NEUTRON STAR EQUATION OF STATE AND UNCERTAINTY ON THE RADIUS DETERMINATION

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Astro-PF
Polish-French collaboration in astrophysics
compstar
NATIONAL SCIENCE CENTRE
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Equation of state

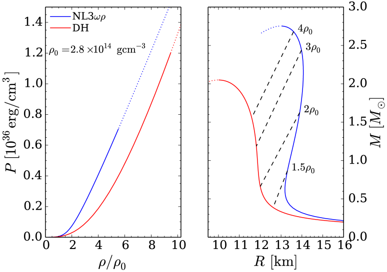
Oertel et al. arXiv:1610.03361 RMP (2017), Haensel et al. book (2007)

Mystery : equation of state (EoS)

- Neutron star matter: many-body system of strongly-interacting particles (e, p, n, μ, more?) at T=0.
- EoS: describes its composition and properties;
- $P(\rho)$ with P the pressure, ρ the energy density.

Mass-radius plot

EoS + TOV equations



Key point

How to constrain the EoS and thus the properties of the nuclear interaction at large densities thanks to NS observations and experiments ?
$\Rightarrow$ mass and radius measurements for example.

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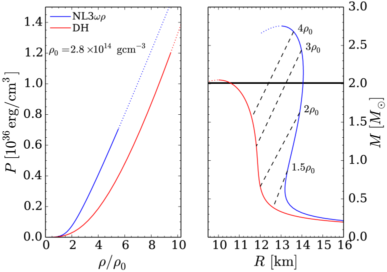

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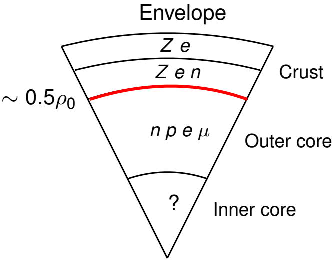
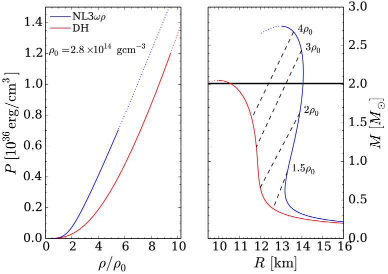
NS structure



Mass-radius plot

EoS + TOV equations



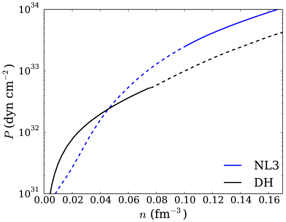
EoS

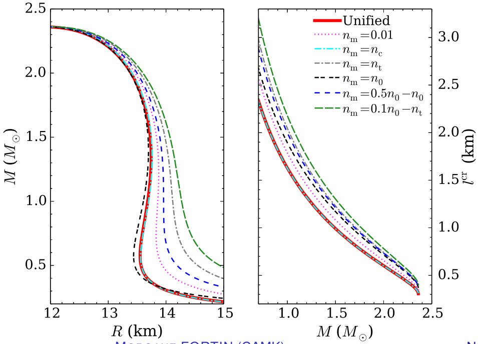
- core: homogeneous mixture
- crust: lattice of neutron rich atomic nuclei → non-uniform.

⇒ many more core EoS than crust EoS.

How to glue an EoS for the core to one for the crust?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, PRC 94 (2016)





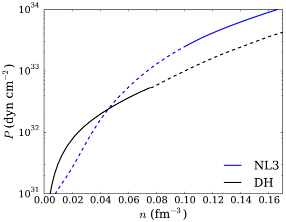
- core glued to BPS+BBP EoS at 0.01 fm^{-3};
- transition at the crossing density between the 2 EoSs;
- transition at the core-crust transition density n_t;
- transition at $n_0 = 0.16$ fm^{-3};
- crust below $0.5n_0$ and core above n_0;
- crust below $0.1n_0$ and core above n_t;
- reference: unified EoS.

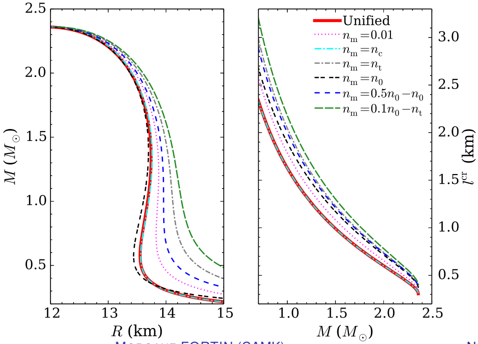
Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim$ 4% (up to $\sim$ 30% on the crust thickness),
- decreases if crust and core EoSs with similar saturation properties.

How to glue an EoS for the core to one for the crust?

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, PRC 94 (2016)





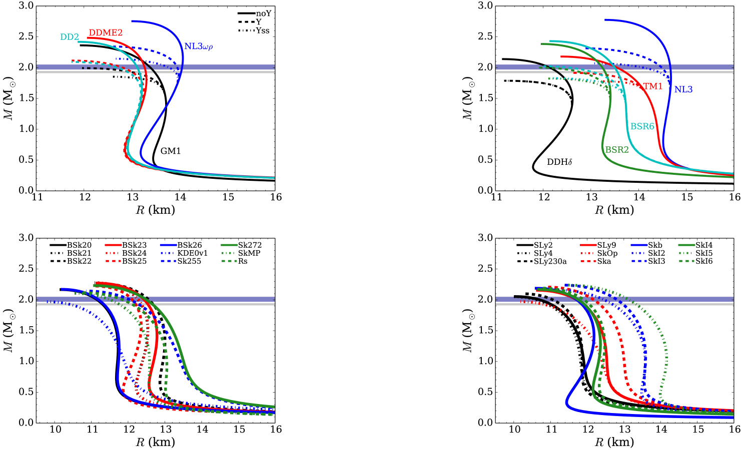
- core glued to BPS+BBP EoS at 0.01 fm^{-3};
- transition at the crossing density between the 2 EoSs;
- transition at the core-crust transition density $n_{\rm t}$;
- transition at $n_0 = 0.16$ fm^{-3};
- crust below $0.5n_0$ and core above n_0;
- crust below $0.1n_0$ and core above $n_{\rm t}$;
- reference: unified EoS.

Uncertainty on R

- due to the treatment of the core-crust transition: up $\sim 4\%$
- with NICER, Athena or LOFT(?): expected precision $\sim 5\%$ ....
- how to, if not solve, at least handle this problem?

1. Unified equations of state

Very few unified EoSs for NSs exist
eg. Douchin & Haensel 01, BSk EoS (Chamel+), Sharma+ 15



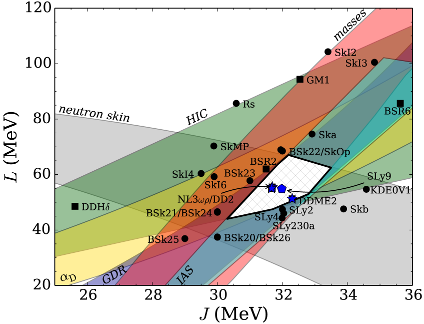
Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, PRC 94 (2016)

33 nucleonic EoSs and 15 hyperonic EoSs
Tables with n, ρ, P as supplemental material to the paper + soon on Compose

1. Unified equations of state

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, PRC 94 (2016)

Nuclear constraints

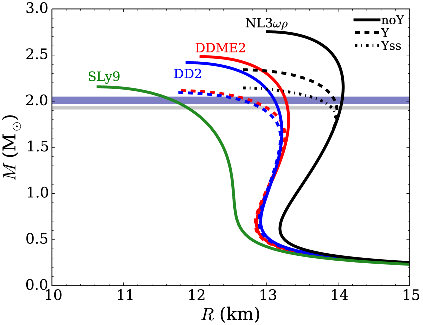


- neutron skin thickness of ^{208}Pb
- heavy ion collisions (HIC)
- electric dipole polarizability α_{D}
- giant dipole resonance of ^{208}Pb
- measured nuclear masses
- isobaric analog states (IAS)

Low-density: $n_{\mathrm{b}} < n_0$

Hebeler et al. ApJ (2013): chiral effective field theory;
Gandolfi et al. PRC (2012): Quantum Monte Carlo technique

Selected EoSs



$R_{1.4} = 13.10 \pm 0.65$ km.

2. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, A&A (2017)

Thickness of a shell in a catalyzed crust

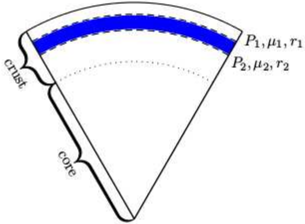
Assuming that in the crust $m \approx M$ and $4\pi r^3 P/mc^2 \ll 1$ in the TOV equation one obtains:

$$\frac{\mathrm{d}P}{\rho + P/c^2} = -GM \frac{\mathrm{d}r}{r^2(1 - 2GM/rc^2)} \ .$$

With

$$\frac{\mathrm{d}P}{\rho c^2 + P} = \frac{\mathrm{d}\mu}{\mu} \qquad \text{one gets} \qquad \frac{\sqrt{1 - 2GM/r_2 c^2}}{\sqrt{1 - 2GM/r_1 c^2}} = \frac{\mu_2}{\mu_1}$$

valid for no jump in the chemical potential.



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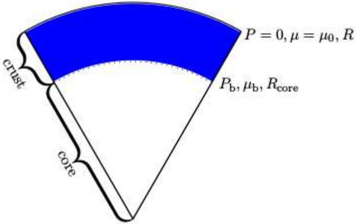
valid for no jump in the chemical potential.
Taking $r_1 = R$ and $r_2 = R_{\rm core}$

$$\frac{\sqrt{1 - 2GM/Rc^2}}{\sqrt{1 - 2GM/R_{\rm core} c^2}} = \frac{\mu_{\rm b}}{\mu_0}$$

with $\mu_0 = \mu(P = 0) = 930.4$ MeV - minimum energy per nucleon of a bcc lattice of ^{56}Fe and $\mu_{\rm b}$ at the core-crust transition.



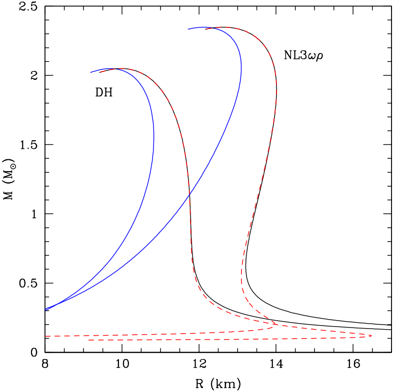
2. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, A&A (2017)

- All you need is . . . : the core EOS down to a chosen density $n_{\rm b}$ with $\mu(n_{\rm b}) = \mu_{\rm b}$.
- Obtain the $M(R_{\rm core})$ relation solving the TOV equations.
- Obtain $M(R)$ with $R =$ $R_{\rm core}/\left(1 - (\frac{\mu_{\rm b}^2}{\mu_0^2} - 1)(\frac{R_{\rm core}c^2}{2GM} - 1)\right)$.

Results

- uncertainty in the radius: $\lesssim 0.2\%$ for $M > 1\ M_\odot$
- uncertainty in the crust thickness: $\lesssim 1\%$ for $M > 1\ M_\odot$



Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\rm core})$
Approximate $M(R)$ for $n_{\rm b} = 0.077\ {\rm fm}^{-3}$

2. Approximate formula for the radius and crust thickness

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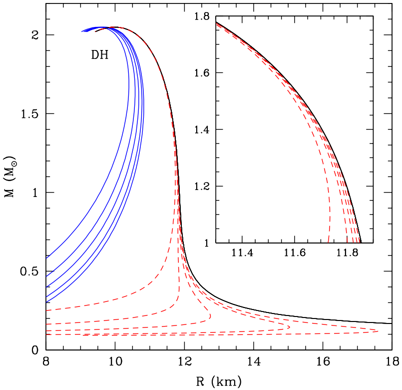
How to choose the core-crust transition density $n_{\rm b}$?

- inversely proportional to L (Horowitz & Piekarewicz 2001)
- Ducoin et al. (2011): for EOSs with $30 \leq L \leq 120$ MeV, obtain: $0.06 \lesssim n_{\rm b} \lesssim 0.10$ fm^{-3}

$\Rightarrow$ $n_{\rm b} \simeq n_0/2 = 0.08$ fm^{-3}



Solution of the TOV equation with a unified EoS
TOV solution for the core $M(R_{\rm core})$
Approximate $M(R)$ for $n_{\rm b} = 0.16, 0.13, 0.11, 0.09, 0.077$ fm^{-3} from left to right.

2. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, A&A (2017)

Thickness of a shell in an accreted crust

For a catalyzed crust

$$\frac{\sqrt{1-2GM/r_2c^2}}{\sqrt{1-2GM/r_1c^2}}=\frac{\mu_2}{\mu_1}$$

For an accreted crust

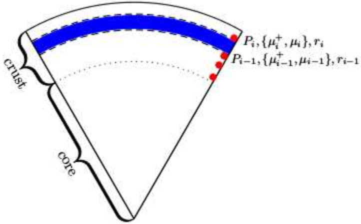
$$\frac{\sqrt{1-\frac{2GM}{Rc^2}}}{\sqrt{1-\frac{2GM}{R_1c^2}}}=\frac{\mu_1^+}{\mu_0}$$

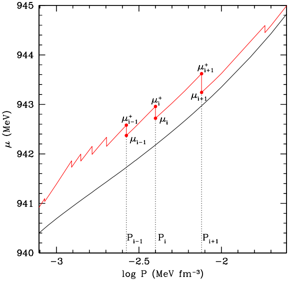
$$\dots$$

$$\frac{\sqrt{1-\frac{2GM}{R_ic^2}}}{\sqrt{1-\frac{2GM}{R_{i+1}c^2}}}=\frac{\mu_{i+1}^+}{\mu_i}$$

$$\dots$$

$$\frac{\sqrt{1-\frac{2GM}{R_nc^2}}}{\sqrt{1-\frac{2GM}{R_{\rm core}c^2}}}=\frac{\mu_{\rm b}}{\mu_n}$$





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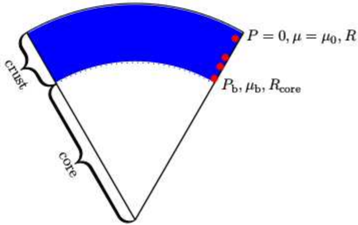


For a catalyzed crust

$$\frac{\sqrt{1 - 2GM/r_2c^2}}{\sqrt{1 - 2GM/r_1c^2}} = \frac{\mu_2}{\mu_1}$$

For an accreted crust

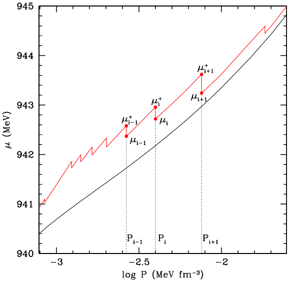
$$\frac{\sqrt{1 - \frac{2GM}{Rc^2}}}{\sqrt{1 - \frac{2GM}{R_\mathrm{core}c^2}}} = \frac{\mu_1^+}{\mu_1} \cdot \frac{\mu_2^+}{\mu_2} \cdots \frac{\mu_i^+}{\mu_i} \cdots \frac{\mu_n^+}{\mu_n} \cdot \frac{\mu_\mathrm{b}}{\mu_0}$$

$$= \frac{\mu_\mathrm{b}}{\mu_0} \cdot \prod_{i=1}^{n} \frac{\mu_i^+}{\mu_i}$$

Energy release at P_i: $Q_i = \mu_i^+ - \mu_i$.

$$\frac{\sqrt{1 - \frac{2GM}{Rc^2}}}{\sqrt{1 - \frac{2GM}{R_\mathrm{core}c^2}}} \simeq \frac{\mu_\mathrm{b}}{\mu_0} \cdot \left(1 + \frac{Q^\mathrm{tot}}{\mu_\mathrm{IC}}\right)$$

with $Q^\mathrm{tot} = \sum_{i=1}^{n} Q_i$ the total energy release in the crust and the mean chemical potential in the inner-crust $\mu_\mathrm{IC} \simeq 941\ \mathrm{MeV}$.



3. Approximate formula for the radius and crust thickness

Zdunik, Fortin, and Haensel, A&A (2017)

Catalyzed vs. accreted crusts

$$\frac{\sqrt{1-\frac{2GM}{Rc^2}}}{\sqrt{1-\frac{2GM}{R_{\rm core}c^2}}} \simeq \frac{\mu_{\rm b}}{\mu_0}\cdot(1+\frac{Q^{\rm tot}}{\mu_{\rm IC}})$$

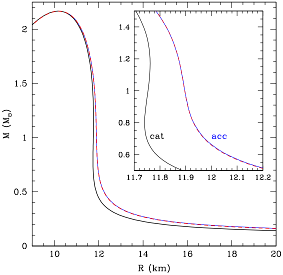
Radius of a star with a catalyzed crust: $R_{\rm cat}$ with an accreted crust $R_{\rm acc}$.

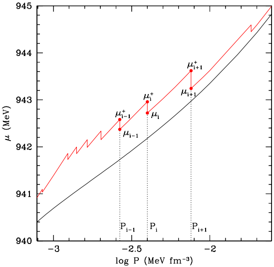
$$R_{\rm acc} = \frac{R_{\rm cat}}{1-(\alpha-1)(\frac{R_{\rm cat}c^2}{2GM}-1)}$$

with $\sqrt{\alpha} \equiv \prod_{i=1}^{n}\frac{\mu_i^+}{\mu_i} = (1+\frac{Q^{\rm tot}}{\mu_{\rm IC}})$.

Difference in the radius between a NS with an accreted crust and a catalyzed crust

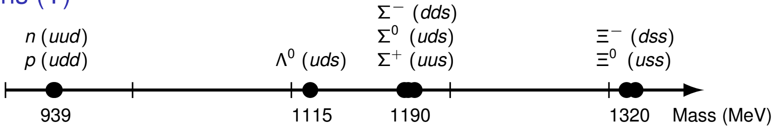
$$\Delta R \simeq 144\,{\rm m}\cdot\left(\frac{Q^{\rm tot}}{2\,{\rm MeV}}\right)\left(\frac{R_{\rm cat}}{10\,{\rm km}}\right)^2\left(\frac{M}{M_\odot}\right)\left(1-\frac{2GM}{R_{\rm cat}c^2}\right)$$





Hyperonic equations of state

Hyperons (Y)



n (uud)
p (udd)

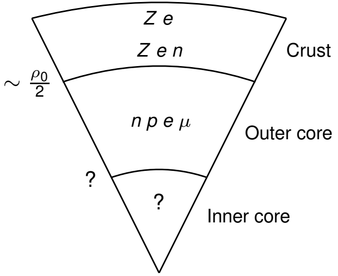
Λ^0 (uds)

Σ^- (dds)
Σ^0 (uds)
Σ^+ (uus)

Ξ^- (dss)
Ξ^0 (uss)

939 1115 1190 1320 Mass (MeV)

Structure



$Z\ e$

$Z\ e\ n$

Crust

$\sim \frac{\rho_0}{2}$

$n\ p\ e\ \mu$

Outer core

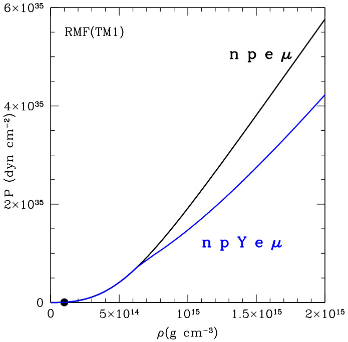
?

?

Inner core

$\rho_0 \simeq 3 \times 10^{14}$ g cm^{-3}

Equation of state



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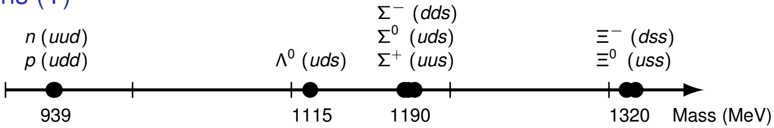
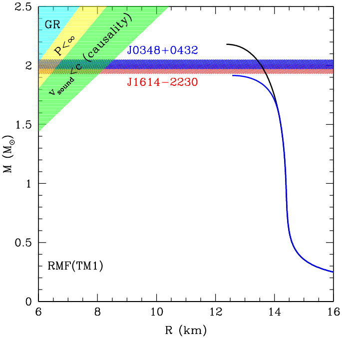
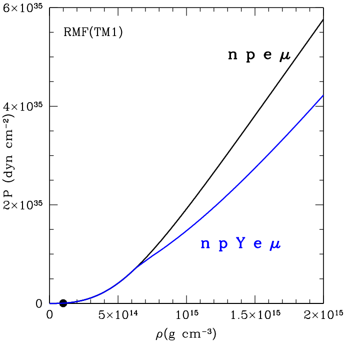


$M - R$ plot

Equation of state





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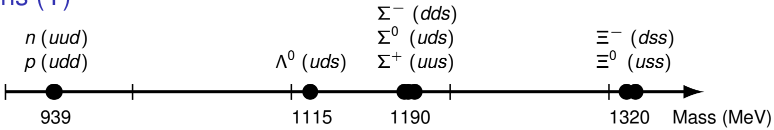

$n\,(uud)$
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$\Lambda^0\;(uds)$

$\Sigma^-\;(dds)$
$\Sigma^0\;(uds)$
$\Sigma^+\;(uus)$

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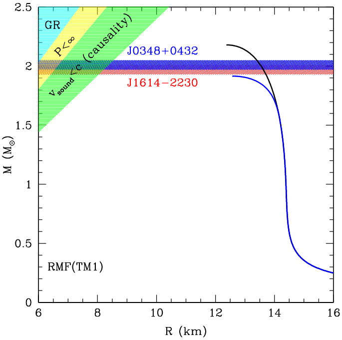
$M - R$ plot



- each EoS has a maximum mass $M_{\rm max}$;
- $M_{\rm max}$ strongly reduced when Y are included;
- consistency with the observations: $M_{\rm max} \geq M_{\rm max}^{\rm obs}$.

Hyperon puzzle

Can hyperons be present in NSs and yet $M_{\rm max} \geq M_{\rm max}^{\rm obs}$ with $M_{\rm max}^{\rm obs} \simeq 2\,M_\odot$?

Hyperonic equations of state

Fortin, Zdunik, Haensel and Bejger, A&A 576 (2015)

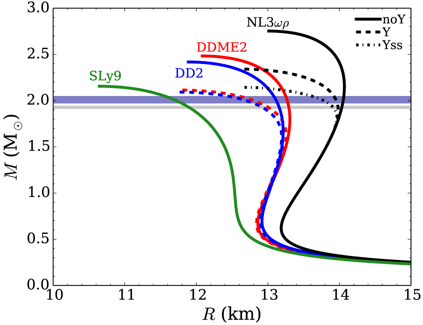
14 hyperonic EoSs consistent with 2 $M_\odot$:

- large radius for hyperonic EoSs correlated with a large pressure at n_0
- over-pressure at n_0 for hyperonic EoSs inconsistent with up-to-date microscopic calculations by Hebeler et al. (2013)

→ 2 $M_\odot$ is reach by compensating the decrease of the pressure at high density due to Y by a large pressure at low density

Fortin et al. PRC 94 (2016)

Hyperonic EoSs consistent with Hebeler et al. constraint and with $M_{\rm max} \geq 2\,M_\odot$.



+ eg. Oertel et al. JPG (2015)

Experimentally calibrated hyperonic EoSs

Fortin, Providência, Vidaña, and Avancini, PRC 95 (2017)

Experimental hypernuclei data

Gal et al., RMP (2016)

- $\sim$ 40 Λ-hypernuclei
  + measurement of binding energy B_Λ
- few Ξ-hypernuclei
  but no measurement of binding energy
- no Σ-hypernuclei
  repulsive Σ-nucleon interaction?
- only one unambiguous ΛΛ-hypernuclei:
  measurement of the bond energy:

$$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 0.67 \pm 0.17 \text{ MeV}.$$

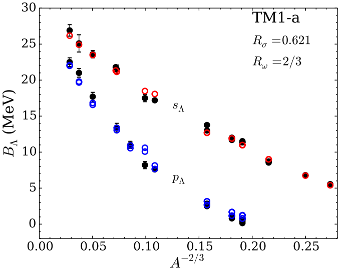
Usual approach to hyperons

Adjust the couplings for the Λ to reproduce:

- the Λ-potential in symmetry nuclear matter $U^N_\Lambda(n_0)$: usually (-30, -28) MeV
- the Λ-potential in pure Λ matter $U^\Lambda_\Lambda(n_0, n_0/5)$: usually (-5, -1) MeV

Modeling of hypernuclei

- for the TM1, TM2$\omega\rho$, NL3, NL3$\omega\rho$, DDME2 RMF models.
- adjust the Λ-couplings to reproduce:
  1. Λ-hypernuclei:



$$U^N_\Lambda(n_0) \in [-36, -30] \text{ MeV}$$

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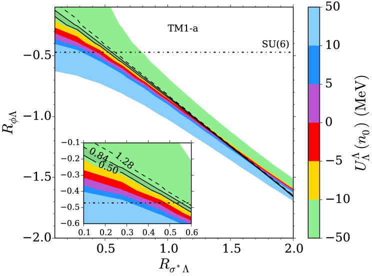
Modeling of hypernuclei

- for the TM1, TM2$\omega\rho$, NL3, NL3$\omega\rho$, DDME2 RMF models.
- adjust the Λ-couplings to reproduce:
  1. Λ-hypernuclei:
  2. ΛΛ-hypernuclei



$$U_\Lambda^\Lambda(n_0) \in [-14, -9]\ \mathrm{MeV}$$
$$U_\Lambda^\Lambda(n_0/5) \in [-7, -5]\ \mathrm{MeV}$$

Experimentally calibrated hyperonic EoSs

Fortin, Providência, Vidaña, and Avancini, PRC 95 (2017)

Experimental hypernuclei data

Gal et al., RMP (2016)

- $\sim$ 40 Λ-hypernuclei
  + measurement of binding energy B_Λ
- few Ξ-hypernuclei
  but no measurement of binding energy
- no Σ-hypernuclei
  repulsive Σ-nucleon interaction?
- only one unambiguous ΛΛ-hypernuclei: measurement of the bond energy:

$$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 0.67 \pm 0.17\ \mathrm{MeV}.$$

Usual approach to hyperons

Adjust the couplings for the Λ to reproduce:

- the Λ-potential in symmetry nuclear matter $U_\Lambda^N(n_0)$: usually (-30, -28) MeV
- the Λ-potential in pure Λ matter $U_\Lambda^\Lambda(n_0, n_0/5)$: usually (-5, -1) MeV

Modeling of hypernuclei

- for the TM1, TM2$\omega\rho$, NL3, NL3$\omega\rho$, DDME2 RMF models.
- adjust the Λ-couplings to reproduce:
  1. Λ-hypernuclei:
  2. ΛΛ-hypernuclei
- build calibrated hyperonic EoSs including the other hyperons and the current experimental uncertainty of their properties

Conclusions

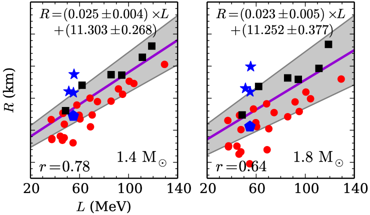
- models are consistent with 2 $M_\odot$
- because lack of constraints on the nuclear model at high density
- thus no solution to the hyperon puzzle at the moment

Nuclear parameters and radii

Fortin, Providência, Raduta, Gulminelli, Zdunik, Haensel, & Bejger, PRC 94 (2016)

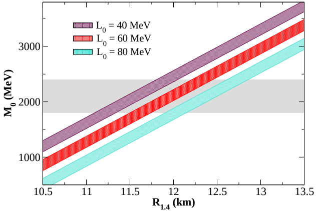
- 33 nuclear models (RMF & Skyrme)
- *L* the slope of the symmetry energy
- correlated to ^{48}Ca and ^{208}Pb neutron skin thickness (PREX & CREX)

also in Horowitz & Piekarewicz works.



Alam, Agrawal, Fortin, Pais, Providência, Raduta, Sulaksono, PRC 94 (2016)

- 44 nuclear models (RMF, Skyrme, & BHF)
- *L* the slope of the symmetry energy
- *M* the slope of the incompressibility.
- gray strip: experimental constraint from giant monopole resonance (De+ PRC 92, 2015)
- $R_{1.4} = 11.09 - 12.86$ km



Conclusions

- Be careful when gluing an EoS for the core to one for the crust:
- Use unified equations of state eg. Fortin+ PRC 94 (2016)
- Approximate formula for $M(R)$ with no crust needed (Zdunik+ A&A, 2017).
- Experimentally calibrated hyperonic EoSs (Fortin+ PRC 95, 2017)
- Hyperonic equations of state are *not* ruled out by the existence of 2 $M_\odot$ neutron stars.
- Correlations between the neutron star radius and nuclear parameters measurable in laboratory (Fortin+ and Alam+ PRC 94, 2016)