Superfluid Hydrodynamics and Entrainment in the Inner Crust of Neutron Stars

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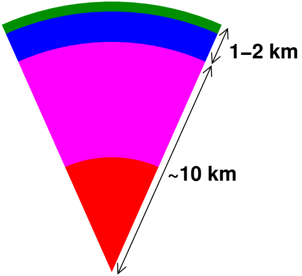
Outline

- Introduction: inner crust, superfluid hydrodynamics, and entrainment
- Microscopic modeling of the entrainment
  - Band structure theory
  - Superfluid hydrodynamics at the microscopic scale
  - Deviations from superfluid hydrodynamics
- Consequences for glitches
- Summary and outlook

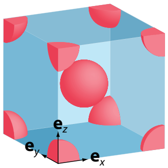
For details, see N. Martin and M. U., Phys. Rev. C 94, 068501 (2016)

The inner crust of neutron stars

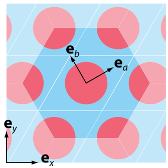


outer crust:	Coulomb lattice of neutron rich nuclei in a degenerate electron gas
inner crust:	superfluid gas of unbound neutrons between the nuclei ("clusters")
outer core:	homogeneous matter: n, p, e^-, (μ^-)
inner core:	hyperons? quark matter?

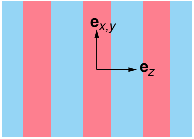
- Crystalline and "pasta" phases to minimize surface + Coulomb energy



BCC crystal (3D)



hexagonal lattice of rods ("spaghetti", 2D)



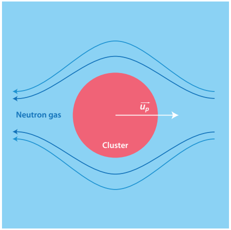
plates ("lasagne", 1D)

Superfluid hydrodynamics at large scales

- Typical temperature: $T \sim 10^8 - 10^9$ K $\sim 10 - 100$ keV
- Critical temperature of the neutron gas: $T_c \sim 1$ MeV
  $\longrightarrow$ neutrons are superfluid
- Order parameter (gap) $\Delta = |\Delta| e^{i\varphi} \neq 0$
- Consider scales large compared to the unit cell ($10 - 100$ fm)
  but small compared to the distance between vortices ($10 - 100$ μm)
- Introduce coarse-grained order parameter $\bar{\Delta}$ and its phase $\bar{\varphi}$
- Superfluid velocity $\vec{u}_n = \dfrac{1}{2m} \vec{\nabla} \bar{\varphi}$
- Effective low-energy theory using $\bar{\varphi}(\vec{r})$ and cluster displacements $\vec{d}(\vec{r})$ as degrees of freedom [Cirigliano, Sharma, and Reddy, PRC (2011)]
- Density of superfluid neutrons?
  $\longrightarrow$ effective theory needs input from a more microscopic approach

Entrainment in the inner crust

- Consider relative motion between the clusters and the neutron gas on a microscopic scale
- Gas has to flow around (or through?) the clusters $\rightarrow$ a certain fraction of neutrons are "entrained" by the protons



- Entrainment can be expressed in terms of effectively "bound" and "superfluid" neutrons or in terms of a cluster effective mass

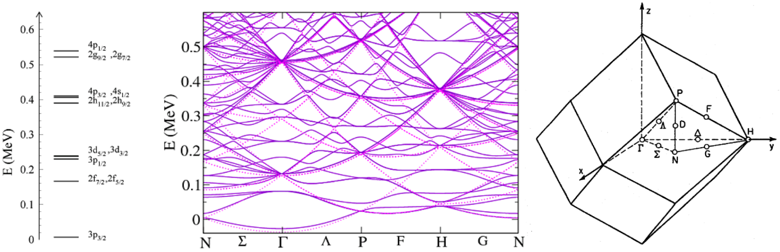
$$N = N^b + N^s \qquad A_{eff} = Z + N^b \qquad \frac{E_{kin}}{V} = \frac{m}{2}\Big((\bar{n}_p + \bar{n}_n^b)\vec{u}_p^{\,2} + \bar{n}_n^s \vec{u}_n^{\,2}\Big)$$

where:
$\vec{u}_p$ = velocity of protons (clusters)
$\vec{u}_n$ = velocity of superfluid neutrons

Microscopic approaches: (a) band-structure theory

[N. Chamel]

- Analogous to band structure theory in solids
- Neutrons in a periodic mean field
  $\longrightarrow$ energy bands α with complicated dispersion relations $\epsilon_\alpha(\vec{k})$



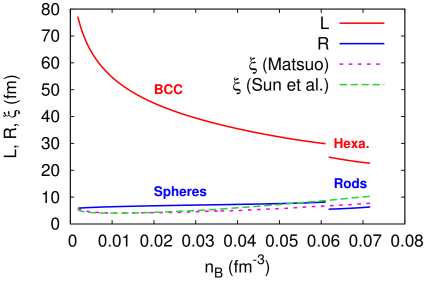
[Figures from Chamel and Haensel, Living Rev. Relativity 11 (2008)]

- Neutron "mobility" obtained from $\dfrac{d\epsilon_\alpha}{dk}$ at the Fermi surface
- Strong entrainment $\longrightarrow$ A_{eff} strongly increased, $\bar{n}_n^s$ strongly reduced

Microscopic approaches: (b) superfluid hydrodynamics

[A. Sedrakian; P. Magierski and A. Bulgac]

- Assumption: coherence length (Cooper pair size) ξ sufficiently small



L = size of unit cell

R = cluster radius

ξ = coherence length in the neutron gas

[Matsuo, PRC 73 (2006); Sun et al., PLB 683 (2010)]

$\longrightarrow$ neutron velocity on a microscopic scale given by $\vec{v}_n(\vec{r}) = \dfrac{1}{2m}\vec{\nabla}\varphi(\vec{r})$

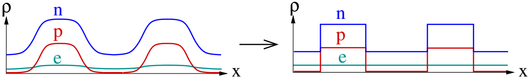
- Analytical results for the case of a single cluster in an infinite gas
- This work: extension to a periodic lattice of clusters

Hydrostatic equilibrium: phase coexistence

- Hydrodynamic approach $\rightarrow$ equilibrium state must satisfy

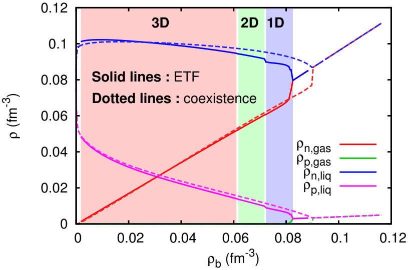
$$P(\vec{r}) = const. , \qquad \mu_a(\vec{r}) = const. \qquad (a = n, p)$$

- Clusters with diffuse surface are approximated by coexisting phases with sharp interface



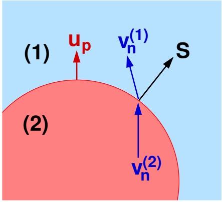
cluster $\hat{=}$ liquid drop
neutron gas $\hat{=}$ vapor

- Cluster and gas densities obtained in the ETF approach satisfy phase coexistence conditions very well



Stationary flow of the clusters through the gas

- Protons bound in clusters $\longrightarrow$ velocity of clusters = velocity of protons $\vec{u}_p$
- No compression $\longrightarrow$ velocity potential $\phi = \varphi/2m$ satisfies: $\triangle\phi = 0$
- Boundary conditions at the cluster-gas interface:
  - Continuity of the phase: $\phi^{(1)} = \phi^{(2)}$
  - Conservation of the neutron current: $n_n^{(1)}(\vec{\nabla}\phi^{(1)} - \vec{u}_p)\cdot\vec{S} = n_n^{(2)}(\vec{\nabla}\phi^{(2)} - \vec{u}_p)\cdot\vec{S}$



- Coarse-grained velocity potential reads $\bar{\phi} = \vec{u}_n \cdot \vec{r}$

  $\longrightarrow$ microscopic velocity potential can be written as

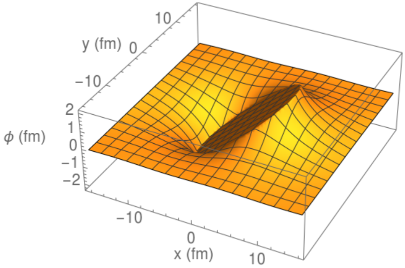
$$\phi(\vec{r};\vec{u}_p,\vec{u}_n) = \vec{u}_n\cdot\vec{r} + \phi(\vec{r};\vec{u}_p - \vec{u}_n,\vec{0})$$

- In the rest frame of the superfluid neutrons, $\phi(\vec{r};\vec{u}_p - \vec{u}_n,\vec{0})$ is periodic

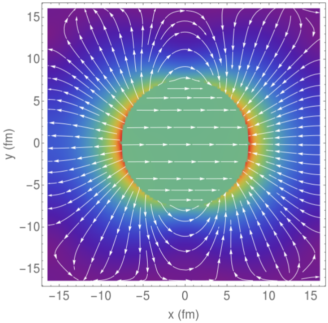
Uniform flow in the BCC lattice (3D)

- Example: $n_B = 0.049$ fm^{-3}

velocity potential ϕ



stream lines and speed $|\vec{v}_n|$



- Flow in the cluster is slower than motion of the cluster itself ($|\vec{v}_n^{(2)}| < |\vec{u}_p|$)

  $\rightarrow$ gaz neutrons flow through the cluster

  $\rightarrow$ $A_{eff} < A$ $\rightarrow$ impact on phonon velocities, heat capacity, cooling...

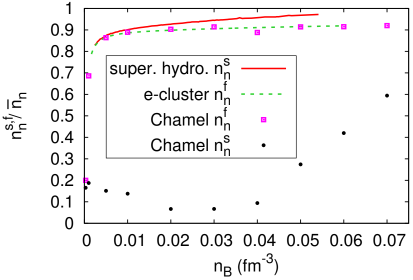
Superfluid density

- Distinguish densities of energetically free ($\bar{n}_n^f$) and superfluid ($\bar{n}_n^s$) neutrons

- free neutron density in good agreement with band structure theory [N. Chamel, PRC (2012)]

- superfluid fraction obtained from hydrodynamics is much larger than the result of band structure theory

- Possible reasons for this discrepancy?



- Validity of hydrodynamics questionable: ξ of the same order as R

- Band structure theory does not account for the strong pairing [cf. also Watanabe & Pethick (2017)]

  in superconductors: spacing between bands $\gg$ pairing gap Δ

  in the inner crust: spacing between bands $\lesssim$ pairing gap Δ

Estimate of the deviation from hydrodynamics

- Problem: coherence length $\xi \sim$ cluster size R

  $\rightarrow$ deviations from $\vec{v}_n = \frac{1}{2m}\vec{\nabla}\varphi$ [Migdal (1959)]

- Same situation as in atomic nuclei

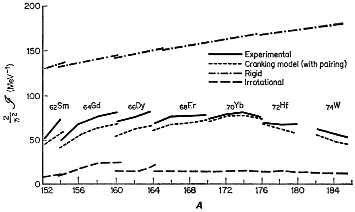
- Nuclear moments of inertia lie in between the irrotational-flow and the rigid-body values

- Assume that only a fraction $\delta \leq 1$ of the neutrons in the cluster are superfluid

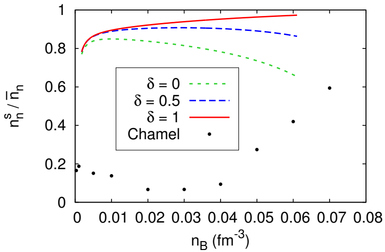
- in the equations, replace

$$n_n^{(2)} \rightarrow \delta n_n^{(2)}, \quad n_p^{(2)} \rightarrow n_p^{(2)} + (1-\delta) n_n^{(2)}$$

- Even with $\delta = 0$, the superfluid neutrons in the gas are enough to produce a large superfluid fraction

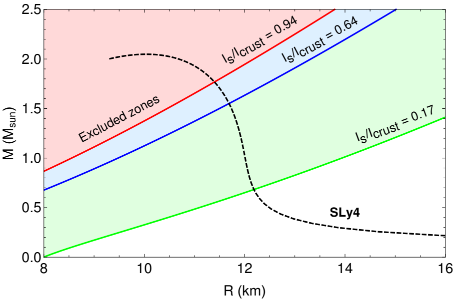


[figure: Rowe book (1970)]



Consequences for glitches

- Strong glitch activity of Vela pulsar
  $\rightarrow$ superfluid must contribute $\gtrsim 1.6\,\%$ to the total moment of inertia I of the star



- With the superfluid fraction of band theory ($I_s/I_{crust} = 0.17$), Vela mass would have to be $\lesssim 0.7 M_{sun}$
  [Andersson, PRL 109; Chamel PRL 110]

- Suggestions to solve this "Vela glitch puzzle":
  - Superfluidity in the core? [Andersson PRL 109]
  - New equation of state that gives a thicker crust? [Piekarewicz, PRC 90]

- Take the superfluid fraction of the present model
  $\rightarrow$ observed glitches can be conciliated with Vela mass of up to $1.7 M_{sun}$ if $\delta = 1$ ($I_s/I_{crust} = 0.94$) or still $1.5 M_{sun}$ if $\delta = 0$ ($I_s/I_{crust} = 0.64$)

Summary

- Entrainment of neutrons by the clusters in the inner crust
  $\longrightarrow$ cluster effective mass, superfluid fraction
- Superfluid hydrodynamics on a microscopic scale
  $\longrightarrow$ entrainment much weaker than in band structure theory
  $\longrightarrow$ deviations expected because of small cluster size
- Possible solution to the Vela glitch puzzle

Outlook

- Extension to oscillations (long-wavelength effective theory)
  $\longrightarrow$ coupling between superfluid phonons and lattice vibrations
- Consequences for transport properties (cooling)
- Check hydrodynamics by comparing it with QRPA calculations
- Temperature effects? Zero-point motion of the clusters?