Partial frequency redistribution in cyclotron lines of neutron stars

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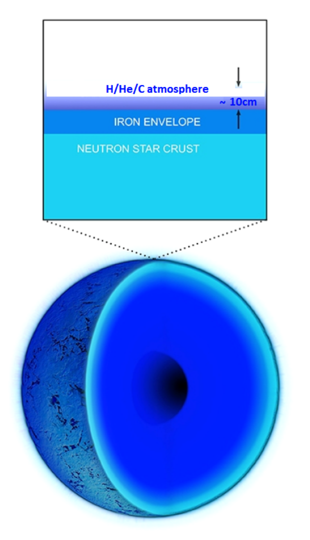
Why to study cyclotron lines?

1. Detailed models of radiation transfer in view of new missions: XARM, ATHENA, LYNX
2. Structure of outer layers of NS atmospheres (Wasserman et al., 1989; Garasyov et al. 2012)
3. Radiation pressure effects in NS atmospheres and magnetospheres
4. Modeling the outbursts in accreting X-ray binaries (Rothschild et al. 2017)

Atmospheric structure



- Wide range of temperatures and magnetic field strengths
- Scattering dominated over absorption $\gamma_a/\gamma_{sc} \lesssim 10^{-4} - -10^{-6}$
- The opacity of magnetized plasma is very high near the frequencies $\omega = n\omega_B$, where $\omega_B = eB/(mc)$.

$$\frac{\sigma_{cyc}}{\sigma_T} \sim \frac{1}{\alpha\beta_T}\frac{mc^2}{\hbar\omega_B} \approx 4 \cdot 10^5 \frac{1}{\sqrt{T_{100\,eV}}B_{12}}$$

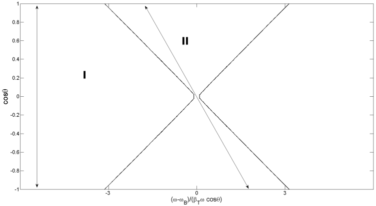
- Peculiar resonance scattering, qualitatively different from scattering in atomic lines

Frequency redistribution during scattering

Two main reasons: finite width of the resonance and Doppler broadening

- Complete frequency redistribution
- Partial redistribution
- Coherent scattering

Basic view of cyclotron scattering



Two types of photons: photons in the line's core with $(\omega - \omega_B)/(\beta_T \omega_B) \lesssim 3$ and photons in the wings with $(\omega - \omega_B)/(\beta_T \omega_B) > 3$.

I. Photons in the wing scatters almost coherently

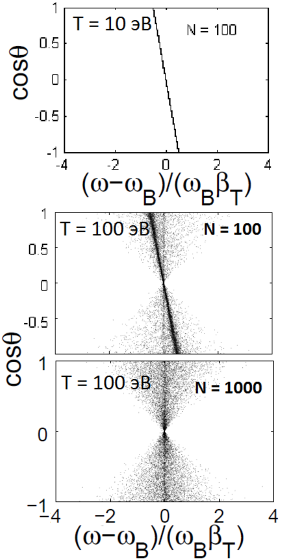
$$\omega' \approx \omega$$

II. Photons from the core usually follow the relation

$$\omega'(1 - \beta \cos\theta') = \omega(1 - \beta \cos\theta).$$

$\beta = v/c$ – velocity of some particular electron, $\beta_T = \sqrt{T/mc^2}$ – typical or 'thermal' velocity. θ – angle between the magnetic field and the wave vector.

Trapping of resonant photons in the line core



Resonance condition. Nonrelativistic approximation. Quasicoherent scattering (Zheleznyakov, Litvinchuk, 1987)

$$\omega(1 - \beta\cos\theta) = \omega_{\rm B}.$$

$$\beta_* = (\omega - \omega_{\rm B})/(\omega\cos\theta)$$

$$(\omega, \theta) \Longleftrightarrow \left(\frac{\omega - \omega_{\rm B}}{\omega\cos\theta}, \theta\right).$$

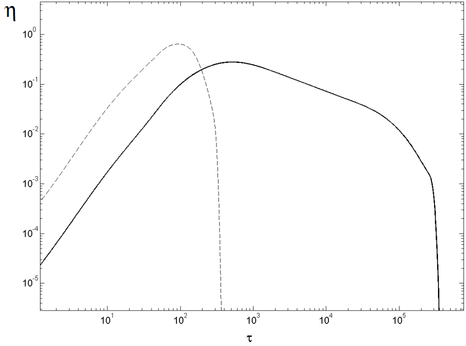
Mildly relativistic approximation

$$\omega\left(1 - \beta\cos\theta + \frac{\beta^2}{2}\right) = \omega_{\rm B}.$$

Two resonance velocities:

$$\beta_{1,2} = \cos\theta \pm \sqrt{\cos^2\theta - 2\left(1 - \frac{\omega_{\rm B}}{\omega}\right)}.$$

Importance of relativistic effects



Relative fraction of photons emitted at optical depth τ in the emergent spectra. Solid line — with redistirbution effects; dashed — without (quasicoherent scattering). Atmospheric parameters: $T = 50\,\text{eV}$, $\gamma/\omega_B = 10^{-6}$, $P_{abs}/P_{sc}(\tau = 1) = 10^{-6}$.

Some analytical results

Garasev et al. MNRAS 2016a, Garasyov et al. A&A 2011

- Escape probability: $P_{esc} \approx \frac{\beta_T}{\tau\sqrt{8 \ln \tau}}$
- Redistribution escape probability > diffusion probability $(1/\tau^2) \rightarrow$, $\tau > \beta_T/4 = \tau*$
- $\tau* < \tau_{th}$:

$$\frac{B}{B_{cr}} \gtrsim \cdot 10^{-4} \frac{1}{\sqrt{g_{14} T_{50eV}}}$$

- Quantum recoil important if

$$\frac{B}{B_{cr}} \gtrsim \beta_T = 0.01/\sqrt{T_{50eV}}$$

Application to the radiation transfer problem

Radiation transfer equation:

$$\cos\theta \frac{dI}{dz} = -(\mu + \kappa)I + \int R(\theta', \omega' \to \theta, \omega) I(\theta', \omega', z) d\theta' d\omega' + \epsilon. \qquad (1)$$

where μ, κ — scattering and absorption coefficients, ϵ — emission rate, $R(\theta', \omega' \to \theta, \omega)$ — the frequency redistribution function. For cyclotron line R have very narrow peak. The location of this peak strongly dependent on the angles, which complicates usage of standard methods. However, it is possible to construct approximate redistribution function R' for some fixed angle of propagation:

$$\pm\cos\theta_0 \frac{dI}{dz} = -(\mu' + \kappa')I + \int R'(\omega' \to \omega) I(\omega') d\omega' + \epsilon' .$$

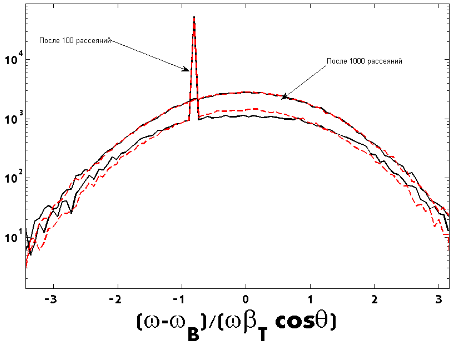
Approximate redistribution function

The R' must fulfill to some properties:
1. $\int R'(\omega \to \omega')d\omega' = \mu'(\omega)$ — normalization
2. $R'(\omega \to \omega') = \left(\frac{\omega'}{\omega}\right)^3 \exp\left(\frac{\hbar\omega - \hbar\omega'}{kT}\right) R'(\omega' \to \omega)$ — the detailed balance condition.
The simplest form for that function:

$$R'(\omega \to \omega') = P_{\mathbf{wing}}(\omega, \omega')\delta(\omega - \omega') + (1 - P_{\mathbf{wing}}(\omega, \omega'))\left[(1 - P_{\mathbf{res}})\delta(\omega' + \omega - 2\omega_B) + P_{\mathbf{res}} * \mu(\omega)\mu(\omega').\right]$$

First term describes coherent scattering, the second one models the relativistic redistribution, while the third describes the total redistribution in the limit of large number of scatterings. P_{wing} describes the scattering in the wings of line, it could be found in (Kneer, 1975), $P_{res} \approx \beta_T$ describes the probability of noncoherent scattering for photon from the line's core.

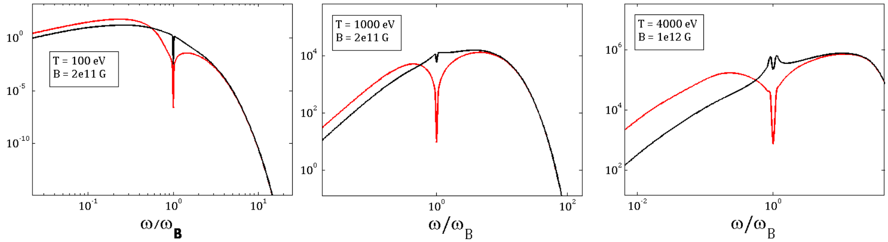
Exact vs. Approximate Redistribution function



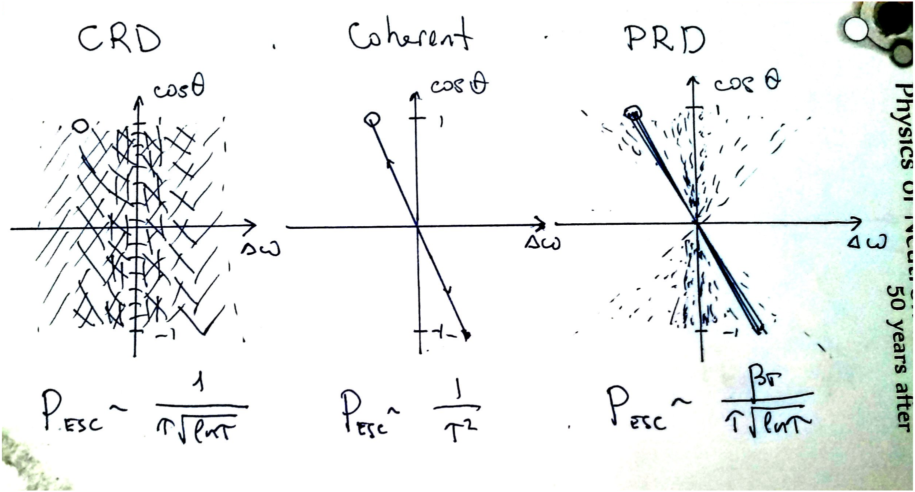
Domparision of redistribution of a bunch of photons with initial frequency $\omega = \omega_B(1 - 0.8\beta_T)$ and initial angle $\cos\theta = 1$ calculated for exact RF (black curves) and approximated RF (red curves).

$T = 100eV$

Preliminary calculations



Summary



CRD
Coherent
PRD
cos θ
Δω
1
-1
$P_{ESC} \sim \frac{1}{\tau\sqrt{\ln \tau}}$
$P_{ESC} \sim \frac{1}{\tau^2}$
$P_{ESC} \sim \frac{\beta_T}{\tau\sqrt{\ln \tau}}$