How nucleon gets its mass

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1. Quantum Chromodynamics: the theory of strong interactions

2. Chiral symmetry of strong interactions and its spontaneous breaking

3. Vacuum in a microscope

4. Bound states, including pentaquarks?

Scales in strong interactions

'Strong interactions' are about everything from π's to ^{238}U. Masses are measured in MeV:

$$m_e = 0.511\,\mathrm{MeV}, \quad m_\pi = 140\,\mathrm{MeV}, \quad m_N = 940\,\mathrm{MeV}$$

Distances and sizes are measured in $\mathrm{fm} = 10^{-15}\,\mathrm{m}$:

$$r_\pi \approx r_N \approx 1\,\mathrm{fm} \qquad [\text{Bohr radius (atom size)} = \ 5 \cdot 10^4\ \mathrm{fm}\,]$$

Quarks

$$\text{flavours :} \quad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{pmatrix} Q = \frac{2}{3} \\ Q = -\frac{1}{3} \end{pmatrix} \quad \times \quad 3 \text{ colours} \Rightarrow \mathrm{QCD}$$

$m_u \approx 4\,\mathrm{MeV}$, $m_d \approx 7\,\mathrm{MeV}$, $m_s \approx 150\,\mathrm{MeV}$ - light quarks

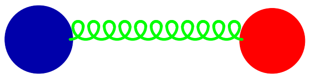
$m_c \approx 1250\,\mathrm{MeV}$, $m_b \approx 4500\,\mathrm{MeV}$, $m_t \approx 150,000\,\mathrm{MeV}$ - heavy quarks

Gluons

Eight spin-1 massless particles, similar to photons, but contrary to photons they are self-interacting

Bound states are called hadrons – from the Greek word meaning “strong”.

Heavy-quark bound states are similar to the H atom, bound by the Coulomb force due to 1-gluon exchange:



Bound states with light quarks are totally different from bound states we are used to.

$m_u \approx 4\,\mathrm{MeV}, \quad m_d \approx 7\,\mathrm{MeV}$ *but*

$$\left.\begin{array}{cc} \text{proton} & p \approx uud \\ \text{neutron} & n \approx udd \end{array}\right\} m_N = 940\,\mathrm{MeV}$$

How come the nucleon is 50 times heavier its constituents?

Quantum Chromodynamics (QCD)

The microscopic theory of strong interactions is encoded in a one-line QCD Lagrangian [Fritzsch, Gell-Mann and Leutwyler (1972)]:

$$\mathcal{L} = \bar{\psi}_f \left(i\gamma_\mu \nabla_\mu - m_f\right) \psi_f - \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu},$$

$$\nabla_\mu = \frac{\partial}{\partial x_\mu} - i A^a_\mu t^a, \qquad a = 1, 2 \ldots 8,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu + f^{abc} A^b_\mu A^c_\nu.$$

ψ_f are quark fields, $u, d, s, \ldots,$ A^a_μ is the gluon field, $F^a_{\mu\nu}$ is the gluon field strength. $t^{1,2,..,8}$ are 3×3 traceless Hermitian matrices; $[t^a t^b] = i\, f^{abc}\, t^c$.

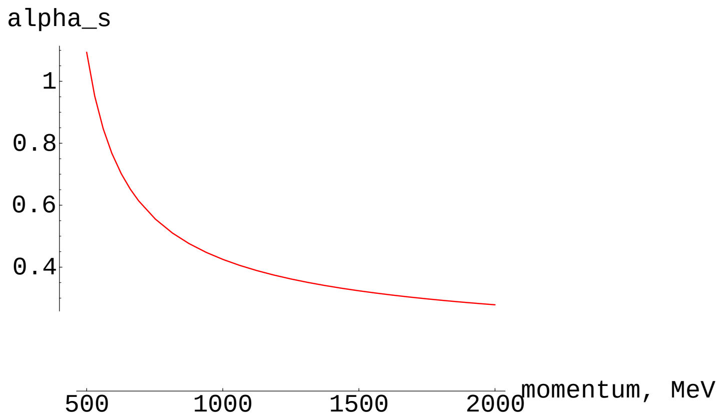
The critical circumstance that makes QCD different from "eight electrodynamics" is the gluon self-interacting term $\sim A^2$ in the field strength. Its necessity follows from the requirement of **local gauge invariance**

$$A^a_\mu t^a \to U^\dagger (A^a_\mu t^a) U + i U^\dagger \partial_\mu U, \qquad U \in SU(3).$$

'Dimensional transmutation'

$\alpha_s = \frac{g^2}{4\pi}$ is the analog of $\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$ but it 'runs' as a function of the characteristic momentum:

$$\alpha_s(p) = \frac{2\pi}{9\,\ln\frac{p}{\Lambda} + \frac{32}{9}\ln\ln\frac{p^2}{\Lambda^2} + O\left(\frac{1}{\ln\frac{p}{\Lambda}}\right)}$$



All dimensionfull observables come out as combinations of the UV cutoff μ and the coupling constant given at that cutoff, $g^2(\mu)$.

$$\Lambda = \mu\,\exp\left(-\frac{2\pi}{9\alpha_s(\mu)}\right)\left(\frac{4\pi}{9\alpha_s(\mu)}\right)^{\frac{32}{81}}(1 + O(\alpha_s(\mu))\,.$$

μ is similar to $\frac{1}{a}$ where a is the lattice spacing, and $\frac{1}{\Lambda}$ is similar to the **correlation length** which is exponentially larger than the lattice spacing. Λ is called 'the QCD scale parameter' and measured experimentally.

Chiral symmetry of strong interactions

Consider an idealization: $m_u = m_d = m_s = 0$, called **chiral limit**. In this (idealized) world interaction of quarks with gluons conserves quark helicity or chirality: left-polarized u, d, s quarks remain left-polarized forever. In addition, u, d, s quarks are interchangeable.

Mathematically, the QCD lagrangian is invariant under $U(3)_L \times U(3)_R$ separate rotations of left- and right-polarized quarks,

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \rightarrow \begin{pmatrix} * & * & * \\ * & A & * \\ * & * & * \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L ,$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_R \rightarrow \begin{pmatrix} * & * & * \\ * & B & * \\ * & * & * \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R$$

A and B are 3×3 unitary matrices, 9+9=18 parameters!

The invariance under chiral rotations is called the chiral symmetry of strong interactions.

Since $L \leftrightarrow R$ in the mirror image,

unbroken chiral symmetry of QCD imply that
ALL STATES WITH OPPOSITE PARITY
HAVE EQUAL MASSES

In reality

$$
\begin{array}{llll}
\rho & \text{meson} & (J^{PC}=1^{--}) & m=770\,\mathrm{MeV} \\
a_1 & \text{meson} & (J^{PC}=1^{+-}) & m=1250\,\mathrm{MeV} \\
 & & & \text{difference} \approx \underline{500\,\mathrm{MeV}}
\end{array}
$$

For the nucleons the splitting is even larger,

$$
\begin{array}{lll}
N & (\frac{1}{2}^{+}) & m=940\,\mathrm{MeV} \\
N^* & (\frac{1}{2}^{-}) & m=1535\,\mathrm{MeV} \\
 & & \text{difference} \approx \underline{600\,\mathrm{MeV}}
\end{array}
$$

The difference is too large to be explained by nonzero current quark masses ($m_u=4\,\mathrm{MeV},\; m_d=7\,\mathrm{MeV}$).

$\Longrightarrow$ chiral symmetry is **spontaneously** broken.
$\Longrightarrow$ pions are light [=(pseudo)-Goldstone bosons]
$\Longrightarrow$ nucleons are heavy.

Were chiral symmetry unbroken, it would be the other way: pions would be heavy while nucleons would be very light.

95% of the visible mass around us is due to the spontaneous chiral symmetry breaking.
It is the basic event in QCD

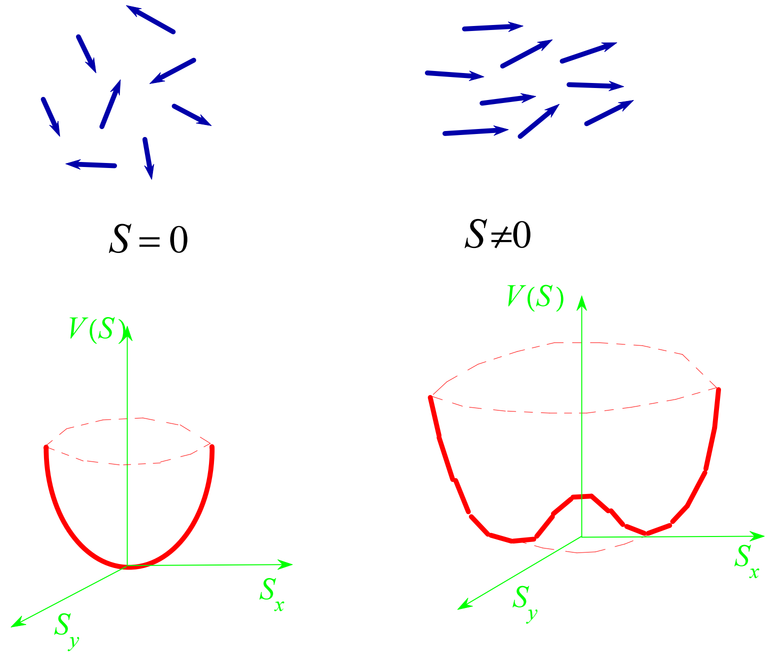
Spontaneous breaking of any continuous symmetry is marked by a nonzero order parameter, in this case called quark condensate $<\bar{u}u>=<\bar{d}d>=<\bar{s}s>\neq 0$.

Out of 18 symmetries 9 are spontaneously broken. It means that there must be nine (pseudo) massless Goldstone bosons. These are the pseudoscalar mesons:

$$\pi^+ \sim \bar{d}u, \quad \pi^- \sim \bar{u}d, \quad \pi^0 \sim \frac{\bar{u}u - \bar{d}d}{\sqrt{2}}, \quad \eta \sim \frac{\bar{u}u + \bar{d}d - 2\bar{s}s}{\sqrt{6}},$$

$$K^+ \sim \bar{s}u, \quad K^0 \sim \bar{s}d, \quad K^-, \quad \bar{K}^0, \quad \eta' \sim \frac{\bar{u}u + \bar{d}d + \bar{s}s}{\sqrt{3}}$$

Analogy: Spontaneous magnetization



Order parameter $< S_x > \neq 0$.

Symmetry with respect to rotation,

$$\begin{aligned} S_x &\rightarrow S_x \cos\phi + S_y \sin\phi, \\ S_y &\rightarrow -S_x \sin\phi + S_y \cos\phi \end{aligned}$$

is broken! As a result, there is one Goldstone or massless or gapless excitation in the system. The energy is $\mathcal{E} = \frac{1}{2}(\nabla\phi)^2$.

If magnetic field $\mathbf{B}$ is added, rotational symmetry is broken from the start. The "Mexican hat" is tilted, and the energy gets a term

$$\begin{aligned} \mathcal{E}_{\mathrm{pot}} &= -\mu(\mathbf{B}\cdot\mathbf{S}) = -\mu|B| < S > \cos\phi \\ &= \mu|B| < S > (-1 + \tfrac{1}{2}\phi^2) \end{aligned}$$

such that the Goldstone excitation gets a mass

$$m_G^2 = \mu|B| < S >$$

Notice that m_G is **square root** of $|B|$.

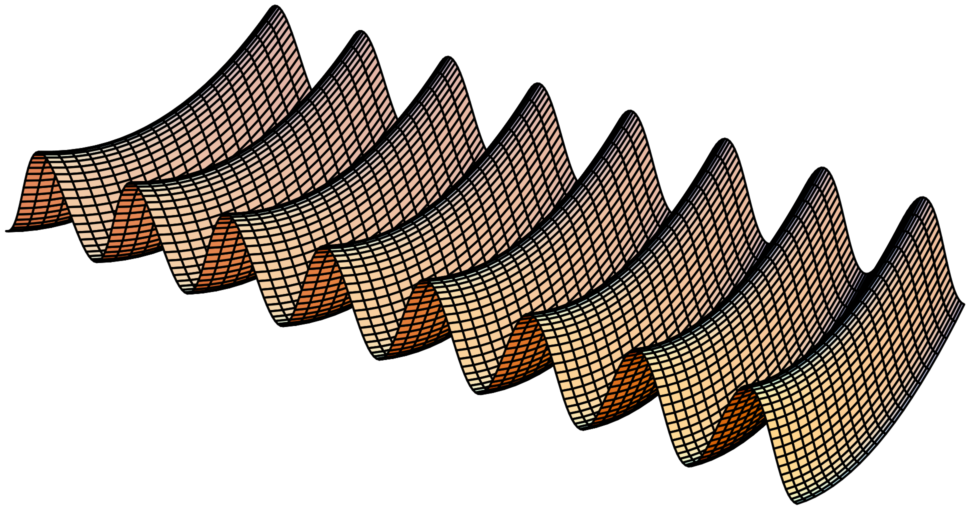
Recall nonzero $m_{u,d,s}$ – analogs of magnetic field $\overrightarrow{B}$ which breaks symmetry explicitly. It lifts the degeneracy of the bottom of the valley, and makes Goldstone bosons massive, although still light (Gell-Mann–Oakes–Renner formula):

$$\begin{aligned} m_\pi^2 &= \frac{1}{F^2}(m_u + m_d) < \bar{q}q >= (140\,\mathrm{MeV})^2 \\ m_K^2 &= \frac{1}{F^2}(m_s + m_{u(d)}) < \bar{q}q >= (495\,\mathrm{MeV})^2 \end{aligned}$$

$$\Longrightarrow \quad m_s \simeq 140\,\mathrm{MeV}, \quad m_d \simeq 7\,\mathrm{MeV}, \quad m_u \simeq 4\,\mathrm{MeV}$$

Microscopic mechanism of spontaneous chiral symmetry breaking

QCD deals with fluctuating gluon $A_i^a(x)$ and quark $\psi^{\alpha f}(x)$ fields. A fundamental fact [Faddeev, Jackiw and Rebbi (1976)] is that the potential energy of gluon fields is a periodic function in one particular direction in the functional Hilbert space:



INSTANTON = large fluctuation of the gluon field $A_i^a(x, it)$ which tunnels from one minimum to the neighbour one, with minimal action [Belavin, Polyakov, Schwartz and Tiupkin (1975), Gribov (1976), 't Hooft (1976)]

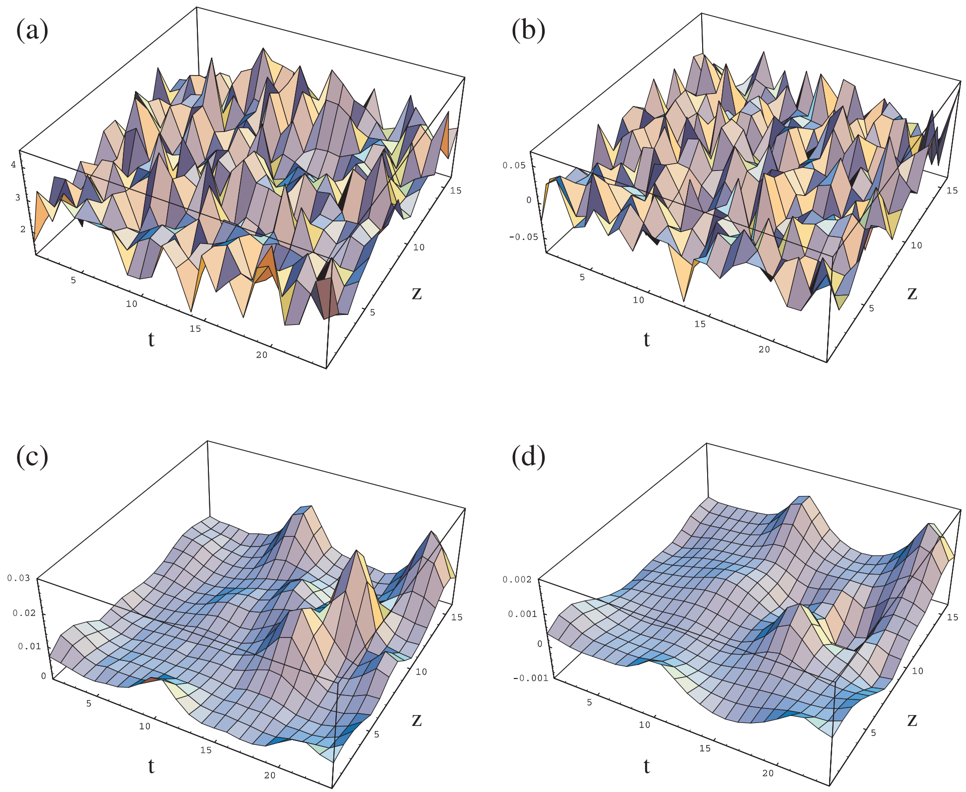
Instanton fluctuations are characterized by their position in space-time z_μ, its spatial size ρ and its orientation in colour space O (all in all, 12 'coordinates').

The probability for an instanton fluctuation to happen is, roughly, given by the WKB tunneling probability:

$$
\begin{aligned}
\begin{array}{l}\text{Tunneling}\\ \text{amplitude}\end{array} \quad &\sim \quad e^{-\text{Action}} \\
&= \quad \exp\left(-\frac{1}{4g^2}\int d^4x\, F_{\mu\nu}^2\right) \\
&= \quad \exp\left(-\frac{2\pi}{\alpha_s}\right)
\end{aligned}
$$

It is non-analytic in the coupling constant and hence instantons are missed in all orders of the perturbation theory! However, propagation of electrons in metals is also a tunneling phenomena (non-analytic in the coupling constant α), but we would get nowhere in understanding metals without taking this tunneling into account!

Instantons are clearly seen in lattice simulations of the gluon vacuum. Smearing out zero-point fluctuations of the gluon field in the vacuum reveals much more smooth configurations – instantons and anti-instantons:



upper plots: fluctuating gluon vacuum;
lower plots: same configuration but after smearing
left: action density in the $t-z$ plane;
right: topological charge density.
[M.-C. Chu, J. Grandy, S. Huang, J. Negele (1994)]

Average size of instantons: $\bar{\rho} \simeq 0.36$ fm, average separation between instantons: $\bar{R} \simeq 0.89$ fm.

'Instanton vacuuum' is the assumption that the QCD partition function is dominated by large non-perturbative fluctuations of the gluon field (instantons), plus perturbative oscillations about them. Since we do not know beforehand what is the number of instantons and instantons, we have to sum over all numbers, hence it

$$\mathcal{Z} = \sum_{N_\pm} \frac{1}{N_+! N_-!} \prod \int d^4z \, dO \, \frac{d\rho}{\rho^5} \, (\rho\Lambda)^b \, \exp(-U_{int}).$$

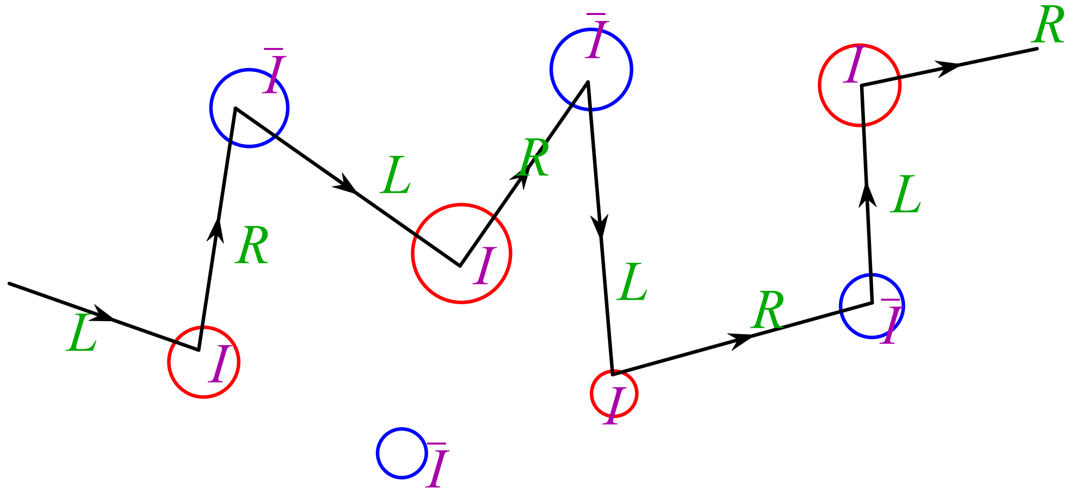
z: instanton centers, ρ: sizes, O: colour orientations. It resembles a theory of an interacting liquid (in 4 dimensions)– hence the "instanton liquid". Its basic properties were established from the Feynman variational principle [D.D. and Petrov (1984), D.D., Polyakov and Weiss (1995)]:

$$\bar{\rho} \simeq \left.\frac{0.48}{\Lambda_{\overline{\rm MS}}}\right|_{\Lambda=280\,{\rm MeV}} = 0.35\,{\rm fm}, \qquad \bar{R} \simeq \left.\frac{1.35}{\Lambda_{\overline{\rm MS}}}\right|_{\Lambda=280\,{\rm MeV}} = 0.95\,{\rm fm}.$$

Important: The "dimensional transmutation" has already happened at this point $\Longrightarrow$ all dimensionfull quantities, like m_N, $<\bar{\psi}\psi>$ etc. are henceforth expressed through $\Lambda_{\rm QCD}$

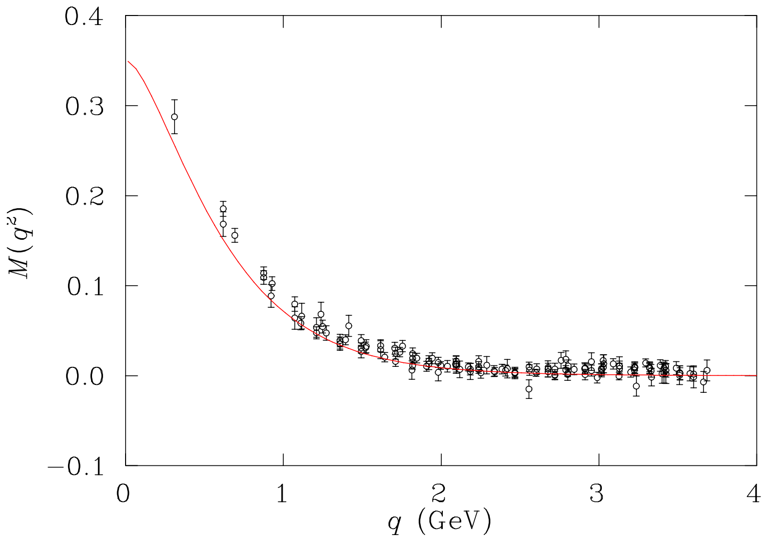
How do instantons break chiral symmetry ?

We now switch in light u, d, s quarks into the random instanton ensemble. The basic property is that quarks are bound by instantons with exactly zero 'energy' ['t Hooft (1976)]. These localized states are called quark zero modes; they have definite helicity: "$+$" for quarks sitting on instantons, "$-$" for anti-instantons.

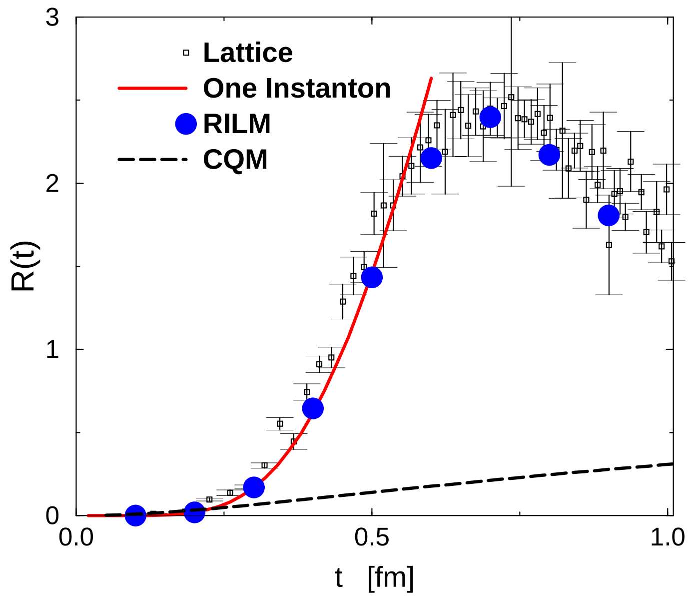


Because of the quantum-mechanical overlap, quarks are hopping from one (anti-) instanton to another, and the would-be zero modes get delocalized. Each time quarks jump from one instanton to another, they change the helicity. It is a non-perturbative effect [D.D. and Petrov (1986)]. [It is similar to the appearance of Anderson conductivity in a material with random impurities].

Propagating through the random instanton medium quarks get a dynamical mass $M(p)$.



Dynamical quark mass $M(p)$ measured on a lattice [Bowler et al. (2003)] compared to the instanton prediction D.D. and Petrov (1986)].



Helicity-flip correlation function from lattice simulations [Faccioli and DeGrand (2003)] compared to the instanton prediction (blue dots).

Mathematically, there are three independent formalisms yielding identical results:

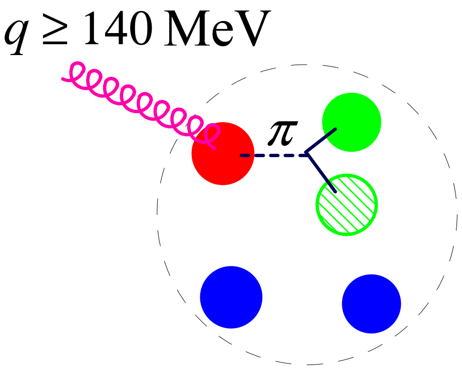
- Diagonalization of the would-be zero modes $\Longrightarrow$ Random Matrix Theory
- Averaging of the quark propagator in the medium
- Averaging first over the medium and getting an effective low-energy four-quark interaction

Some results from the calculations:

$$
\begin{aligned}
<\bar{\psi}\psi> &= -\frac{\text{const.}}{\bar{R}^2\bar{\rho}} \simeq -(255\,\text{MeV})^3 \\
M(0) &= \frac{\pi\bar{\rho}}{\bar{R}^2 N_c} \simeq 345\,\text{MeV} \\
F_\pi &= \frac{\bar{\rho}}{\bar{R}^2}\sqrt{\log\frac{\bar{R}}{\bar{\rho}}} \simeq 100\,\text{MeV}\ \text{vs.}\ 94\,\text{MeV (exper)} \\
m_{\eta'} &= \frac{\text{const.}}{\bar{\rho}} \simeq 980\,\text{MeV}\ \text{vs.}\ 958\,\text{MeV (exper)}\ldots
\end{aligned}
$$

Bound states of light u, d, s quarks

"Nucleons are made of three quarks" $=$ kindergarten physics! They are made **not only** of three quarks but plus an indefinite number of quark-antiquark pairs.



Uncertainty principle at work: When one attempts to measure the quark position in the nucleon to an accuracy better than the pion Compton wave length of $1.4\,\mathrm{fm}$ one produces a pion, i.e. a new $Q\bar{Q}$ pair. But the nucleon size is $\leq 1\,\mathrm{fm}$! Hence, the **description of nucleons as "made of" precisely three quarks is senseless.**

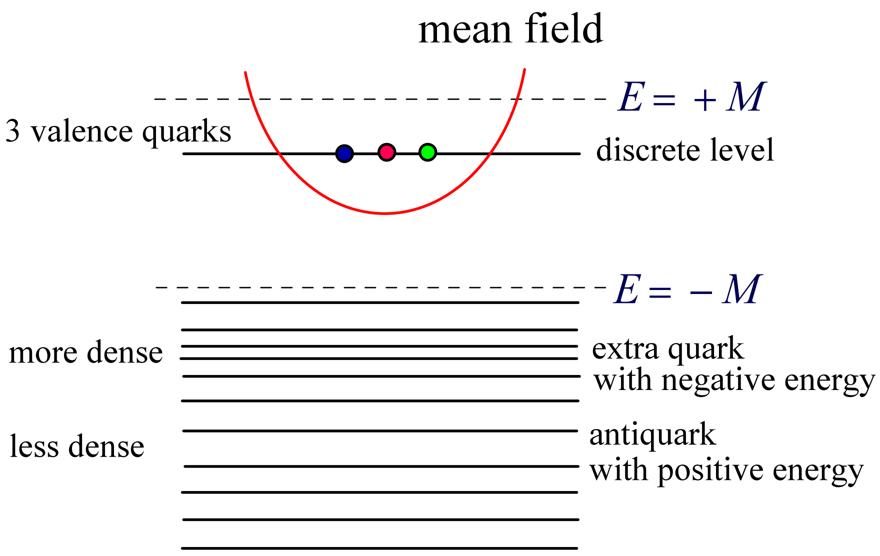
Failures of the "three quarks" picture:

- Measured: Three quarks carry only 0.3 ± 0.1 of the nucleon spin
- Measured: $\frac{m_u+m_d}{2} < N|\bar{u}u + \bar{d}d|N > = \ 67 \pm 6\,\mathrm{MeV}$

  From 3 quarks: $\frac{4\,\mathrm{MeV}+7\,\mathrm{MeV}}{2} \times 3 = 17.5\,\mathrm{MeV}$
- ...

Relativistic Mean Field Approximation *or* the Chiral Quark Soliton Model

[D.D. and Petrov + Pobylitsa (1986)]



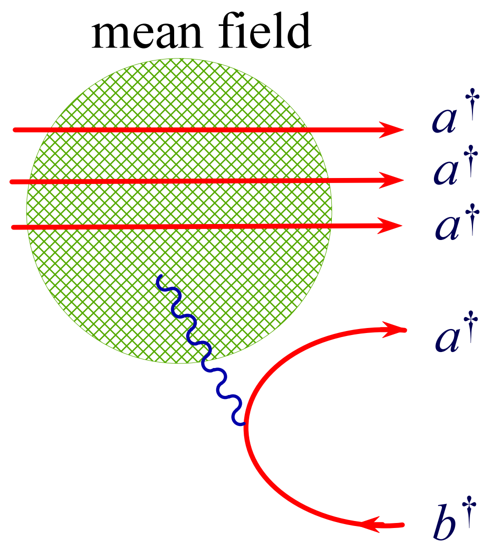


Figure 1: A schematic view of baryons. There are three "valence" quarks at a discrete energy level created by the mean field, and the negative-energy Dirac continuum distorted by the mean field, as compared to the free one.

Figure 2: Equivalent view of baryons, where the polarized Dirac sea is presented as $Q\bar{Q}$ pairs. More than 30% of the time the nucleon has 5,7,... quarks inside.

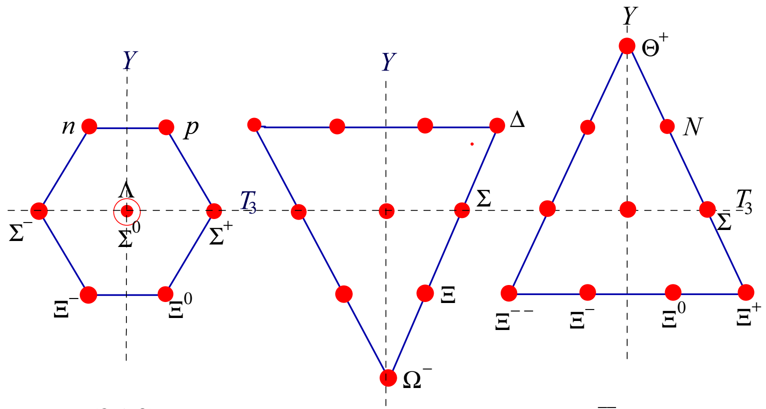
$$\text{Baryon mass} \; = N_c \; (E_{\rm lev}[\pi(x)] + E_{\rm sea}[\pi(x)]) \;\Longleftarrow\; \text{minimize it in } \pi(x)\,!$$

Self-consistent pion field in the nucleon:

$$M(0) = 345\,\mathrm{MeV} \Longrightarrow \begin{cases} E_{\rm lev} \approx 200\,\mathrm{MeV} & \text{quarks are tightly bound and relativistic!} \\ M_N \approx 1000\,\mathrm{MeV} & \text{OK!} \end{cases}$$

In the chiral limit $m_u = m_d = m_s$ the mean chiral field is degenerate in $SU(3)$ rotations of u, d, s quarks. The quantization of those rotations leads to the following baryon multiplets:

- octet with spin $\frac{1}{2}$
- decuplet with spin $\frac{3}{2}$
- anti-decuplet with spin $\frac{1}{2}$



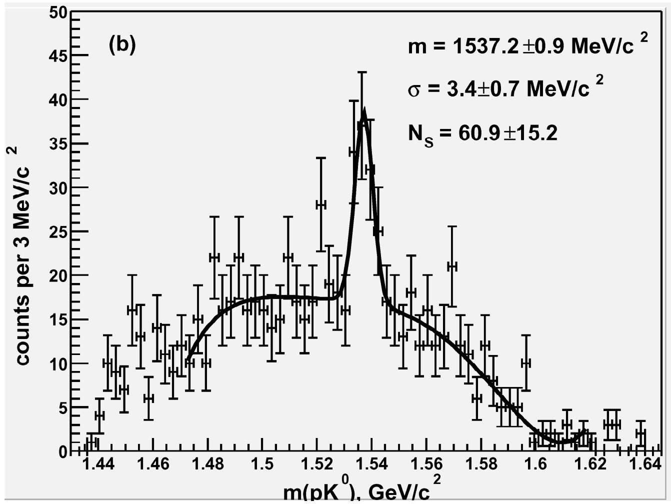
Only the first two multiplets **can be** composed from three quarks but in fact have an admixture of additional $Q\bar{Q}$ pairs.
The anti-decuplet **can not** be composed of three quarks but starts with an additional $Q\bar{Q}$ pair, hence the **pentaquark.**

D.D., Victor Petrov (PNPI) and Maxim Polyakov (Bochum):
Exotic Anti-Decuplet of Baryons: Prediction from Chiral Solitons, *Z. Phys.* (1997)

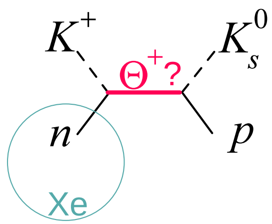
Abstract

We predict an exotic baryon (having spin $1/2$, isospin 0 and strangeness $+1$) with a relatively low mass of about $1530\,\mathrm{MeV}$ and total width of less than $15\,\mathrm{MeV}$. It seems that this region of masses has avoided thorough searches in the past.

In the end of 2002 two groups [Nakano *et al.* at SPring-8, Osaka, and Dolgolenko *et al.* at ITEP, Moscow] reported observing a resonance signal with mass $1537 \pm 2\,\mathrm{MeV}$ and width $< 9\,\mathrm{MeV}$. Subsequently the resonance was observed in a dozen of other experiments and not observed in another dozen. The only direct **formation** experiment 'sees' the resonance with a statistical significance of 7 standard deviations:



Distribution in the invariant mass of the $K^0 p$ system where the new pentaquark Θ^+ has been predicted. [Barmin *et al.* (2006)]



Summary

1. 95% of the nucleons mass is due to the Spontaneous Chiral Symmetry Breaking. It implies that quarks get a large dynamically-generated mass, and that pseudoscalar mesons π, K, η are (pseudo) Goldstone bosons.

2. The microscopic mechanism of the Spontaneous Chiral Symmetry Breaking is presumably due to **instantons** – random large fluctuations of the gluon field in the vacuum.

3. Because of the presence of light pions, the bound states of light quarks are very unusual: there are always additional $Q\bar{Q}$ pairs floating around.

4. $> 30\%$ of the time nucleons are "pentaquarks". From this point of view it is not unexpected that there must exist baryons that have five quarks inside $\sim 100\%$ of the time. However, experimental evidence for such explicit pentaquark states is still controversial.