Journal of Physics: Conference Series **461** (2013) 012012 doi:10.1088/1742-6596/461/1/012012

The influence of core superfluidity on the neutron stars long-term rotation evolution.

D P Barsukov[1,2], O A Goglichidze[1] and A I Tsygan[1]

[1]Ioffe Physical-Technical Institute of the Russian Academy of Sciences, st. Polytekhnicheskaya, 26, Saint-Petersburg, 194021, Russian Federation.
[2]Saint-Petersburg State Polytechnical University, st. Polytekhnicheskaya, 29, Saint-Petersburg, 195251, Russian Federation.

Abstract. We investigate the evolution of neutron star rotation taking into account the superfluidity of the neutrons in the neutron star core. The neutron star is treated as a two-component system consisting of a charged component (including the crust and the core protons, electrons and normal neutrons) and a core superfluid neutron component. The components are supposed to interact through the mutual friction force. We assume that the charged component rotates rigidly. The neutron superfluid velocity field is calculated directly from linearized hydrodynamical equations. It is shown that the superfluid core accelerates the evolution of inclination angle and makes all pulsars evolve to orthogonal state. But as it is known from observations the rate of the angle evolution is not very high. Therefore, a small size of superfluid cores is more likely. These facts may allow to examine superfluid models.

1. Introduction

Neutron stars cores consist mostly of the neutrons with small fractions of protons, electrons and probably more exotic particles like hyperons. The neutrons in some regions of the star must be in superfluid state at that time as the protons must be in superconducting state. Superfluidity may appear in the rotational dynamics in several ways and several time-scales. Present work is devoted to studying how the superfluidity affects the long-term rotation evolution.

2. Superfluid hydrodynamics

It is well known that for the uncharged superfluids like neutron liquid the velocity field $\vec{v}$ must satisfy the equation $\operatorname{rot}\vec{v} = 0$. However, such superfluids can rotate by forming an array of vortices. The velocity field near each vortex has the form:

$$\vec{v} = \frac{\hbar}{m_{cp}\tilde{r}}\vec{e}_\phi, \tag{1}$$

where m_{cp} is the mass of Cooper pair, $\tilde{r}$ is the distance from the vortex core, $\vec{e}_\phi$ is the azimuthal unit vector. Vorticity is located only in the vortex cores, in which superfluidity breaks. However, one can average $\vec{v}$ over the volume greater than the intervortex space. This procedure allows to use the hydrodynamical equations instead vortices dynamics consideration even when the rotating superfluids are studied (see [1] for example).

In a mixture of superfluids so-called entrainment effect takes place (see [2]). For the neutron-proton mixture it leads to the following formulas:

$$\vec{J}_n = (\rho_n - \rho_{nn} - \rho_{np})\vec{v}_{ex} + \rho_{nn}\vec{v}_n + \rho_{np}\vec{v}_p, \tag{2}$$
$$\vec{J}_p = (\rho_p - \rho_{pp} - \rho_{pn})\vec{v}_{ex} + \rho_{pp}\vec{v}_p + \rho_{pn}\vec{v}_n, \tag{3}$$
$$\vec{p}_n = (\rho_n - \rho_{nn} - \rho_{np})\vec{v}_{ex} + (\rho_{nn} + \rho_{np})\vec{v}_n, \tag{4}$$
$$\vec{p}_p = (\rho_p - \rho_{pp} - \rho_{pn})\vec{v}_{ex} + (\rho_{pp} + \rho_{pn})\vec{v}_p, \tag{5}$$

where $\vec{J}_\alpha$ is the mass current of the α constituent ($\alpha = p, n$), $\vec{p}_\alpha$ are the proton and neutron momentum densities, $\vec{v}_{ex}$ is the thermal excitation velocity (it is assumed that the excitation velocity is the same for both fluids), $\rho_{\alpha\beta}$ is the mass density matrix. It can be shown that $\rho_{np} = \rho_{pn}$. The superfluid velocities here are determined just like for the ordinary superfluids:

$$\vec{v}_n = \frac{\hbar}{2m_n}\nabla S_n, \quad \vec{v}_p = \frac{\hbar}{2m_p}\nabla S_p - \frac{e}{m_p c}\vec{A}, \tag{6}$$

where S_α are the phases of the complex order parameters, $\vec{A}$ is the electromagnetic vector potential. So in the superfluid mixtures the mass currents corresponding to each fluid are no longer parallel to they velocities and don't equal to corresponding momentum density. Note, however, that

$$\vec{J}_n + \vec{J}_p = \vec{p}_n + \vec{p}_p. \tag{7}$$

3. The Model

We treat a neutron star core as a two-component system. The first component consists of the superfluid neutrons and moves with smooth averaged velocity $\vec{v}_s$. Protons, electrons, and normal neutrons are coupled to each other on the small time-scales and form the second component which we denote as "charged". All particles forming the charged component are supposed to move with the same velocity $\vec{v}_c$:

$$\vec{v}_e = \vec{v}_{ex} = \vec{J}_p/\rho_p = \vec{v}_c. \tag{8}$$

The star crust rotates with angular velocity $\vec{\Omega}$ which is identified with the observed angular velocity of the pulsar. The external torque $\vec{K}$ acts on the crust. We will consider the torques depending only on $\vec{\Omega}$ and the magnetic field configuration which supposed to be frozen in the crust. In this case, the torque $\vec{K}$ is a very slowly evolving vector in the frame co-rotating with the crust.

Except the entrainment effect, superfluid and charged components interact through the so-called mutual friction, which arises when the charged component particles are scattered by the neutron vortices. The system of hydrodynamical equations describing the mixture of superfluid neutrons, superconducting protons and degenerate electrons taking into account the neutron-proton entrainment, mutual friction and gravitational force was developed by Mendell and Lindblom (see [3] and [4]). We are based on these equations.

Introducing $\vec{u}_\alpha = \vec{v}_\alpha - [\vec{\Omega} \times \vec{r}]$ which is a velocity field of the α constituent measured in the frame co-rotating with the crust, we suppose that

$$\Omega r \gg u_s \gg u_c, \tag{9}$$

so we keep only the linear in u_s terms and neglect all terms containing u_c. The second part of inequality (9) requires a sufficiently effective physical mechanism which damps the differential motions of the charged component and connect this component to the crust. It can be ensure by viscosity or by the magnetic field (the last requires the second type superconductivity for the protons). Terms containing rot $\vec{\lambda}_n$ in Mendell-Linblom equations arise from vortices self-acting. These terms do not participate in transferring the angular momentum from the superfluid component to the charged one, so in the linear approximation they can be ignored. We also neglect the gravitational potential perturbation.

Vector $\dot{\vec{\Omega}}$ as well as the external torque $\vec{K}$ evolves very slowly in co-rotation frame in compare with the mutual friction time-scales Thus, the long-term evolution corresponds to the quasistationary solution. It means that at the large time-scales

$$\partial_t \vec{u}_s \approx [\vec{\Omega} \times \vec{u}_s] - \left([\vec{\Omega} \times \vec{r}] \cdot \nabla \right) \vec{u}_s, \tag{10}$$

and equations for $\vec{u}_s$ take the form:

$$2\Omega \frac{\varrho^2}{\rho_{pp}\rho_s} \left(\delta [\vec{e}_z \times \vec{u}_s] - \beta \vec{e}_z \times [\vec{e}_z \times \vec{u}_s] \right) + \nabla \tilde{\mu}_1 = -[\dot{\vec{\Omega}} \times \vec{r}] \tag{11}$$

$$\operatorname{div} \left(\frac{\varrho^2}{\rho_{pp}} \vec{u}_s \right) = 0, \tag{12}$$

$$\left[\frac{\varrho^2}{\rho_{pp}} (\vec{u}_s \cdot \vec{r}) \right]_{r = r_{core}} = 0, \tag{13}$$

where $\delta = (1 - \beta')$, β and β' are the mutual friction coefficients, $\varrho^2 = \rho_{nn}\rho_{pp} - \rho_{np}^2$, r_{core} is the radius of sphere in which the superfluidity breaks, $\dot{\vec{\Omega}} = d_t \vec{\Omega}$, $\tilde{\mu}_1 = -\frac{1}{2}[\vec{\Omega} \times \vec{r}]^2 + \tilde{\mu}_n + \Phi_G$, $\tilde{\mu}_n$ is the neutrons chemical potential, Φ_G is the non-perturbed gravitational potential. Note that all mass densities here are not perturbed too.

The solution satisfying equations (11), (12), and (13) has the form

$$\vec{u}_s = \frac{\rho_{pp}\rho_s}{\varrho^2} [\vec{\varpi} \times \vec{r}] - \frac{\rho_{pp}\rho_s}{\varrho^2} \frac{\dot{\Omega}_{||}}{2\Omega} \frac{\delta - \beta\psi}{\delta^2 + \beta^2} \vec{\tilde{r}} + \vec{e}_z \frac{\rho_{pp}}{\varrho^2} \frac{\dot{\Omega}_{||}}{2\Omega} \int\limits_0^z \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left(\tilde{r}^2 \rho_s \frac{\delta - \beta\psi}{\delta^2 + \beta^2} \right) dz', \tag{14}$$

$$\vec{\varpi} = -\frac{\dot{\Omega}_{||}}{2\Omega} \frac{\beta + \delta\psi}{\delta^2 + \beta^2} - \frac{\beta}{\delta^2 + \beta^2} \frac{\dot{\vec{\Omega}}_\perp}{\Omega} + \frac{\delta}{\delta^2 + \beta^2} \left[\vec{e}_z \times \frac{\dot{\vec{\Omega}}_\perp}{\Omega} \right], \tag{15}$$

$$\tilde{\mu}_1 = -yz\dot{\Omega}_\perp - \dot{\Omega}_{||} \int\limits_0^{\tilde{r}} \psi(\tilde{r}') \tilde{r}' d\tilde{r}', \tag{16}$$

$$\psi(\tilde{r}) = \left[\int\limits_0^{z_b} \frac{\rho_s \delta}{\delta^2 + \beta^2} dz' \right] \left[\int\limits_0^{z_b} \frac{\rho_s \beta}{\delta^2 + \beta^2} dz' \right]^{-1}, \; z_b = \sqrt{r_{core}^2 - \tilde{r}^2}. \tag{17}$$

Here r_{core} is the radius of the superfluid core. The definitions of used coordinates and vectors are given in fig. 1.

In general case, the flow has a complex form. It consists of a differential rotation about the local axis $\vec{\varpi}$ and a poloidal flow. The last, however, is a consequence of the dependence of β and β' on the r and it vanishes if we make β and β' to be constant. Neglecting, in addition, the entrainment effect ($\rho_{pp}\rho_s/\varrho^2 \approx 1$), we obtain a rigidly rotating superfluid.

Protons of course are not required to be superconducting everywhere inside the superfluid core. Entrainment vanishes where the superconductivity breaks (see [5]). Obtained solution, however, remains to be valid there. One just needs to replace $\frac{\rho_{pp}\rho_s}{\varrho^2} \to 1$ and $\frac{\rho_{pp}}{\varrho^2} \to \frac{1}{\rho_s}$. Except this formally replacements the superconductivity breaking apparently leads to decrease by several orders of magnitude of the mutual friction coefficients.

4. The equations of motion

Possessing the expression for the superfluid velocity field, one can calculate the angular momentum transfer rate from superfluid core to the charged component. From the angular momentum conservation law we have

$$\dot{\vec{M}} = \vec{K}, \tag{18}$$

where $\vec{M}$ is the full angular momentum of the star which obviously is a sum of angular momenta of each component, so one can write

$$\vec{M} = \vec{M}_c + \vec{M}_s = \int \vec{r} \times (\vec{p}_c + \vec{p}_s) dV = \int \vec{r} \times \vec{J}_c dV + \int \vec{r} \times \vec{J}_s dV \tag{19}$$

Here we have used (7). By the assumption $\vec{J}_c \approx \rho_c[\vec{\Omega} \times \vec{r}]$ and $\vec{J}_s \approx \rho_s[\vec{\Omega} \times \vec{r}] + \frac{\varrho^2}{\rho_{pp}} \vec{u}_s$. Using (10), (14), (15), we obtain:

$$I_c \dot{\vec{\Omega}} = S_1 I_s \dot{\vec{\Omega}} - S_2 I_s \dot{\vec{\Omega}}_{||} + S_3 I_s [\vec{e}_z \times \dot{\vec{\Omega}}] + \vec{K}, \tag{20}$$

where

$$S_1 = \frac{8\pi}{3I_s} \int \frac{\delta\beta' - \beta^2}{\delta^2 + \beta^2} \rho_s r^4 dr, S_2 = \frac{8\pi}{3I_s} \int \frac{\delta}{\delta^2 + \beta^2} \rho_s r^4 dr, \tag{21}$$

$$S_3 = \frac{8\pi}{3I_s} \int \frac{\beta}{\delta^2 + \beta^2} \rho_s r^4 dr, \ I_\alpha = \frac{8\pi}{3} \int_0^R \rho_\alpha r^4 dr. \tag{22}$$

If we introduce basis vectors $\vec{\varepsilon}_x$, $\vec{\varepsilon}_y$, and $\vec{\varepsilon}_z$, anchored in the star crust, where $\vec{\varepsilon}_z = \vec{m}/m$ and vectors $\vec{\varepsilon}_x$ and $\vec{\varepsilon}_y$ are perpendicular to $\vec{\varepsilon}_m$ and to each other, the orientation of $\vec{\Omega}$ can be determined by two angles χ and φ_Ω (see fig. 2). Angle χ is the pulsar inclination angle, and the variation of φ_Ω relates with star precession. Without making any additional restriction external torque $\vec{K}$ can be represented as

$$\vec{K} = K_\Omega \vec{e}_z + K_m \vec{\varepsilon}_z + K_\perp [\vec{e}_z \times \vec{\varepsilon}_z] \tag{23}$$

Equation (20) can be solved for $\dot{\vec{\Omega}}$ and rewritten as three scalar equations:

$$\dot{\Omega} = \frac{K_\Omega + K_m \cos\chi}{I_s + I_c}, \tag{24}$$

$$\dot{\chi} = -\frac{1}{\Omega} \frac{(I_c + S_1 I_s) K_m - S_3 I_s K_\perp}{(I_c + S_1 I_s)^2 + S_3^2 I_s^2} \sin\chi, \tag{25}$$

$$\dot{\varphi}_\Omega = -\frac{1}{\Omega} \frac{(I_c + S_1 I_s) K_\perp + S_3 I_s K_m}{(I_c + S_1 I_s)^2 + S_3^2 I_s^2}. \tag{26}$$

Up to now the developed formalism does not require any specification of the profiles of the mutual friction coefficients and the mass densities as well as K_Ω, K_m and $K_\perp$ can be arbitrary small in magnitude functions. Next we apply the particular models (simplistic at some points) in order to demonstrate that the choice of the model may significantly affect the rate of the rotation evolution.

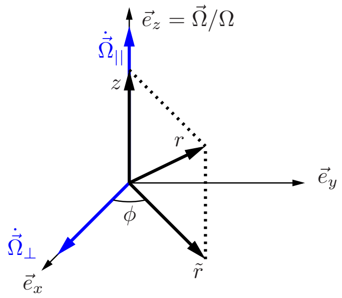

Figure 1. Coordinates and vectors used in section 3.

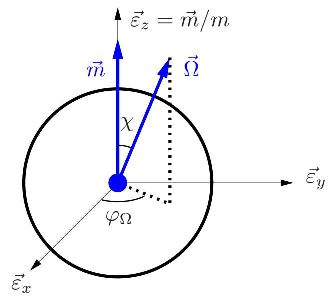

Figure 2. Coordinates and vectors used in section 4.

5. Inclination angle evolution

We suppose that the most effective mutual friction mechanism is based on the electrons scattering on the vortices magnetic field proposed by Alpar and Sauls (see [6]).

$$\beta = \frac{\sigma}{\sigma^2+1}, \quad \beta' = \frac{\sigma^2}{\sigma^2+1}, \quad \sigma = 1.3\times 10^{-2}\frac{x}{1-x}\,(x\rho_{14})^{1/6}\left(\frac{m_p}{m_p^*}\right)^{1/2}\left(\frac{\rho_{np}}{\rho_{pp}}\right)^2, \tag{27}$$

where $x = \rho_c/\rho$, $\rho_{14} = \rho/10^{-14}$ g cm^{-3}, m_p^* is the effective proton mass.

Superfluid mass density matrix $\rho_{\alpha\beta}$ with taking into account temperature effects has been calculated by Gusakov and Haensel (see [5]). Densities $\rho_{\alpha\beta}$ depend on the parameters $\tau_\alpha = T/T_{c\alpha}$, where T is the core temperature, $T_{c\alpha}$ are the temperatures of the protons and neutrons phase transition temperature. We model $T_{c\alpha}$ profiles by the parabolas (see fig 3):

$$\log_{10}\left(\frac{T_{c\alpha}}{T_{c\alpha}^{max}}\right) = -4\log_{10}^2\left(\frac{\rho}{\rho_\alpha^{max}}\right). \tag{28}$$

It is not quite realistic of course but the variation of T_{cn}^{max} or T_{cp}^{max} allows us to demonstrate the sensitivity of the results to the choice of critical temperature profiles. We determine r_{core} as a radius of the sphere on which $T = T_{cn}$ and suppose that $T = 5\times 10^7$K.

As for the torque $\vec{K}$, we use the one proposed by Barsukov, Polyakova and Tsygan (see [7] for the detail) for which $K_\Omega = -K_0$, $K_m = K_0(1-\alpha(\chi,\varphi_\Omega))\cos\chi$, $K_\perp = K_0 R\cos\chi$, $K_0 = \frac{2\Omega^3 m^2}{3c^3}$, $R = \frac{9c}{10\Omega r_{ns}}$, $\alpha(\chi,\varphi_\Omega)$ is the function ~ 1 which describes the structure of the small-scale magnetic field in vicinities of neutron star magnetic poles.

One can average the equations over the precession period and divide (25) over (26), taking into account that $P = 2\pi/\Omega$ is the pulsar period:

$$\frac{d\chi}{dP} \approx -\frac{1}{P}\frac{I_c(I_c+I_s)}{I_c^2+S_3^2I_s^2}\frac{\sin\chi\cos\chi}{\sin^2\chi+\bar{\alpha}(\chi)\cos^2\chi}\left[1-\bar{\alpha}(\chi)-S_3\frac{I_s}{I_c}\frac{9}{20\pi}\left(\frac{c}{r_{ns}}\right)P\right]. \tag{29}$$

The evolution trajectories obtained for different initial χ and P and different T_n^{max} are given in figures 4, 5 and 6.

6. Discussion

The presence of the superfluid core increases the rate of inclination angle evolution and makes all pulsars evolve to the orthogonal state. This facts together with observations may allow to examine superfluid models.

We do not take into account any pinning phenomena which may play significant role in neutron star rotational dynamics. Also we do not take into account the thermal evolution of the neutron star. We will include these factors in our future developments.

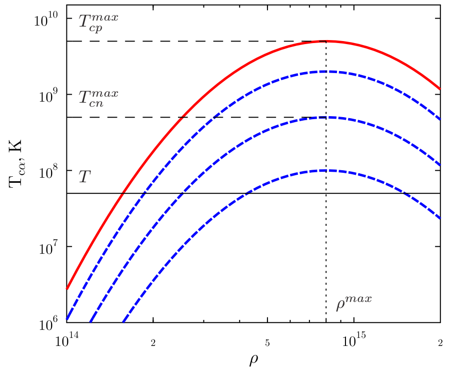

Figure 3. Critical temperature profiles for protons (solid line) and neutrons (dashed lines) used in the calculations. $\rho_n^{max} = \rho_p^{max} = 8 \times 10^{14}\mathrm{g/cm^3}$, $T_{cp}^{max} = 5 \times 10^9$K, $T_{cp}^{max} = 1,\ 5,\ 20 \times 10^8$K.

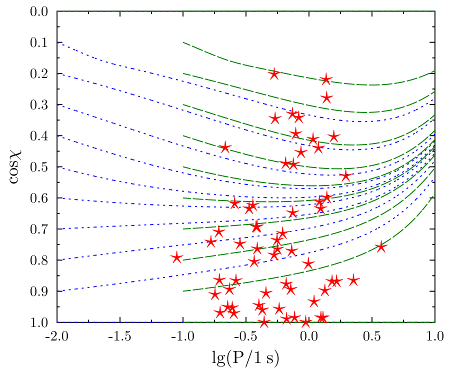

Figure 4. The evolution trajectories obtained for non-dipolarity parameter $\nu = 0.5$ and $T_{cn}^{max} = 1 \times 10^8$K ($I_s/I_c = 0.08$) for pulsars with initial periods equal to 10 msec (dotted lines) and 100 msec (dashed lines). Stars demonstrate the observation data for 62 pulsars from [8].

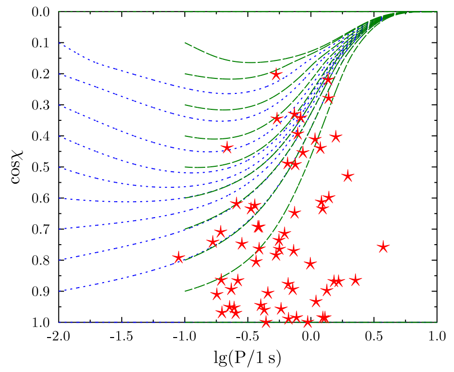

Figure 5. The same as before but $T_n^{max} = 5 \times 10^8$K ($I_s/I_c \approx 0.5$)

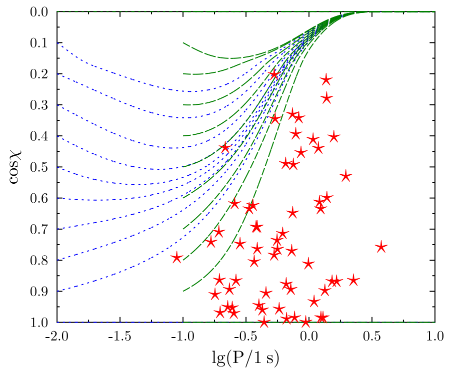

Figure 6. The same as before but $T_n^{max} = 2 \times 10^9$K ($I_s/I_c \approx 1$)

Acknowledgements
The authors are grateful to M E Gusakov and E M Kantor for discussion. This work was supported by the Russian Foundation for the Basic Research (project 13-02-00112), the Program of the State Support for Leading Scientific Schools of the Russian Federation (grant NSh-4035.2012.2) and Ministry of Education and Science of Russian Federation (Agreement No.8409, 2012 and contract No. 11.G34.31.0001).

References
[1] Baym G, Chandler E 1983 *Low Temp. Phys.* **50** 57
[2] Andreev A F, Bashkin E P 1976 *JETP*, **42** 164
[3] Mendell G Lindblom L 1991 *An. of Phys.*, **205** 110
[4] Mendell G 1991 *Astrophys. J.* **380** 515
[5] Gusakov E M, Haensel P *Nucl. Phys. A* **761** 333
[6] Alpar M A, Langer S A, Sauls J A 1984 *Astrophys. J.* **282** 533
[7] Barsukov D P, Polyakova P I, Thygan A I 2009, *Astron. Rep.* **53** 1146
[8] Malov I F 2004 *Radio Pulsars* (Moskow: Nauka) [in Russian]