Superfluid Hydrodynamics and Entrainment in the Inner Crust of Neutron Stars

Michael Urban (IPN Orsay)

in collaboration with Noël Martin
Outline

▶ Introduction: inner crust, superfluid hydrodynamics, and entrainment

▶ Microscopic modeling of the entrainment
  ▶ Band structure theory
  ▶ Superfluid hydrodynamics at the microscopic scale
  ▶ Deviations from superfluid hydrodynamics

▶ Consequences for glitches

▶ Summary and outlook

For details, see N. Martin and M. U., Phys. Rev. C 94, 068501 (2016)
The inner crust of neutron stars

- **outer crust:** Coulomb lattice of neutron rich nuclei in a degenerate electron gas
- **inner crust:** superfluid gas of unbound neutrons between the nuclei ("clusters")
- **outer core:** homogeneous matter: $n$, $p$, $e^-$, ($\mu^-$)
- **inner core:** hyperons? quark matter?

- Crystalline and "pasta" phases to minimize surface + Coulomb energy

- BCC crystal (3D)
- Hexagonal lattice of rods ("spaghetti", 2D)
- Plates ("lasagne", 1D)
Superfluid hydrodynamics at large scales

- Typical temperature: $T \sim 10^8$ – $10^9$ K $\sim$ 10 – 100 keV
- Critical temperature of the neutron gas: $T_c \sim 1$ MeV
  $\rightarrow$ neutrons are superfluid
- Order parameter (gap) $\Delta = |\Delta| e^{i\varphi} \neq 0$
- Consider scales large compared to the unit cell (10 – 100 fm)
  but small compared to the distance between vortices (10 – 100 μm)
- Introduce coarse-grained order parameter $\bar{\Delta}$ and its phase $\bar{\varphi}$
- Superfluid velocity $\vec{u}_n = \frac{1}{2m} \vec{\nabla} \bar{\varphi}$
- Effective low-energy theory using $\bar{\varphi}(\vec{r})$ and cluster displacements $\bar{d}(\vec{r})$ as
degrees of freedom [Cirigliano, Sharma, and Reddy, PRC (2011)]
- Density of superfluid neutrons?
  $\rightarrow$ effective theory needs input from a more microscopic approach
Entrainment in the inner crust

- Consider relative motion between the clusters and the neutron gas on a microscopic scale.

- Gas has to flow around (or through?) the clusters → a certain fraction of neutrons are “entrained” by the protons.

- Entrainment can be expressed in terms of effectively “bound” and “superfluid” neutrons or in terms of a cluster effective mass.

\[ N = N^b + N^s \]
\[ A_{\text{eff}} = Z + N^b \]
\[ \frac{E_{\text{kin}}}{V} = \frac{m}{2} \left( (\bar{n}_p + \bar{n}_n^b)\vec{u}_p^2 + \bar{n}_n^s\vec{u}_n^2 \right) \]

where:
\[ \vec{u}_p = \text{velocity of protons (clusters)} \]
\[ \vec{u}_n = \text{velocity of superfluid neutrons} \]
Microscopic approaches: (a) band-structure theory

[N. Chamel]

- Analogous to band structure theory in solids
- Neutrons in a periodic mean field
  - energy bands $\alpha$ with complicated dispersion relations $\epsilon_\alpha(\vec{k})$

- Neutron “mobility” obtained from $ \frac{d\epsilon_\alpha}{dk} $ at the Fermi surface
- Strong entrainment $\rightarrow A_{eff}$ strongly increased, $\bar{n}_n^s$ strongly reduced

[Figures from Chamel and Haensel, Living Rev. Relativity 11 (2008)]
Microscopic approaches: (b) superfluid hydrodynamics

[A. Sedrakian; P. Magierski and A. Bulgac]

- Assumption: coherence length (Cooper pair size) $\xi$ sufficiently small

$$\vec{v}_n(\vec{r}) = \frac{1}{2m} \vec{\nabla} \varphi(\vec{r})$$

$\xi = \text{coherence length in the neutron gas}$

$\xi (\text{Matsuo})$  $\xi (\text{Sun et al.})$

$L = \text{size of unit cell}$

$R = \text{cluster radius}$

$\rightarrow$ neutron velocity on a microscopic scale given by $\vec{v}_n(\vec{r})$

- Analytical results for the case of a single cluster in an infinite gas

- This work: extension to a periodic lattice of clusters
Hydrostatic equilibrium: phase coexistence

- Hydrodynamic approach → equilibrium state must satisfy

\[ P(\vec{r}) = \text{const.}, \quad \mu_a(\vec{r}) = \text{const.} \quad (a = n, p) \]

- Clusters with diffuse surface are approximated by coexisting phases with sharp interface

Cluster and gas densities obtained in the ETF approach satisfy phase coexistence conditions very well
Stationary flow of the clusters through the gas

- Protons bound in clusters \(\rightarrow\) velocity of clusters = velocity of protons \(\vec{u}_p\)

- No compression \(\rightarrow\) velocity potential \(\phi = \varphi/2m\) satisfies: \(\triangle \phi = 0\)

- Boundary conditions at the cluster-gas interface:
  - Continuity of the phase: \(\phi^{(1)} = \phi^{(2)}\)
  - Conservation of the neutron current:
    \[
    n_n^{(1)}(\nabla \phi^{(1)} - \vec{u}_p) \cdot \vec{S} = n_n^{(2)}(\nabla \phi^{(2)} - \vec{u}_p) \cdot \vec{S}
    \]

- Coarse-grained velocity potential reads \(\overline{\phi} = \vec{u}_n \cdot \vec{r}\)
  \(\rightarrow\) microscopic velocity potential can be written as
  \[
  \phi(\vec{r}; \vec{u}_p, \vec{u}_n) = \vec{u}_n \cdot \vec{r} + \phi(\vec{r}; \vec{u}_p - \vec{u}_n, \vec{0})
  \]

- In the rest frame of the superfluid neutrons, \(\phi(\vec{r}; \vec{u}_p - \vec{u}_n, \vec{0})\) is periodic
Uniform flow in the BCC lattice (3D)

- Example: $n_B = 0.049$ fm$^{-3}$

  velocity potential $\phi$

  stream lines and speed $|\vec{v}_n|$

- Flow in the cluster is slower than motion of the cluster itself ($|\vec{v}_n^{(2)}| < |\vec{u}_p|$)
  
  → gaz neutrons flow through the cluster

  → $A_{eff} < A$ → impact on phonon velocities, heat capacity, cooling…
Superfluid density

- Distinguish densities of energetically free ($\bar{n}_f^n$) and superfluid ($\bar{n}_s^n$) neutrons

- Free neutron density in good agreement with band structure theory [N. Chamel, PRC (2012)]

- Superfluid fraction obtained from hydrodynamics is much larger than the result of band structure theory

- Possible reasons for this discrepancy?

- Validity of hydrodynamics questionable: $\xi$ of the same order as $R$

- Band structure theory does not account for the strong pairing [cf. also Watanabe & Pethick (2017)]

  in superconductors: spacing between bands $\gg$ pairing gap $\Delta$

  in the inner crust: spacing between bands $\lesssim$ pairing gap $\Delta$
Estimate of the deviation from hydrodynamics

- Problem: coherence length $\xi \sim \text{cluster size } R$
  $\rightarrow$ deviations from $\vec{v}_n = \frac{1}{2m} \vec{\nabla} \varphi$ [Migdal (1959)]

- Same situation as in atomic nuclei

- Nuclear moments of inertia lie in between the irrotational-flow and the rigid-body values

- Assume that only a fraction $\delta \leq 1$ of the neutrons in the cluster are superfluid

- in the equations, replace
  \[ n_n^{(2)} \rightarrow \delta n_n^{(2)} , \quad n_p^{(2)} \rightarrow n_p^{(2)} + (1 - \delta)n_n^{(2)} \]

- Even with $\delta = 0$, the superfluid neutrons in the gas are enough to produce a large superfluid fraction

[figure: Rowe book (1970)]
Consequences for glitches

- Strong glitch activity of Vela pulsar \( \rightarrow \) superfluid must contribute \( \gtrsim 1.6\% \) to the total moment of inertia \( I \) of the star

- With the superfluid fraction of band theory \( (I_s/I_{\text{crust}} = 0.17) \), Vela mass would have to be \( \lesssim 0.7M_{\odot} \)  
  [Andersson, PRL 109; Chamel PRL 110]

Suggestions to solve this “Vela glitch puzzle”:

- Superfluidity in the core?  [Andersson PRL 109]
- New equation of state that gives a thicker crust?  [Piekarewicz, PRC 90]

- Take the superfluid fraction of the present model  
  \( \rightarrow \) observed glitches can be conciliated with Vela mass of up to  
  \( 1.7M_{\odot} \) if \( \delta = 1 \) \( (I_s/I_{\text{crust}} = 0.94) \) or still \( 1.5M_{\odot} \) if \( \delta = 0 \) \( (I_s/I_{\text{crust}} = 0.64) \)
Summary

- Entrainment of neutrons by the clusters in the inner crust
  → cluster effective mass, superfluid fraction
- Superfluid hydrodynamics on a microscopic scale
  → entrainment much weaker than in band structure theory
  → deviations expected because of small cluster size
- Possible solution to the Vela glitch puzzle

Outlook

- Extension to oscillations (long-wavelength effective theory)
  → coupling between superfluid phonons and lattice vibrations
- Consequences for transport properties (cooling)
- Check hydrodynamics by comparing it with QRPA calculations
- Temperature effects? Zero-point motion of the clusters?