

# CMBR distortion concerned with recombination of the primordial hydrogen plasma

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CMBR distortion concerned with recombination of the primordial plasma is calculated in frequency band from 1 GHz to 100 GHz in the frame of the standard cosmological model for different values of cosmological density parameters: nonrelativistic matter density  $\Omega_m$  and baryonic matter density  $\Omega_b$ . Comparison of these results with observational data which will be obtained from planned experiments may be used for independent determination of the cosmological parameters  $\Omega_m$  and  $\Omega_b$ .

## 1. Introduction

The Universe was existing at the thermodynamical equilibrium during 100 thousand years after the electron - positron annihilation. When the temperature of the Universe became low enough ( $< 2 \cdot 10^4$  K, it is corresponding to redshift  $z < 7000$ ) because of the cosmological expansion, the recombination of the main elements began (Fig. 1).

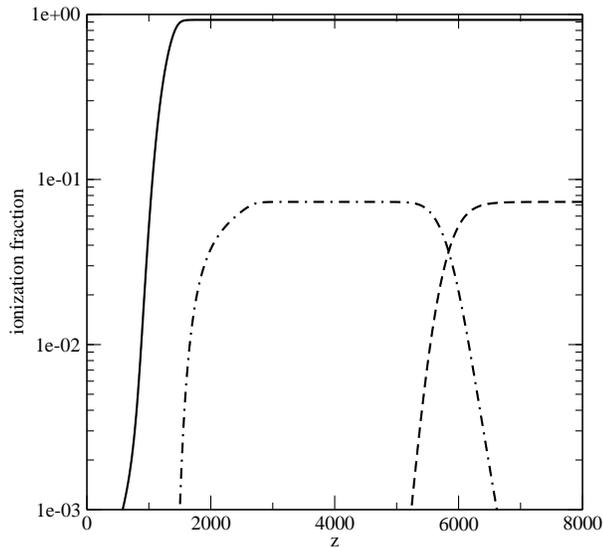


Figure 1: The recombination of the main elements: the solid line corresponds to the recombination of HII, dashed – HeIII, dashed-dot – HeII

This process leads to the birth of non-equilibrium photons. The most of them had survived until the modern epoch and have appeared as lines of the CMBR spectrum (Fig. 2).

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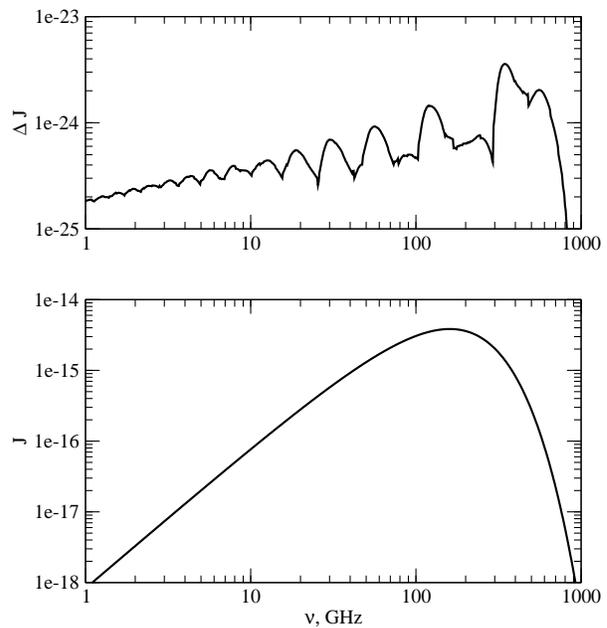


Figure 2: Top picture:  $\Delta J$  is the brightness [erg/cm<sup>2</sup>/sec/Hz/ster] of the non-equilibrium radiation concerned with the primordial hydrogen recombination. Bottom picture:  $J$  [erg/cm<sup>2</sup>/sec/Hz/ster] is the CMBR brightness as a function of frequency.

The measurement of the intensity and the form of these lines would give the information on the values of key cosmological parameters (e.g.  $\Omega_b, \Omega_m$ ).

The calculation of the distortion of the CMBR spectrum requires:

1. The calculation of the level populations of the primordial hydrogen atoms in the presence of a strong radiation (photon per mode  $\eta \gg 1$  for the transitions investigated) which has the Planck spectrum at temperature decreasing with the cosmological time.

2. The modelling of the radiation transfer in the homogeneous expanding environment.

## 2. Physical model

We used the standard cosmological model (see Tab.1). The temperature, the concentration of protons, and the Hubble constant as functions of redshift are:

$$T = T_0(1+z), \quad N_{tot} = N_{p0}(1+z)^3, \quad (1)$$

$$H = H_0 \sqrt{\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_{rel}(1+z)^4} \quad (2)$$

Table 1. The standard cosmological model parameters

Value description	Symbol	Value
total matter	$\Omega_{tot}$	1
space curvature	$\Omega_k$	0
non-relativistic matter	$\Omega_m$	$\sim 0.3$
baryonic matter	$\Omega_b$	$\sim 0.04$
relativistic matter	$\Omega_{rel}$	$\sim 10^{-4}$
vacuum-like energy	$\Omega_\Lambda$	$\sim 0.7$
Hubble constant	$H_0$	70 km/s/Mpc
radiation temperature	$T_0$	$2.725 \pm 0.002$ K
proton concentration	$N_{p0}$	$\sim 2 \cdot 10^{-7}$ cm $^{-3}$

The characteristic values of the cosmological parameters in the recombination epoch are presented in Tab. 2. The knowledge of these values allows us to estimate the relative CMBR temperature distortion concerned with the non-equilibrium radiation at  $i \rightarrow k$  transition ( $h\nu_{ik} \ll k_B T$ ) at frequency  $\nu \simeq \nu_{ik}/(1+z)$  by the formula:

$$\frac{\Delta T}{T} = \frac{c^3}{8\pi\nu_{ik}^3} \frac{1+z}{\Delta z} \frac{h\nu_{ik}}{k_B T(z)} N_{tot}(z) \theta_{ik} \quad (3)$$

where  $\Delta z$  is the duration of recombination epoch,  $\theta_{ik}$  is an element of the matrix, so-called ‘‘matrix of efficiency of radiative transitions’’ (ERT-matrix was introduced by Bernstein et al. [1]). Their values are  $10^{-2} - 10^{-4}$  for  $i, k = 10 - 40$  correspondingly (see [2]).

Table 2

Redshift	$z$	800 - 1600
Temperature	$T$	2200 - 4100 K
Proton concentration	$N_p$	100 - 700 cm $^{-3}$

For example, according to formula (3), the relative distortion of the CMBR temperature concerned with  $17 \rightarrow 16$  transition (redshifted frequency  $\simeq 1.14$  GHz,  $\theta \simeq 5 \cdot 10^{-3}$ ) is  $\Delta T/T = 2.5 \cdot 10^{-8}$ .

Formula (3) does not take into account the following facts:

1. There are overlaps of the line profiles because the cosmological expansion leads to the formation of the

wide line profile with the relative frequency width about 0.3.

2. The variation of the recombination rate leads to the formation of a non-rectangular line profile.

3. The elements of the ERT-matrix  $\theta_{ik}$  depend on redshift. They vary during recombination epoch within 20%.

The points mentioned above made us to calculate the formation of the CMBR distortion numerically.

## 3. Initial equations

### 3.1. Description of population behavior

We considered the hydrogen atoms in the excited states with principal quantum number  $n \geq 2$  ([1], [2]). The transitions into the ground state are considered separately because the optical depth for these transitions is much more than one ([3]).

For the description of the behavior of the hydrogen atoms we used the quasi-stationary approximation of the kinetic equations averaged over the angular momentum:

$$\sum_{k=1}^{\infty} A_{ki} x_k - \sum_{k=1}^{\infty} A_{ik} x_i + \alpha_i N_e x_p - \beta_i x_i - J_i = 0 \quad (4)$$

where  $x_i = N_i/N_{tot}$  is the relative population of the state with principal quantum number  $n = i + 1$  (subscript  $i = 1$  corresponds to the first excited state),  $N_i$  is the population of state  $i$  (the number of atoms in state  $i$  per volume unit),  $N_e$  is the free electron concentration,  $x_p$  is the free proton fraction,  $A_{ik}$  is the coefficient of  $i \rightarrow k$  transition ( $A_{ii} = 0$ ),  $\alpha_i$  is the coefficient of recombination to state  $i$ ,  $\beta_i$  is the coefficient of ionization from state  $i$ . Value  $J_i$  is the rate of uncompensated transitions from state  $i$  to the ground state of hydrogen atom. Value  $J_i$  is an independent parameter and to be described below.

In this work, the free electron concentration  $N_e(t)$  and the free proton fraction  $x_p(t)$  are also independent parameters. These functions are the solution of the cosmological recombination problem. For the first time this problem has been solved by Peebles [3], Zel’dovich et al. [4] for hydrogen primordial plasma. The problem of the recombination of the hydrogen-helium primordial plasma has been solved by Seager et al. [5] who used the model based on [3].

We have reproduced the results of investigations [3] and [5] (see Fig. 3).

To solve equations (4) the following symbols are convenient:  $X$  is the vector of the values  $x_i$ ,  $B$  is the vector of the values  $b_i = \alpha_i N_e x_p$ ,  $J$  is the vector of the values  $J_i$ ,  $Q$  is the transition matrix which has the elements given by the following expressions:

$$Q_{ik} = A_{ki}, \quad i \neq k; \quad Q_{ii} = - \left( \sum_{k=1}^{\infty} A_{ik} + \beta_i \right) \quad (5)$$

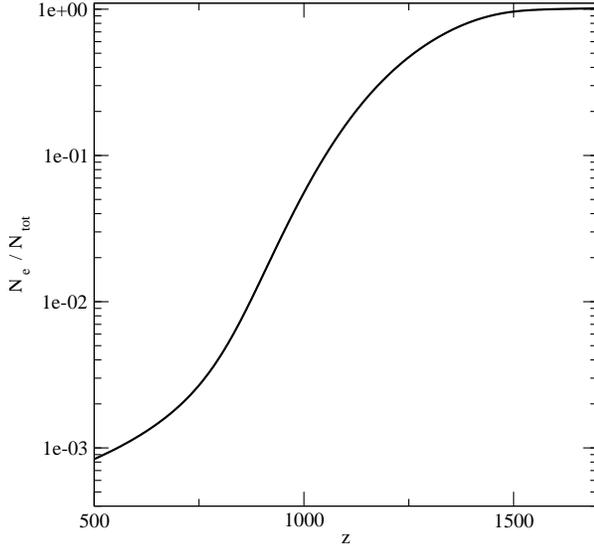


Figure 3: The ionization fraction of the hydrogen plasma as a function of redshift

These definitions allow us to rewrite equations (4):

$$Q(t)X - J(t) + B(t) = 0 \quad (6)$$

The solution of this equation can be presented in the form  $X = X^0 + \Delta X$ , where  $X^0$  is the equilibrium values of the populations given by the Boltzmann distribution,  $\Delta X$  is the corrections concerned with the uncompensated transitions into the ground state. Value  $X^0$  is the solution of the equation

$$Q(t)X^0 + B(t) = 0, \quad (7)$$

then  $\Delta X$  is the solution of the equation

$$Q(t)\Delta X = J(t). \quad (8)$$

The explicit form of  $\Delta X$  value is

$$\Delta X = Q^{-1}(t)J(t) \quad (9)$$

The method mentioned above (equations 4 - 9) requires preliminary determination of  $J$  value. The exact calculation of this value demands of the joint modelling of:

1. The recombination process.
2. The population behavior, taking two-quantum decay of high-excited states into account.
3. The Lyman-radiation transport for each line  $n \rightarrow 1$ .

In this paper we used the following approximation: all uncompensated transitions occur only from the first excited state due to two-quantum decay of state 2S and  $L_\alpha$ -quantum out of a line profile ([3], [4]). Thus we can write  $J_i = |\dot{x}_p| \delta_{1i}$  ( $\delta_{ik}$  is the Kroneker symbol).

This definition allows us to write the following expression for the population corrections:

$$\Delta x_i = (Q^{-1})_{i1} |\dot{x}_p| \quad (10)$$

System (8) has been solved by the Gauss method with the choice of the leading element. The equilibrium values have been determined by the Boltzmann distribution.

To show a convergence of calculations with a variation of the level number, we used three models of the hydrogen atom: 40-level, 80-level, and 160-level. The convergence is quite satisfactory for our aim (see Fig.7). The standard calculations are performed for 80-level model.

### 3.2. Description of radiation behavior

In order to describe the radiation behavior, we used the kinetic equation for radiation:

$$\begin{aligned} \frac{\partial \eta}{\partial t} - H\nu \frac{\partial \eta}{\partial \nu} = \\ = \frac{c^3}{8\pi\nu^2} N_{tot} \sum_{i=1}^{\infty} \sum_{k=0}^{i-1} (A_{ik}x_i - A_{ki}x_k) \Psi_{ik}(\nu) \end{aligned} \quad (11)$$

where  $\eta(t, \nu)$  is the number of photon per mode as a function of time and frequency,  $\Psi_{ik}(\nu)$  is the line profile normalized by the expression  $\int \Psi_{ik}(\nu) d\nu = 1$ .

The function  $\Psi_{ik}(\nu)$  has the maximum at frequency  $\nu = \nu_{ik}$  and differs noticeably from zero in the frequency range  $[\nu_d, \nu_u]$ , where  $\nu_d \simeq \nu_{ik} - 3\Delta\nu_D$ ,  $\nu_u \simeq \nu_{ik} + 3\Delta\nu_D$ ,  $\Delta\nu_D \simeq 3 \cdot 10^{-5} \nu_{ik}$  is the thermal line width.

The solution of the equation (11) can be presented as:

$$\eta(t, \nu) = \eta_0(t, \nu) + \sum_{i=2}^{\infty} \sum_{k=1}^i \Delta\eta_{ik}(t, \nu) \quad (12)$$

where  $\eta_0(t, \nu)$  is the solution of the homogeneous equation corresponding to equation (11),  $\Delta\eta_{ik}(t, \nu)$  is the partial solution of the equation:

$$\begin{aligned} \frac{\partial \Delta\eta_{ik}}{\partial t} - H\nu \frac{\partial \Delta\eta_{ik}}{\partial \nu} = \\ = \frac{c^3}{8\pi\nu^2} N_{tot} (A_{ik}x_i - A_{ki}x_k) \Psi_{ik}(\nu) \end{aligned} \quad (13)$$

The property of the equation (11) allows us to consider the equation for each line separately then to sum up the partial solutions.

The problem of the behavior of the non-equilibrium radiation can be solved in two stage considered below.

#### 3.2.1. Formation of non-equilibrium radiation

To research the formation of non-equilibrium radiation, we considered the frequency range  $[\nu_d, \nu_u]$  close to central frequency  $\nu_{ik}$  of the line. The quasi-stationary approximation in this range ([3]) is:

$$-H\nu \frac{\partial \Delta\eta_{ik}^s}{\partial \nu} = \frac{c^3}{8\pi\nu^2} N_{tot} (A_{ik}x_i - A_{ki}x_k) \Psi_{ik}(\nu) \quad (14)$$

where  $\Delta\eta_{ik}^s(t, \nu)$  is the partial solution in the range considered. This solution depends on time parametrically. The value  $\Delta\eta_{ik}^s(t, \nu)$  at the lower limit of the range considered sets the boundary condition for the further evolution of the non-equilibrium radiation:  $\Delta\eta_{ik}^d(t) = \Delta\eta_{ik}^s(\nu_d, t)$ . This value can be obtained by the formula based on the general solution of the radiation transport equation in the case of the interaction between radiation and matter at different temperatures ([6]):

$$\Delta\eta_{ik}^d = (\eta_m - \eta_0(\nu_{ik}))(1 - \exp(-\tau_{ik})) \quad (15)$$

where value  $\eta_m = 1/(x_k i^2/x_i k^2 - 1)$  describes the radiation of matter,  $\eta_0$  is the photon number per mode in the flow of the incident quanta. In our case  $\eta_0$  is given by the Planck distribution. The value  $\tau_{ik}$  is the optical depth in the line  $i \rightarrow k$  ([7], [8]):

$$\tau_{ik} = \frac{c^3}{8\pi\nu_{ik}^3} N_{tot} \frac{A_{ik}^s}{H} \left( \frac{i^2}{k^2} x_k - x_i \right) \quad (16)$$

where  $A_{ik}^s$  is the Einstein coefficient of the spontaneous transition ([9], [10]). The formula (16) is obtained by means of the absorption coefficient formula of line  $i \rightarrow k$  ([11]).

We can linearize expression (15) over the optical depth, if it is much less than one ( $\tau_{ik} \ll 1$ ):

$$\Delta\eta_{ik}^d = (\eta_m - \eta_0(\nu_{ik}))\tau_{ik} \quad (17)$$

and using (16) it can be written:

$$\Delta\eta_{ik}^d = \frac{c^3}{8\pi\nu_{ik}^3} N_{tot}(z) H^{-1}(z) (A_{ik}x_i - A_{ki}x_k) \quad (18)$$

The following definition is convenient:

$$\theta_{ik} = J_1^{-1}(A_{ik}x_i - A_{ki}x_k) \quad (19)$$

It can be rewritten (in our model):

$$\theta_{ik} = |\dot{x}_p|^{-1} (A_{ik}\Delta x_i - A_{ki}\Delta x_k) \quad (20)$$

Taking into account the relation  $dz/dt = -H(z)(1+z)$  and expression (20), formula (18) can be rewritten in the form:

$$\Delta\eta_{ik}^d = \frac{c^3}{8\pi\nu_{ik}^3} N_{tot}(z)(1+z) \frac{dx_p}{dz} \theta_{ik} \quad (21)$$

This formula allows us to obtain the following (in the case  $h\nu_{ik} \ll k_B T$ ):

$$\frac{\Delta T}{T} = \frac{c^3}{8\pi\nu_{ik}^3} (1+z) \frac{dx_p}{dz} \frac{h\nu_{ik}}{k_B T(z)} N_{tot}(z) \theta_{ik} \quad (22)$$

For the first time this formula has been obtained in the paper [1] in somewhat other form.

We can estimate  $dx_p/dz$  on its averaged value  $\overline{dx_p/dz} = 1/\Delta z$ . Then we obtain formula (3).

The element of the ERT-matrix  $\theta_{ik}$  means the averaged number of photons emitted in line  $i \rightarrow k$  per one act of uncompensated recombination.

Taking definition (20) and expression (10) into account, the elements of the ERT-matrix are given by the formula:

$$\theta_{ik} = (A_{ik}(Q^{-1})_{i1} - A_{ki}(Q^{-1})_{k1}) \quad (23)$$

It shows that the elements of the ERT-matrix can be calculated independently with the solution of the cosmological recombination problem.

In paper [1] the calculation of the ERT-matrix was necessary because of the approach accepted. In paper [2] the ERT-values were the main result. To compare our result with that of paper [2], ERT-matrix has been calculated in this work (Fig. 4, Fig. 5). The Burgin's results [2] have been reproduced with relative accuracy within 1%.

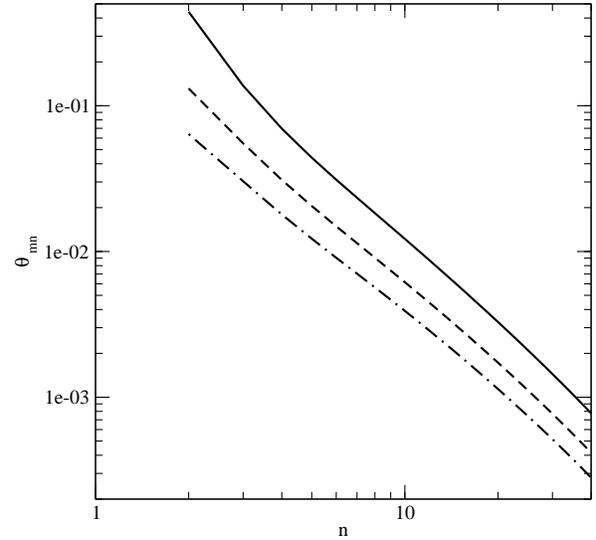


Figure 4: The ERT-elements as functions of the principal quantum number  $n$  of lower level: the solid line corresponds to transition with  $m = n + 1$ , dashed -  $m = n + 2$ , dashed-dot -  $m = n + 3$

### 3.2.2. Radiation transport in homogeneous expanding environment

To investigate the radiation transport in the homogeneous expanding environment, we considered the frequency range  $[0, \nu_d]$ .

Due to the properties of function  $\Psi_{ik}(\nu)$ , equation (11) in this range is:

$$\frac{\partial}{\partial t} \Delta\eta_{ik} - H\nu \frac{\partial}{\partial \nu} \Delta\eta_{ik} = 0 \quad (24)$$

where  $\Delta\eta_{ik}(t, \nu)$  is the partial solution with the boundary condition  $\Delta\eta_{ik}(t, \nu_{ik}) = \Delta\eta_{ik}^d(t)$  (taking  $\nu_{ik} \simeq \nu_d$  into account).

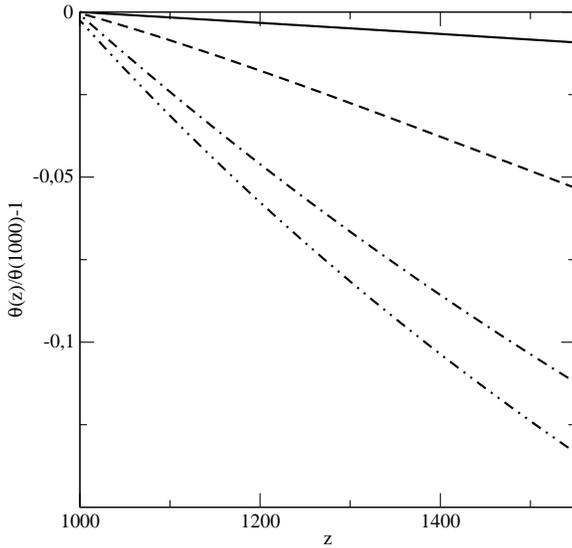


Figure 5: The relative change of ERT-elements during the recombination epoch: from top to bottom the transitions are showed: 3 → 2, 11 → 10, 21 → 20, 31 → 30

Changing from time-functions to redshift-functions, equation (24) takes the form:

$$(1+z)\frac{\partial}{\partial z}\Delta\eta_{ik} + \nu\frac{\partial}{\partial\nu}\Delta\eta_{ik} = 0 \quad (25)$$

with the boundary condition  $\Delta\eta_{ik}(z, \nu_{ik}) = \Delta\eta_{ik}^d(z)$ .

The solution of the equation (25) is:

$$\Delta\eta_{ik}(z, \nu) = \Delta\eta_{ik}^d((1+z)\nu_{ik}/\nu - 1) \quad (26)$$

The number of photons per mode at frequency  $\nu$  in the epoch with redshift  $z$  is given by the formula:

$$\eta(z, \nu) = \eta_0(z, \nu) + \sum_{i=2}^{\infty} \sum_{k=1}^i \Delta\eta_{ik}(z, \nu) \quad (27)$$

The relative distortion  $\Delta T/T = (T_{ex} - T)/T$  of the CMBR temperature is calculated by the formula:

$$T_{ex}(z, \nu) = \frac{h\nu}{k_B \ln(1/\eta(z, \nu) + 1)} \quad (28)$$

#### 4. Features of observation

Many lines cannot be registered because the CMBR is contaminated by quasar radiation and IR-source radiation ([12]). For example, the strongest expected line  $L_\alpha$  is invisible because of dust radiation. Taking into account this background, it is reasonable to calculate the recombination radiation concerned with the transitions with the principal quantum number over ten. The spectrum of this recombination radiation is located in the frequency range observed with the detectors for CMBR measurements.

#### 5. Results

The main result of this paper is the relative CMBR temperature distortion as a function of frequency. This result is shown in Fig.6. We calculated the CMBR distortion with the various sets of the cosmological parameters  $\Omega_m$  and  $\Omega_b$ :  $\{\Omega_m = 0.3; h^2\Omega_b = 0.014, 0.018, 0.022\}$  and  $\{\Omega_m = 0.1, 0.3, 0.5; h^2\Omega_b = 0.018\}$

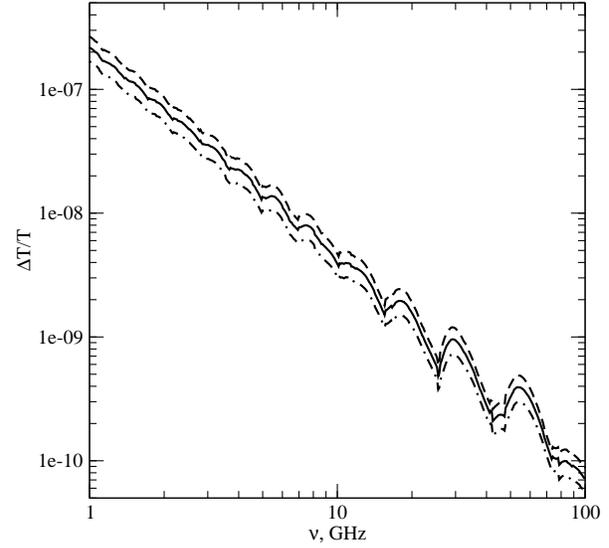


Figure 6: The relative CMBR temperature distortion as a function of frequency. The curves correspond to  $h^2\Omega_b = 0.014; 0.018; 0.022$  from bottom to top. All curves were calculated at  $\Omega_m = 0.3$

The result of  $\Omega_m$ -variation is not presented graphically because of weak dependence of CMBR distortion on  $\Omega_m$ -value. This fact can be explained for the optically thin lines (in our case  $\tau < 10^{-4}$ ) by formula (21). It displays that, the CMBR distortion  $\Delta\eta$  depends on  $\Omega_m$  by only the ionization fraction  $x_p(z)$  which vary with  $\Omega_m$  weakly.

The  $\Omega_b$ -variation shows that CMBR temperature distortion is directly proportional to the baryonic matter density  $\Omega_b$  (Fig.6).

The maximum value of the relative CMBR distortion in the observed range is less than  $3 \cdot 10^{-7}$ . This value is to be observed at the low frequencies.

To resolve the distortion corresponding to different values  $\Omega_b$ , it is necessary to measure the CMBR temperature with absolute accuracy  $\sim 1$  nK.

#### Acknowledgement

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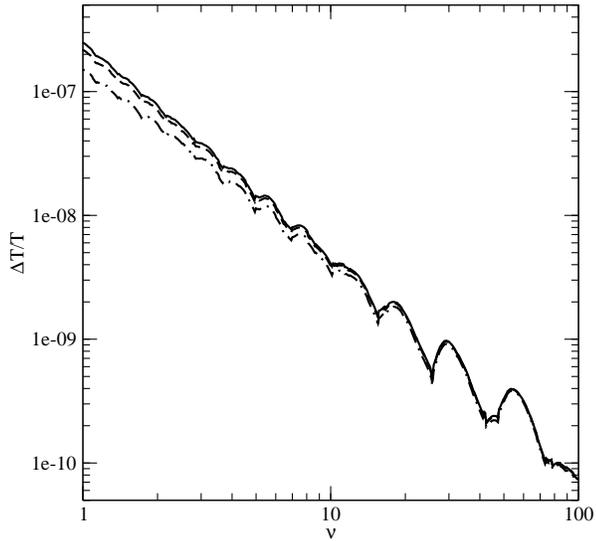


Figure 7: The distortion at variation of the level number of hydrogen atom. The curves correspond to  $n_{max} = 40, 80, 160$  from bottom to top. All curves were calculated at  $\Omega_m = 0.3, h^2\Omega_b = 0.018$

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