ON THE SPECTRUM AND POLARIZATION OF MAGNETAR FLARE EMISSION

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Outline

- Introduction - Magnetars
- Theoretical model
- Numerical implementation
- Conclusions and Future prospects

Spectrum and polarization of magnetar flares – 2
Introduction - Magnetars
Magnetars - Basics

- Magnetars are isolated Neutron Stars powered by their own magnetic energy, observationally identified with SGRs and AXPs
  
  \[ L_X \sim 10^{33} - 10^{36} \text{ erg/s} > \dot{E} \]
  
  \[ B \gtrsim B_{\text{QED}} \text{ (at least inside the star)} \]

\[ B = 3.2 \times 10^{19} \sqrt{P \dot{P}} \text{ G} \]

\[ B \sim 10^{14} \text{ G} \]
Magnetars - Basics

• Magnetars are isolated Neutron Stars powered by their own magnetic energy, observationally identified with SGRs and AXPs
  
  \[- L_X \sim 10^{33} \sim 10^{36} \text{ erg/s} > \dot{E} \]
  
  \[- B \geq B_{\text{QED}} \text{ (at least inside the star)} \]

• Persistent emission
  
  – Soft X-ray spectrum (0.5 \sim 10 \text{ keV})
    \[ \text{BB + PL (or BB + BB)} \]
  
  – Additional PL for \( E \geq 20 \text{ keV} \)
Magnetars - Basics

- Bursting activity
  - Giant flares
    \[ \Delta t_{\text{spike}} \sim 0.1 - 1 \text{ s} \]
    \[ \Delta t_{\text{tail}} \sim 10^2 - 10^3 \text{ s} \]
    \[ E = 10^{44} - 10^{47} \text{ erg} \]
  - Intermediate flares
    \[ \Delta t \sim 1 - 10^2 \text{ s} \]
    \[ E \sim 10^{41} - 10^{43} \text{ erg} \]
  - Short bursts
    \[ \Delta t \sim 0.01 - 1 \text{ s} \]
    \[ E \sim 10^{36} - 10^{41} \text{ erg} \]
Theoretical magnetar model

Twist of the external magnetic field

- Twist angle:

\[ \Delta \phi_{N-S} = 2 \lim_{\theta \to 0} \int_{\theta}^{\pi/2} \frac{B_\phi}{\sin \theta B_\theta} \, d\theta \]

- Giant flares and short bursts are related to the plastic deformation of the crust (or to magnetic reconnection)

- Occurrence of RCS

Photon polarization

- In the presence of strong magnetic fields photons are polarized in two normal modes:
  - **X-mode**: (photon electric field oscillates perpendicular to both \( k \) and \( B \))
  - **O-mode**: (photon electric field oscillates in the \( kB \) plane)
Photon polarization

- In the presence of strong magnetic fields photons are polarized in two normal modes

- A convenient way to describe polarized radiation is through the Stokes parameters (that are additive)

\[
\begin{align*}
I &= A_x A_x^* + A_y A_y^* = a_x^2 + a_y^2 \\
Q &= A_x A_x^* - A_y A_y^* = a_x^2 - a_y^2 \\
U &= A_x A_y^* + A_y A_x^* = 2a_x a_y \cos(\varphi_x - \varphi_y) \\
V &= i(A_x A_y^* - A_y A_x^*) = 2a_x a_y \sin(\varphi_x - \varphi_y)
\end{align*}
\]

\[
E = A(z)e^{i(k_0 z - \omega t)}
\]

\[
A = (A_x, A_y) = (a_x e^{-i\varphi_x}, a_y e^{-i\varphi_y})
\]
Theoretical model
Trapped fireball model

- Alfvén pulses injected by crustal displacements dissipate into a magnetically confined electron-positron plasma
Trapped fireball model

- Alfvén pulses injected by crustal displacements dissipate into a magnetically confined electron-positron plasma
- We assumed the fireball plasma as a pure-scattering medium, restricting our calculations to the Thomson limit

\[
\frac{d^2 \sigma}{d \epsilon' d \Omega'} = \frac{3}{8\pi} \sigma_T (1 - \mu_B^2)(1 - \mu'_B^2) \delta(\epsilon' - \epsilon)
\]

\[
\frac{d^2 \sigma}{d \epsilon' d \Omega'} \bigg|_{oo} = \frac{3}{8\pi} \sigma_T \left( \frac{\epsilon}{\epsilon_B} \right)^2 \mu_B^2 \cos^2(\phi_B - \phi'_B) \delta(\epsilon' - \epsilon)
\]

\[
\frac{d^2 \sigma}{d \epsilon' d \Omega'} \bigg|_{ox} = \frac{3}{8\pi} \sigma_T \left( \frac{\epsilon}{\epsilon_B} \right)^2 \mu'_B^2 \cos^2(\phi_B - \phi'_B) \delta(\epsilon' - \epsilon)
\]

\[
\frac{d^2 \sigma}{d \epsilon' d \Omega'} \bigg|_{xo} = \frac{3}{8\pi} \sigma_T \left( \frac{\epsilon}{\epsilon_B} \right)^2 \mu'_B^2 \cos^2(\phi_B - \phi'_B) \delta(\epsilon' - \epsilon)
\]

\[
\frac{d^2 \sigma}{d \epsilon' d \Omega'} \bigg|_{xx} = \frac{3}{8\pi} \sigma_T \left( \frac{\epsilon}{\epsilon_B} \right)^2 \sin^2(\phi_B - \phi'_B) \delta(\epsilon' - \epsilon)
\]

\[\epsilon_B = m_e c^2 \frac{B}{B_{QED}}\]

We assumed \( \epsilon \ll \epsilon_B \)
Second order processes

• Thermal bremsstrahlung
  – $e^- - e^- (e^+ - e^+)$ bremsstrahlung strongly suppressed for particle energies $< 300$ keV ($\sim 0.01 \sigma_T$, Haug, 1975)
  – $e^- - e^+$ bremsstrahlung (slightly enhanced with respect to $e^- - p^+$) becomes negligible above $\epsilon = 1$ keV and $kT = 10$ keV ($< 0.5 \sigma_T$, Svenson, 1982)
Second order processes

- Thermal bremsstrahlung

- Photon splitting
  - Assuming weak dispersive effects (Stoneham, 1979) the only allowed channel is that of $X \rightarrow OO$, for which, in the non-relativistic regime ($\varepsilon \ll m_e c^2$) and for $B \leq B_{\text{QED}}$

$$\sigma_{sp}(X \rightarrow OO) \propto \left(\frac{B}{B_{\text{QED}}} \sin \theta_{Bk}\right)^6 \left(\frac{\varepsilon}{m_e c^2}\right)^5 \ll \sigma_T$$

  - In the strong field limit ($B > B_{\text{QED}}$) the splitting amplitude decreases exponentially (as $\exp(-B/B_{\text{QED}})$)
Second order processes

- Thermal bremsstrahlung
- Photon splitting
- Double Compton scattering
  - At energies $\varepsilon \ll kT$ and far from the cyclotron resonance photons are injected in the fireball at a rate (Lightman, 1981)

$$Q \approx \frac{4\alpha_F}{3\pi} \frac{\sigma_T}{m_e^2 c^4} \frac{\exp(\varepsilon/kT) - 1}{\varepsilon^3} \left[f_{\text{Pl}}(\varepsilon, T) - f(\varepsilon)\right]I$$

  - At higher energies scattering establishes a Bose-Einstein distribution $f_{\text{BE}}(\varepsilon, T)$, but with small chemical potential (see Lyubarsky, 2002)

$$\ln \frac{\mu + \varepsilon_0}{\varepsilon_0} \ll 0.5 \left(\frac{10B_{\text{QED}}}{B}\right)^2$$
Radiative transfer
Radiative transfer

- We solved the radiative transfer equation in the geometrically thin surface layer of the fireball (see Yang & Zhang, 2015), divided into a number of patches.
Radiative transfer

- We solved the radiative transfer equation in the geometrically thin surface layer of the fireball (see Yang & Zhang, 2015), divided into a number of patches.

- Assuming the patch dimension small enough the problem can be solved in the plane-parallel approximation.

\[
\mu_z = \mu_{Bk}\mu_B - \sqrt{(1 - \mu_{Bk}^2)(1 - \mu_B^2)}\cos\phi_{Bk}
\]
Radiative transfer equation (RTE)

• Under these assumptions the RTE for the photons polarized in the two modes assumes a simple form

\[
\frac{d n_0}{d \tau} = - \left[ 1 - \mu_{Bk}^2 + \frac{3 \mu_{Bk}^2}{4} \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \right] n_0(\alpha) + \frac{3}{8\pi} \int_{4\pi} \left[ (1 - \mu_{Bk}^2)(1 - \mu_{Bk}'^2) n_0(\alpha') + \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \mu_{Bk}^2 \cos^2(\phi_{Bk} - \phi_{Bk}') n_X(\alpha') \right] d\Omega'
\]

\[
\frac{d n_X}{d \tau} = - \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 n_X(\alpha) + \frac{3}{8\pi} \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \int_{4\pi} \left[ \sin^2(\phi_{Bk} - \phi_{Bk}') n_X(\alpha') + \mu_{Bk}'^2 \cos^2(\phi_{Bk} - \phi_{Bk}') n_0(\alpha) \right] d\Omega'
\]
Radiative transfer equation (RTE)

- Under these assumptions the RTE for the photons polarized in the two modes assumes a simple form

- Owing to the suppression factor of the X-mode photon cross sections, the propagation in the fireball medium is quite different according to the polarization mode

\[
\tau_O \sim n_e \sigma_O H \approx \tau \\
\tau_X \sim n_e \sigma_X H \approx \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \tau
\]

\[\sigma_i = \sigma_{iO} + \sigma_{iX}\]
Radiative transfer equation (RTE)

- Under these assumptions the RTE for the photons polarized in the two modes assumes a simple form.
- Owing to the suppression factor of the X-mode photon cross sections, the propagation in the fireball medium is quite different according to the polarization mode.

Introduction

Theoretical model

Implementation & Results

Conclusions

\[ \tau_O \sim n_e \sigma_O H \approx \tau \]
\[ \tau_X \sim n_e \sigma_X H \approx \varepsilon \]
\[ \tau_x = \tau_{O} + \tau_{X} \]

\[ \varepsilon = 10 \text{ keV} \]
\[ \varepsilon = 50 \text{ keV} \]
\[ \varepsilon = 100 \text{ keV} \]

\[ H \]

\[ \tau_0 \sim 1 \]
Radiative transfer equation (RTE)

- Under these assumptions the RTE for the photons polarized in the two modes assumes a simple form.
- Owing to the suppression factor of the X-mode photon cross sections, the propagation in the fireball medium is quite different according to the polarization mode.
- Hence we solved the photon transport in terms of the Rosseland mean optical depth $\tau_R$.

\[
\langle \sigma_X \rangle_R = \frac{4\pi^2}{5} \sigma_T \left( \frac{kT B_{QED}}{m_e c^2 B} \right)^2
\]

Temperature distribution (see Lyubarsky, 2002)

\[
T = T_b \sqrt{1 + \frac{3}{4} \tau_R}
\]
Radiative transfer equation (RTE)

• Under these assumptions the RTE for the photons polarized in the two modes assumes a simple form

• Owing to the suppression factor of the X-mode photon cross sections, the propagation in the fireball medium is quite different according to the polarization mode

• Hence we solved the photon transport in terms of the Rosseland mean optical depth $\tau_R$ for the X-mode photons

$$\tau_R = \frac{4\pi^2}{5} \left( \frac{kT_b B_{\text{QED}}}{m_e c^2 B} \right)^2 \sigma_T \int n_e ds = R(B)\tau$$

$$\tau_O = \frac{1}{R(B)} \tau_R$$

$$\tau_X = \frac{5}{4\pi^2} \left( \frac{\varepsilon}{kT_b} \right)^2 \tau_R$$
Numerical implementation and results
Assumptions

- Optical depth in the fireball atmosphere:
  \[ \tau_R = 1000 \text{ (at the base)} - 0 \text{ (at the top)} \]
Assumptions

- Optical depth in the fireball atmosphere:
  \( \tau_R = 1000 \) (at the base) \(- 0 \) (at the top)

- Photon energy \( \varepsilon = 1 - 100 \) keV

- Temperature distribution \( T = T_b \sqrt{1 + 0.75 \tau_R} \) \((T_b = 10 \) keV\)

- Dipolar \( B \)-field with \( B_p = 2 \times 10^{14} \) G

- \( R_{\text{max}} = 2R_{\text{NS}} \) \((R_{\text{NS}} = 10 \) km\)
Integration of the RTE

- $\Lambda$-iteration method (Runge-Kutta 4th order integration routine)

- Output quantities

$$n_O(\tau_R, \varepsilon, \mu_{Bk}, \phi_{Bk}) \quad n_X(\tau_R, \varepsilon, \mu_{Bk}, \phi_{Bk})$$

or

$$n_O(\tau_R, \varepsilon, \mu_Z, \phi_Z) \quad n_X(\tau_R, \varepsilon, \mu_Z, \phi_Z)$$
Integration of the RTE 
Spectrum and polarization of magnetar flares – 12

• Λ-iteration method (Runge-Kutta 4th order integration routine)

<table>
<thead>
<tr>
<th>Output quantities</th>
<th>n₀, τ R, ε, μ B k, φ B k</th>
</tr>
</thead>
<tbody>
<tr>
<td>or</td>
<td>n₀, τ R, ε, μ z, φ z</td>
</tr>
</tbody>
</table>

Introduction

Theoretical model

Implementation & Results

Conclusions
Introduction

Theoretical model

Implementation & Results

Conclusions

Spectrum and polarization of magnetar flares

- $\Lambda$-iteration method (Runge-Kutta 4th order integration routine)

Output quantities $n_n$, $\tau_R$, $\epsilon$, $\mu_B$, $\phi_B$ or $n_n$, $\tau_R$, $\epsilon$, $\mu_z$, $\phi_z$


$B = 10^{13}$ G

$\theta_B = 90^\circ$

$B = 10^{14}$ G

$\theta_B = 45^\circ$

$B = 10^{13}$ G

$\theta_B = 90^\circ$

$B = 10^{14}$ G

$\theta_B = 45^\circ$
Ray-tracing code

• The contributions from all the patches in view are summed together in a ray tracing code
  - $\chi$ angle between $\ell$ and $\Omega$
  - $\xi$ angle between $b_{dip}$ and $\Omega$
  - $\psi$ angle between $u$ and $X$
Ray-tracing code

• The contributions from all the patches in view are summed together in a ray tracing code

• Fireball visibility

Terminator equation: \( \hat{z} \cdot \hat{\ell} = 0 \)
Ray-tracing code

- The contributions from all the patches in view are summed together in a ray tracing code
- Fireball visibility
- Different emission geometries
Ray-tracing code

- The contributions from all the patches in view are summed together in a ray tracing code.
- Fireball visibility.
- Different emission geometries.
- QED and geometrical effects on the polarization observables.

Adiabatic propagation

Stokes parameter rotation

Mode decoupling
Ray-tracing code

• The contributions from all the patches in view are summed together in a ray tracing code

• Fireball visibility

• Different emission geometries

• QED and geometrical effects on the polarization observables

• Plasma contributions to the dielectric tensor are assumed to be negligible wrt the vacuum terms (vacuum resonance effects are relevant at $\varepsilon \lesssim 1\,\text{keV}$ only, Lyubarsky, 2002)
Spectral analysis

- Phase-averaged spectrum

  - The total spectrum is nearly superimposed to the X-mode component
  - O-mode photon flux strongly suppressed
  - High intrinsic polarization degree
Spectral analysis

- Phase-averaged spectrum
- Spectral fit

\[ f(\varepsilon) = \frac{A_1 \varepsilon^3}{\exp(\varepsilon/kT_1) - 1 + \frac{A_2/A_1}{\exp(\varepsilon/kT_2) - 1}} \]

<table>
<thead>
<tr>
<th>( \chi )</th>
<th>( \xi )</th>
<th>( T_1 ) (keV)</th>
<th>( T_2 ) (keV)</th>
<th>( R_2/R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model a</td>
<td>60°</td>
<td>30°</td>
<td>1.67</td>
<td>9.11</td>
</tr>
<tr>
<td>Model b</td>
<td>60°</td>
<td>30°</td>
<td>1.67</td>
<td>9.12</td>
</tr>
<tr>
<td>Model a</td>
<td>90°</td>
<td>0°</td>
<td>1.67</td>
<td>9.18</td>
</tr>
<tr>
<td>Model b</td>
<td>90°</td>
<td>0°</td>
<td>1.67</td>
<td>9.14</td>
</tr>
</tbody>
</table>

Observations of SGR 1900+14 (Israel et al., 2008)

\[ T_s = 4.8 \pm 0.3 \text{ keV} \quad T_h = 9.0 \pm 0.3 \text{ keV} \]

\[ R_h/R_s = 0.19 \pm 0.03 \]
Spectral analysis

- Phase-averaged spectrum
- Spectral fit
- Pulse profile

Model a

Model b
Polarization signal

- Phase-resolved polarization fraction

![Graphs showing phase-resolved polarization fraction for two models, Model a and Model b. The graphs illustrate how the polarization fraction (PF) drops at higher photon energies, where the O-mode contribution becomes more important.]

- The PF drops at higher photon energies (where the O-mode contribution becomes more important).
Polarization signal

- Phase-resolved polarization fraction

- The PF drops at higher photon energies (where the O-mode contribution becomes more important)

- Photons coming from the constant-$\phi$ cuts are generally more polarized than those coming from the torus
Polarization signal

• Phase-resolved polarization fraction
• Phase-resolved polarization angle

• No substantial differences between model a and model b
• The polarization angle behavior is related to the viewing geometry and the magnetic field topology
Polarization signal

- Phase-resolved polarization fraction
- Phase-resolved polarization angle
- Phase-averaged polarization observables
Conclusions and Future prospects
Conclusions

• Method for modelling the spectral and polarization properties of radiation emitted during magnetar flares from a steady trapped-fireball

• Magnetic (Thomson) scattering is the dominant source of opacity

• Simulated spectra are well fitted by two BB as suggested by observations (but the hypothesis by Israel et al., 2008 of the two different O- and X-mode photospheres is not supported)

• The model can reproduce the pulse profiles observed in intermediate/giant flare decay tails (tuning both viewing and emission geometries)
• Radiation is expected to be highly polarized (in the extraordinary mode)