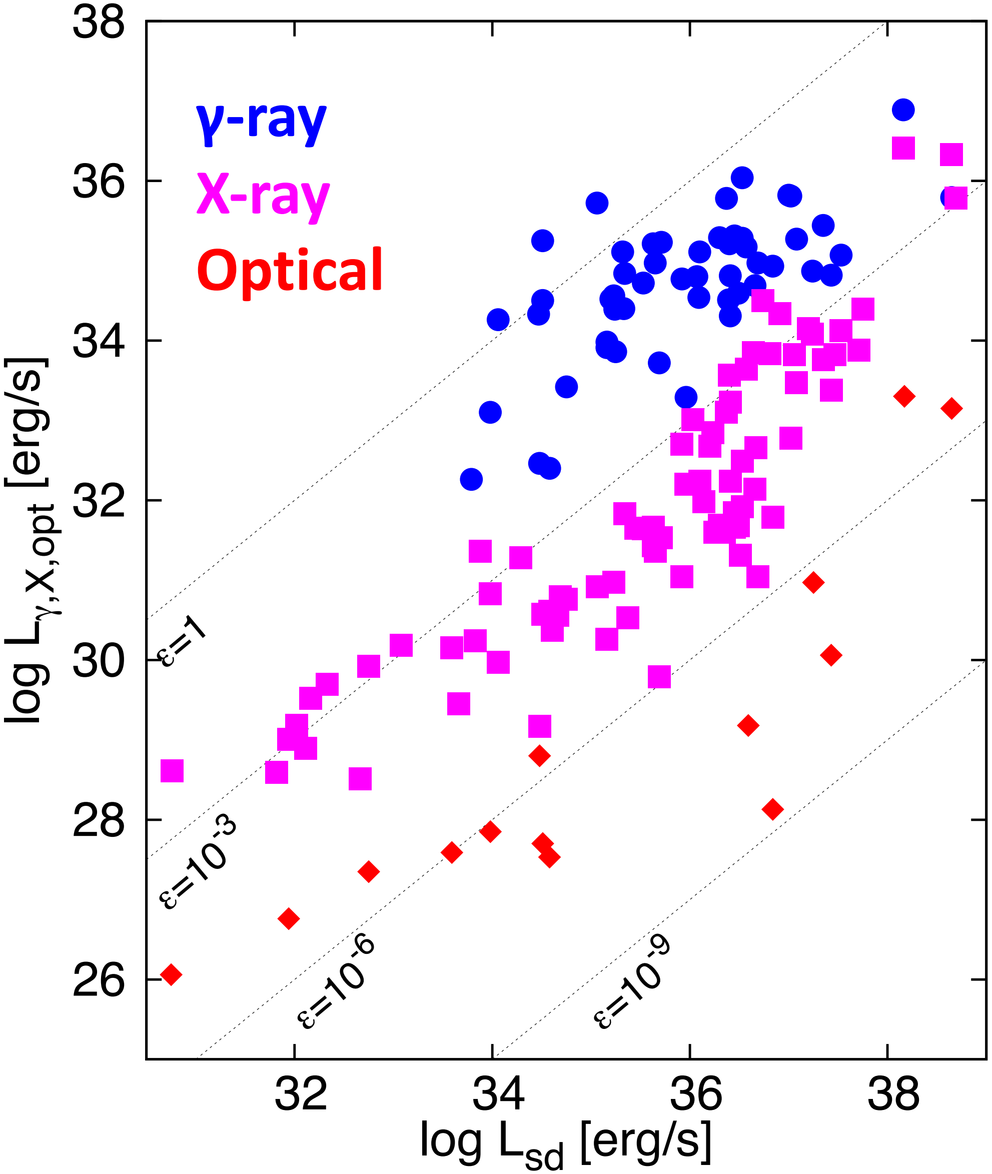
Luminosity of Synchrotron Radiation in Pulsar Magnetosphere

Shota Kisaka (Aoyama Gakuin University)
Shuta J. Tanaka (Konan University)

Kisaka & Tanaka (2017) ApJ 833 76

X-ray and Optical Emission



- GeV γ-ray is considered to be emitted by accelerated particles.
- X-ray and optical emission are considered as the emission from secondary $e^{\pm}$.
- X-ray and optical emission, and the combination with γ-ray emission provide valuable information to understand the pair cascade process in the magnetosphere.

Data : Abdo+13, Hou+ 14, Ackermann+ 15, Kargaltsev & Pavlov 08, Kargaltsev+12, Posselt+ 12, Acero+ 13, Prinz & Becker 15, Kuiper & Hermsen 15, Szary+ 17, Hermsen+17, Zavlin & Pavlov 04, Mignani+ 08, 10, 16

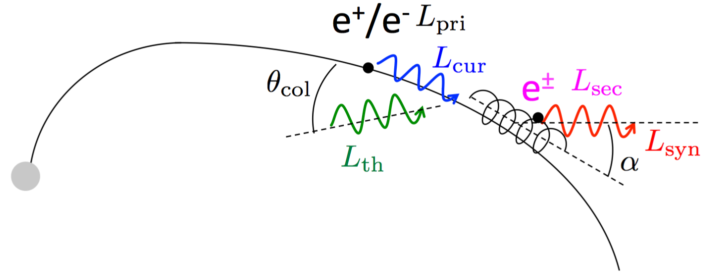
L_{sd} and distance are taken from ATNF Pulsar Catalog 1.56 (Manchester+ 05)

Model

Model

- Primary particles emit curvature radiation (CR).
- Synchrotron radiation (SR) is emitted by secondary particles through **magnetic (Bγ)** or **photon-photon (γγ)** pair creation.

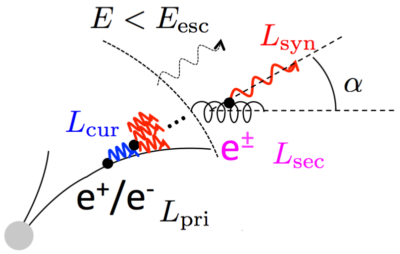
γγ scenario



Assumptions

- Physical quantities are described **by a function of radius *r*.**
- Maximum luminosity is limited **by the spin-down luminosity**.
- Non-thermal X-ray and optical components are caused **by synchrotron radiation.**
- **Dipole structure** dominates near the light cylinder.
- Seed photons are **thermal X-rays from the heated polar cap**.

Bγ scenario



Observed values

$P, \dot{P},$ $L_{\rm pc},$ $E_{\rm pc},$ $E_{\rm cur},$ $\nu_{\rm obs}$

$(L_{\rm sd}, B_{\rm s})$ $(\gamma\gamma)$ $(\gamma\gamma)$

Model parameters

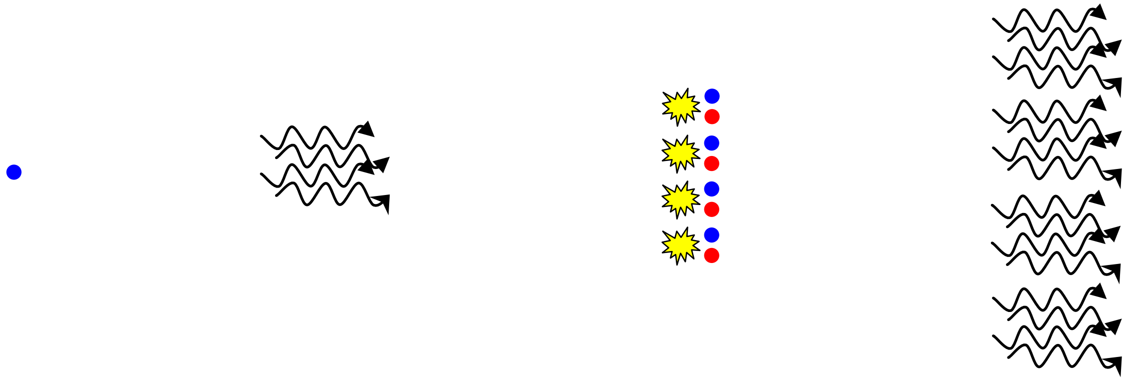
η

$\alpha_0, \zeta_{\rm B}$ (only non-dipole case)

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

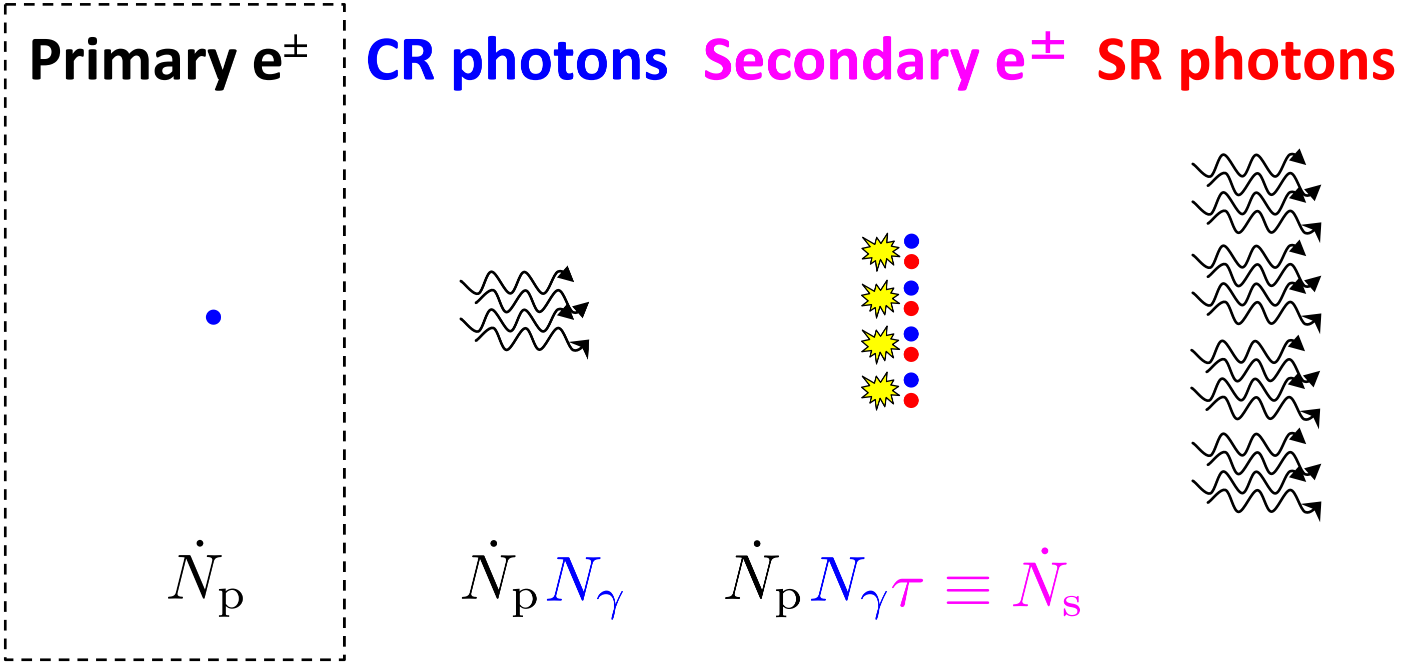
Primary $e^\pm$ **CR photons** **Secondary $e^\pm$** **SR photons**



$\dot{N}_{\rm p}$ $\dot{N}_{\rm p} N_\gamma$ $\dot{N}_{\rm p} N_\gamma \tau \equiv \dot{N}_{\rm s}$

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e± **CR photons** **Secondary e±** **SR photons**

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p} N_\gamma$ $\dot{N}_{\rm p} N_\gamma \tau \equiv \dot{N}_{\rm s}$

#-flux of primary e±

$$\dot{N}_{\rm p} = \frac{\eta L_{\rm sd}}{\gamma_{\rm p} m_{\rm e} c^2}$$

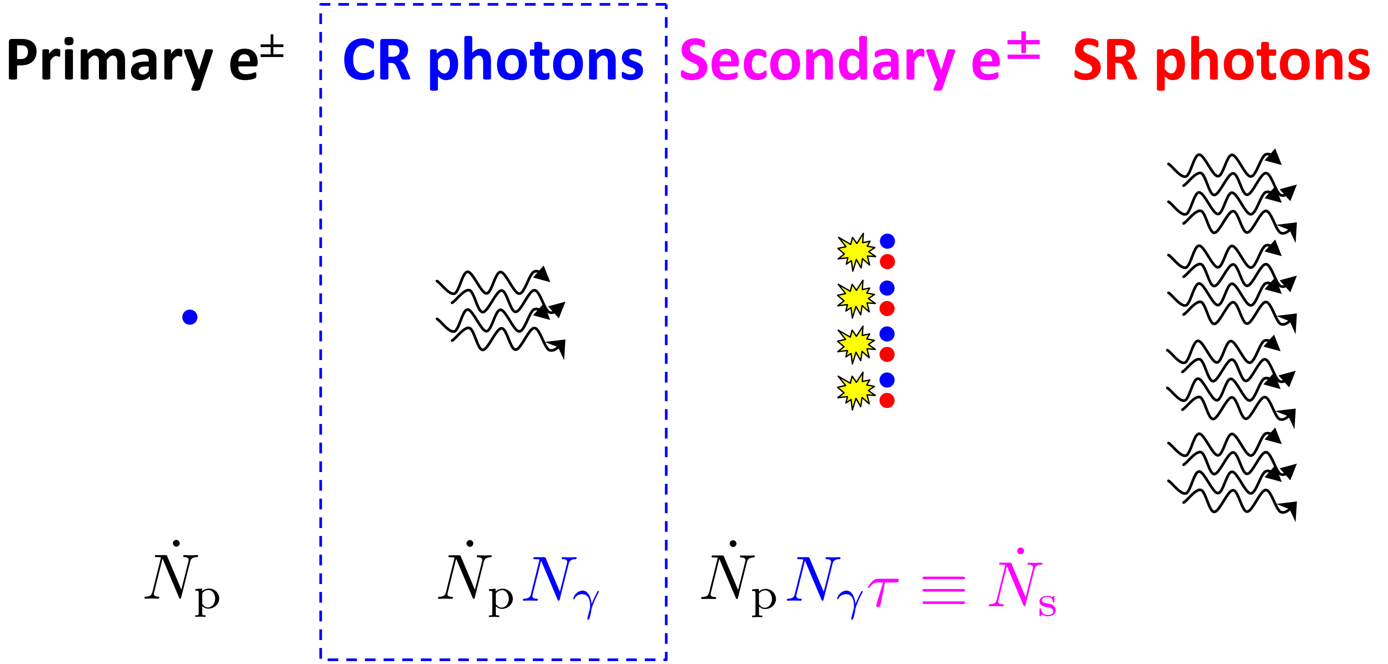
η : Energy conversion efficiency

Lorentz factor of primary e±

$$\gamma_{\rm p} = \begin{cases} \gamma_{\rm p}(E_{\rm cur}) = \left(\frac{4\pi}{0.87} \frac{E_{\rm cur}}{h} \frac{R_{\rm cur}}{c}\right)^{1/3} & (\gamma\gamma) \\ \gamma_{\rm p,max} = \frac{e\Delta V}{m_{\rm e} c^2} & (B\gamma) \end{cases}$$

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary $e^{\pm}$ **CR photons** **Secondary $e^{\pm}$** **SR photons**

$\dot{N}_{\rm p}$

$\dot{N}_{\rm p} N_\gamma$

$\dot{N}_{\rm p} N_\gamma \tau \equiv \dot{N}_{\rm s}$

# of CR photons for a particle

$$N_\gamma \sim \frac{P_{\rm cur}}{E_{\rm cur}} \min\{t_{\rm cool,cur}, t_{\rm ad}\}$$

$$N_\gamma \propto \gamma_{\rm p} E_{\rm cur}^{-1} \quad (t_{\rm cool,cur} < t_{\rm ad})$$

CR power

$$P_{\rm cur} = \frac{2e^2 c}{3R_{\rm cur}^2}\gamma_{\rm p}^4$$

CR cooling timescale

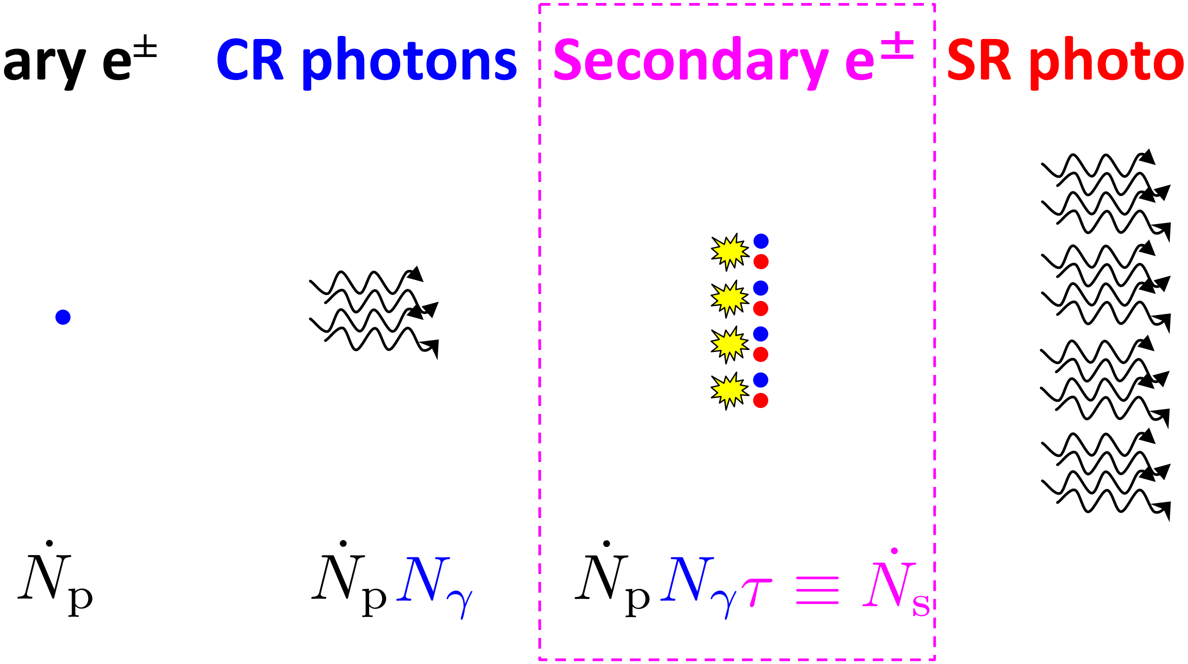
$$t_{\rm cool,cur} \sim \frac{\gamma_{\rm p} m_{\rm e} c^2}{P_{\rm cur}}$$

Advection timescale

$$t_{\rm ad} \sim \frac{r}{c}$$

Synchrotron Luminosity (γγ)

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e± **CR photons** **Secondary e±** **SR photons**

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p}N_\gamma$ $\dot{N}_{\rm p}N_\gamma\tau \equiv \dot{N}_{\rm s}$

#-flux of secondary e±

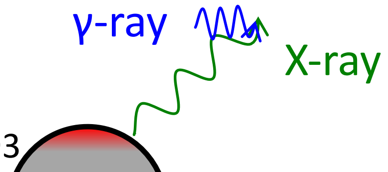
$$\dot{N}_{\rm s} \sim 2\dot{N}_{\rm p}N_\gamma \times \begin{cases} \min\{\tau_{\gamma\gamma}, 1\} & (\gamma\gamma) \\ \frac{E_{\rm cur}}{E_{\rm esc}} & (B\gamma) \end{cases}$$

$L_{\rm pc} = 10^{-3}L_{\rm sd}$

Becker & Trümper 97

$T_{\rm pc} = 10^{6.5}K$

Halpern& Ruderman 93



$E_{\rm pc} = 2.8kT_{\rm pc}$

Optical depth (γγ)

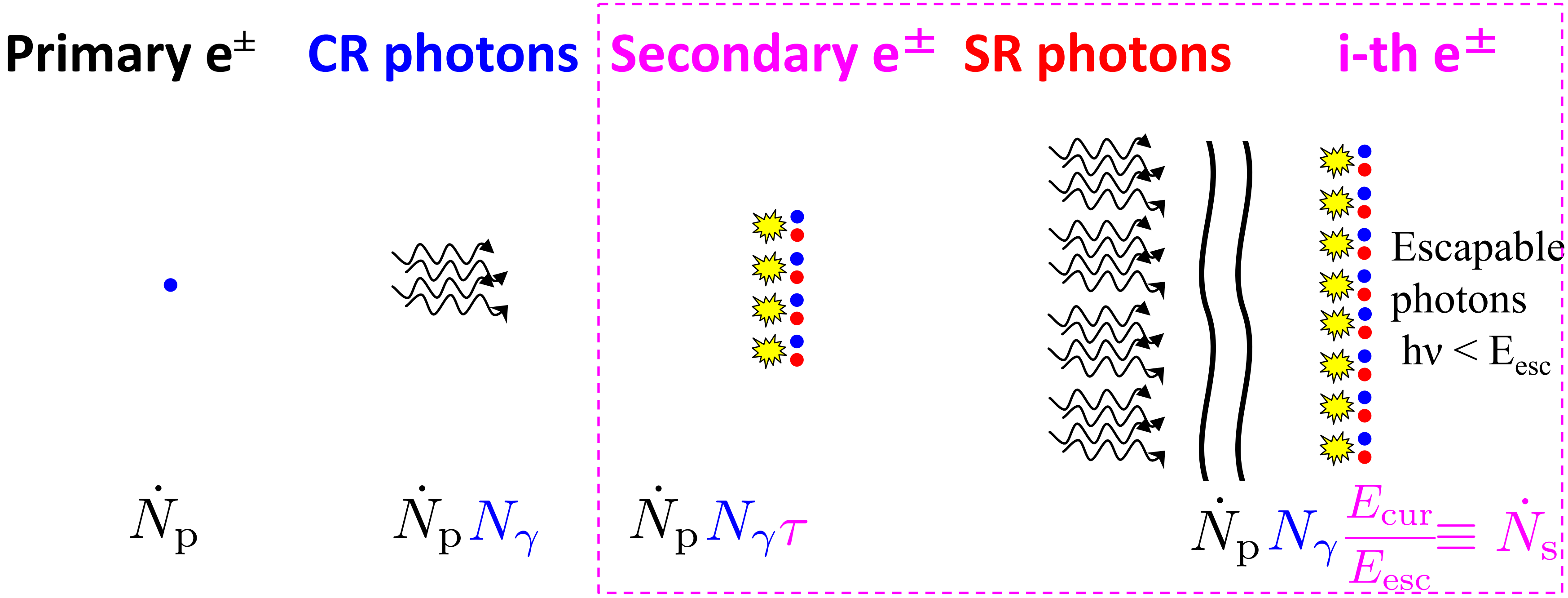
$$\tau_{\gamma\gamma} \sim \frac{L_{\rm pc}}{4\pi r^2 cE_{\rm pc}}\sigma_{\gamma\gamma}(1-\cos\theta_{\rm col})r$$

Collision angle

$$1-\cos\theta_{\rm col} \sim \frac{1}{2}\left(\frac{r}{R_{\rm cur}}\right)^2 \sim \frac{1}{2}\frac{r}{R_{\rm lc}}$$

Synchrotron Luminosity (Bγ)

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e± **CR photons** **Secondary e±** **SR photons** **i-th e±**

Escapable photons hν < $E_{\rm esc}$

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p} N_\gamma$ $\dot{N}_{\rm p} N_\gamma \tau$ $\dot{N}_{\rm p} N_\gamma \frac{E_{\rm cur}}{E_{\rm esc}} \equiv \dot{N}_{\rm s}$

#-flux of secondary e±

$$\dot{N}_{\rm s} \sim 2 \dot{N}_{\rm p} N_\gamma \times \begin{cases} \min\{\tau_{\gamma\gamma}, 1\} & (\gamma\gamma) \\ \frac{E_{\rm cur}}{E_{\rm esc}} & (B\gamma) \end{cases}$$

Escapable photon energy

$$E_{\rm esc} = 2 m_{\rm e} c^2 \chi_{\rm min} \frac{B_{\rm q}}{B_\perp}$$

$$\dot{N}_{\rm p} \propto \gamma_{\rm p}^{-1}$$

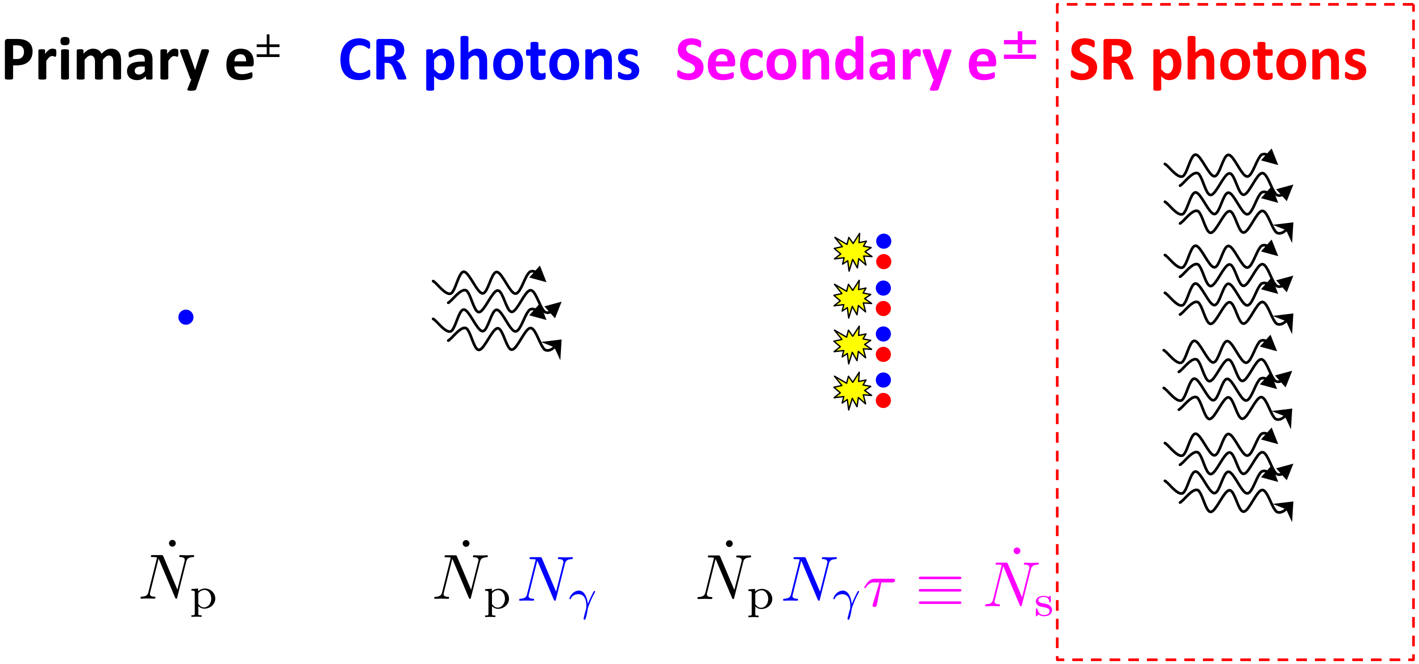
$$N_\gamma \propto \gamma_{\rm p} E_{\rm cur}^{-1}$$

$$B_\perp \sim B\alpha \qquad \chi_{\rm min} = 1/15$$

Ruderman & Sutherland 75

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e± **CR photons** **Secondary e±** **SR photons**

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p} N_\gamma$ $\dot{N}_{\rm p} N_\gamma \tau \equiv \dot{N}_{\rm s}$

SR power

$$P_{\rm syn} = \frac{2e^4 B^2 \alpha^2}{3c^3 m_{\rm e}^2} \gamma_{\rm s,syn}^2$$

SR cooling timescale

$$t_{\rm cool,syn} \sim \frac{\gamma_{\rm s,syn} \alpha m_{\rm e} c^2}{P_{\rm syn}}$$

Lorentz factor of SR emitting e±

$$\gamma_{\rm s,syn}(\nu_{\rm obs}) = \sqrt{\frac{4\pi}{0.87} \nu_{\rm obs} \frac{m_{\rm e} c}{eB\alpha}}$$

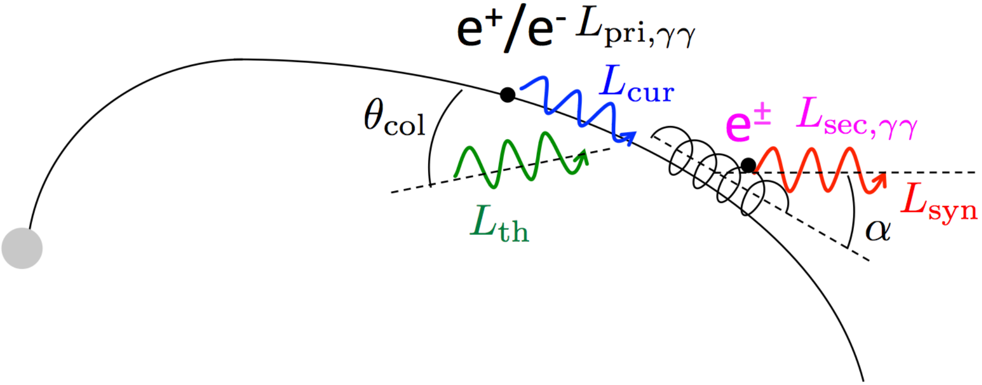
Advection timescale

$$t_{\rm ad} \sim \frac{r}{c}$$

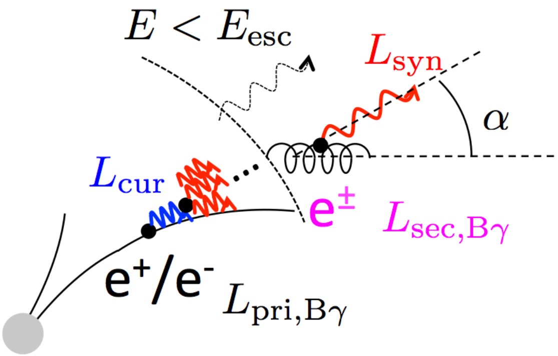
Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

γγ scenario



Bγ scenario



Pitch angle

$$\alpha \sim \begin{cases} \sqrt{r/R_{\rm lc}} & ({\rm dipole}) \\ \alpha_0 (\leq 1) & ({\rm non-dipole}) \end{cases}$$

Curvature radius

$$R_{\rm cur} \sim \begin{cases} \sqrt{r R_{\rm lc}} & ({\rm dipole}) \\ r & ({\rm non-dipole}) \end{cases}$$

Magnetic field strength

$$B(r) \equiv \zeta_{\rm B} B_{\rm dip}(r), \quad (\zeta_{\rm B} \geq 1)$$

Constraints

- **Energy of secondary e±**

$$\gamma_{\rm s,pair} > \gamma_{\rm s,syn}$$

$$\gamma_{\rm s,syn} = \gamma_{\rm s}(\nu_{\rm obs})$$

$$\gamma_{\rm s,pair} = \gamma_{\rm s}(E_{\rm cur})$$

- **SR condition** O'Dell & Sartori 70, Rudak & Dyks 99

$$\gamma_{\rm s,syn}\alpha > 1$$

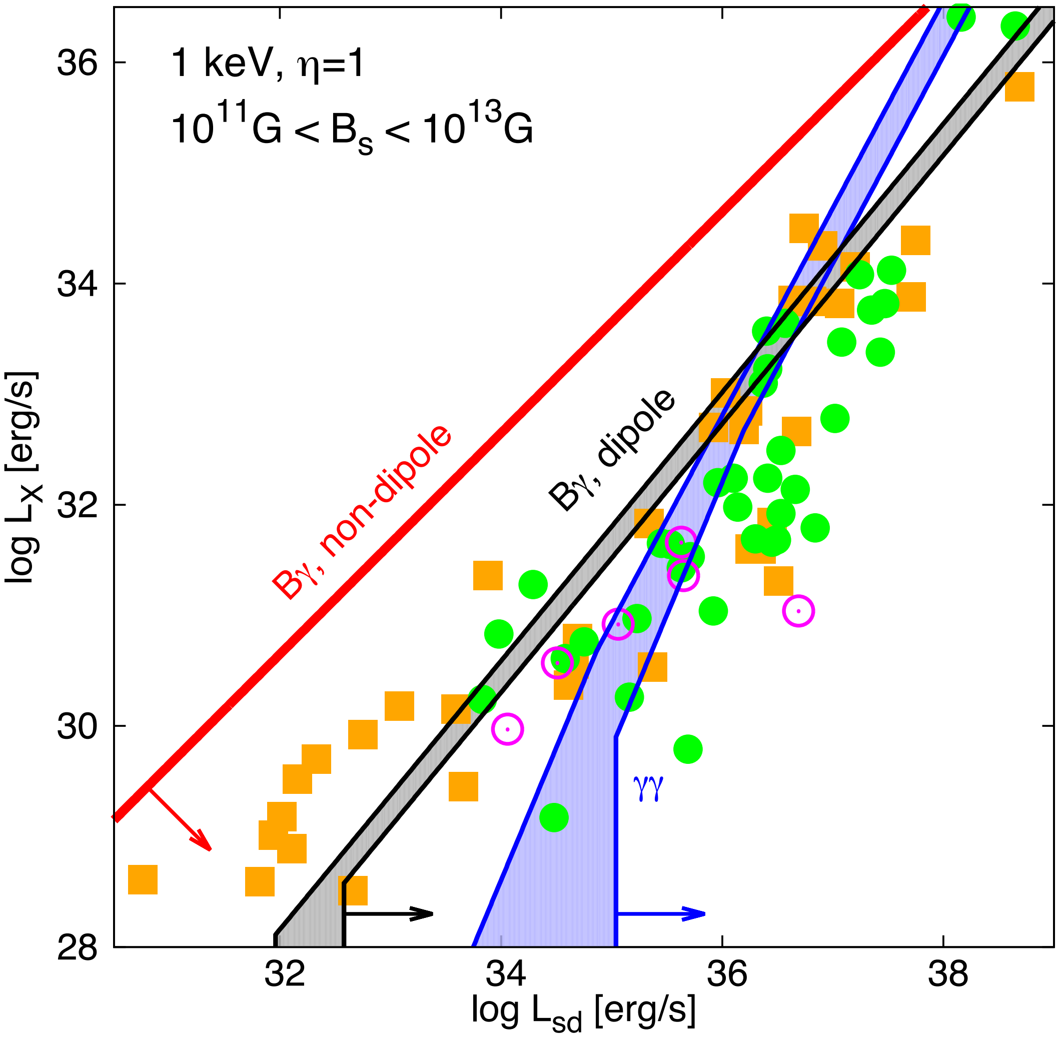
- **Pair production threshold**

$$(1-\cos\theta_{\rm col})E_{\rm pc}E_{\rm cur} > 2(m_{\rm e}c^2)^2$$ (γγ)

$$\frac{E_{\rm cur}}{2m_{\rm e}c^2}\frac{B_\perp}{B_{\rm q}} > \chi_{\rm min}\left(=\frac{1}{15}\right)$$ (Bγ)

$L_{syn} - L_{sd}$ Plots

$\eta = 1$



●: γ-ray detected radio-loud pulsars

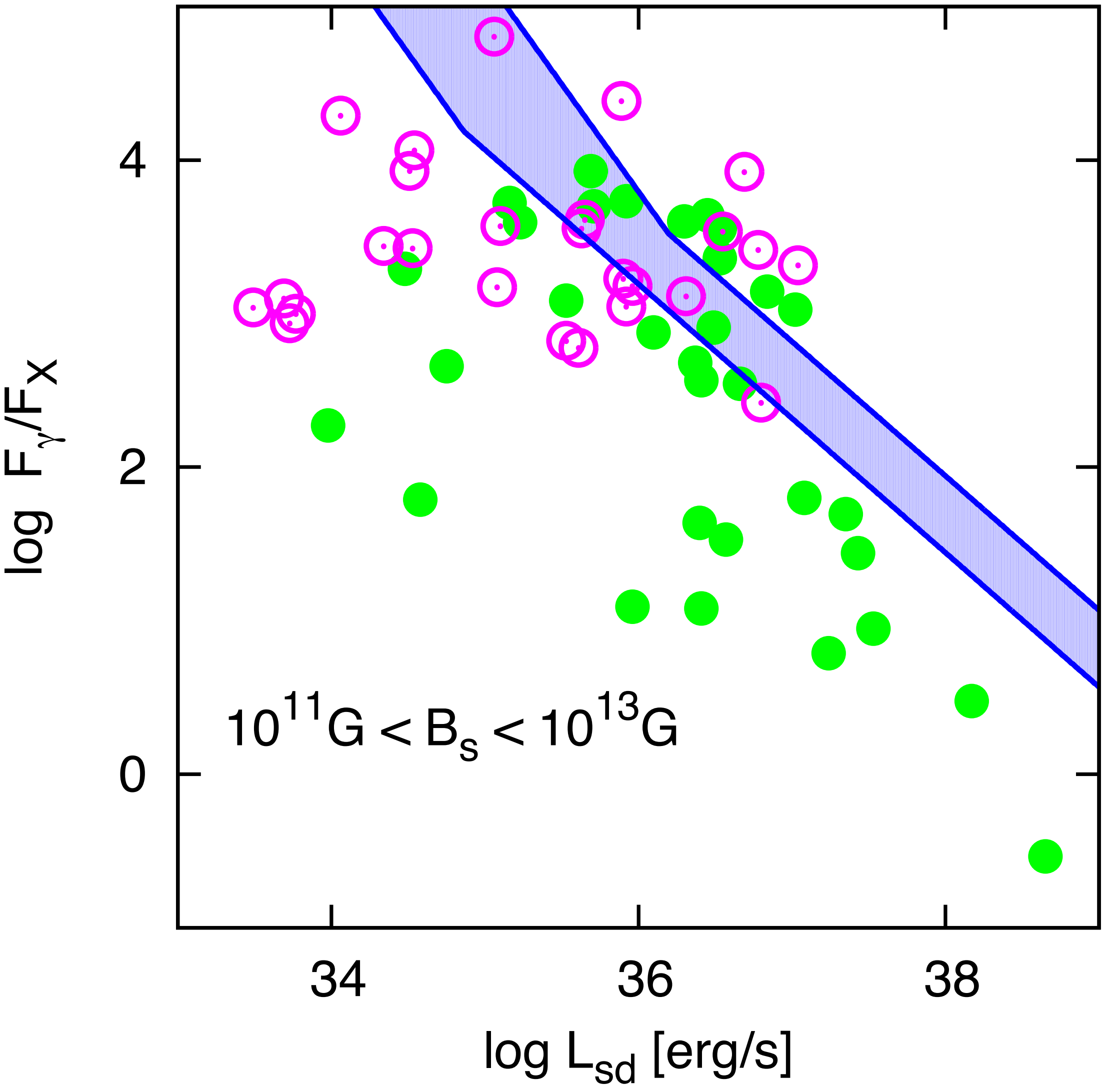
○: γ-ray detected radio-quiet pulsars

■: Other pulsars

In γγ scenario, the X-ray luminosity of pulsars with $L_{sd} < 10^{35}$ erg s^{-1} is higher than the SR luminosity even if η = 1.

Even if Bγ scenario with dipole field, the X-ray luminosity of pulsars with $L_{sd} < 10^{33}$ erg s^{-1} is higher than the SR luminosity.

Flux Ratio (F_γ/F_X)



● : γ-ray detected radio-loud pulsars

○ : γ-ray detected radio-quiet pulsars

Luminosity of CR

$$L_{\rm cur} = \eta L_{\rm sd}$$

$$(t_{\rm cool,cur} < t_{\rm ad})$$

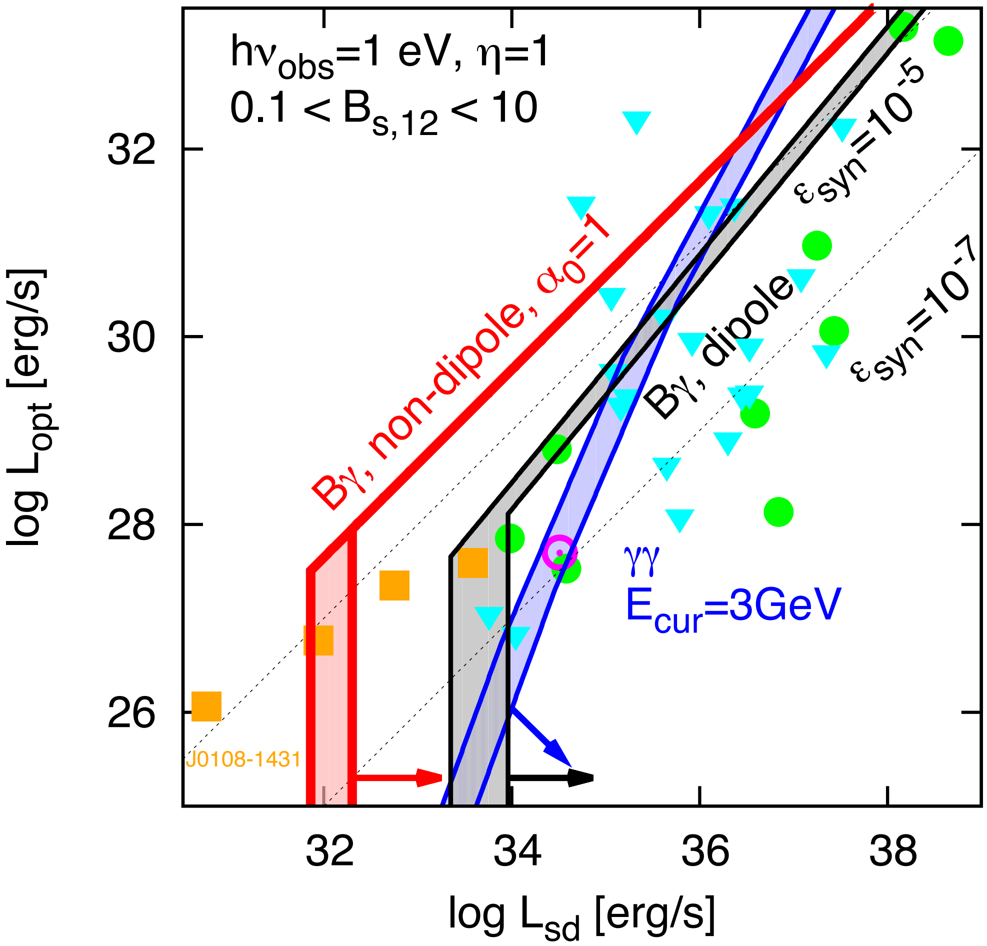
Flux ratio dose not depend on η.

Thermal radiation from entire surface could increase $\tau_{\gamma\gamma}$

Low-L_{sd} pulsars require additional emission mechanisms in X-ray.

L_{syn} – L_{sd} Plots

$\eta = 1$

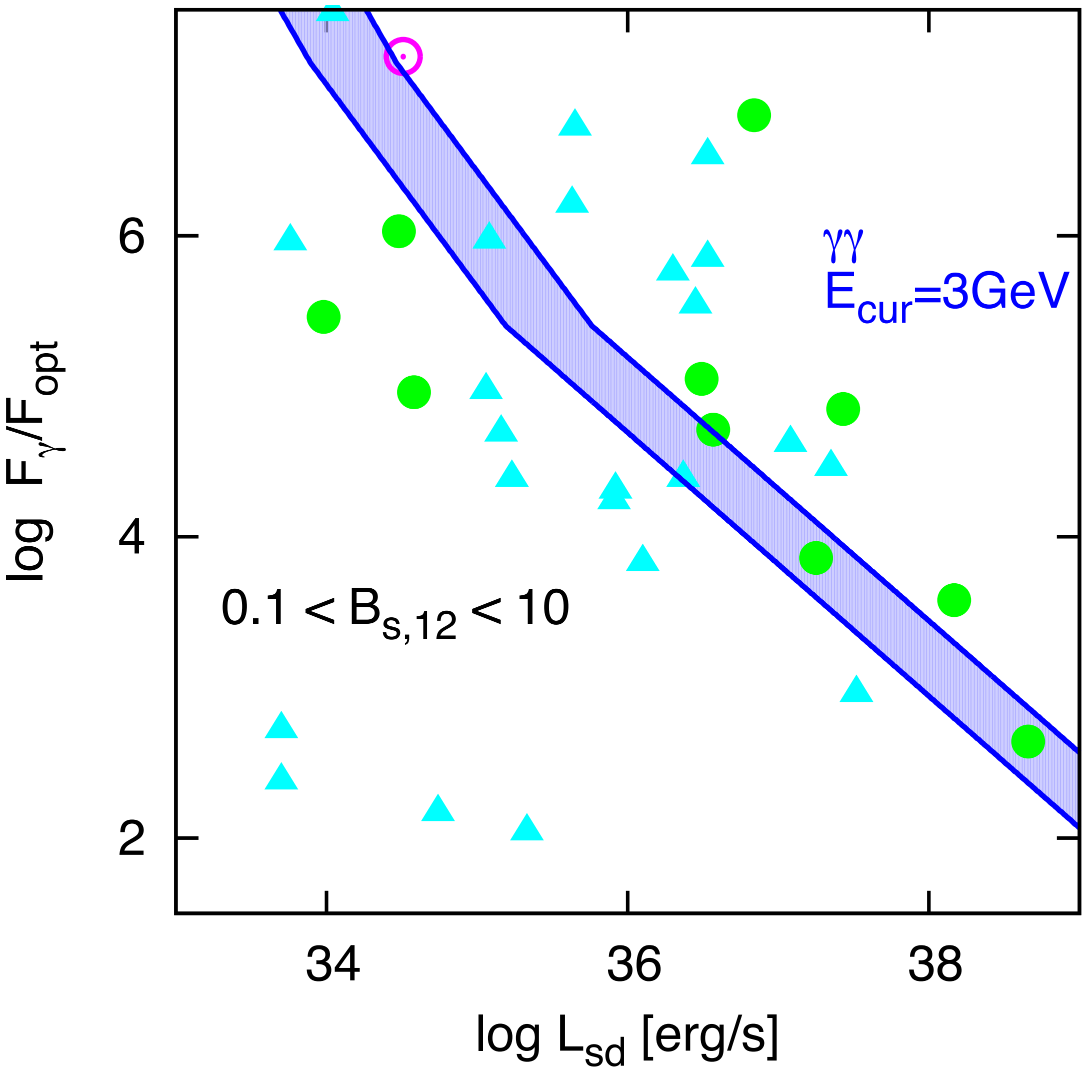


In γγ scenario, the optical luminosity of pulsars with $L_{sd} < 10^{35}$ erg s^{-1} is also higher than the SR luminosity.

For pulsars with $L_{sd} < 10^{32}$ erg s^{-1}, Bγ process cannot work at the region $\hbar\omega_{gyro} < 1$eV even in the non-dipole B-field.

●: γ-ray detected radio-loud pulsars
○: γ-ray detected radio-quiet pulsars
■: Other pulsars
▼: Observed upper limit

Flux Ratio (F_γ/F_{opt})



●: γ-ray detected radio-loud pulsars

○: γ-ray detected radio-quiet pulsars

▼: Observed upper limit

Luminosity of CR

$$L_{\rm cur} = \eta L_{\rm sd}$$
$$(t_{\rm cool,cur} < t_{\rm ad})$$

Flux ratio dose not depend on η.

Low-L_{sd} pulsars require additional emission mechanisms in X-ray and optical.

Discussion

- **Multiple accelerators ?**

  e.g., Yuki & Shibata 12
  Petrova 13
  Marelli+ 14
  Philippov+ 15

- **Another energy source ?**

  e.g., Dissipation of magnetic field ?

- **Resonant Compton scattering ?**

  e.g., Zhang & Harding 00

- **Another mechanism to give the pitch angle ?**

  e.g., Machabeli & Usov 79

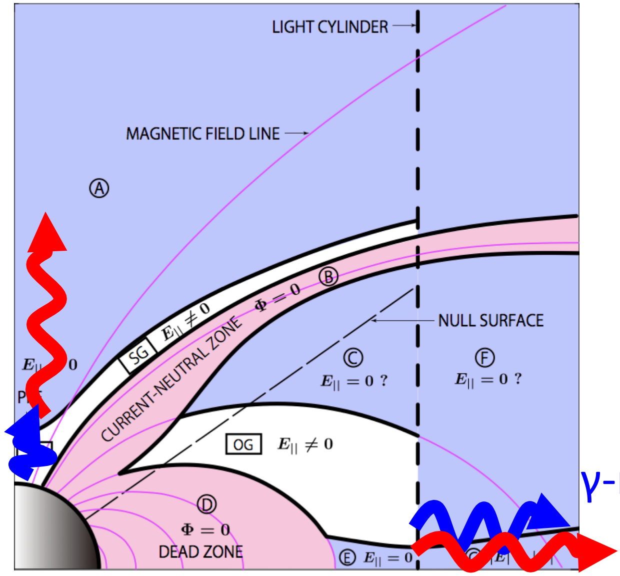
- **Small pitch angle SR ?**

  e.g., Epstein 73, SK & Tanaka 14

- **Emission from inwardly moving e$^{\pm}$?**

  e.g., Wang+ 13, but see also SK & Tanaka 15

$L_{\rm sd} \lesssim 10^{35} {\rm erg\ s}^{-1}$



X-ray
optical ?

Absorbed
γ-ray

γ-ray

X-ray, optical

Summary

- We analytically calculate the luminosity of synchrotron radiation from secondary $e^{\pm}$ in the pulsar magnetosphere.

- Since the energy conversion efficiency η should be close to unity, significant fraction of electromagnetic energy should convert to the particle energy in the magnetosphere.

- In $\gamma\gamma$ scenario, the observed non-thermal X-ray luminosity exceed the upper limit for γ-ray pulsars with $L_{sd} < 10^{35}$ erg s^{-1}. In addition, the flux ratio F_{γ}/F_{X} significantly lower than the synchrotron emission model. Other emission mechanisms or multiple accelerators are required for low-L_{sd} pulsars.

Synchrotron Luminosity (γγ)

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

The most optimistic case (cooling time << advection time)

Radiation efficiency

$$\epsilon_{\rm syn} \equiv \frac{L_{\rm syn}}{L_{\rm sd}} \sim \eta \tau_{\gamma\gamma} \frac{\gamma_{\rm s,syn} \alpha}{\gamma_{\rm s,pair}}$$

Energy conversion efficiency

$$\eta \equiv \frac{\dot{N}_{\rm p} \gamma_{\rm p} m_{\rm e} c^2}{L_{\rm sd}}$$

$$\gamma_{\rm s,syn} = \gamma_{\rm s}(\nu_{\rm obs})$$

$$\gamma_{\rm s,pair} = \gamma_{\rm s}(E_{\rm cur}) = \frac{E_{\rm cur}}{2 m_{\rm e} c^2}$$

Synchrotron Luminosity (Bγ)

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

The most optimistic case (cooling time << advection time)

Radiation efficiency

$$\epsilon_{\rm syn} \equiv \frac{L_{\rm syn}}{L_{\rm sd}} \sim \eta \frac{\gamma_{\rm s,syn}\alpha m_{\rm e}c^2}{E_{\rm esc}}$$

Energy conversion efficiency

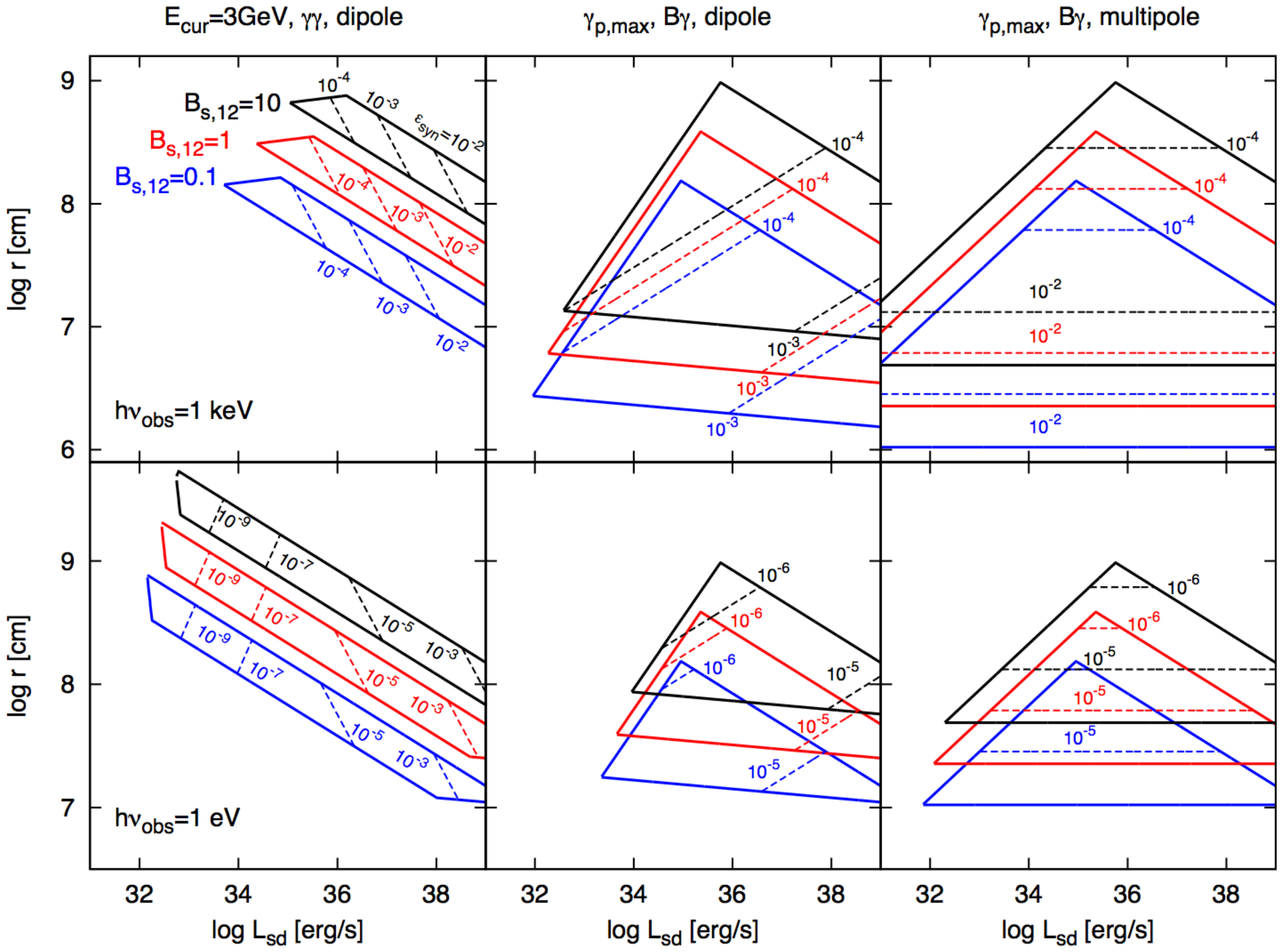
$$\eta \equiv \frac{\dot{N}_{\rm p}\gamma_{\rm p}m_{\rm e}c^2}{L_{\rm sd}}$$

$$\gamma_{\rm s,syn} = \gamma_{\rm s}(\nu_{\rm obs})$$

$$E_{\rm esc} = 2m_{\rm e}c^2\chi_{\rm min}\frac{B_{\rm q}}{B_{\perp}}$$

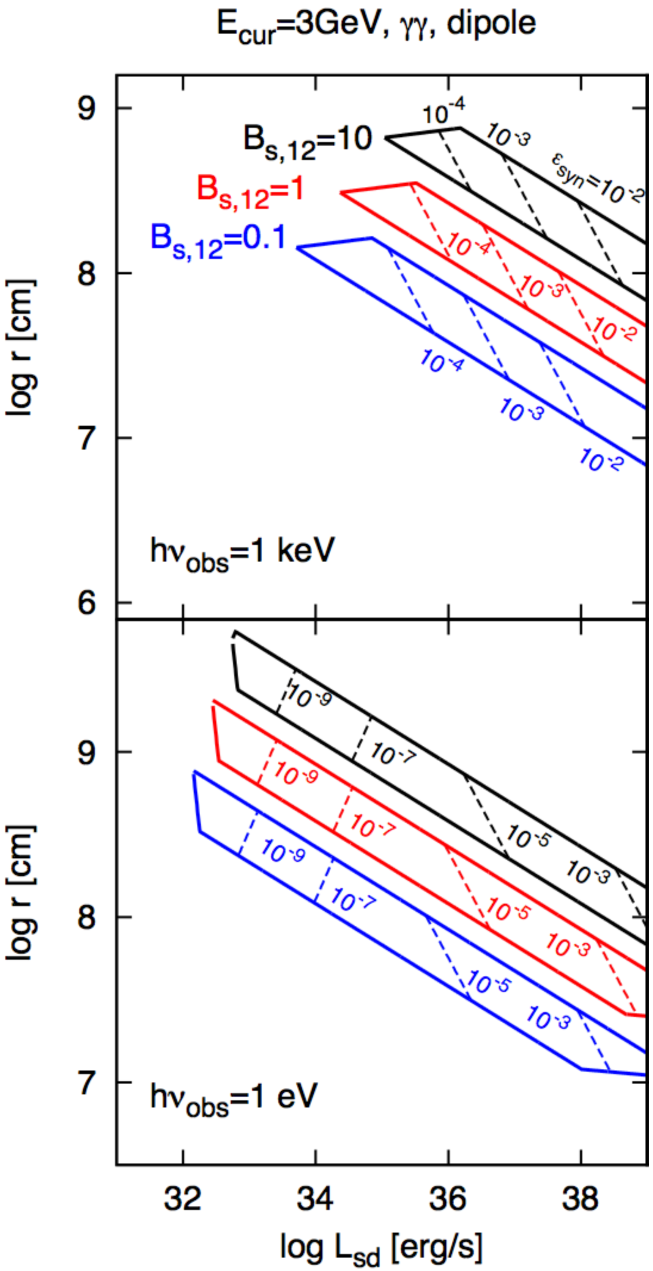
Emission Region

$\eta = 1$



Emission Region

$\eta = 1$



Upper boundary

Light cylinder

$$R_{\rm lc} = Pc/2\pi$$

Secondary $e^{\pm}$ energy

$$\gamma_{\rm s,pair} > \gamma_{\rm s,syn}$$

Lower boundary

Pair creation threshold

$$(1 - \cos\theta_{\rm col}) E_{\rm pc} E_{\rm cur} > 2(m_{\rm e}c^2)^2$$

The maximum SR luminosity is given at r = $R_{\rm lc}$ (even the model extends to r > $R_{\rm lc}$).

Emission Region

$\eta = 1$

Upper boundary

Light cylinder

$$R_{\rm lc} = Pc/2\pi$$

Pair creation threshold

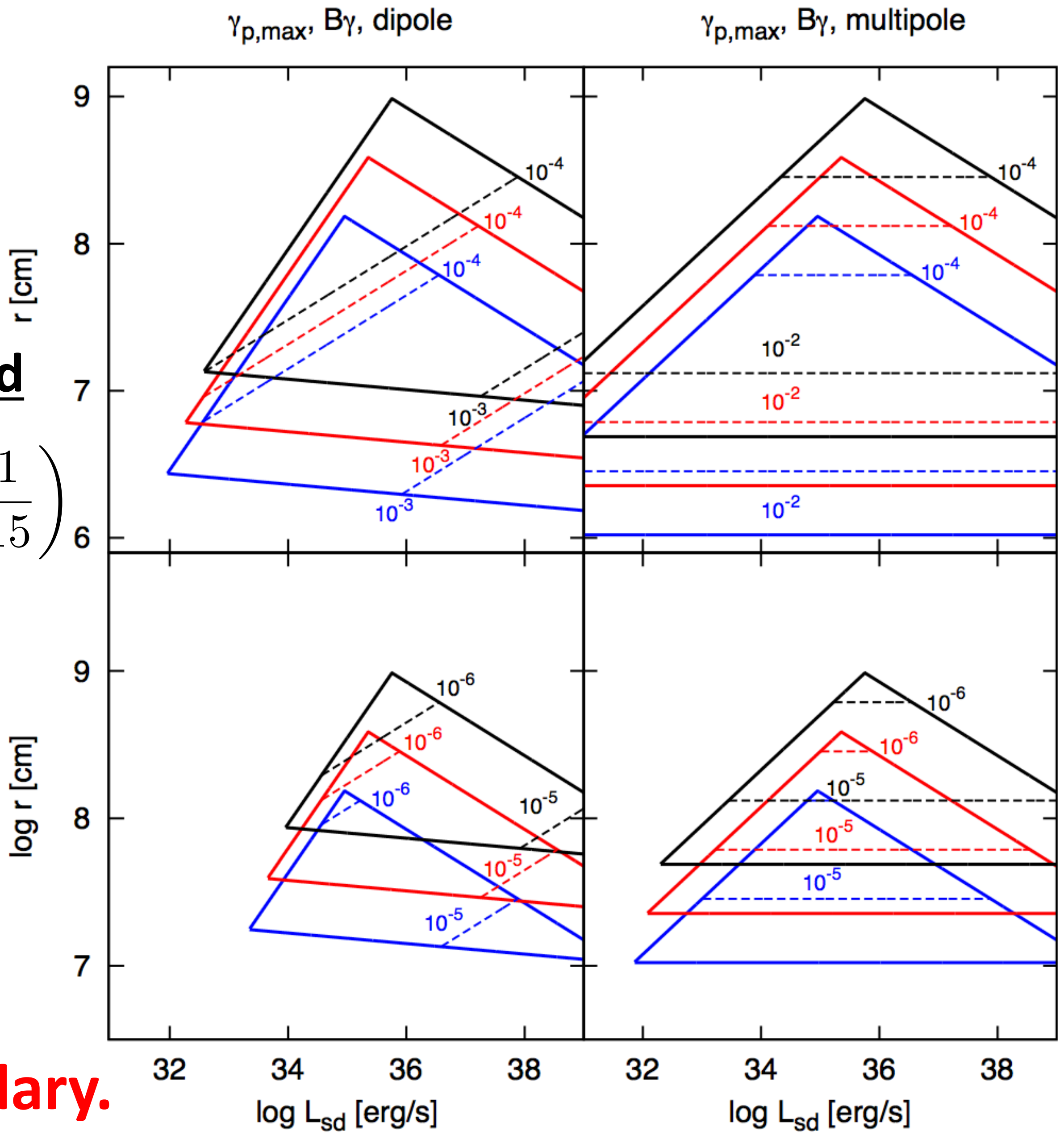
$$\frac{E_{\rm cur}}{2m_{\rm e}c^2}\frac{B_\perp}{B_{\rm q}} > \chi_{\rm min}\left(=\frac{1}{15}\right)$$

Lower boundary

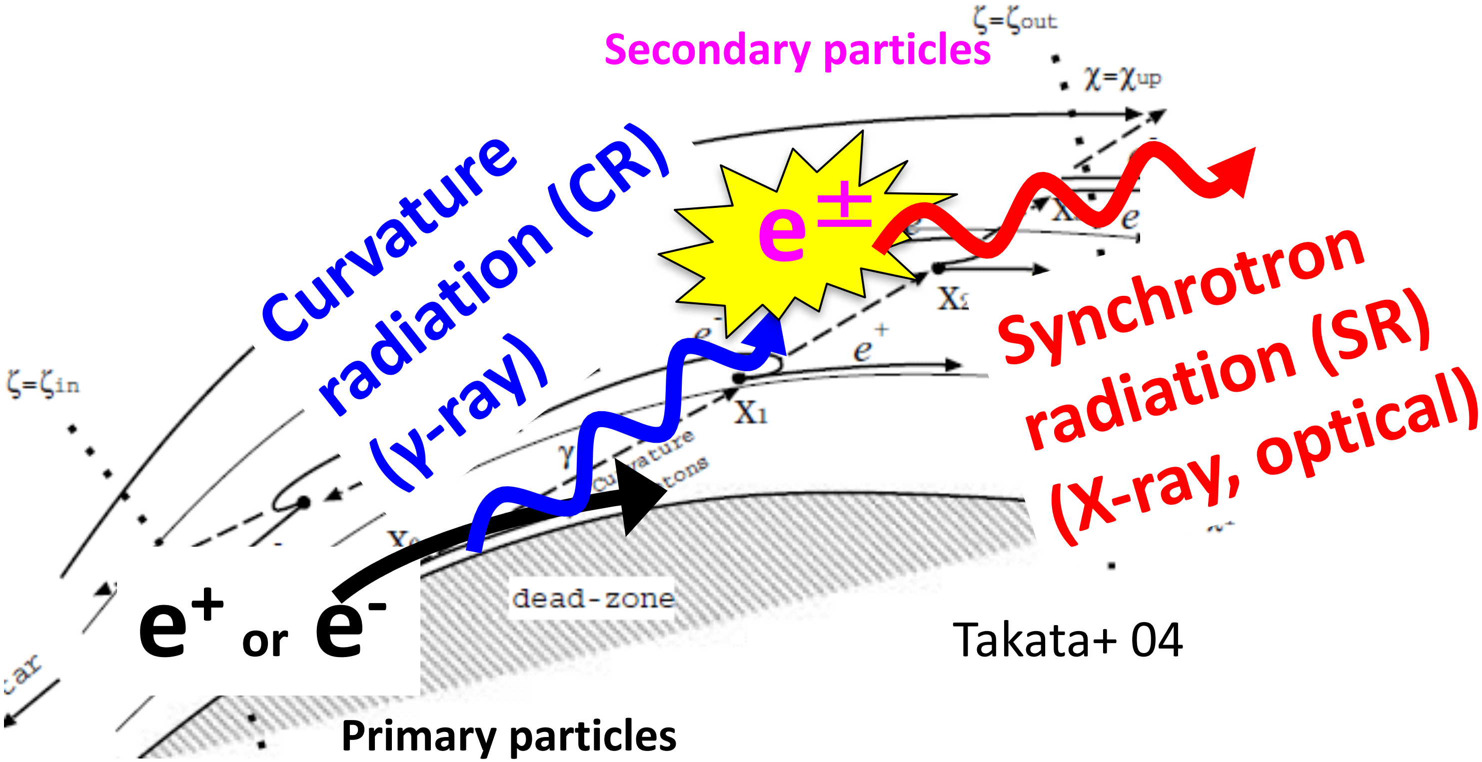
SR condition

$$\gamma_{\rm s,syn}\alpha > 1$$

The maximum SR luminosity is given at the lower boundary.



Synchrotron Radiation from Pulsars



Takata+ 04

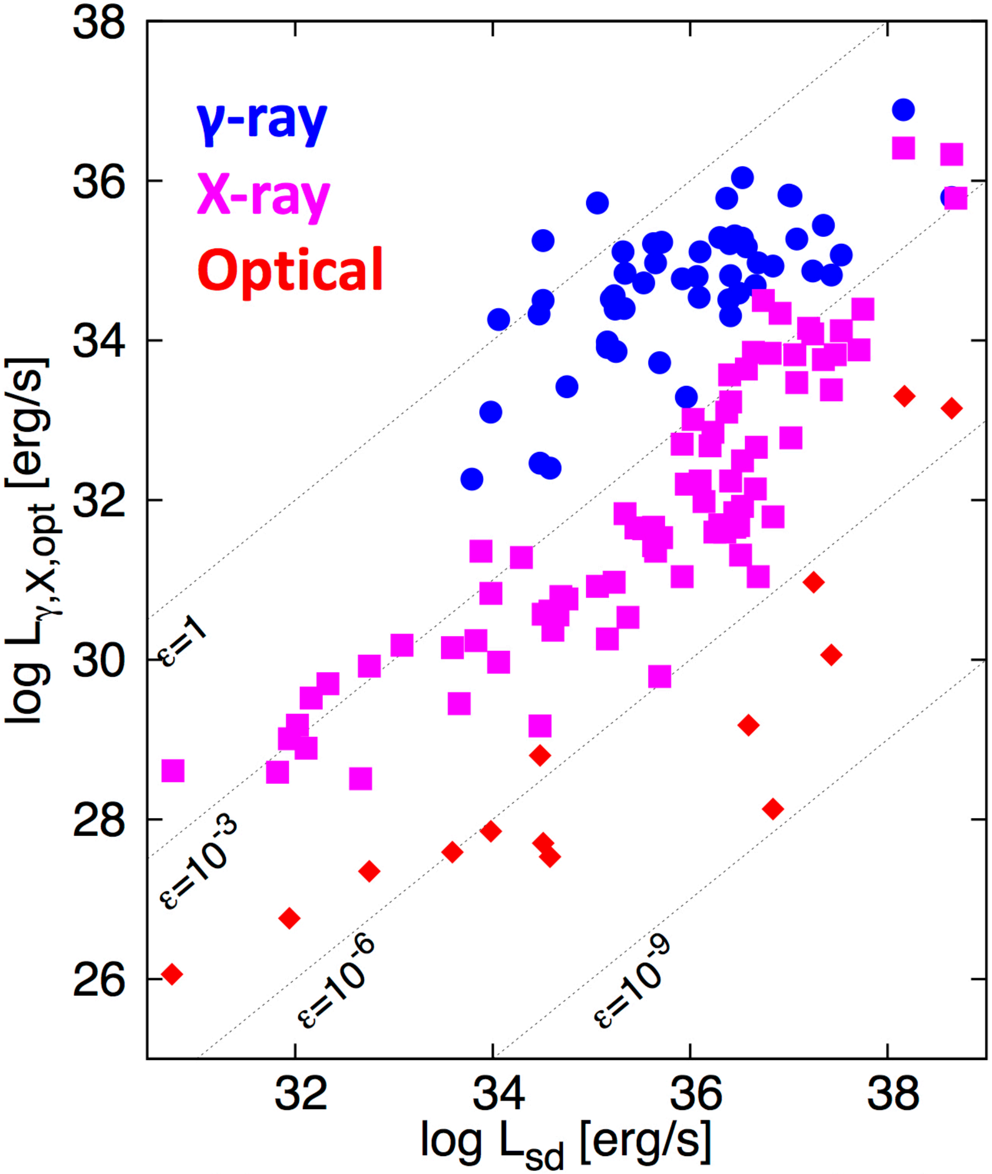
We derive the luminosity of the synchrotron radiation and compare its with the observed non-thermal luminosity to investigate the pair cascade process in the magnetosphere.

Luminosity of Synchrotron Radiation in Pulsar Magnetosphere

Shota Kisaka (Aoyama Gakuin University)
Shuta J. Tanaka (Konan University)

Kisaka & Tanaka (2017) ApJ 833 76

X-ray and Optical Emission



- GeV γ-ray is considered to be emitted by accelerated particles.
- X-ray and optical emission are considered as the emission from secondary $e^{\pm}$.
- X-ray and optical emission, and the combination with γ-ray emission provide valuable information to understand the pair cascade process in the magnetosphere.

Data : Abdo+13, Hou+ 14, Ackermann+ 15, Kargaltsev & Pavlov 08, Kargaltsev+12, Posselt+ 12, Acero+ 13,
Prinz & Becker 15, Kuiper & Hermsen 15, Szary+ 17, Hermsen+17, Zavlin & Pavlov 04, Mignani+ 08, 10, 16
L_{sd} and distance are taken from ATNF Pulsar Catalog 1.56 (Manchester+ 05)

Model

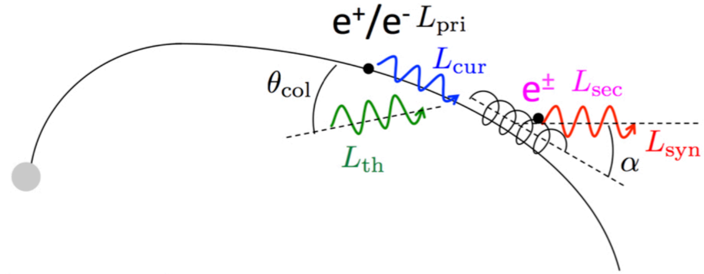
Model

- Primary particles emit curvature radiation (CR).
- Synchrotron radiation (SR) is emitted by secondary particles through **magnetic (Bγ)** or **photon-photon (γγ)** pair creation.

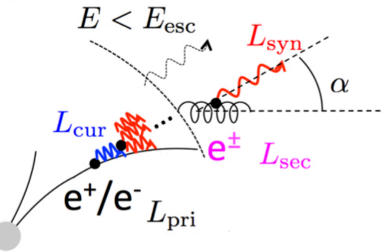
Assumptions

- Physical quantities are described **by a function of radius *r*.**
- Maximum luminosity is limited **by the spin-down luminosity**.
- Non-thermal X-ray and optical components are caused **by synchrotron radiation.**
- **Dipole structure** dominates near the light cylinder.
- Seed photons are **thermal X-rays from the heated polar cap**.

γγ scenario



Bγ scenario



Observed values

$P, \dot{P},$ $L_{\rm pc}, E_{\rm pc},$ $E_{\rm cur},$ $\nu_{\rm obs}$

$(L_{\rm sd}, B_{\rm s})$ $(\gamma\gamma)$ $(\gamma\gamma)$

Model parameters

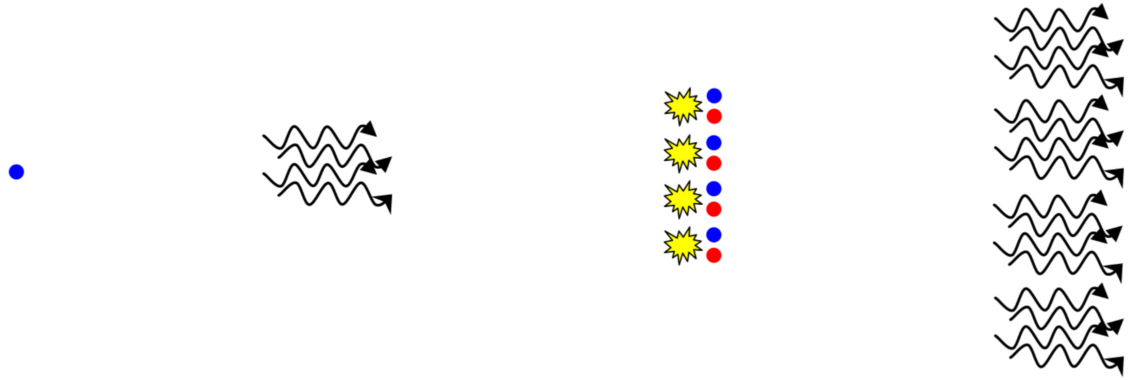
η

$\alpha_0, \zeta_{\rm B}$ (only non-dipole case)

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

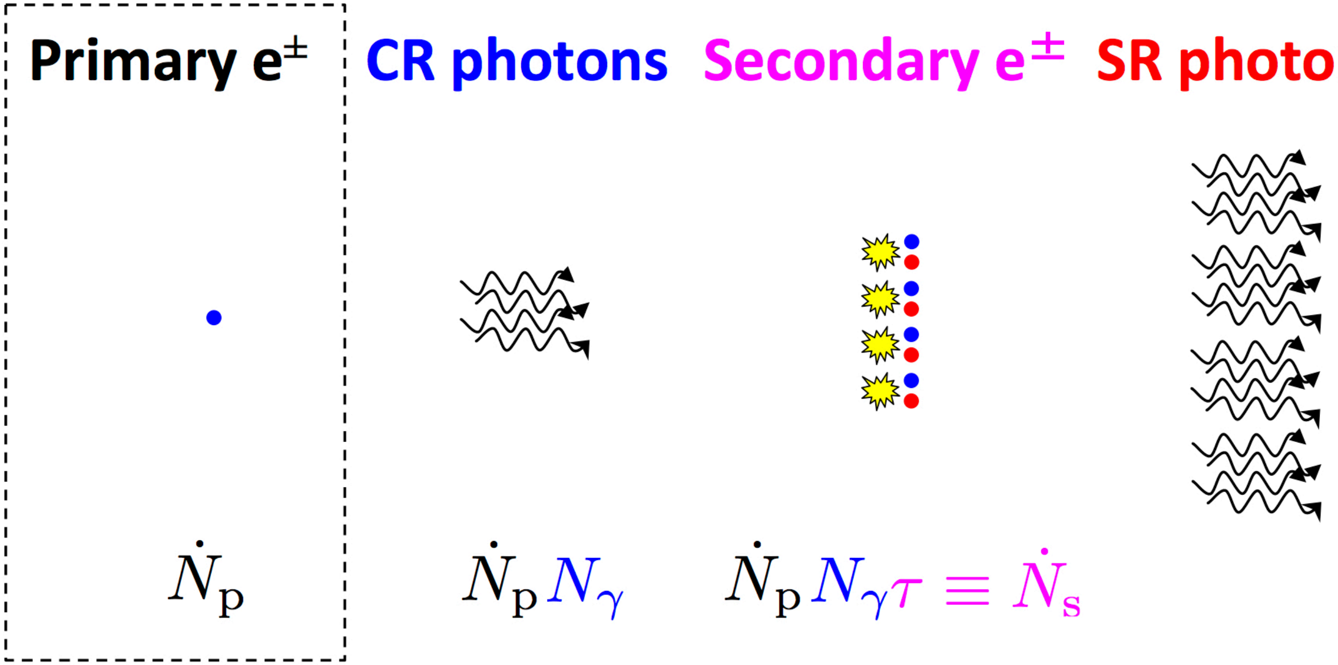
Primary e$^\pm$ **CR photons** **Secondary e$^\pm$** **SR photons**



$\dot{N}_{\rm p}$ $\dot{N}_{\rm p}N_\gamma$ $\dot{N}_{\rm p}N_\gamma\tau \equiv \dot{N}_{\rm s}$

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e$^{\pm}$ **CR photons** **Secondary e$^{\pm}$** **SR photons**

$\dot{N}_{\rm p}$

$\dot{N}_{\rm p} N_{\gamma}$

$\dot{N}_{\rm p} N_{\gamma}\tau \equiv \dot{N}_{\rm s}$

#-flux of primary e$^{\pm}$

$$\dot{N}_{\rm p} = \frac{\eta L_{\rm sd}}{\gamma_{\rm p} m_{\rm e} c^2}$$

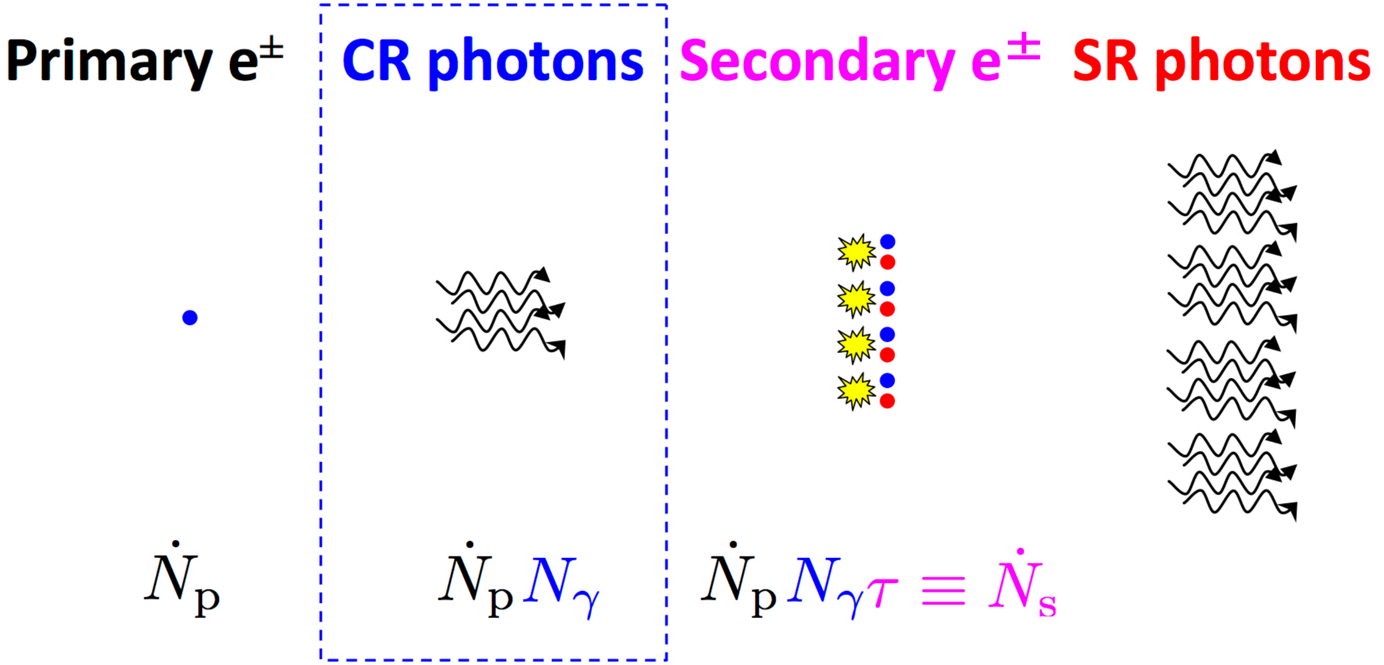
η : Energy conversion efficiency

Lorentz factor of primary e$^{\pm}$

$$\gamma_{\rm p} = \begin{cases} \gamma_{\rm p}(E_{\rm cur}) = \left(\frac{4\pi}{0.87}\frac{E_{\rm cur}}{h}\frac{R_{\rm cur}}{c}\right)^{1/3} & (\gamma\gamma) \\ \gamma_{\rm p,max} = \frac{e\Delta V}{m_{\rm e}c^2} & (B\gamma) \end{cases}$$

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e± **CR photons** **Secondary e±** **SR photons**

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p}N_\gamma$ $\dot{N}_{\rm p}N_\gamma\tau \equiv \dot{N}_{\rm s}$

# of CR photons for a particle

$$N_\gamma \sim \frac{P_{\rm cur}}{E_{\rm cur}}\min\{t_{\rm cool,cur}, t_{\rm ad}\}$$

$$N_\gamma \propto \gamma_{\rm p}E_{\rm cur}^{-1} \quad (t_{\rm cool,cur} < t_{\rm ad})$$

CR power

$$P_{\rm cur} = \frac{2e^2c}{3R_{\rm cur}^2}\gamma_{\rm p}^4$$

CR cooling timescale

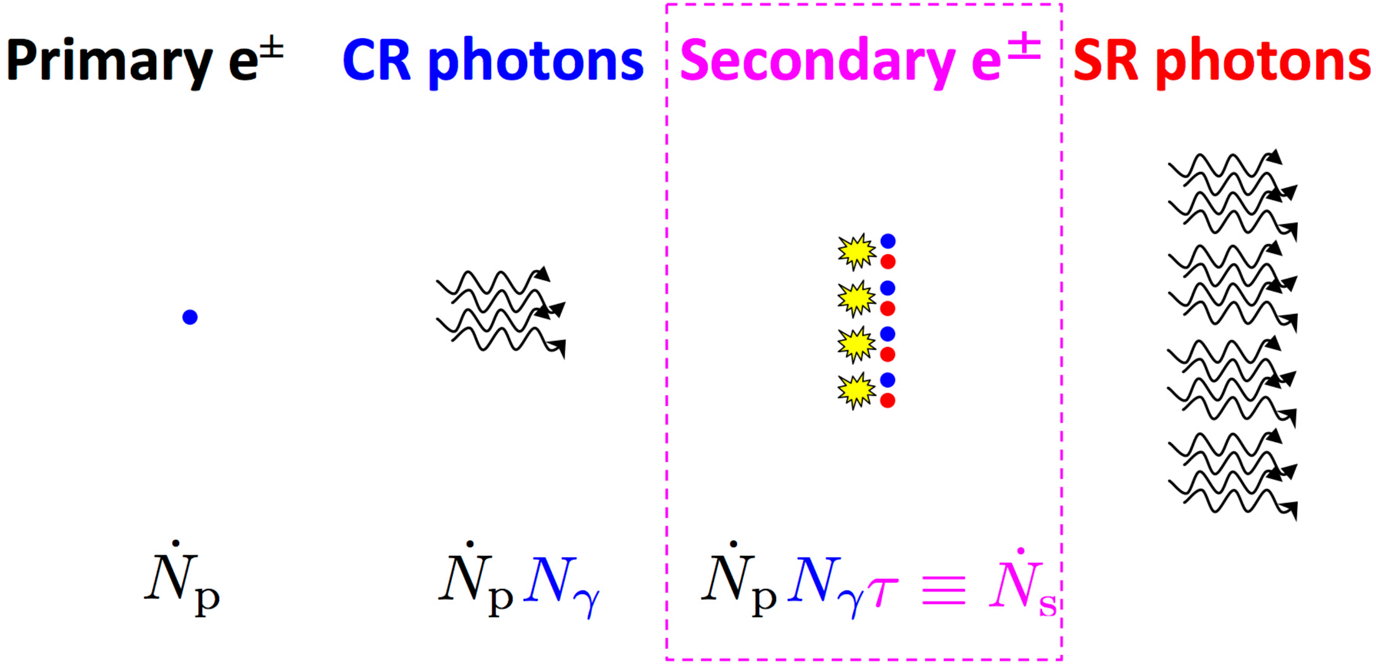
$$t_{\rm cool,cur} \sim \frac{\gamma_{\rm p}m_{\rm e}c^2}{P_{\rm cur}}$$

Advection timescale

$$t_{\rm ad} \sim \frac{r}{c}$$

Synchrotron Luminosity (γγ)

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



#-flux of secondary e$^\pm$

$$\dot{N}_{\rm s} \sim 2\dot{N}_{\rm p} N_\gamma \times \begin{cases} \min\{\tau_{\gamma\gamma}, 1\} & (\gamma\gamma) \\ \frac{E_{\rm cur}}{E_{\rm esc}} & (B\gamma) \end{cases}$$

Optical depth (γγ)

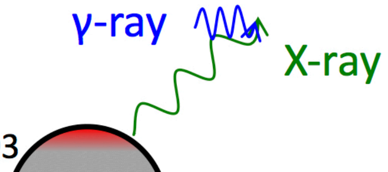
$$E_{\rm pc} = 2.8kT_{\rm pc}$$

$$\tau_{\gamma\gamma} \sim \frac{L_{\rm pc}}{4\pi r^2 c E_{\rm pc}} \sigma_{\gamma\gamma}(1 - \cos\theta_{\rm col}) r$$

Collision angle

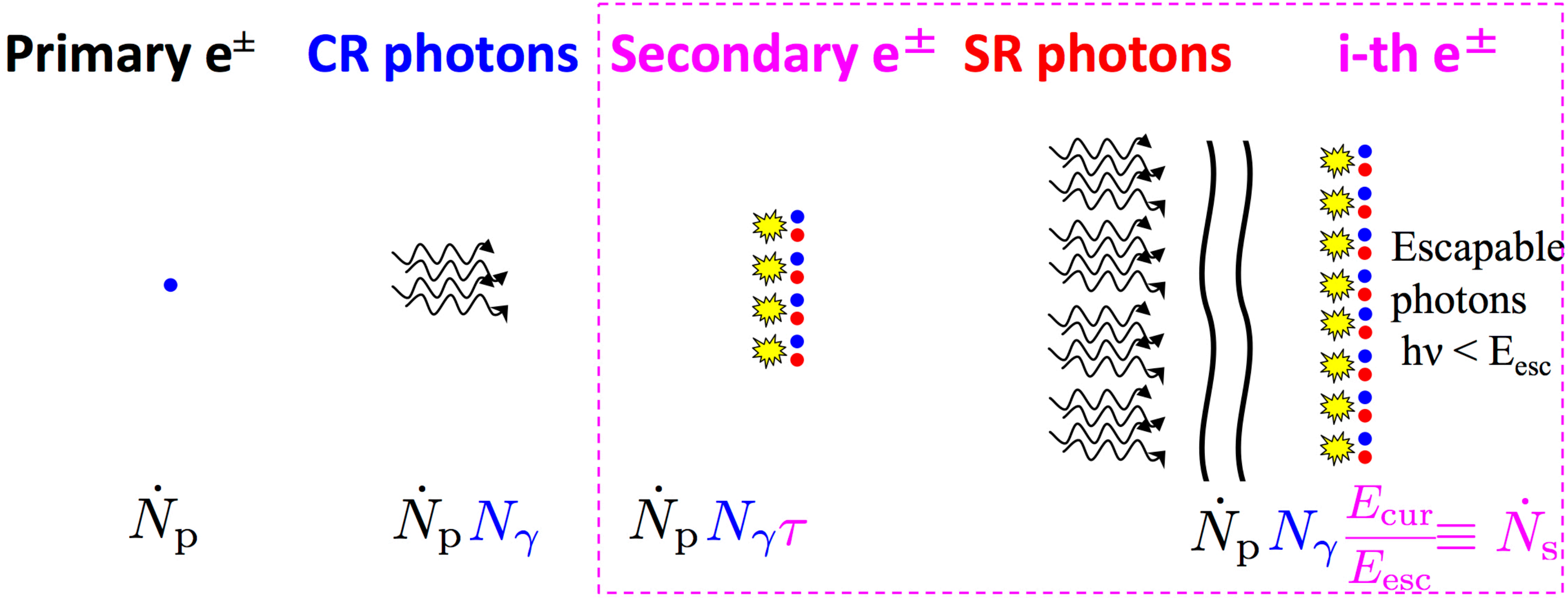
$$1 - \cos\theta_{\rm col} \sim \frac{1}{2}\left(\frac{r}{R_{\rm cur}}\right)^2 \sim \frac{1}{2}\frac{r}{R_{\rm lc}}$$

$L_{\rm pc} = 10^{-3} L_{\rm sd}$
Becker & Trümper 97
$T_{\rm pc} = 10^{6.5} K$
Halpern& Ruderman 93



Synchrotron Luminosity (Bγ)

$$L_{\rm syn} = P_{\rm syn} \dot{N}_{\rm s} \min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e$^{\pm}$ **CR photons** **Secondary e$^{\pm}$** **SR photons** **i-th e$^{\pm}$**

Escapable photons $h\nu < E_{\rm esc}$

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p} N_\gamma$ $\dot{N}_{\rm p} N_\gamma \tau$ $\dot{N}_{\rm p} N_\gamma \frac{E_{\rm cur}}{E_{\rm esc}} \equiv \dot{N}_{\rm s}$

#-flux of secondary e$^{\pm}$ **Escapable photon energy**

$$\dot{N}_{\rm p} \propto \gamma_{\rm p}^{-1}$$

$$N_\gamma \propto \gamma_{\rm p} E_{\rm cur}^{-1}$$

$$\dot{N}_{\rm s} \sim 2\dot{N}_{\rm p} N_\gamma \times \begin{cases} \min\{\tau_{\gamma\gamma}, 1\} & (\gamma\gamma) \\ \frac{E_{\rm cur}}{E_{\rm esc}} & (B\gamma) \end{cases}$$

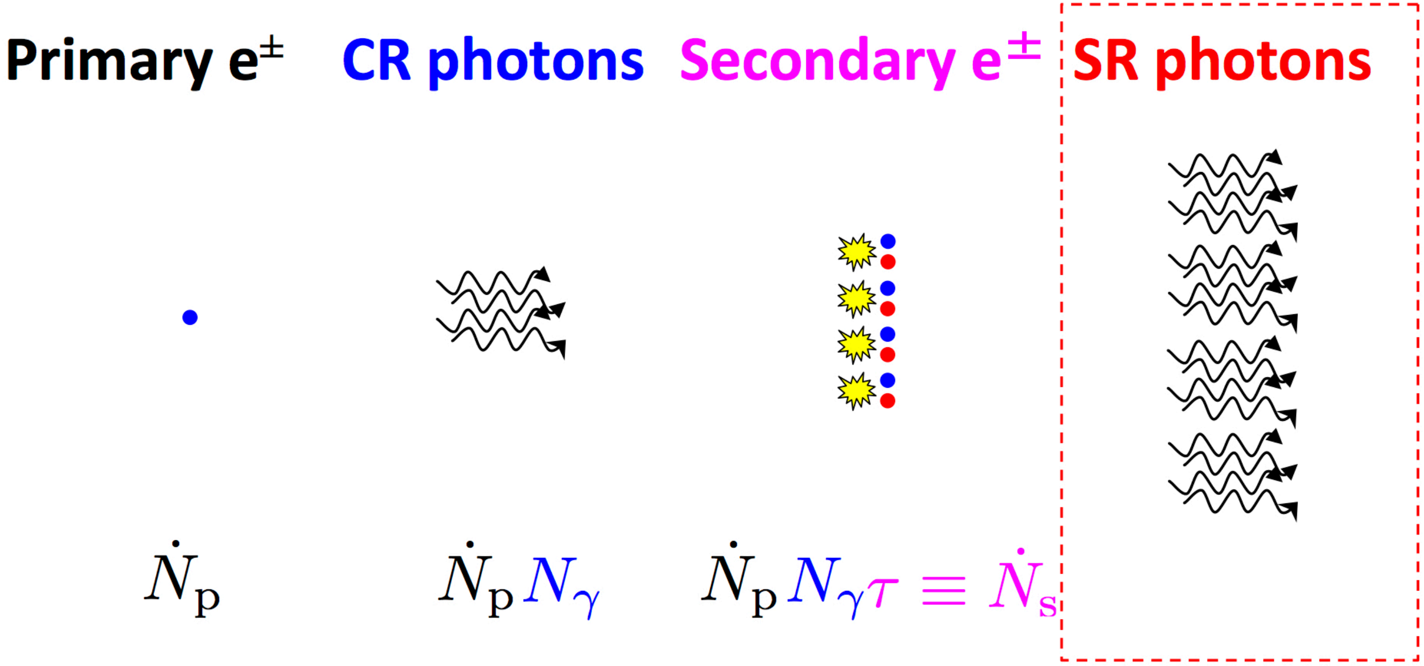
$$E_{\rm esc} = 2 m_{\rm e} c^2 \chi_{\rm min} \frac{B_{\rm q}}{B_\perp}$$

$$B_\perp \sim B\alpha \qquad \chi_{\rm min} = 1/15$$

Ruderman & Sutherland 75

Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$



Primary e$^{\pm}$ **CR photons** **Secondary e$^{\pm}$** **SR photons**

$\dot{N}_{\rm p}$ $\dot{N}_{\rm p}N_{\gamma}$ $\dot{N}_{\rm p}N_{\gamma}\tau \equiv \dot{N}_{\rm s}$

SR power

$$P_{\rm syn} = \frac{2e^4B^2\alpha^2}{3c^3m_{\rm e}^2}\gamma_{\rm s,syn}^2$$

SR cooling timescale

$$t_{\rm cool,syn} \sim \frac{\gamma_{\rm s,syn}\alpha m_{\rm e}c^2}{P_{\rm syn}}$$

Lorentz factor of SR emitting e$^{\pm}$

$$\gamma_{\rm s,syn}(\nu_{\rm obs}) = \sqrt{\frac{4\pi}{0.87}\nu_{\rm obs}\frac{m_{\rm e}c}{eB\alpha}}$$

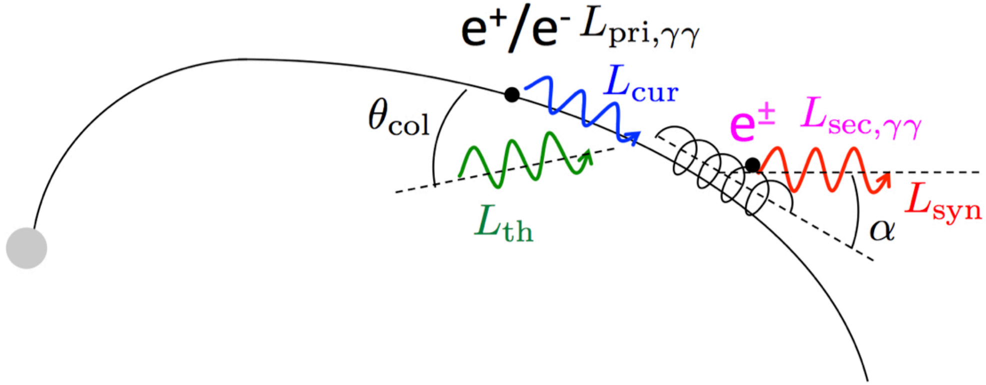
Advection timescale

$$t_{\rm ad} \sim \frac{r}{c}$$

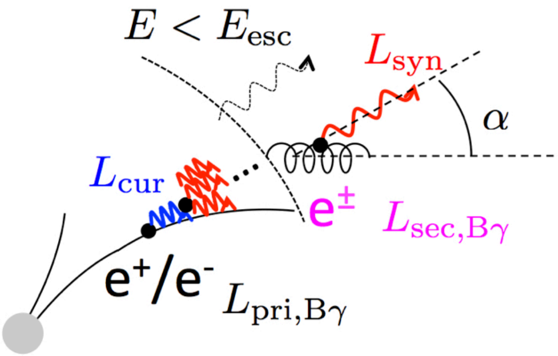
Synchrotron Luminosity

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

γγ scenario



Bγ scenario



Pitch angle

$$\alpha \sim \begin{cases} \sqrt{r/R_{\rm lc}} & ({\rm dipole}) \\ \alpha_0(\leq 1) & ({\rm non-dipole}) \end{cases}$$

Curvature radius

$$R_{\rm cur} \sim \begin{cases} \sqrt{rR_{\rm lc}} & ({\rm dipole}) \\ r & ({\rm non-dipole}) \end{cases}$$

Magnetic field strength

$$B(r) \equiv \zeta_{\rm B}B_{\rm dip}(r), \quad (\zeta_{\rm B} \geq 1)$$

Constraints

- **Energy of secondary e$^{\pm}$**

$$\gamma_{\rm s,pair} > \gamma_{\rm s,syn}$$

$$\gamma_{\rm s,syn} = \gamma_{\rm s}(\nu_{\rm obs})$$

$$\gamma_{\rm s,pair} = \gamma_{\rm s}(E_{\rm cur})$$

- **SR condition** O'Dell & Sartori 70, Rudak & Dyks 99

$$\gamma_{\rm s,syn}\alpha > 1$$

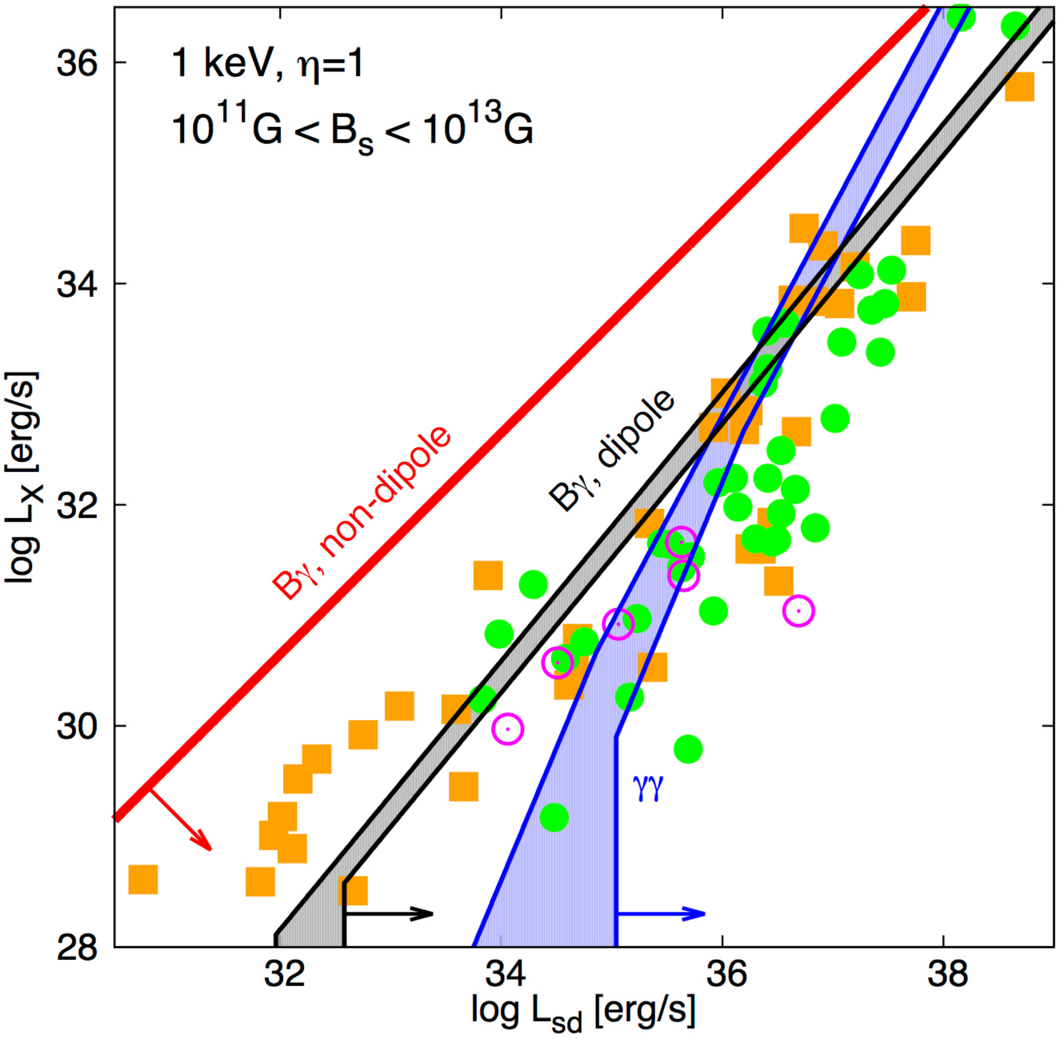
- **Pair production threshold**

$$(1-\cos\theta_{\rm col})E_{\rm pc}E_{\rm cur} > 2(m_{\rm e}c^2)^2$$ (γγ)

$$\frac{E_{\rm cur}}{2m_{\rm e}c^2}\frac{B_{\perp}}{B_{\rm q}} > \chi_{\rm min}\left(=\frac{1}{15}\right)$$ (Bγ)

L_{syn} – L_{sd} Plots

$$\eta = 1$$

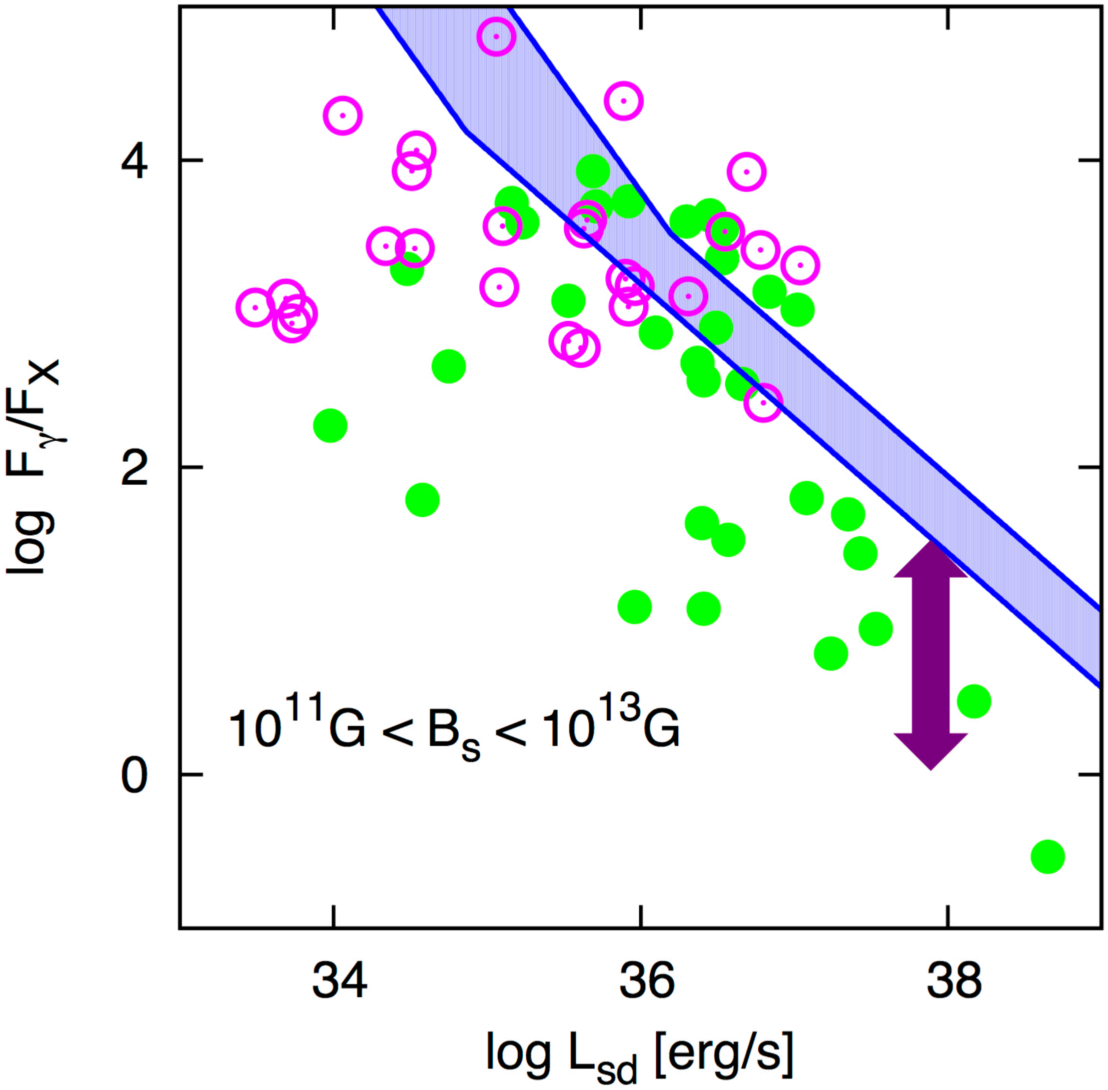


In γγ scenario, the X-ray luminosity of pulsars with $L_{sd} < 10^{35}$ erg s^{-1} is higher than the SR luminosity even if η = 1.

Even if Bγ scenario with dipole field, the X-ray luminosity of pulsars with $L_{sd} < 10^{33}$ erg s^{-1} is higher than the SR luminosity.

● : γ-ray detected radio-loud pulsars
○ : γ-ray detected radio-quiet pulsars
■ : Other pulsars

Flux Ratio (F_γ/F_X)



●: γ-ray detected radio-loud pulsars

○: γ-ray detected radio-quiet pulsars

Luminosity of CR

$$L_{\rm cur} = \eta L_{\rm sd}$$
$$(t_{\rm cool,cur} < t_{\rm ad})$$

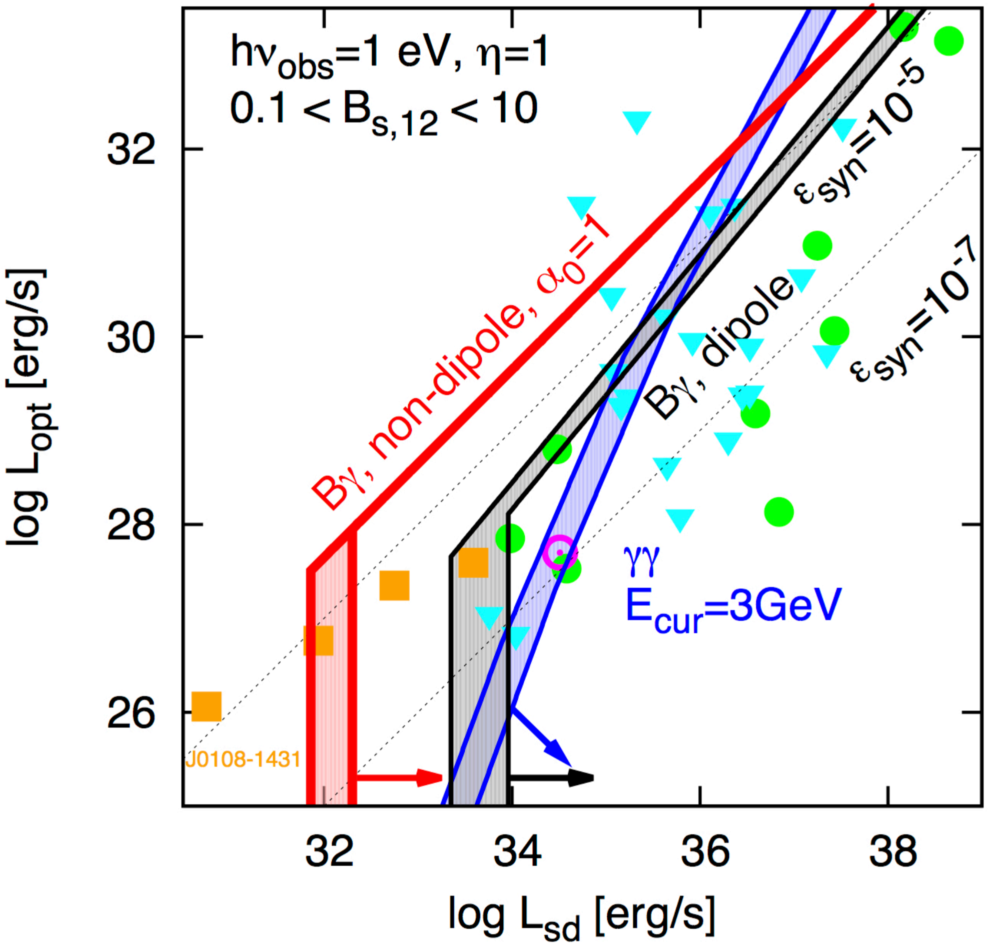
Flux ratio dose not depend on η.

Thermal radiation from entire surface could increase $\tau_{\gamma\gamma}$

Low-L_{sd} pulsars require additional emission mechanisms in X-ray.

L_{syn} – L_{sd} Plots

$\eta = 1$



In γγ scenario, the optical luminosity of pulsars with $L_{sd} < 10^{35}$ erg s^{-1} is also higher than the SR luminosity.

For pulsars with $L_{sd} < 10^{32}$ erg s^{-1}, Bγ process cannot work at the region $\hbar\omega_{gyro} < 1$eV even in the non-dipole B-field.

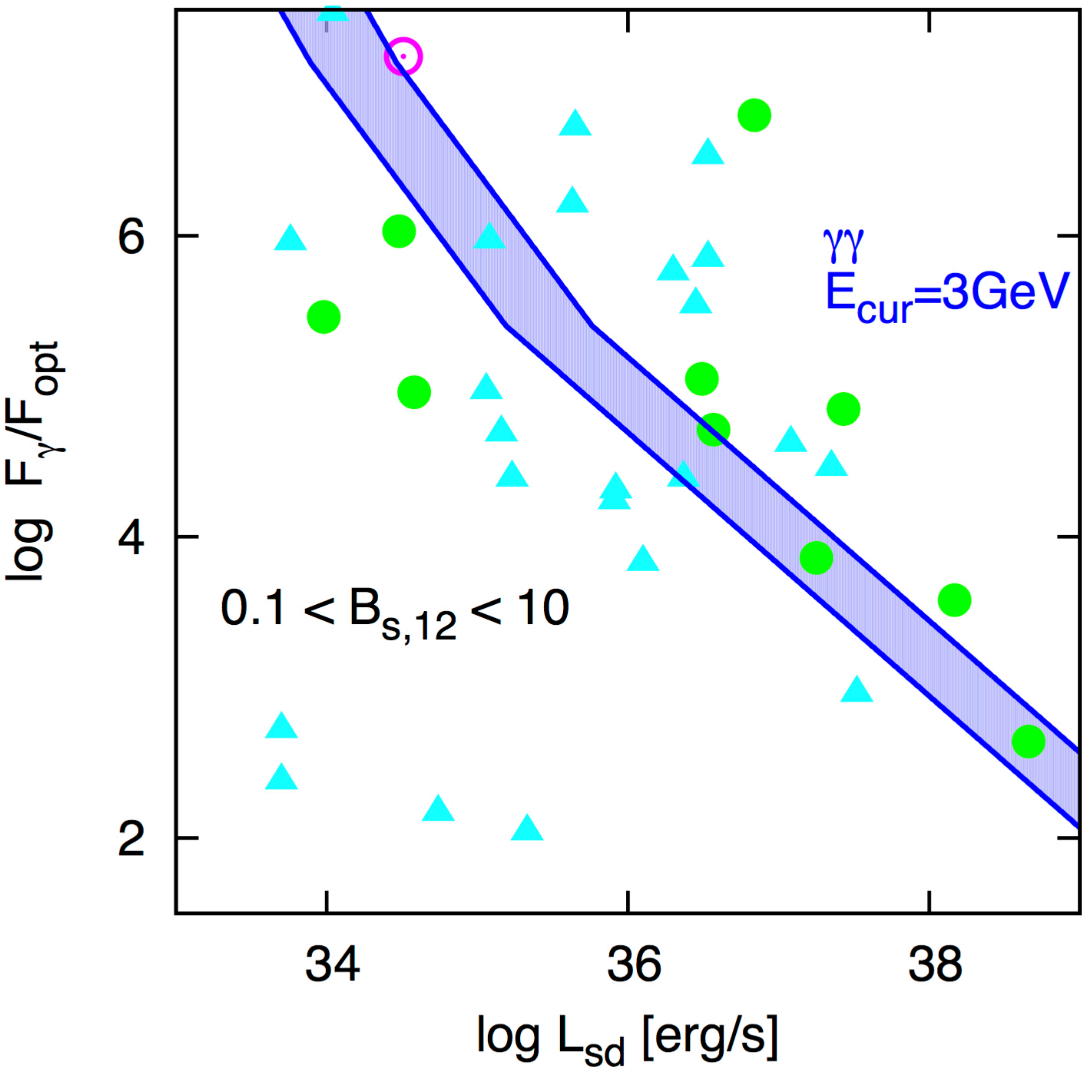
● : γ-ray detected radio-loud pulsars

○ : γ-ray detected radio-quiet pulsars

■ : Other pulsars

▼ : Observed upper limit

Flux Ratio (F_γ/F_{opt})



● : γ-ray detected radio-loud pulsars

○ : γ-ray detected radio-quiet pulsars

▼ : Observed upper limit

Luminosity of CR

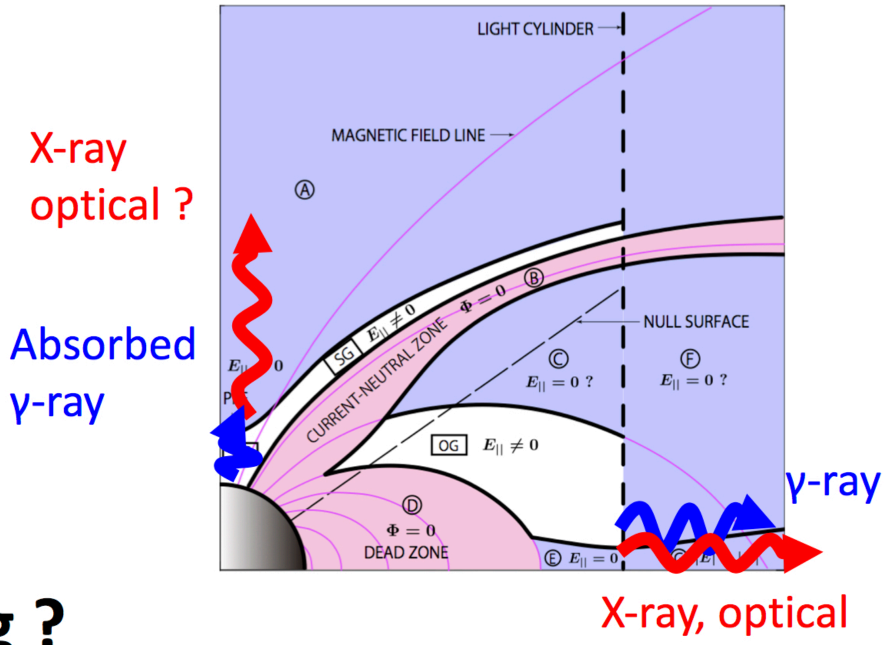
$$L_{\rm cur} = \eta L_{\rm sd}$$
$$(t_{\rm cool,cur} < t_{\rm ad})$$

Flux ratio dose not depend on η.

Low-L_{sd} pulsars require additional emission mechanisms in X-ray and optical.

Discussion

$L_{\rm sd} \lesssim 10^{35} {\rm erg\ s^{-1}}$



- **Multiple accelerators ?**

  e.g., Yuki & Shibata 12
  Petrova 13
  Marelli+ 14
  Philippov+ 15

- **Another energy source ?**

  e.g., Dissipation of magnetic field ?

- **Resonant Compton scattering ?**

  e.g., Zhang & Harding 00

- **Another mechanism to give the pitch angle ?**

  e.g., Machabeli & Usov 79

- **Small pitch angle SR ?**

  e.g., Epstein 73, SK & Tanaka 14

- **Emission from inwardly moving $e^{\pm}$?**

  e.g., Wang+ 13, but see also SK & Tanaka 15

Summary

- We analytically calculate the luminosity of synchrotron radiation from secondary $e^{\pm}$ in the pulsar magnetosphere.

- Since the energy conversion efficiency η should be close to unity, significant fraction of electromagnetic energy should convert to the particle energy in the magnetosphere.

- In γγ scenario, the observed non-thermal X-ray luminosity exceed the upper limit for γ-ray pulsars with $L_{sd} < 10^{35}$ erg s^{-1}. In addition, the flux ratio F_{γ}/F_{X} significantly lower than the synchrotron emission model. Other emission mechanisms or multiple accelerators are required for low-L_{sd} pulsars.

Synchrotron Luminosity (γγ)

$$L_{\rm syn} = P_{\rm syn}\dot{N}_{\rm s}\min\{t_{\rm ad}, t_{\rm cool,syn}\}$$

The most optimistic case (cooling time << advection time)

Radiation efficiency

$$\epsilon_{\rm syn} \equiv \frac{L_{\rm syn}}{L_{\rm sd}} \sim \eta\tau_{\gamma\gamma}\frac{\gamma_{\rm s,syn}\alpha}{\gamma_{\rm s,pair}}$$

Energy conversion efficiency

$$\eta \equiv \frac{\dot{N}_{\rm p}\gamma_{\rm p}m_{\rm e}c^2}{L_{\rm sd}}$$

$$\gamma_{\rm s,syn} = \gamma_{\rm s}(\nu_{\rm obs})$$

$$\gamma_{\rm s,pair} = \gamma_{\rm s}(E_{\rm cur}) = \frac{E_{\rm cur}}{2m_{\rm e}c^2}$$

Synchrotron Luminosity (Bγ)

$$L_{\mathrm{syn}} = P_{\mathrm{syn}} \dot{N}_{\mathrm{s}} \min\{t_{\mathrm{ad}}, t_{\mathrm{cool,syn}}\}$$

The most optimistic case (cooling time << advection time)

Radiation efficiency

$$\epsilon_{\mathrm{syn}} \equiv \frac{L_{\mathrm{syn}}}{L_{\mathrm{sd}}} \sim \eta \frac{\gamma_{\mathrm{s,syn}} \alpha m_{\mathrm{e}} c^2}{E_{\mathrm{esc}}}$$

Energy conversion efficiency

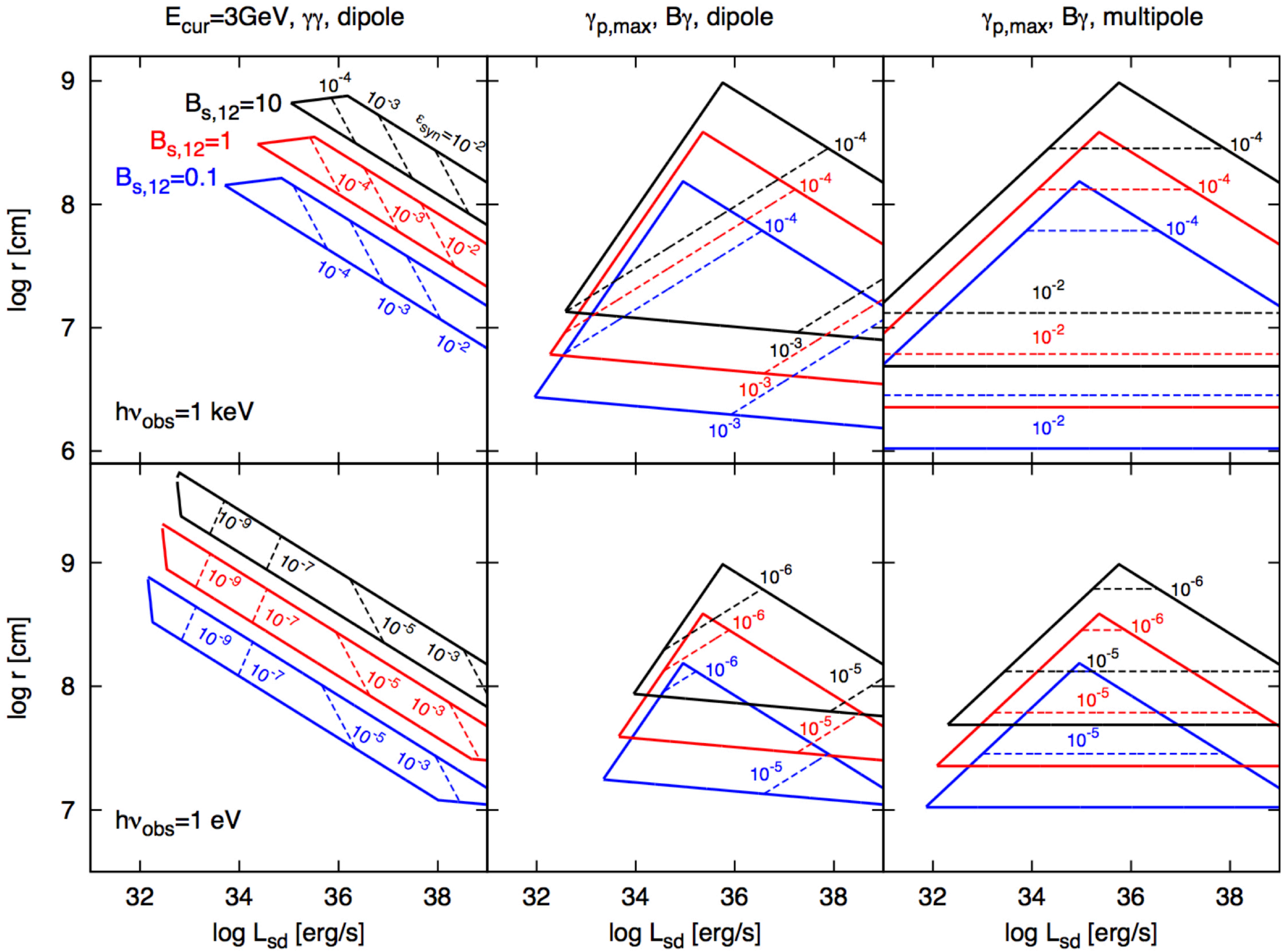
$$\eta \equiv \frac{\dot{N}_{\mathrm{p}} \gamma_{\mathrm{p}} m_{\mathrm{e}} c^2}{L_{\mathrm{sd}}}$$

$$\gamma_{\mathrm{s,syn}} = \gamma_{\mathrm{s}}(\nu_{\mathrm{obs}})$$

$$E_{\mathrm{esc}} = 2 m_{\mathrm{e}} c^2 \chi_{\mathrm{min}} \frac{B_{\mathrm{q}}}{B_{\perp}}$$

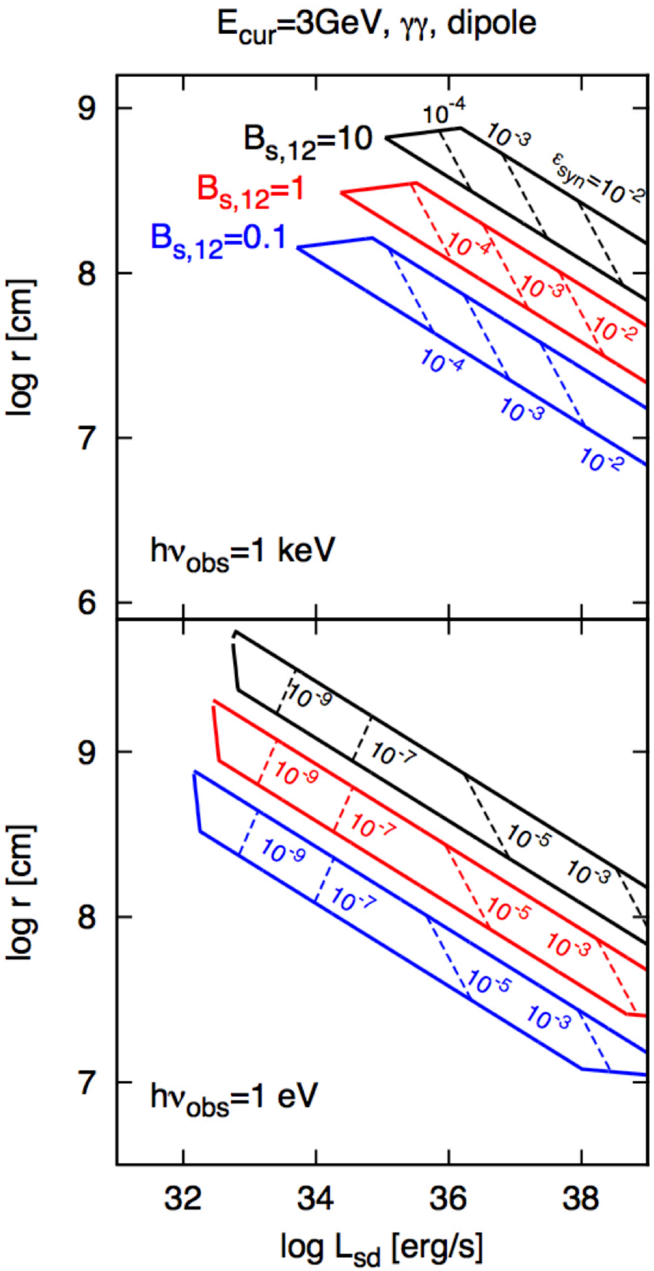
Emission Region

$\eta = 1$



Emission Region

$\eta = 1$



Upper boundary

Light cylinder

$$R_{\rm lc} = Pc/2\pi$$

Secondary $e^{\pm}$ energy

$$\gamma_{\rm s,pair} > \gamma_{\rm s,syn}$$

Lower boundary

Pair creation threshold

$$(1 - \cos\theta_{\rm col}) E_{\rm pc} E_{\rm cur} > 2(m_{\rm e}c^2)^2$$

The maximum SR luminosity is given at r = R_{lc} (even the model extends to r > R_{lc}).

Emission Region

$\eta = 1$

Upper boundary

Light cylinder

$$R_{\rm lc} = Pc/2\pi$$

Pair creation threshold

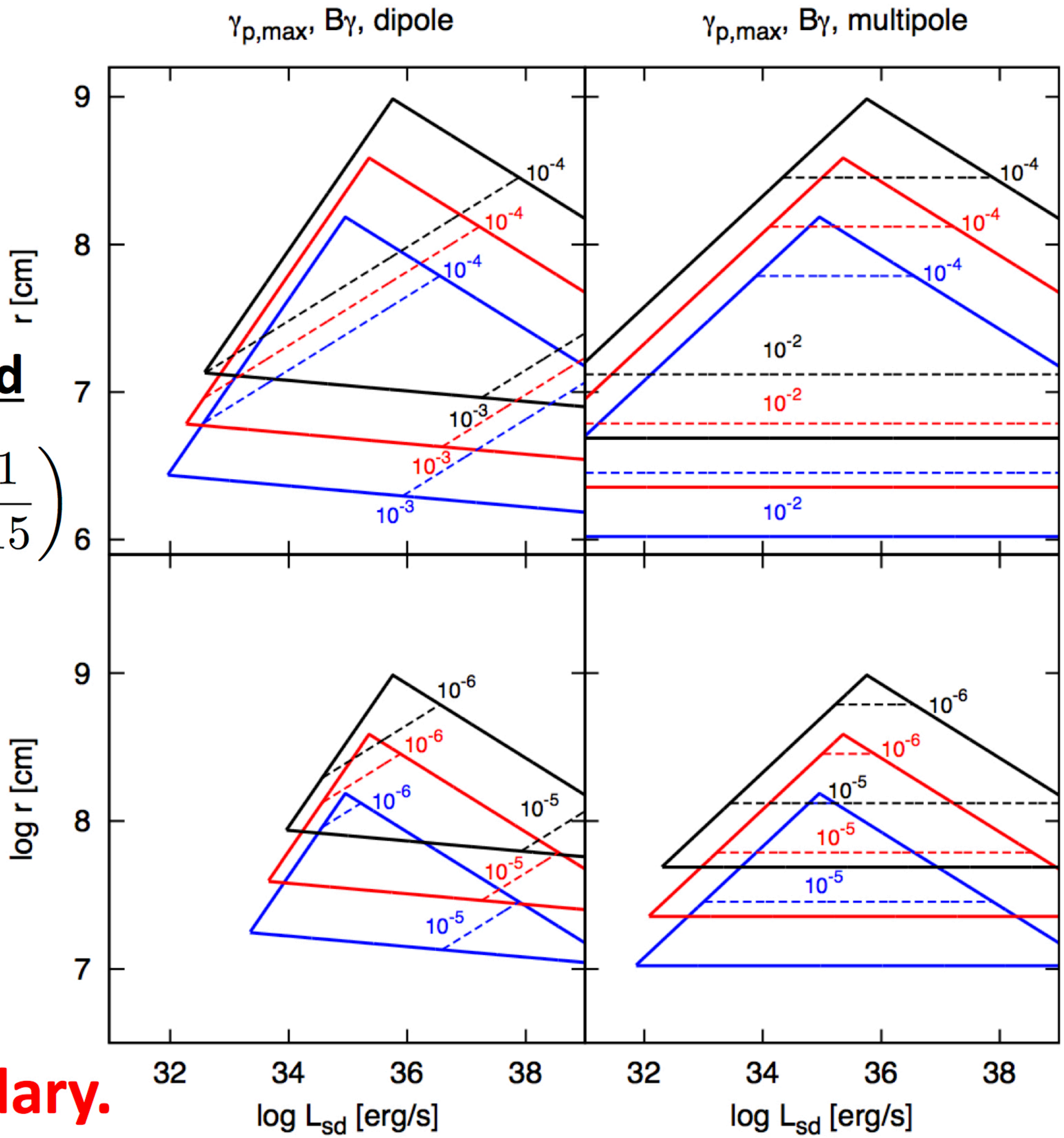
$$\frac{E_{\rm cur}}{2m_{\rm e}c^2}\frac{B_{\perp}}{B_{\rm q}} > \chi_{\rm min}\left(=\frac{1}{15}\right)$$

Lower boundary

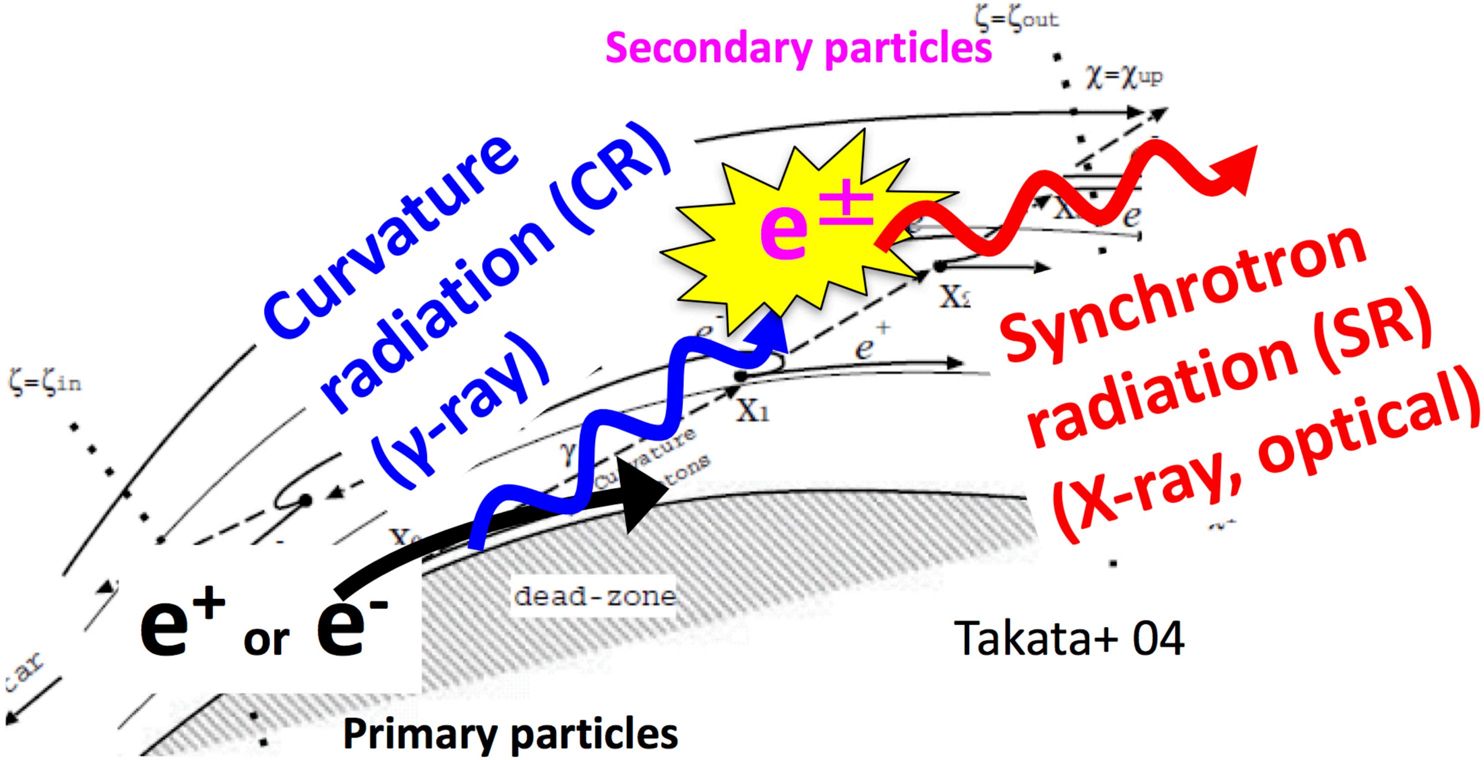
SR condition

$$\gamma_{\rm s,syn}\alpha > 1$$

The maximum SR luminosity is given at the lower boundary.



Synchrotron Radiation from Pulsars



Takata+ 04

We derive the luminosity of the synchrotron radiation and compare its with the observed non-thermal luminosity to investigate the pair cascade process in the magnetosphere.