Equation of State of Neutron Stars
Recent Developments

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Introduction

Measured masses $M_i$ and radii $R_j$ supply information about the EOS of NS
Ideally: measured\{R_i, M_i\} $\implies$ deduced $P(\rho)$
in reality: theoretical $P(\rho) \implies$ predicted $M(R)$

Crucial fact $M_{\text{max}} > 2 M_\odot \approx 3M_{\text{max}}^{\text{FFG}}$
Nuclear forces push up $M_{\text{max}}$ by a factor of about three: strong interaction is dominant in the EOS of NS

Convenient units for density $\varepsilon_0/c^2 = \rho_0 = 2.8 \times 10^{14}$ g cm$^{-3}$, $n_0 = 0.16$ fm$^{-3}$

Density regimes in dense degenerate matter

$\rho \lesssim \rho_0$ NS crust; nuclear theory methods valid, but some uncertainties persist

$\rho_0 \lesssim \rho \lesssim 10\rho_0$ NS cores: uncertainties growing with density. Deconfinement of quarks? Maybe, but in the non-perturbative QCD regime. Fortunately NS exist and are available for measurements!

$10\rho_0 < \rho \lesssim 100\rho_0$ no stars in this density range. Strong coupling QCD plasma, reliable calculations not possible

$\rho > 100 \rho_0$ quark plasma, perturbative QCD enables reliable calculation of the EOS
Nuclear structures and phenomena in terms of $N = n, p$ interacting via pion exchanges (long and intermediate range force), while short-range force components are represented by contact (zero-range) interactions.

Momentum/Energy scales: momenta in the system $Q$, cut-off momentum in the momentum integration $\Lambda$, chiral symmetry restoration $\Lambda_{\chi}$. Few-body observables calculated using diagrammatic perturbative expansion ordered by $(Q/\Lambda)^k$

$$\hat{H}(\Lambda) = \hat{T} + \hat{V}_{2N}(\Lambda) + \hat{V}_{3N}(\Lambda) + \ldots$$

Simplest many-body systems: pure neutron matter (PNM)

<table>
<thead>
<tr>
<th>$k$</th>
<th>LO</th>
<th>$(Q/\Lambda)^0$</th>
<th>$c_1$</th>
<th>NLO</th>
<th>$(Q/\Lambda)^2$</th>
<th>$c_1, c_2$</th>
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</thead>
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<tr>
<td>0</td>
<td></td>
<td>$c_1$</td>
<td></td>
<td>2</td>
<td>$(Q/\Lambda)^2$</td>
<td>$c_1, c_2$</td>
</tr>
<tr>
<td>3</td>
<td>N^2LO</td>
<td>$(Q/\Lambda)^3$</td>
<td></td>
<td>4</td>
<td>$(Q/\Lambda)^4$</td>
<td>2BF + 3BF accounted for</td>
</tr>
<tr>
<td>4</td>
<td>N^3LO</td>
<td>$(Q/\Lambda)^4$</td>
<td>2BF + 3BF + 4BF accounted for</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Calculations are "parameter free" except for $\Lambda$; constants $c_1, c_2, \ldots$ determined fitting ChEFT results to experimental measurements.

Calculation of $E(n_b)$ of PNM with

$$\hat{H}(\Lambda) = \hat{T} + \hat{V}_{2N}(\Lambda) + \hat{V}_{3N}(\Lambda) + \hat{V}_{4N}(\Lambda)$$

$\Lambda = 500 - 700$ MeV

$\equiv 2.53 - 3.55$ fm$^{-1}$

Remember: $\Lambda_\chi \simeq 1$ GeV

$E_{4N}(n_0) \simeq -0.18$ MeV

- small compared to

uncertainties in $E$ from $c_3, c_4, \ldots$
B-fields coupled to M-fields, static solutions in the mean-field approximation \( \Rightarrow \) EOS. **Puzzle:** how to make RMF simultaneously consistent with nuclear and astrophysical constraints?

**Way I** - minimal structure of \( \mathcal{L} \) but \( g_{BM} \) and \( m_M \) scaled with scalar meson field \( \sigma \) (Kolomeitsev, Maslov, Voskresensky) or \( g_{BM} \) are density dependent (Typel et al.)

**Way II** - more meson fields and higher order couplings - RMF with quartic terms added

**Kolomeitsev et al. N-matter**

Universal scaling of masses with \( f = \frac{g^* \sigma N}{m_N} \sigma \) assumed, where \( g^* \sigma N = g \sigma N \chi \sigma N (\sigma) \)

\[
S_N(f) = S_M(f) = 1 - f
\]

\( \mathcal{L} \) is quadratic in \( \omega_\mu, \rho_\mu^i, \phi_\mu \) so that the equations of motion for vector fields are trivial

What we have to solve is \( \partial E[n_n, n_p, f]/\partial f = 0 \)
Kolomeitsev et al. NHΔ-matter

Universal scaling of masses with $f$, empirical $U_H$ to get $g_{\sigma H}$ at $n_0$, scaling using SU(6) for the vector mesons. $M_{\text{max}} > 2 M_\odot$ if $\phi_\mu$ (coupled to $H$ only) is included.

Very low strangeness per baryon ($\sim 1\%$) even at $M_{\text{max}}$.

Features of the EOS
(a) $n_b \lesssim 4n_0$ SNM rather soft to satisfy nuclear constraints
(b) $n_b \gtrsim 4n_0$ PNM stiff to get $2 M_\odot$

No problem with $\Delta^-$ provided $U_\Delta = -50 \ldots -100$ MeV
$\Delta$ fraction is large but resulting decrease of $M_{\text{max}}$ is small

$\Delta$ and $H$ puzzles are solved with using appropriate scaling of coupling constants and masses, respecting nuclear physics and $2M_\odot$ constraints.

Typel et al. NH matter

Density dependence of coupling constants via $n_b = \sqrt{j_b^\mu j_b^\mu}$. Single baryon fluid assumed. Successful parametrization - DD2 model.
Relativistic Mean Field - solving the puzzles -3

\[ P \left[ 10^{36} \text{erg/cm}^3 \right] \]

\[ \varepsilon / \varepsilon_0 \]

\[ R \ [\text{km}] \]

\[ N \]
\[ H_{\phi\sigma} \]
\[ \Delta - U_\Delta = -100 \text{ MeV} \]
\[ H_{\phi\sigma}\Delta \]
\[ \text{DD2} \]
\[ \text{DD2 H} \]

\[ 4n_0 \]
\[ 3n_0 \]
\[ 2.5n_0 \]
Hyperonic EOS with Dirac-Brueckner-Hartree-Fock theory

Katayama & Sato (2015)

Lorentz invariant many-body theory with One-Boson-Exchange Bonn "potentials". Self-consistent self-energies instead of $U_B \rightarrow \Sigma_B^S, \Sigma_B^V$. Density dependence of $\Sigma_N^V$ results in no need for strong 3BF to fit nuclear matter. Multiply coupled Bethe-Salpeter equations for four G-matrices. Complexity much higher than for the non-relativistic BHF theory.

Numerical calculations - N matter

Calculations for Bonn A,B,C OBE nucleon interactions - saturation parameters very poor. Decisive point: Bonn A $\rightarrow$ Bonn A* via $g_{N\sigma} \rightarrow g_{N\sigma}^* (\Sigma_N^S)$ (analogously to Kolomeitsev et al. 2016)

Enhancement of $g_{N\sigma}^*$ by 2% is sufficient to get correct saturation parameters

Numerical calculations - NH matter

Bonn A* applied to NH matter using SU(6) to get coupling of H to vector mesons

Experimental $U_{\Lambda,\Sigma,\Xi}$ potential wells $\rightarrow g_{H\sigma}^*$

Universal repulsion comes from $\Sigma_B^V$ density dependence
Table: Configurations with $M_{\text{max}}$ for different versions of A* DBHF models

<table>
<thead>
<tr>
<th>Version</th>
<th>$R_{M_{\text{max}}}$ (km)</th>
<th>$n_c$ ($n_0$)</th>
<th>$M_{\text{max}}$ (M$_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM6</td>
<td>12.6</td>
<td>4.9</td>
<td>2.44</td>
</tr>
<tr>
<td>B5M6</td>
<td>13.1</td>
<td>4.75</td>
<td>2.03</td>
</tr>
<tr>
<td>B8M6</td>
<td>13.1</td>
<td>4.7</td>
<td>2.04</td>
</tr>
<tr>
<td>NM6</td>
<td>13.1</td>
<td>4.6</td>
<td>2.08</td>
</tr>
</tbody>
</table>
New unified EOS -1

Based on the same nuclear interaction model for core & crust. Consistent description of the crust-core transition, relevant for the determination of the NS radius, \( M_{\text{crust}} \), \( I_{\text{crust}} \)

### Energy Density Functional (EDF) method

*Kohn-Sham (1965)*: For an \( A \)-body system there exists a reference system without correlations with orthonormal orbitals \( \varphi_q^i (q = n, p, \ldots) \) which yields actual \( E \), actual density \( n_q(\vec{r}) \),...

\[ E_{\text{eff}} \] depends on the uncorrelated densities

\[ E_{\text{eff}}[n_q, \ldots] = \min \implies \varphi_q^i \]

\[ E = E_{\text{bulk}}^N + E_{\text{surf}}^N + E_{s-o}^N + \ldots + E_{\text{Coul}} + E_{\text{lept}} \]

### BCPM (Barcelona-Catania-Paris-Madrid) EDF *Sharma et al. 2015*

\( E_{\text{bulk}}^N \) - from \( v_{12}(2\text{BF:Argonne18}) + \tilde{v}_{12}(3\text{BF:UIX}) \) two parameters in UIX adjusted to fit \(-16.0 \text{ MeV} \) and \( 0.16 \text{ fm}^{-3} \) \([c_1, c_2]\). Three parameters in the

\( E_{\text{surf}}^N \) \([c_3, c_4, c_5]\). Strength of spin-orbit term \( E_{s-o}^N \) \([c_6]\).
*Sharma et al. 2015 - continued*

Optimized fit to 467 known spherical nuclei $\rightarrow$ values of $c_3, c_4, c_5, c_6$

r.m.s. deviation $\sigma_{E_{\text{bind}}} = 1.3$ MeV

Gives EOS very similar to *Douchin & Haensel 2001* (SLy4), similar $M(R)$ but:

(a) $Z$ in the inner crust $\sim 30$ instead of $\sim 40$ of DH2001;

(b) pasta phases present - while no pastas in DH2001. These differences are probably due to the differences in $E_{\text{surf}}^N$

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*Unified EOS based on RMF models - Fortin et al. 2016*

Euler-Lagrange equations for $\vec{r}$-dependent fields from $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{\text{lept}}$.

Static solutions - only scalar $\sigma$ and $\nu = 0$ components of vector fields. Sources $(n, p)$ fixed $\rightarrow$ M-fields equations of motion

**9 unified RMF EOS calculated.** Presence of pastas - depends on the surface energy (should not be too high!)

Advantage: precise determination of $R(M), I_{\text{crust}}, M_{\text{crust}}$
New unified EOS - 3

![Graph showing the pressure (P) and mass (M) as functions of the ratio of energy density (ε) to its zero (ε₀) and radius (R)].
Baryons have finite size but they are treated as point-like objects (i.e. like quarks and leptons). In the **Excluded Volume** (ExVol) model they are treated (in the rest frame) as non-penetrable spheres of radius $r_N (N = n, p)$

**Historical Remark.** In the perturbative approach (e.g. BHF) ExVol effect is obtained using **hard core** interaction of range $2r_N$ such that $V_{ij} (r_{ij} < 2r_N) = +\infty$. Many hard-core 2BFs in 1960-1970. Example: Reid hard-core with $2r_N = 0.4 \text{ fm}$

**RMF+ExVol - Typel et al. 2016**

**Available volume fraction** $\Phi = \Phi_{NR} = 1 - n_b v$, while correct is $\Phi = 1 - n_b^{(s)} v$

$$
\Phi_{NR} = \frac{1}{2} \frac{4\pi}{3} (2r_N)^3 = 4V_N
$$

Minimal coupling RMF model involving $\sigma, \omega^\mu, \rho^\mu_i$

$$
P_{\text{had.}} (\mu_n, \mu_p) = \frac{1}{\Phi} \sum_N P_N + P_M \\
E_{\text{had.}} (\mu_n, \mu_p) = -P_{\text{had.}} + \sum_N \mu_N n_N
$$
Start with successful RMF DD2 model with density-dependent coupling constants (Typel & Wolter 2009). Implement ExVol to get DD2-EV, still fitting the same saturation parameters. "Non-relativistic" $\Phi_{NR}$ with large $v = 2.86 \text{ fm}^3$ corresponds to $r_N = 0.55 \text{ fm}$ and this means hard-core radius of 1.1 fm.

$\Phi_{NR} = 0$ at $n_{crit} = 0.35 \text{ fm}^{-3}$, pressure explodes there to $+\infty$ and therefore $N\rightarrow Q$ has to occur before this singularity.

This (not-so-fine) tuning results in high-mass twins because

$$\frac{\rho_Q}{\rho_N} > \frac{3}{2} \left( 1 + \frac{P_{NQ}}{\rho_N c^2} \right)$$

an unstable segment of $M(R)$ separates N-stars from more compact NQ-stars, twins with $M \approx 2M_\odot$. 
\Phi\textsubscript{NR} makes DD2-EV extremely stiff.
Excluded volume effects and high-mass twins -3

Using physical $\Phi$ would make the EOS softer and then the twin-NQ stars would most probably go away...

Based on unpublished results of J.L. Zdunik (2015)
**Conclusion**

**Some developments since 2014, but progress is very slow...**

Precise EOS for pure neutron matter for $\rho \leq \rho_0$. ChEFT calculations show that 4BF contribution at $\rho_0$ is negligibly small.

No hope to calculate precise EOS using ChEFT for $\rho > \rho_0$ because of slow convergence at supra-nuclear densities.

Progress: Many new unified EOS are now available; some EOS-dispersion in the inner crust and in the outer NS core.

Scaling of in-medium nuclear force with scalar quantities makes EOS more flexible: this allows H-puzzle and $\Delta$-puzzle to be solved.

Imposing strict astrophysical and nuclear physics constraints is hard to fulfil but possible.

Relativistic Excluded Volume procedure for the hadron EOS avoids drastic stiffening and unphysical infinite pressure: should be carried out . . .