The apparent decay of pulsar magnetic fields

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1 Estimating pulsar magnetic fields (B)

2 Estimating pulsar ages (t)

3 Analyzing the apparent B(t)

4 Observational selection and the intrinsic B(t)

5 Interpretation and conclusions
Estimating the magnetic fields

- Spin-down of a spherical isolated NS:

\[ P \dot{P} = \frac{4\pi^2 R^6}{I c^3} \cdot B^2 \cdot (k_0 + k_1 \sin^2 \alpha) \]

- The magnetic field strength (at the magnetic equator):

\[ B = \sqrt{\frac{c^3}{4\pi^2}} \times \sqrt{\frac{I(M)}{R^3(M)}} \times \sqrt{\frac{P \dot{P}}{k_0 + k_1 \sin^2 \alpha}} \]

\[ \Omega = \frac{2\pi}{P} \]
Estimating the magnetic fields

The logarithm of the magnetic field:

\[ \log B^{(\text{eos})}(P, \dot{P}, M, \alpha) = \log B_{\text{md}}(P, \dot{P}) + \Delta_B^{(\text{eos})}(M, \alpha) \]

*additive correction depending on the accepted EOS, NS mass and obliquity*

Standard «magneto-dipolar» formula:

\[ B_{\text{md}}(P, \dot{P}) = \sqrt{\frac{3 I_0 c^3}{8 \pi^2 R_0^6 \cdot \sin^2 \alpha_0}} P \dot{P} = 3.2 \times 10^{19} \sqrt{P \dot{P}} \text{ Gs} \]

\[ R_0 = 10 \text{ km}, \ I_0 = 10^{45} \text{ g} \cdot \text{cm}^2, \ \alpha_0 = 90^\circ \]
Correction $\Delta_B$ constituents

- $L_{sd} \propto B^2(k_0 + k_1 \sin^2 \alpha)$
  $k_0 \approx 1, \quad k_1 \approx 1.4$
  Spitkovsky, astro-ph/0603147
  Philippov, Tchekhovskoy & Li, 1311.1513

- Distribution of the magnetic angles is based on the data by Rankin (1993) with their parametrization by Zhang et al. (2003)

- Isotropic $\alpha$ was also checked out.

![Graph showing distribution of magnetic angles with data points and curves representing ZJM03 approximation and isotropic obliquity compared to Rankin (1993) data for 149 pulsars.]
NS masses and equation(s) of state

NS initial masses by OF16
M/M\(_{\odot}\) ~ normal(1.49, 0.19)
Özel & Freire, 1603.02698

\[ I, 10^{45} \text{ g cm}^2 \]

\[ \Delta_{B}^{(\text{eos})} \text{ for an orthogonal rotator} \]

\[ R, \text{ km} \]

\[ \text{Mass in } M_{\odot} \]

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$\Delta_B$ distribution(s)

Distributions averages:
\[ \langle \Delta_B^{(\text{eos})} \rangle \approx -0.55 \ldots -0.25 \text{ dex} \]

Distributions widths:
\[ \sigma \left[ \Delta_B^{(\text{eos})} \right] \approx 0.06 \ldots 0.09 \text{ dex} \]

Mixture PDF for all 22 considered EOSs:
\[
p(\Delta_B^*) = \sum w_i p(\Delta_B|\text{eos}) \Rightarrow \Delta_B^* \approx -0.37 \pm 0.10 \quad \text{or} \quad B^* \approx \frac{3}{7} B_{\text{md}}
\]
Properties of $\Delta_B$

- $\Delta_B(M, \alpha)$ does not correlate with $B_{md}(P, \dot{P})$

- $<\Delta_B>$ is the same for all pulsars (when a particular EOS is adopted)

- $\sigma[\Delta_B]$ can be considered as a formal precision of the magnetic field estimation for an individual pulsar

AB, Astashenok, G. Beskin, 1702.00018
Pulsar ages

- 22 PSR-SNR associations with ages within $10^3...10^5$ years (mostly from Popov & Turolla, 1204.0632 and Gill & Heyl, 1305.0930);

- 54 pulsars with kinematic ages within $10^5...10^8$ years (mostly from Noutsos+, 1301.1265; also: Chmyreva, G. Beskin, AB, 1203.2836; Tetzlaff+, 1401.4678)
  - 36 pulsars with «well-constrained» $t_{\text{kin}}$
  - 18 pulsars with «low-precision» $t_{\text{kin}}$

Figures from Noutsos+ 2013
76 pulsars

![Graph](image)

- **SNR ages**: Blue squares
- **Well-constrained kinematic ages**: Black circles
- **Low-precision kinematic ages**: Open circles
- **Known normal pulsars**: Dots

**Axes:**
- **Period derivative [s/s]**
- **Period [s]**

**Labels:**
- $10^{13} Gs$
- $3 \times 10^{13} Gs$
- $10^6 \text{ yr}$
- $3 \times 10^{12} Gs$
- $10^7 \text{ yr}$
- $B_{rad} = 3 \times 10^{11} Gs$
- $10^8 \text{ yr}$
Bayesian fit by the model:

\[ B(t) = B_0 \left(1 + \frac{t}{t_d}\right)^{-\beta} \]

The shape and the slope of the cloud are independent on the choice of EOS.
Apparent field evolution

\[ B(t) \propto t^{-1/5} \]

\[ 0.204 \pm 0.036 \]

\[ 12.5 \pm 0.2 \]

\[ 2.9 \pm 1.2 \]

\[ 0.30 \pm 0.03 \]
Population synthesis

- Based on the best model derived by Faucher-Giguere & Kaspi, astro-ph/0512585 (including initial $P_0$, $B_0$, luminosities, kinematics etc.), death line equation: $\dot{P} = \left(2.82 \times 10^{-17} \text{ sec}^{-3}\right) \cdot P^3$

- Some extensions of the model:
  - Direct modeling of pulsar beam direction to the observer and pulse width $W_{10}$. Assume beam half-width $\rho = 5.7^\circ P^{-1/2}$
  - Modeling of the magnetic angle evolution (from isotropic $\alpha_0$)
  - Spin-down + field decay: $P \dot{P} \propto B_0^2 \left(1 + \frac{t}{10^3 \text{ yr}}\right)^{-2\beta_0} \cdot \left[1 + 1.4 \sin^2 \alpha(t)\right]$
  - $\alpha$-dependend death line: $\cos \alpha < \left(\frac{P}{\text{ sec}}\right)^{15/7} \left(\frac{B}{10^{12} \text{ Gs}}\right)^{-8/7}$

Beskin, Gurevich & Istomin 1993
(see also Arzamasskiy, Beskin & Pirov, 1612.04820)
Modeling the observational selection

Modeling $\log B - \log t$ for synthetic pulsars assuming a particular $\beta_0$

Extracting and fit the independent samples of 76 pulsars with

$$B = B_0 \left(1 + \frac{t}{10^3 \text{ yr}}\right)^{-\beta}$$

Plot the distribution of simulated $\beta$
Selectional effects (for $\beta_0 = 0$)

- Death line exist and luminosity is period-dependent
- No death line and luminosity is period-dependent
- No death line and constant luminosity
Intrinsic field decay

\[ \beta_0 = -0.12 \pm 0.07 \]
\[ P(\beta_0 > 0) = 0.032 \]
\[ \beta_{0,\text{ABP17}} = -0.27 \pm 0.08 \]
\[ P(\beta_{0,\text{ABP17}} > 0) = 6 \times 10^{-4} \]

Intrinsic slope \( \beta_0 \) in
- \([-0.33...0.09] @ 3\sigma \) C.L. for an \( \alpha \)-independent death line
- \([-0.51...-0.03] @ 3\sigma \) C.L. for an \( \alpha \)-dependent death line
Observerd $B(t)$ rejects: moderate decay with $\beta > 0.1$ (at $3\sigma$ confidence)

$Q_{\text{imp}} \propto \sum_i \left( Z_i^2 - \langle Z \rangle \right)^2$

Igoshev & Popov, 1507.07962
$Q_{\text{imp}} = 0.05$
with the Hall attractor

Igoshev & Popov, 1507.07962
$Q_{\text{imp}} = 0.05$
no Hall attractor

Vigano+, 1306.2156
$Q_{\text{imp}} = 25$

Vigano+, 1306.2156
$Q_{\text{imp}} = 100$
Apparent $B(t)$ does not **formally** reject @ $< 3.2\%$ C.L.:

- neither the absence of the field evolution ($\beta = 0$) nor its moderate growth ($\beta < -0.1$) (but Pons+, astro-ph/0607583; Kaspi, 1005.0876; Xie & Zhang, 1110.3869; Pons, Vigano & Geppert, 1209.2273; Igoshev, Popov, 1407.6269 etc.)
Conclusions

- The timing-based value of $B$ can be estimated with $\sim$15-30% accuracy for a given pulsar within the state-of-the-art constrains on the NS masses, magnetic angles and EOS.

- $B_{md} = 3.2 \times 10^{19} \sqrt{\frac{PP}{}}$ Gs is good, but biased. Use your favorite EOS when calculating $B$, or adopt $B^* = 3B_{md}/7$ at least.

- There is a significant apparent trend $B(t) \propto t^{-1/5}$, which is consistent with the field decay models that assume a low amount of impurities in a NS crust.

- Within the assumptions made in the modeling, the trend also does not reject a systematic moderate field growth at the Myr-timescales.
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Thank you! :-}
Timing noise effects

\[ \dot{P}_{\text{obs}} = \dot{P}(1 + \varepsilon) \] – functional relationship between \( P, \frac{dP}{dt} \) и \( B \) can be broken temporally: \(|\varepsilon| < 1\) is exist.

\[ \Delta\varepsilon = \frac{1}{2} \log(1 - \varepsilon) \] – an additional correction to the \( \log B \)

- Slow variations of \( B, \alpha \) or \( I \rightarrow \varepsilon \equiv 0 \) (Pons+, 1209.2273; Arzamasskiy+, 1504.06626; Hamil+, 1608.01383)

- Additional component in the braking torque \( \rightarrow \varepsilon < 0.8 \neq 0 \) (AB, G. Beskin, Karpov, 1105.5019)
B(t) trend including timing noise

\[
\log B = \log B_{md} + \Delta_B^{Bsk21} + \Delta_\epsilon
\]

\[
\epsilon = 0.8 \sin \varphi
\]

\[
\varphi \sim \text{uniform}(0, 2\pi)
\]
\[ P \dot{P} = \eta_{GR}(x) \times \frac{4\pi^2 R^6}{Ic^3} B^2 \]

– aligned rotator in a curved space-time

(Gralla, Lupsasca, Philippov, 1604.04625)

\[ \eta_{GR}(x) \approx \frac{0.42 \cdot x^6}{[x(x + 2) + 2 \ln(1 - x)]^2} \]

\[ x = \frac{2GM}{Rc^2} \]

\[ \Delta_{GR}^{(eos)}(x) = -\frac{1}{2} \log \eta_{GR}(x) \]