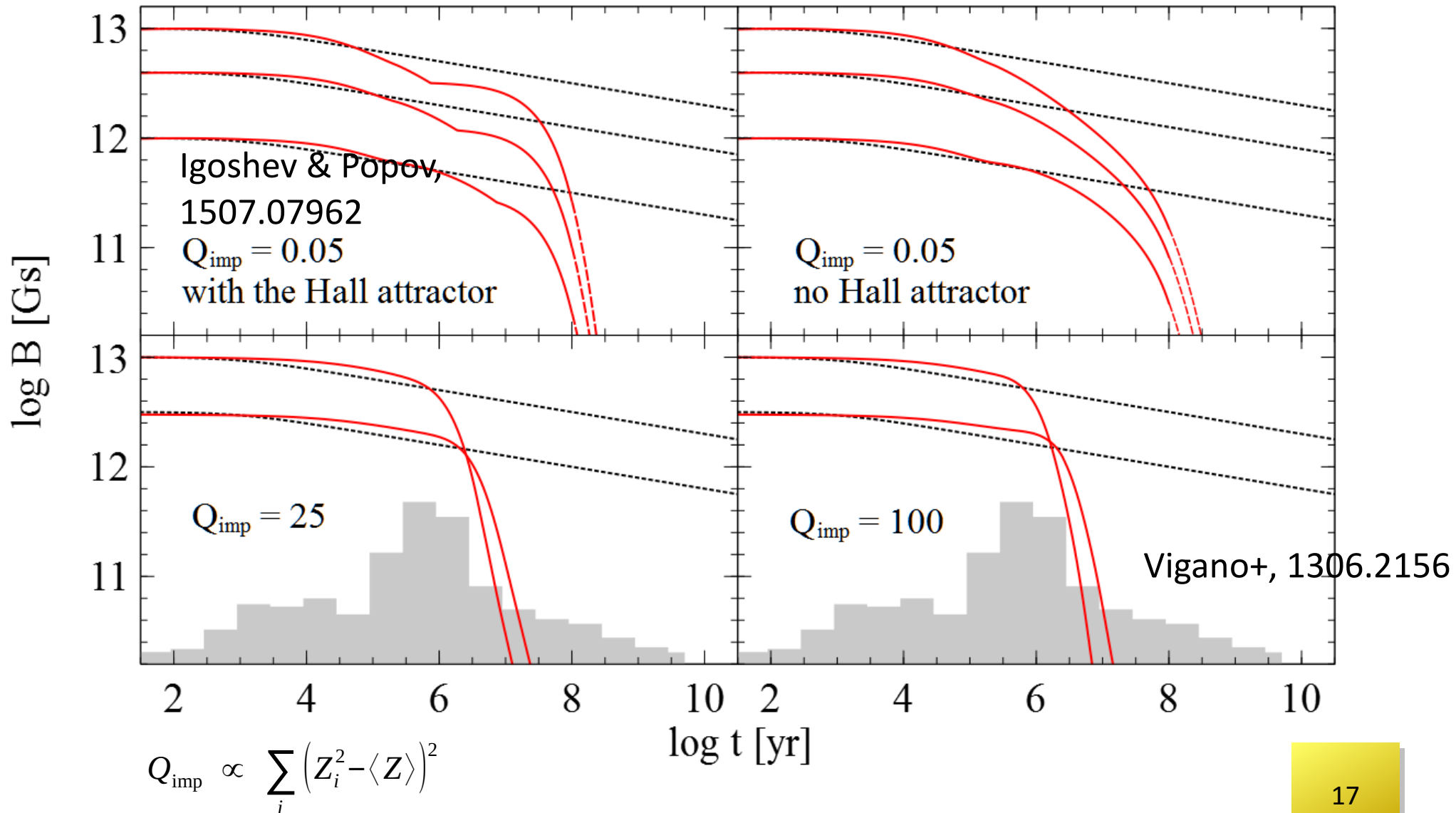
Intrinsic slope  $\beta_0$  in

- $[-0.33 \dots 0.09] @ 3\sigma$  C.L. for an  $\alpha$ -independend death line
- $[-0.51 \dots -0.03] @ 3\sigma$  C.L. for an  $\alpha$ -dependend death line



# Discussion

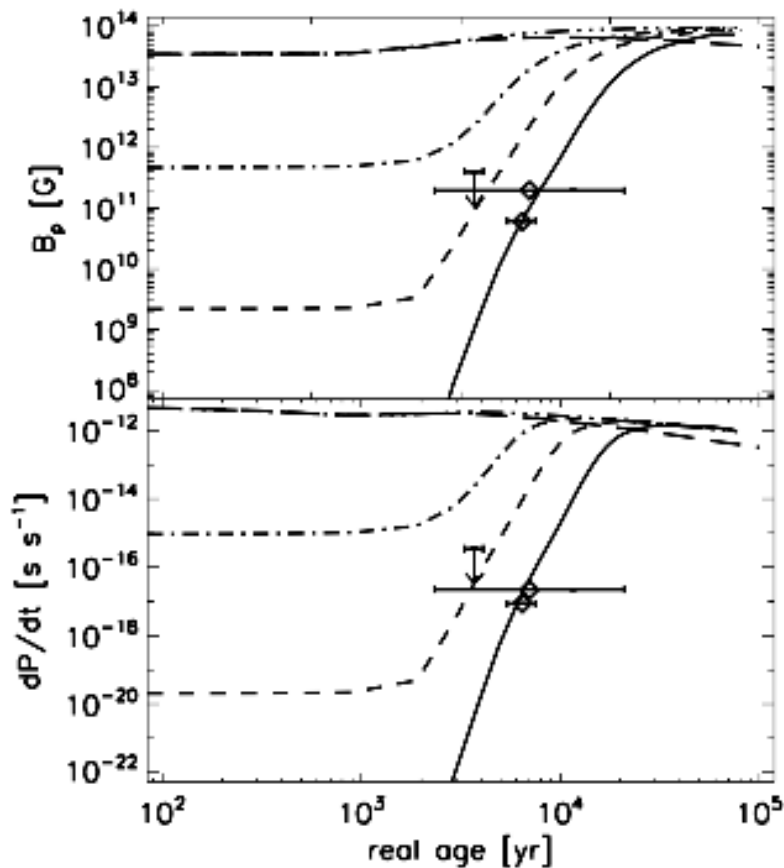
Observed  $B(t)$  rejects:  
 moderate decay with  $\beta > 0.1$  (at  $3\sigma$  confidence)



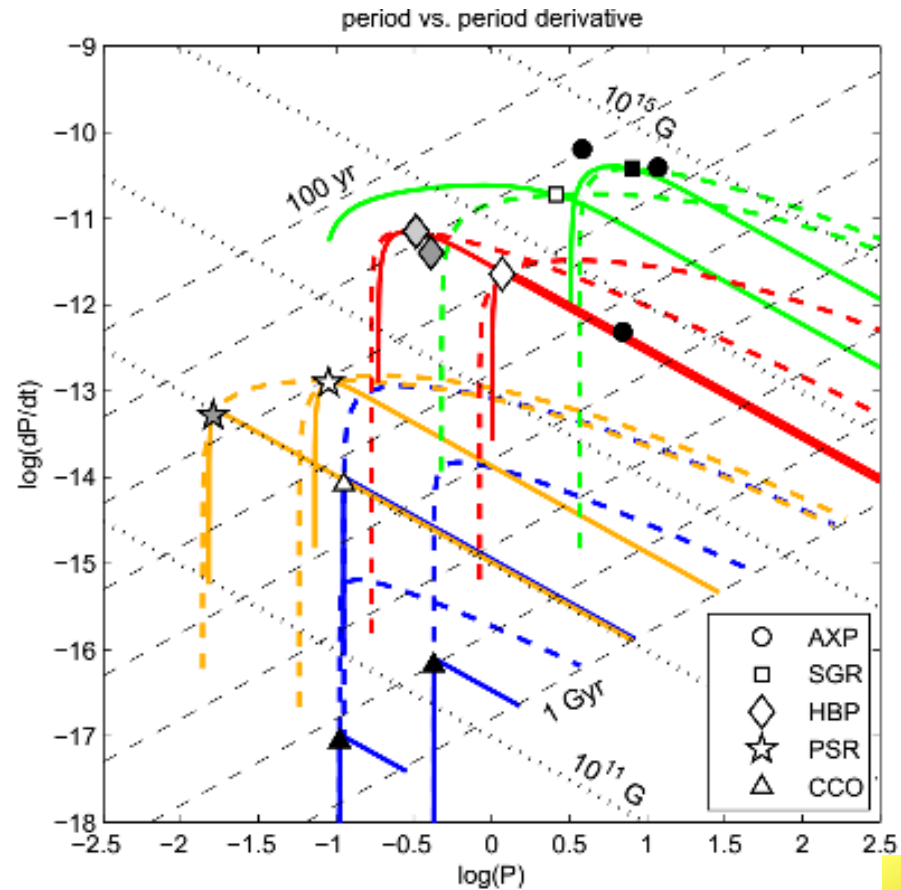
# Discussion

Apparent  $B(t)$  does not **formally** reject @ < 3.2% C.L.:

- neither the absence of the field evolution ( $\beta = 0$ ) nor its moderate growth ( $\beta < -0.1$ ) (**but** Pons+, astro-ph/0607583; Kaspi, 1005.0876; Xie & Zhang, 1110.3869; Pons, Viganò & Geppert, 1209.2273; Igoshev, Popov, 1407.6269 etc.)



Viganò & Pons, 1206.2014



Rogers & Safi-Harb, 1601.00949

# Conclusions

- The timing-based value of  $B$  can be estimated with  $\sim 15\text{-}30\%$  accuracy for a given pulsar within the state-of-the-art constraints on the NS masses, magnetic angles and EOS
- $B_{\text{md}} = 3.2 \times 10^{19} \sqrt{P\dot{P}}$  Gs is good, but biased. Use your favorite EOS when calculating  $B$ , or adopt  $B^* = 3B_{\text{md}}/7$  at least.
- There is a significant apparent trend  $B(t) \propto t^{-1/5}$ , which is consistent with the field decay models that assume a low amount of impurities in a NS crust.
- Within the assumptions made in the modeling, the trend also does not reject a systematic moderate field growth at the Myr-timescales.

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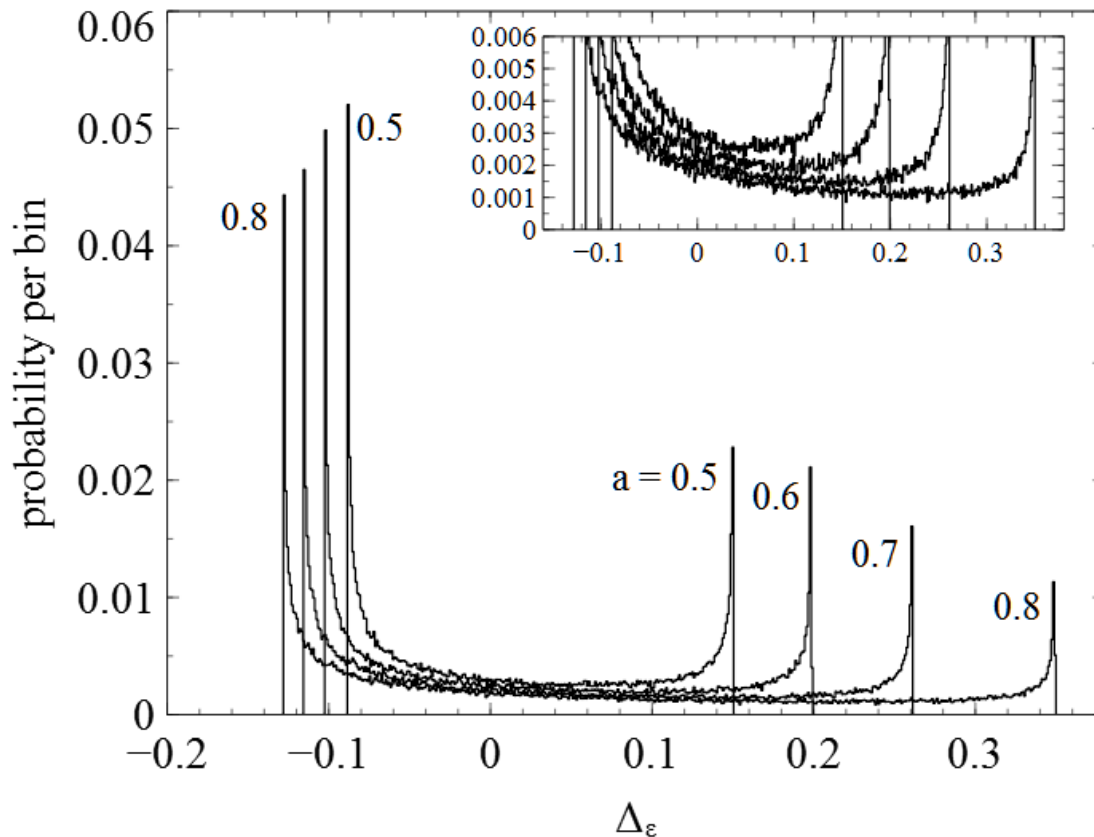
**Thank you! :-)**

# Timing noise effects

$\dot{P}_{\text{obs}} = \dot{P}(1 + \varepsilon)$  – functional relationship between  $P$ ,  $dP/dt$  и  $B$   
 can be broken temporally:  $|\varepsilon| < 1$  is exist.

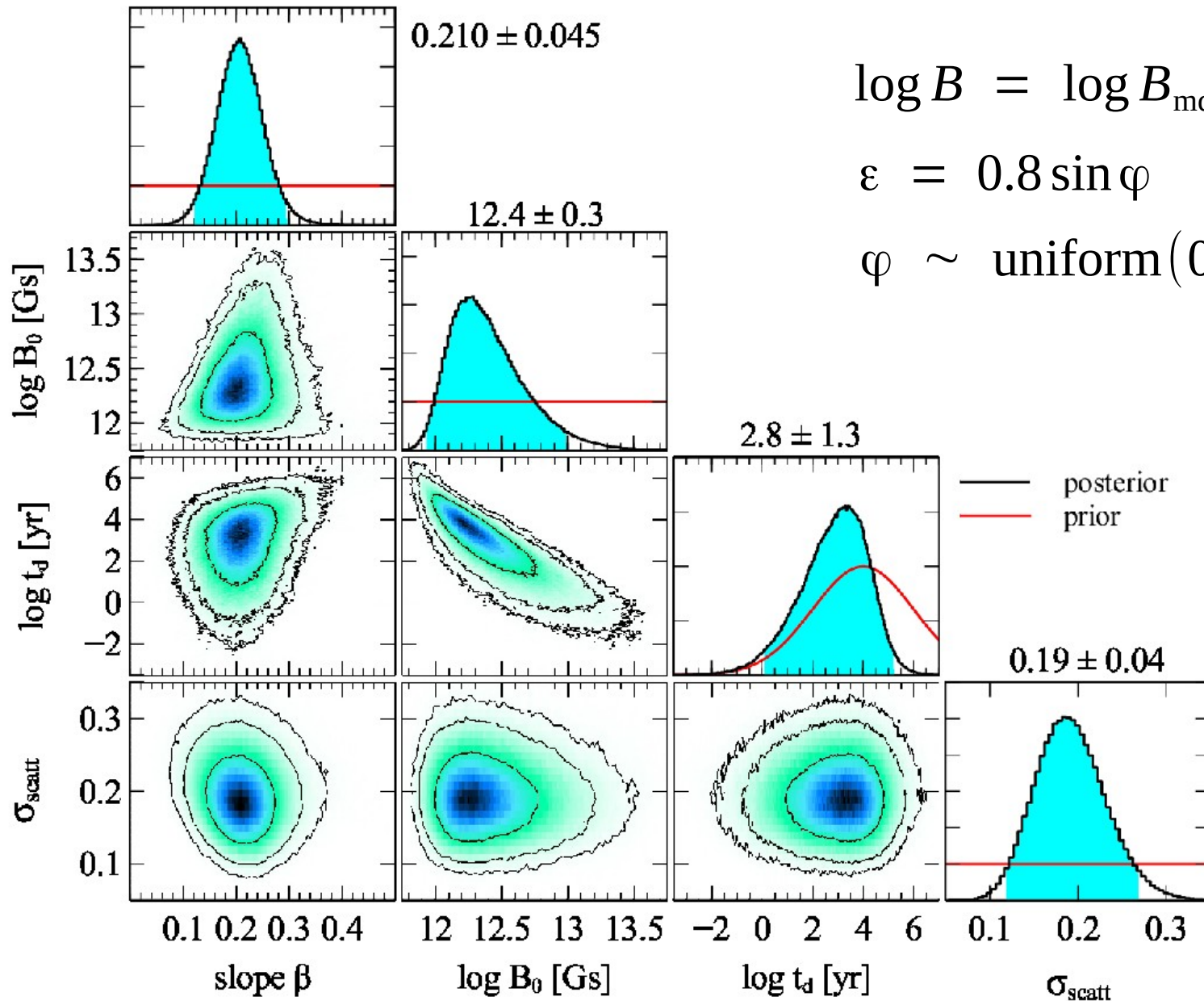


$\Delta_\varepsilon = \frac{1}{2} \log(1 - \varepsilon)$  – an additional correction to the  $\log B$



- Slow variations of  $B$ ,  $\alpha$  or  $I \rightarrow$   
 $\varepsilon \equiv 0$  (Pons+, 1209.2273;  
 Arzamasskiy+, 1504.06626; Hamil+,  
 1608.01383)
- Additional component in the  
 braking torque  $\rightarrow$   
 $|\varepsilon| < 0.8 \neq 0$   
 (AB, G. Beskin, Karpov, 1105.5019)

# B(t) trend including timing noise



$$\log B = \log B_{\text{md}} + \Delta_B^{\text{BSk21}} + \Delta_\varepsilon$$

$$\varepsilon = 0.8 \sin \varphi$$

$$\varphi \sim \text{uniform}(0, 2\pi)$$

# GR effects

$$P \dot{P} = \eta_{\text{GR}}(x) \times \frac{4\pi^2 R^6}{I c^3} B^2 \quad \text{-- aligned rotator in a curved space-time}$$

(Gralla, Lupsasca, Philippov, 1604.04625)

$$\eta_{\text{GR}}(x) \approx \frac{0.42 \cdot x^6}{[x(x+2) + 2 \ln(1-x)]^2}$$

$$x = 2GM/Rc^2$$



$$\Delta_{\text{GR}}^{(\text{eos})}(x) = -\frac{1}{2} \log \eta_{\text{GR}}(x)$$

