Equation of State of Neutron Stars

Recent Developments

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Introduction

Measured masses M_i and radii R_j supply information about the EOS of NS
Ideally: $\mathbf{measured}\{R_i, M_i\} \Longrightarrow \mathbf{deduced}\ P(\rho)$
in reality: $\mathbf{theoretical}\ P(\rho) \Longrightarrow \mathbf{predicted}\ M(R)$

Crucial fact $\boxed{M_{\max} > 2\ \mathrm{M}_{\odot} \approx 3M_{\max}^{\mathrm{FFG}}}$ **Nuclear forces push up $M_{\max}$ by a factor of about three: strong interaction is dominant in the EOS of NS**

Convenient units for density $\varepsilon_0/c^2 = \rho_0 = 2.8 \times 10^{14}\ \mathrm{g\ cm^{-3}}$, $n_0 = 0.16\ \mathrm{fm^{-3}}$

Density regimes in dense degenerate matter

$\rho \lesssim \rho_0$ **NS crust; nuclear theory methods valid, but some uncertainties persist**

$\rho_0 \lesssim \rho \lesssim 10\rho_0$ **NS cores: uncertainties growing with density.** Deconfinement of quarks? Maybe, but in the non-perturbative QCD regime. Fortunately NS exist and are available for measurements!

$10\rho_0 < \rho \lesssim 100\rho_0$ no stars in this density range. Strong coupling QCD plasma, reliable calculations not possible

$\rho > 100\ \rho_0$ quark plasma, perturbative QCD enables reliable calculation of the EOS

Chiral EFT for few- and many-nucleon systems

Nuclear structures and phenomena in terms of $N = n, p$ interacting via pion exchanges (long and intermediate range force), while short-range force components are represented by contact (zero-range) interactions.

Momentum/Energy scales: momenta in the system Q, cut-off momentum in the momentum integration Λ, chiral symmetry restoration Λ_χ. Few-body observables calculated using diagrammatic perturbative expansion ordered by $(Q/\Lambda)^k$

$$\text{EFT Hamiltonian } \hat{H}(\Lambda) = \hat{T} + \hat{V}_{2N}(\Lambda) + \hat{V}_{3N}(\Lambda) + \ldots$$

Simplest many-body systems: pure neutron matter (PNM)

$\boxed{k = 0 \;\; \mathbf{LO} \;\; (Q/\Lambda)^0 \quad c_1}$ $\boxed{k = 2 \;\; \mathbf{NLO} \;\; (Q/\Lambda)^2 \quad c_1, c_2}$ reminds effective range theory of 1940s!

$\boxed{k = 3 \quad \mathbf{N^2LO} \;\; (Q/\Lambda)^3}$ 2BF + 3BF accounted for

$\boxed{k = 4 \quad \mathbf{N^3LO} \;\; (Q/\Lambda)^4}$ 2BF+3BF+4BF accounted for

EOS from Chiral Effective Field Theory

Calculations are "parameter free" except for Λ; constants $c_1, c_2, ...$ determined fitting ChEFT results to experimental measurements

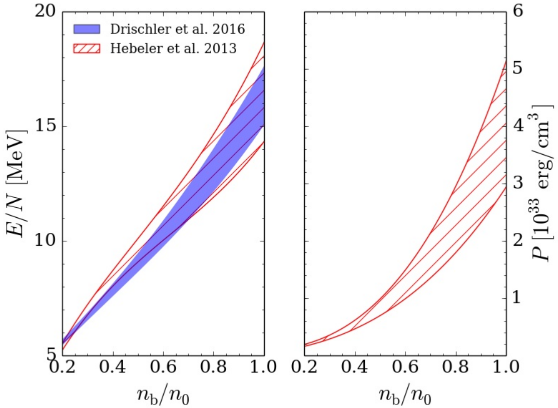
Calculation of $E(n_{\rm b})$ of PNM with $\boxed{\hat{H}(\Lambda) = \hat{T} + \hat{V}_{2N}(\Lambda) + \hat{V}_{3N}(\Lambda) + \hat{V}_{4N}(\Lambda)}$

$\Lambda = 500 - 700$ MeV

$\equiv 2.53 - 3.55$ fm^{-1}

Remember: $\Lambda_\chi \simeq 1$ GeV
$E_{4N}(n_0) \simeq -0.18$ MeV

- small compared to uncertainties in E from $c_3, c_4, ...$



Relativistic Mean Field - solving the puzzles -1

B-fields coupled to M-fields, static solutions in the mean-field approximation $\Longrightarrow$ EOS. **Puzzle: how to make RMF simultaneously consistent with nuclear and astrophysical constraints?**

Way I - minimal structure of $\mathcal{L}$ but g_{BM} and m_M scaled with scalar meson field σ (Kolomeitsev, Maslov, Voskresensky) or g_{BM} are density dependent (Typel et al.)

Way II - more meson fields and higher order couplings - RMF with quartic terms added

Kolomeitsev et al. N-matter

Universal scaling of masses with $f = \frac{g*_{\sigma N}}{m_N}\sigma$ assumed, where $g*_{\sigma N} = g_{\sigma N}\chi_{\sigma N}(\sigma)$

$$\boxed{\mathcal{S}_N(f) = \mathcal{S}_M(f) = 1 - f}$$

$\mathcal{L}$ is quadratic in ω_μ, ρ^i_μ, ϕ_μ so that the equations of motion for vector fields are trivial

What we have to solve is $\boxed{\partial E[n_n, n_p, f]/\partial f = 0}$

Relativistic Mean Field - solving the puzzles -2

Kolomeitsev et al. NHΔ-matter

Universal scaling of masses with f, empirical U_H to get $g_{\sigma H}$ at n_0, scaling using SU(6) for the vector mesons. $M_{\rm max} > 2\ {\rm M}_\odot$ if ϕ_μ (coupled to H only) is included

Very low strangeness per baryon ($\sim 1\%$) even at $M_{\rm max}$.

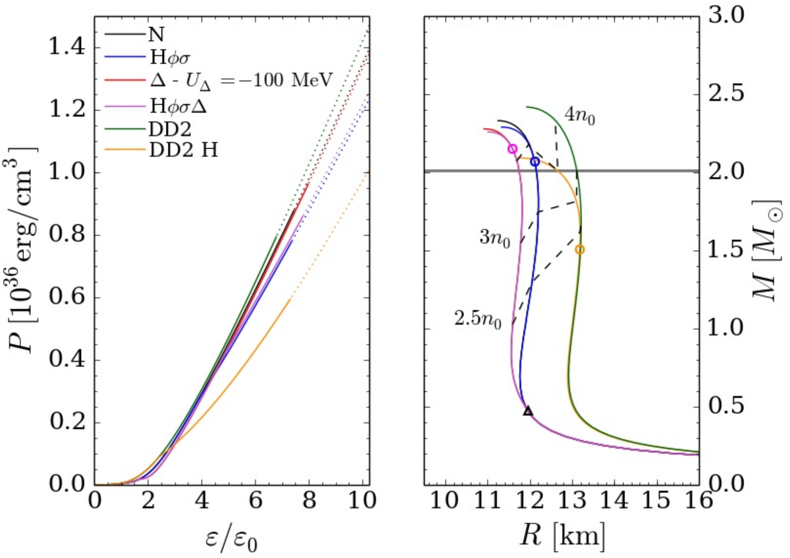
Features of the EOS
(a) $n_{\rm b} \lesssim 4n_0$ SNM rather soft to satisfy nuclear constraints
(b)$n_{\rm b} \gtrsim 4n_0$ PNM stiff to get $2\ {\rm M}_\odot$

No problem with Δ^- provided $U_\Delta = -50 \ldots -100$ MeV
Δ fraction is large but resulting decrease of $M_{\rm max}$ is small

Δ and H puzzles are solved with using appropriate scaling of coupling constants and masses, respecting nuclear physics and $2{\rm M}_\odot$ constraints

Typel et al. NH matter

Density dependence of coupling constants via $\boxed{n_{\rm b} = \sqrt{j_{{\rm b}\mu} j_{\rm b}{}^{\mu}}}$. Single baryon fluid assumed. Successful parametrization - DD2 model


N
Hϕσ
Δ - U_Δ = −100 MeV
HϕσΔ
DD2
DD2 H
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
$P\ [10^{36}\mathrm{erg/cm}^3]$
$\varepsilon/\varepsilon_0$
$4n_0$
$3n_0$
$2.5n_0$
3.0
2.5
2.0
1.5
1.0
0.5
0.0
10
11
12
13
14
15
16
R [km]
$M\ [M_\odot]$

Hyperonic EOS with Dirac-Brueckner-Hartree-Fock theory

Katayama & Sato (2015)

Lorentz invariant may-body theory with One-Boson-Exchange Bonn "potentials". Self-consistent self-energies instead of $U_B \longrightarrow \Sigma_B^{\rm s}, \Sigma_B^{\rm v}$. Density dependence of $\Sigma_N^{\rm v}$ results in no need for strong 3BF to fit nuclear matter. Multiply coupled Bethe-Salpeter equations for four G-matrices. Complexity much higher than for the non-relativistic BHF theory.

Numerical calculations - N matter

Calculations for Bonn A,B,C OBE nucleon interactions - saturation parameters very poor. Decisive point: Bonn A $\longrightarrow$ Bonn A* via $\boxed{g_{N\sigma} \longrightarrow g_{N\sigma}^*(\Sigma_N^{\rm s})}$ (analogously to *Kolomeitsev et al. 2016*)
Enhancement of $g_{N\sigma}^*$ by 2% is sufficient to get correct saturation parameters

Numerical calculations - NH matter

Bonn A* applied to NH matter using SU(6) to get coupling of H to vector mesons
Experimental $U_{\Lambda,\Sigma,\Xi}$ potential wells $\longrightarrow g_{H\sigma}^*$
Universal repulsion comes from $\Sigma_B^{\rm v}$ density dependence

EOS with DBHF theory - NS with hyperon cores

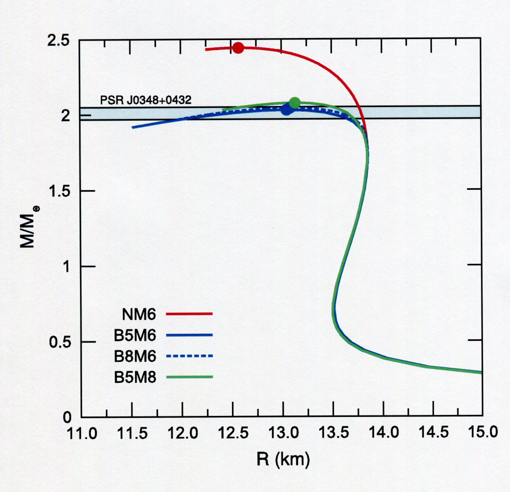


Table: Configurations with $M_{\max}$ for different versions of A* DBHF models

Version	$R_{M_{\max}}$ (km)	n_c (n_0)	$M_{\max}$ ($M_\odot$)
NM6	12.6	4.9	2.44
B5M6	13.1	4.75	2.03
B8M6	13.1	4.7	2.04
NM6	13.1	4.6	2.08

New unified EOS -1

Based on the same nuclear interaction model for core & crust. Consistent description of the crust-core transition, relevant for the determination of the NS radius, $M_{\rm crust}$, $I_{\rm crust}$

Energy Density Functional (EDF) method

Kohn-Sham (1965): For an A-body system there exists a reference system without correlations with orthonormal orbitals $\varphi_i^q (q = n, p, ...)$ which yields actual E, actual density $n_q(\vec{r})$,...

$E^{\rm eff}$ depends on the uncorrelated densities $n_q = \sum_{i,\sigma} |\varphi_i|^2$, $t_q = \sum_{i,\sigma} |\nabla\varphi_i|^2$, . . .

$\boxed{E^{\rm eff}[n_q, ...] = min \Longrightarrow \varphi_i^q}$

$$\boxed{E = E^{\rm N}_{\rm bulk} + E^{\rm N}_{\rm surf} + E^{\rm N}_{\rm s-o} + \ldots + E_{\rm Coul} + E_{\rm lept}}$$

BCPM (Barcelona-Catania-Paris-Madrid) EDF *Sharma et al. 2015*

$E^{\rm N}_{\rm bulk}$ - from v_{12}(2BF:Argonne18) + $\widetilde{v}_{12}$(3BF:UIX) two parameters in UIX adjusted to fit -16.0 MeV and $0.16\ {\rm fm}^{-3}$ $\boxed{c_1, c_2}$. Three parameters in the $E^{\rm N}_{\rm surf}$ $\boxed{c_3, c_4, c_5}$. Strength of spin-orbit term $E^{\rm N}_{\rm s-o}$ $\boxed{c_6}$.

New unified EOS - 2

Sharma et al. 2015 - continued

Optimized fit to 467 known spherical nuclei $\longrightarrow$ values of c_3, c_4, c_5, c_6

r.m.s. deviation $\sigma_{E_{\rm bind}} = 1.3$ MeV

Gives EOS very similar to *Douchin & Haensel 2001*(SLy4), similar $M(R)$ but:

(a) Z in the inner crust ~ 30 instead of ~ 40 of DH2001;

(b) pasta phases present - while no pastas in DH2001. These differences are probably due to the differences in $E^{\rm N}_{\rm surf}$

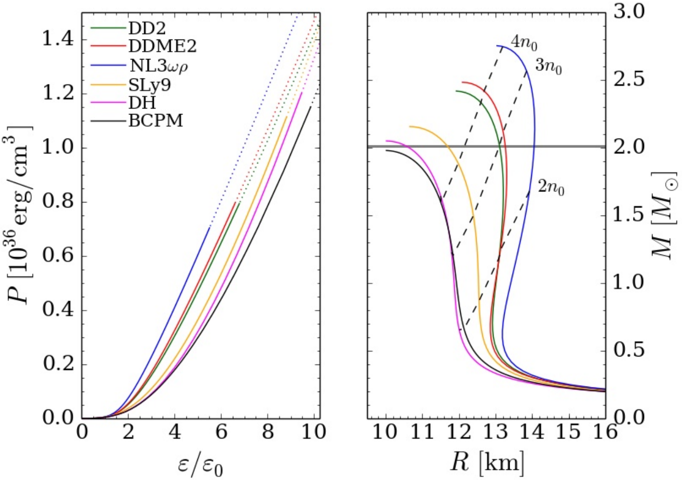
Unified EOS based on RMF models - *Fortin et al. 2016*

Euler-Lagrange equations for $\vec{r}$-dependent fields from $\mathcal{L} = \mathcal{L}_{\rm N} + \mathcal{L}_{\rm M} + \mathcal{L}_{\rm lept}$. Static solutions - only scalar σ and $\nu = 0$ components of vector fields. Sources (n, p) fixed $\longrightarrow$ M-fields equations of motion

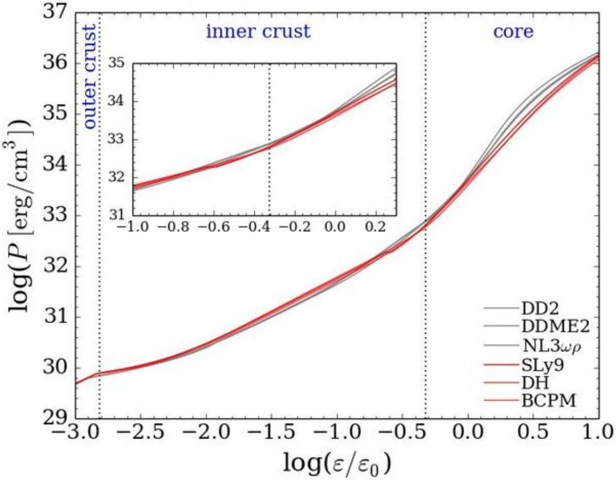
9 unified RMF EOS calculated. Presence of pastas - depends on the surface energy (should not be too high!)

Advantage: precise determination of $R(M)$, $I_{\rm crust}$, $M_{\rm crust}$

New unified EOS - 3



New unified EOS - 4



Finite size of nucleons and the baryon core EOS

Baryons have finite size but they are treated as point-like objects (i.e. like quarks and leptons). In the **Excluded Volume** (ExVol) model they are treated (in the rest frame) as non-penetrable spheres of radius $r_N (N = n, p)$

Historical Remark. In the perturbative approach (e.g. BHF) ExVol effect is obtained using **hard core** interaction of range $2r_N$ such that $V_{ij}(r_{ij} < 2r_N) = +\infty$. Many hard-core 2BFs in 1960-1970. Example: Reid hard-core with $2r_N = 0.4$ fm

RMF+ExVol - *Typel et al. 2016*

Φ= available volume fraction $\quad n_N = \frac{\Phi}{3\pi^3} k^3_{\mathrm{F}N} \quad n_{\mathrm{b}} = \sqrt{j^{\mu}_{\mathrm{b}} j_{\mathrm{b}\mu}}$ (nucleons - single fluid)

$\Phi_{_{\mathrm{NR}}} = 1 - n_{\mathrm{b}} v$, while correct is $\Phi = 1 - n_{\mathrm{b}}^{(\mathrm{s})} v$ $\quad \boxed{v = \frac{1}{2}\frac{4\pi}{3}(2r_N)^3 = 4V_N}$

Minimal coupling RMF model involving $\sigma, \omega^\mu, \rho^\mu_i$

$$P_{\mathrm{had.}}(\mu_n, \mu_p) = \frac{1}{\Phi}\sum_N P_N + P_{\mathrm{M}} \qquad \mathcal{E}_{\mathrm{had.}}(\mu_n, \mu_p) = -P_{\mathrm{had.}} + \sum_N \mu_N n_N$$

Start with successful RMF DD2 model with density-dependent coupling constants (*Typel & Wolter 2009*). Implement ExVol to get DD2-EV, still fitting the same saturation parameters. "Non-relativistic" $\Phi_{\rm NR}$ with large $v = 2.86~\rm fm^3$ corresponds to $r_N = 0.55~\rm fm$ and this means hard-core radius of $1.1~\rm fm$

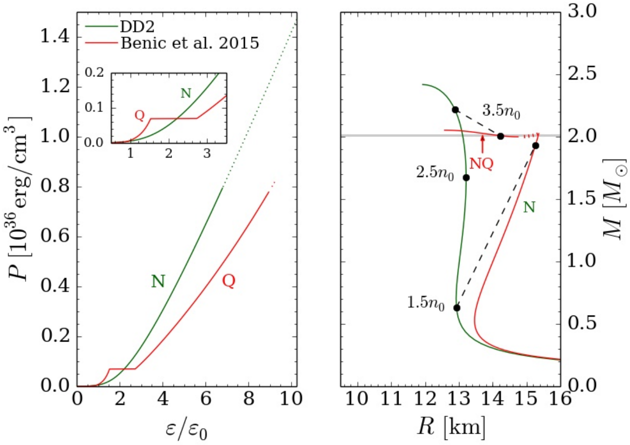
$\Phi_{\rm NR} = 0$ at $n_{\rm crit} = 0.35~\rm fm^{-3}$, pressure explodes there to $+\infty$ and therefore N$\longrightarrow$Q has to occur before this singularity

This (not-so-fine) tuning results in **high-mass twins** because

$$\frac{\rho_{\rm Q}}{\rho_{\rm N}} > \frac{3}{2}(1 + \frac{P_{\rm NQ}}{\rho_{\rm N} c^2})$$

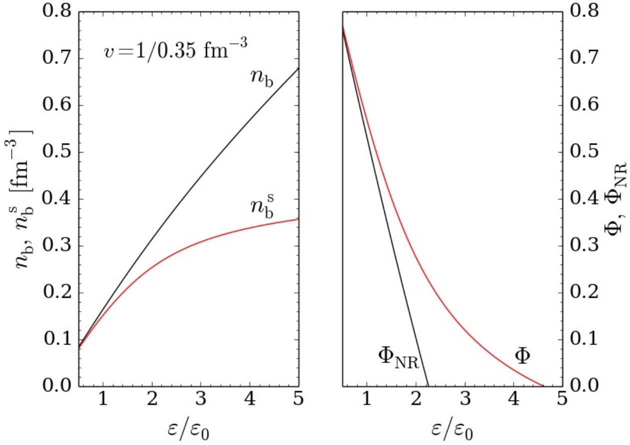
an unstable segment of $M(R)$ separates N-stars from more compact NQ-stars, twins with $M \simeq 2\rm M_\odot$

$\Phi_{\rm NR}$ makes DD2-EV extremely stiff.



Excluded volume effects and high-mass twins -3

Using physical Φ would make the EOS softer and then the twin-NQ stars would most probably go away...



Based on unpublished results of J.L. Zdunik (2015)

Conclusion

Some developments since 2014, but progress is very slow...

Precise EOS for pure neutron matter for $\rho \leq \rho_0$. ChEFT calculations show that 4BF contribution at ρ_0 is negligibly small

No hope to calculate precise EOS using ChEFT for $\rho > \rho_0$ because of slow convergence at supra-nuclear densities

Progress: Many new unified EOS are now available; some EOS-dispersion in the inner crust and in the outer NS core

Scaling of in-medium nuclear force with scalar quantities makes EOS more flexible: this allows H-puzzle and Δ-puzzle to be solved

Imposing strict astrophysical and nuclear physics constraints is hard to fulfil but possible

Relativistic Excluded Volume procedure for the hadron EOS avoids drastic stiffening and unphysical infinite pressure: should be carried out . . .