Hydromagnetic instability in differentially rotating flows

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We study the stability of compressible differentially rotating flows in the presence of the magnetic field, and we show that the compressibility profoundly alters the previous results for a magnetized incompressible flow. The necessary condition of newly found instability can be easily satisfied in various flows in laboratory and astrophysical conditions and reads $B_s B_\varphi \Omega' \neq 0$, where B_s and B_φ are the radial and azimuthal components of the magnetic field, $\Omega' = d\Omega/ds$ with s being the cylindrical radius. Contrary to the well-known magnetorotational instability that occurs only if Ω decreases with s, the instability considered in this paper may occur at any sign of Ω'. The instability can operate even in a very strong magnetic field that entirely suppresses the standard magnetorotational instability. The growth time of instability can be as short as a few rotation periods.

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INTRODUCTION

Instabilities caused by differential rotation of a magnetized gas may play an important role in enhancing transport processes in various astrophysical bodies and laboratory experiments. It is well known since the classical papers by Velikhov [1] and Chandrasekhar [2] that a differentially rotating flow with a negative angular velocity gradient and a weak magnetic field is unstable to the magnetorotational instability. This instability has been analyzed in detail in the astrophysical context (see [3–5]) because it can be responsible for transport of the angular momentum in various objects ranging from accretion disks to galaxies. In accretion disks, this instability is also well studied by numerical simulations in both linear and nonlinear regimes. Simulations of this instability in accretion disks (see, e.g., [6–8]) show that the generated turbulence can enhance substantially the angular momentum transport.

Astrophysical applications of the magnetorotational instability have created great interest in trying to study this instability in the laboratory [9–12]. The experiments, however, are complicated because very large rotation rates should be achieved. Recently, Hollerbach and Rüdiger [13] argued that the rotation rate can be substantially decreased adding an azimuthal field. It is known since the paper by Tayler [14] that an azimuthal field produces a strong destabilizing effect and, as a result of this additional destabilization, the critical Reynolds number in experiment can be reduced.

On the other hand, the magnetorotational instability is not the only instability that operates in differentially rotating magnetized flows. For example, even a weak axial dependence of the angular velocity can result in a double diffusive instability that is often called the Goldreich-Schubert-Fricke instability (e.g., [15,16]). Note that many previous stability analyses have adopted the Boussinesq approximation, and have therefore neglected the effect of compressibility. This is allowed if the magnetic field strength is essentially subthermal, and the sound speed is much greater than the Alfvén velocity, $c_s \gg c_A$, but often this cannot be realized in real astrophysical conditions and in many numerical simulations. An attempt to consider the effect of compressibility on the magnetorotational instability was undertaken by Blaes and Balbus [17] in the context of astrophysical disks. The authors considered a very simplified case of the wave vector parallel to the rotation axis and a vanishing radial magnetic field. As a result, the most interesting physics has been lost in this study since only the standard magnetorotational instability operates in this simple geometry.

In this paper, we show that an instability different from the standard magnetorotational instability may occur in a compressible differentially rotating magnetized flow. This instability appears for any differential rotation and may occur if the magnetic field has nonvanishing radial and azimuthal components. The instability can arise even in a sufficiently strong magnetic field that suppresses the magnetorotational instability. Stability analysis done in this paper will hopefully prove to be a useful guide to understanding various numerical simulations that explore the nonlinear development of instabilities and their effects on the resulting turbulent state of rotating magnetized flows.

BASIC EQUATIONS AND DISPERSION RELATION

We work in cylindrical coordinates (s, φ, z) with the unit vectors $(\vec{e}_s, \vec{e}_\varphi, \vec{e}_z)$. The equations of compressible magnetohydrodynamics (MHD) read

$$\dot{\vec{v}} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{\vec{\nabla} p}{\rho} + \vec{g} + \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B}, \qquad (1)$$

$$\dot{\rho} + \vec{\nabla} \cdot (\rho \vec{v}) = \vec{0}, \qquad (2)$$

$$\dot{p} + \vec{v} \cdot \vec{\nabla} p + \gamma p \vec{\nabla} \cdot \vec{v} = \vec{0}, \qquad (3)$$

$$\dot{\vec{B}} - \vec{\nabla} \times (\vec{v} \times \vec{B}) + \eta \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{0}, \qquad (4)$$

$$\vec{\nabla} \cdot \vec{B} = \vec{0}. \tag{5}$$

Our notation is as follows: ρ and $\vec{v}$ are the density and fluid velocity, respectively; p is the gas pressure; $\vec{g}$ is gravity that can be important in astrophysical applications; $\vec{B}$ is the magnetic field; η is the magnetic diffusivity; and γ is the adiabatic index. For the sake of simplicity, the flow is assumed to be isothermal.

The basic state on which the stability analysis is performed is assumed to be quasistationary with the angular velocity $\Omega = \Omega(s)$ and $\vec{B} \neq \vec{0}$. Generally, a quasistationary basic state cannot be achieved for any differentially rotating magnetic configuration, therefore we discuss in more detail when this assumption can be satisfied. We assume that gas is in hydrostatic equilibrium in the basic state. Then

$$\frac{\vec{\nabla} p}{\rho} = \vec{D} + \frac{1}{4\pi\rho}(\vec{\nabla} \times \vec{B}) \times \vec{B}, \quad \vec{D} = \vec{g} + \Omega^2 \vec{s}. \tag{6}$$

Generally, a geometry of the magnetic field can be rather complex, and our study primarily addresses such complex magnetic configurations in which the radial and azimuthal field components are presented. The presence of a radial magnetic field and differential rotation in the basic state can lead to the development of the azimuthal field. Nevertheless, the basic state can be considered in some cases as quasistationary despite the development of the toroidal field.

For example, if the magnetic Reynolds number is large (η is small), then one can obtain from Eq. (4) that the azimuthal field grows approximately linearly with time,

$$B_\varphi(t) = B_\varphi(0) + s\Omega' B_s t, \tag{7}$$

where $\Omega' = d\Omega/ds$, and $B_\varphi(0)$ is the azimuthal field at $t=0$. As long as the second term on the right-hand side (r.h.s.) is small compared to the first one, and

$$t \ll \tau_\varphi = \frac{1}{s\Omega'} \frac{B_\varphi(0)}{B_s}, \tag{8}$$

stretching of the azimuthal field does not affect significantly the basic state; τ_φ is the characteristic time scale of generation of B_φ. As a result, the basic state can be treated as quasistationary during the time $t \ll \tau_\varphi$. If $B_\varphi(0)/B_s \gg 1$, then steady state can be maintained during a relatively long time before the generated azimuthal field begins to influence the basic state. We will show that the growth time of instability can be shorter than τ_φ in many cases of interest.

If the magnetic Reynolds number is moderate, then stretching of the azimuthal field from B_s by differential rotation can be compensated by Ohmic dissipation, and the basic state can be quasistationary as well. Then, we have from Eq. (4) the following condition of steady state:

$$[\vec{\nabla} \times (\vec{v} \times \vec{B})]_\varphi = \eta [\vec{\nabla} \times (\vec{\nabla} \times \vec{B})]_\varphi \tag{9}$$

or

$$\left(\Delta - \frac{1}{s^2}\right) B_\varphi = -\frac{s}{\eta} \Omega' B_s. \tag{10}$$

The generated toroidal field is typically stronger than the radial field by a factor of the order of the magnetic Reynolds number. This simple model applies only in the case of moderate Reynolds number since the generation of a very strong toroidal field could lead to instabilities of the basic state caused, for example, by magnetic buoyancy or reconnection. Note that a quasistationary basic state with nonvanishing radial and azimuthal field components can be achieved in other models as well. For example, if the angular velocity depends on both the s and z coordinates, then changes in B_φ caused by stretching from the radial and vertical field components due to radial and vertical shear, respectively, can balance each other in such a way that B_φ will be steady state. In fact, there is no principal difference for instability which mechanism is responsible for maintaining a quasistationary basic configuration. The only important point for our model is the presence of the magnetic field with nonvanishing radial and azimuthal components, but such magnetic configurations are rather common in astrophysics (galactic and accretion disks, stellar radiative zones, oceans of accreting neutron stars, etc.)

We consider the stability of axisymmetric short-wavelength perturbations with the spacetime dependence $\propto \exp(\sigma t - i\vec{k} \cdot \vec{r})$, where $\vec{k} = (k_s, 0, k_z)$ is the wave vector, $|\vec{k} \cdot \vec{r}| \gg 1$. Small perturbations will be indicated by subscript 1, while unperturbed quantities will have no subscript. Then, to the lowest order in $|\vec{k} \cdot \vec{r}|^{-1}$ the linearized MHD equations read

$$\sigma \vec{v}_1 + 2\vec{\Omega} \times \vec{v}_1 + \vec{e}_\varphi s\Omega' v_{1s} = \frac{i\vec{k} p_1}{\rho} - \frac{i}{4\pi\rho}(\vec{k} \times \vec{B}_1) \times \vec{B}, \tag{11}$$

$$\sigma \rho_1 - i\rho(\vec{k} \cdot \vec{v}_1) = \vec{0}, \tag{12}$$

$$\sigma p_1 - i\gamma p(\vec{k} \cdot \vec{v}_1) = \vec{0}, \tag{13}$$

$$\sigma \vec{B}_1 = \vec{e}_\varphi s\Omega' B_{1s} - i(\vec{B} \cdot \vec{k})\vec{v}_1 + i\vec{B}(\vec{k} \cdot \vec{v}_1), \tag{14}$$

$$\vec{k} \cdot \vec{B}_1 = \vec{0}. \tag{15}$$

We neglect Ohmic dissipation in the induction equation because the inverse Ohmic decay time scale is small for many cases of interest in astrophysics.

Generally, the dispersion relation for Eqs. (11)–(15) is rather complicated and, in this paper, we consider only a particular case when the wave vector of perturbations is perpendicular to $\vec{B}$, $\vec{k} \cdot \vec{B} = \vec{0}$. This case, being mathematically much simpler, illustrates very well the main qualitative features of the new magnetic shear-driven instability. Besides, the standard magnetorotational instability does not operate in this case because its growth rate is proportional to $\vec{k} \cdot \vec{B}$. Therefore, the difference between instabilities is seen most clearly if $\vec{k} \cdot \vec{B} = \vec{0}$.

In the case $\vec{k}\cdot\vec{B}=\vec{0}$, Eqs. (11)–(15) may be combined after some algebra into a fifth-order dispersion relation,

$$\sigma^5+\sigma^3(\omega_0^2+\Omega_e^2)+\sigma^2\omega_{B\Omega}^3+\sigma\mu\Omega_e^2\omega_0^2+\mu\Omega_e^2\omega_{B\Omega}^3=0, \tag{16}$$

where we denote

$$\Omega_e^2=2\Omega(2\Omega+s\Omega'),\quad \omega_0^2=k^2(c_s^2+c_m^2),\quad \mu=k_z^2/k^2,$$

$$c_m^2=\frac{B^2}{4\pi\rho},\quad c_s^2=\frac{\gamma p}{\rho},\quad \omega_{B\Omega}^3=\frac{k^2B_\varphi B_s s\Omega'}{4\pi\rho}.$$

This equation describes five nontrivial modes that exist in a rotating magnetized flow if $\vec{k}\cdot\vec{B}=\vec{0}$.

In the nonmagnetic case, $\vec{B}=\vec{0}$, Eq. (16) yields

$$\sigma^4+(\omega_s^2+\Omega_e^2)\sigma^2+\mu\omega_s^2\Omega_e^2=0, \tag{17}$$

where $\omega_s=kc_s$ is the frequency of sound waves. The solution is

$$\sigma_{1,2}^2=-\frac{1}{2}(\omega_s^2+\Omega_e^2)\pm\sqrt{\frac{1}{4}(\omega_s^2+\Omega_e^2)^2-\mu\omega_s^2\Omega_e^2}. \tag{18}$$

Instability arises only if the well-known Rayleigh criterion is fulfilled, $\Omega_e^2<0$. In this case, the inertial mode is unstable, which corresponds to the upper sign. The sound mode that corresponds to the lower sign is always stable.

To have an idea about the properties of the dispersion equation (16), we can consider a particular case of flow with $\Omega\propto s^{-2}$. Then, $\Omega_e^2=0$ and we have from Eq. (16)

$$\sigma^3+\sigma\omega_0^2+\omega_{B\Omega}^3=0. \tag{19}$$

The solutions of this equation are

$$\sigma_1=u+v,\quad \sigma_{2,3}=-\frac{1}{2}(u+v)\pm\frac{i\sqrt{3}}{2}(u-v), \tag{20}$$

where

$$(u,v)=\left(-\frac{\omega_{B\Omega}^3}{2}\pm\sqrt{\frac{\omega_{B\Omega}^6}{4}+\frac{\omega_0^6}{27}}\right)^{1/3}. \tag{21}$$

One of the roots has a positive real part (instability) if $u+v\neq 0$. The latter condition is equivalent to $\omega_{B\Omega}^3\neq 0$, which is the criterion of instability in this simple case. It is clear from this simple example that the quantity $\omega_{B\Omega}$ plays a crucial role for stability of magnetized compressible flows.

CRITERIA AND GROWTH RATE OF INSTABILITY

The conditions under which Eq. (16) has unstable solutions can be obtained by making use of the Routh-Hurwitz theorem (see [18,19]). In the case of the dispersion equation of a fifth order, the Routh-Hurwitz criteria are written, for example, in [20]. According to these criteria, Eq. (16) has unstable solutions if one of the following inequalities is fulfilled:

$$\mu\Omega_e^2\omega_{B\Omega}^3<0,\quad \omega_{B\Omega}^3>0,\quad (\omega_{B\Omega}^3)^2<0. \tag{22}$$

These inequalities yield the criterion of instability

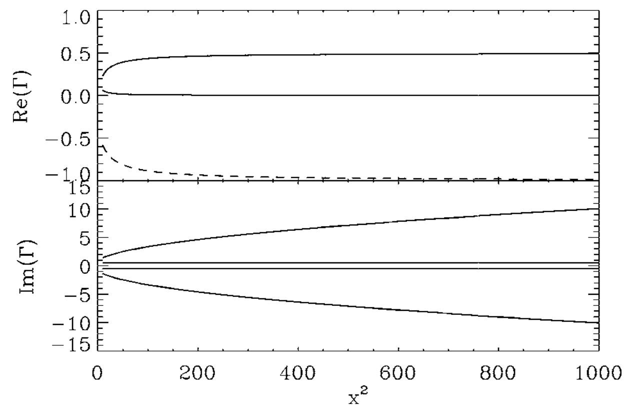

FIG. 1. The dependence of the real and imaginary parts of Γ on x^2 for $\mu=0.3$, $\xi=0.1$, and $\zeta=0.1$. Solids lines show the growth rate and frequency of complex roots, and the dashed line corresponds to the real root.

$$\omega_{B\Omega}^3\neq 0. \tag{23}$$

Apart from differential rotation, this criterion requires nonvanishing radial and azimuthal field components. The vertical component of $\vec{B}$ is unimportant for criterion (23), and the instability may occur even in a plane parallel magnetic field with components only in radius and azimuth. The direction of $\vec{B}$ and the sign of Ω' are insignificant, and the instability may occur for both the inward and outward decreasing angular velocity. Note that this is in contrast with the magnetorotational instability that can arise only if $\Omega'<0$. Another important difference is that the magnetorotational instability is suppressed by a sufficiently strong field, whereas the instability given by Eq. (23) can arise even in a very strong field.

To calculate the growth rate in the general case, it is convenient to introduce dimensionless quantities

$$\Gamma=\frac{\sigma}{\Omega_e},\quad \xi=\frac{1}{x^2}\frac{\omega_0^2}{\Omega_e^2},\quad \zeta=\frac{1}{x^2}\frac{\omega_{B\Omega}^3}{\Omega_e^3},\quad x=ks \tag{24}$$

(we assume that $\Omega_e^2>0$). Note that the parameters ξ and ζ do not depend on the wave vector. Then, Eq. (16) becomes

$$\Gamma^5+\Gamma^3(1+\xi x^2)+\Gamma^2\zeta x^2+\Gamma\mu\xi x^2+\mu\zeta x^2=0. \tag{25}$$

This equation was solved numerically for different μ, ξ, and ζ by computing the eigenvalues of the matrix whose characteristic polynomial is given by Eq. (16) (see [21] for details).

In Fig. 1, we plot the dependence of the real and imaginary parts of Γ on x for $\mu=0.3$, $\xi=0.1$, and $\zeta=0.1$. The solid lines show the growth rate and frequency for complex conjugate roots, and the dashed line for a real root. As mentioned, there should be no instability in the incompressible limit because all the considered perturbations are stable with respect to the standard magnetorotational instability. Our calculations, however, clearly indicate that some roots have a positive real part and, hence, there should exist a new shear-driven instability in the compressible flow with $\zeta\neq 0$. There are two pairs of unstable complex conjugate roots and one real stable root with negative Re Γ. In the considered domain

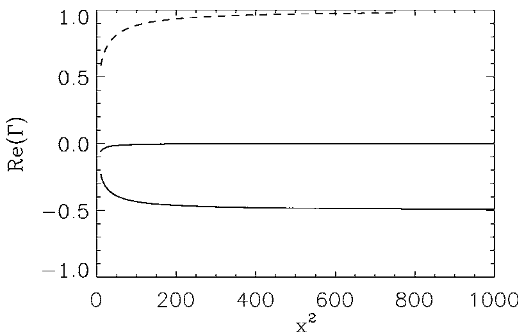

FIG. 2. The dependence of the growth rate on x^2 for $\mu=0.3$, $\xi=0.1$, and $\zeta=-0.1$. The solid and dashed lines correspond to complex and real roots, respectively.

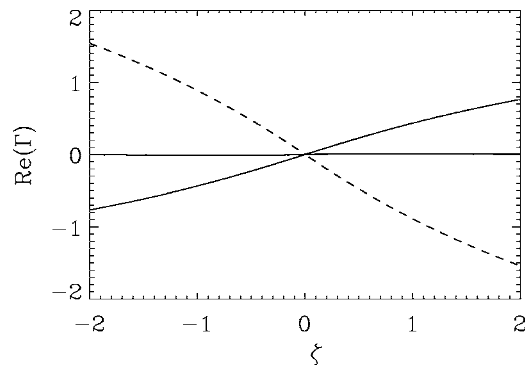

FIG. 3. The dependence of $\mathrm{Re}\,\Gamma$ on ζ for $\mu=0.3$, $x^2=10$, and $\xi=1$. Solid and dashed lines correspond to complex and real roots, respectively.

of parameters, $\mathrm{Im}\,\Gamma$ for complex roots is typically $\sim$10–30 times greater than $\mathrm{Re}\,\Gamma$ except the region of not very large $x^2 \sim 10$–50, where they are of the same order of magnitude (but still $\mathrm{Im}\,\Gamma > \mathrm{Re}\,\Gamma$). One pair of unstable roots has a very small growth rate $\sim 10^{-4}\Omega_e$, but another one grows much faster. For these roots, the growth rate is $\approx 0.5\Omega_e$ and varies very slowly with the wavelength of perturbations. Note that calculations for other values of the parameters show that typically $\mathrm{Re}\,\Gamma \sim 0.5$ if $\xi \sim \zeta$, but $\mathrm{Re}\,\Gamma$ becomes smaller if $\xi \gg \zeta$. This is qualitatively clear because the case $\xi \gg \zeta$ corresponds to the incompressible limit when the considered instability is substantially suppressed.

In Fig. 2, we plot the dependence of $\mathrm{Re}\,\Gamma$ on x for the case $\mu=0.3$, $\xi=0.1$, and $\zeta=-0.1$. We do not plot $\mathrm{Im}\,\Gamma$ because this dependence does not differ much from what is shown in Fig. 1. The change of sign alters qualitatively the behavior of roots. If ζ is negative, then all oscillatory modes are stable ($\mathrm{Re}\,\Gamma < 0$) but the real mode becomes unstable. This conclusion is completely consistent with our analytical consideration of Eq. (14). It is worth mentioning that calculations for other μ, ξ, and ζ also indicate that this sort of behavior is rather general, and the nonoscillatory mode is typically unstable for negative ζ whereas the oscillatory modes are unstable for positive ζ. Like the previous case, the growth rate depends weakly on the wavelength except in the region $x^2 < 200$, where this dependence is stronger. The characteristic value of the growth rate is larger for negative ζ and reaches $\approx \Omega_e$ for $x^2 \geq 200$. Note that this is typical also for other values of μ and ξ and that a nonoscillatory mode (negative ζ) grows faster than oscillatory modes (positive ζ) for the same $|\zeta|$.

Figure 3 illustrates the behavior of roots as functions of the parameter ζ for fixed value of x. It is seen that $\mathrm{Re}\,\Gamma$ vanishes for both oscillatory and nonoscillatory modes when ζ goes to zero. Since $\zeta \propto \omega_{B\Omega}$, the instability occurs only if $\omega_{B\Omega} \neq 0$ in complete agreement with the criterion (13). As usual, the real root is positive (instability) at $\zeta < 0$ whereas the oscillatory roots have positive real parts at $\zeta > 0$. For the same $|\zeta|$, the growth rate is larger for negative ζ.

DISCUSSION

To summarize then, we have considered the instability caused by differential rotation of compressible magnetized gas. To illustrate the main qualitative features of the instability associated to compressibility and shear, we analyzed a particular case of perturbations with the wave vector $\vec{k}$ perpendicular to the magnetic field $\vec{B}$. In this case, the standard magnetorotational instability, well-studied in incompressible fluids (see, e.g., [1,2,5]), does not occur because its growth rate is proportional to $(\vec{k}\cdot\vec{B})$. Nevertheless, even perturbations with $\vec{k}\cdot\vec{B}=\vec{0}$ turn out to be unstable if the necessary condition of the new instability, $\zeta \propto B_s B_\varphi \Omega' \neq 0$, is satisfied.

In our stability analysis, we assume that the basic state is quasistationary. This assumption can be fulfilled in many cases of astrophysical interest despite the development of the azimuthal field from the radial one due to differential rotation. For instance, if the magnetic Reynolds number is large, then the time scale of generation of the toroidal field is $\sim \tau_\varphi$ if $B_\varphi(0) > B_s$. Instability can be considered in a quasistationary approximation if its growth time is shorter than τ_φ. As is seen from Eq. (21), the growth rate of instability in the case of a strong compressibility can be roughly estimated as $\omega_{B\Omega}$. Then, the condition of quasistationarity reads $\omega_{B\Omega} \gg 1/\tau_\varphi$, or

$$kc_{As} > s\Omega' \left(\frac{B_s}{B_\varphi(0)} \right)^2. \qquad (26)$$

Since the left-hand side (l.h.s.) of this equation is proportional to k but the r.h.s. does not depend on k, there always exists the range of k for which Eq. (26) can be satisfied and the basic state is quasistationary.

The considered instability is related basically to shear and compressibility of a magnetized gas. In the incompressible limit that corresponds to $c_s \to \infty$ or $\omega_0^2 \to \infty$, we have from Eq. (16)

$$\sigma(\sigma^2 + \mu\Omega_e^2) = 0, \qquad (27)$$

and the instability disappears. This is a principal difference from other well-known instabilities caused by differential rotation such as the Rayleigh or magnetorotational instabilities. Note that an attempt to consider instability associated with compressibility of differentially rotating magnetized gas has been undertaken by Blaes and Balbus [17]. These authors, however, analyzed only the unperturbed configuration in which the magnetic field has vertical or azimuthal components, but such configurations are stable in accordance with our criterion (23).

The properties of the considered instability are very different from those of other instabilities that can occur in cylindrical magnetized flows. The necessary condition of the instability (23) can be satisfied for both outward increasing and decreasing $\Omega(s)$, whereas the magnetorotational instability occurs only if $\Omega(s)$ decreases with s. The found instability operates only if the basic magnetic configuration is relatively complex with nonvanishing radial and azimuthal field components while the standard magnetorotational instability can arise also if both these components are vanishing and only $B_z \neq 0$.

This new instability can be either oscillatory or nonoscillatory, depending on the sign of ζ, whereas the standard magnetorotational instability is always nonoscillatory. Typically, the considered instability is nonoscillatory if $\zeta < 0$ and oscillatory if $\zeta > 0$. One more important difference is associated with the dependence on the magnetic field strength. A sufficiently strong magnetic field, satisfying the inequality $(\vec{k}\cdot\vec{B})^2 > 8\pi\rho s\Omega|\Omega'|(k_z^2/k^2)|$, completely suppresses the standard magnetorotational instability. On the contrary, the instability discovered in our study cannot be suppressed even in very strong magnetic fields, as is seen from the criterion (23). All this comparison allows us to claim that our analysis demonstrates the presence of the new instability in compressible cylindrical flow.

The growth rate of the newly found instability can be rather large and reach $\sim\Omega_e$. Basically, the growth rate is larger for nonoscillatory modes which are unstable if $\omega_{B\Omega}^3 < 0$. The growth rate depends on compressibility, being smaller for a low compressibility. The incompressible limit (Boussinesq approximation) corresponds to $c_s \gg c_A$, and the considered instability is inefficient in this limit because of a low growth rate. However, in the case of a strong field with $c_A \sim c_s$ when the Boussinesq approximation does not apply, the instability can be much more efficient than the magnetorotational instability.

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