

Seismic signatures of strange stars with crust

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ABSTRACT

We study acoustic oscillations (eigenfrequencies, velocity distributions, damping times) of normal crusts of strange stars. These oscillations are very specific because of huge density jump at the interface between the normal crust and the strange matter core. The oscillation problem is shown to be self-similar. For a low (but non-zero) multipolarity l , the fundamental mode (without radial nodes) has a frequency of ~ 300 Hz and mostly horizontal oscillation velocity; other pressure modes have frequencies $\gtrsim 20$ kHz and almost radial oscillation velocities. The latter modes are similar to radial oscillations (having approximately the same frequencies and radial velocity profiles). The oscillation spectrum of strange stars with crust differs from the spectrum of neutron stars. If detected, acoustic oscillations would allow one to discriminate between strange stars with crust and neutron stars and constrain the mass and radius of the star.

Key words: stars: neutron – stars: oscillations.

1 INTRODUCTION

Strange stars are hypothetical compact stars which are built entirely or almost entirely of strange quark matter (containing light u , d and s quarks and possibly electrons). The hypothesis of strange stars is based on the idea of Witten (1984) that the strange quark matter is the absolutely stable form of matter even at zero pressure. The hypothesis cannot be definitely confirmed or refuted by available theories and experimental data. Strange stars are attracting permanent attention [see e.g. a recent review of Weber (2005)]. One cannot exclude that at least some stars, which are currently thought to be neutron stars, are in fact strange stars. The theories predict the existence of bare strange stars (with the strange matter extended to the very surface) and strange stars with a thin crust of normal matter extended no deeper than the neutron drip point (e.g. Alcock, Fahri & Olinto 1986). The normal crust may occur, for instance, due to accretion of normal matter on to a bare strange star.

Strange stars of masses M much lower than the ‘canonical’ neutron star mass, $M \sim 1.4 M_{\odot}$, are predicted to have radii $R \ll 10$ km, essentially smaller than the radii of neutron stars. However, strange stars with $M \sim 1.4 M_{\odot}$ have nearly the same radii, $R \sim 10$ km, as typical neutron stars.

We will show that strange stars with crusts could be potentially distinguished from neutron stars by their oscillation spectra. Oscillations of strange stars have been analysed by a number of authors. Yip, Chu & Leung (1999) studied three types of quadrupole oscillations of strange stars and hybrid neutron stars. They found some differences in oscillation frequencies of such stars. They also demonstrated that the damping times of oscillations are sensitive to the model of strange matter. Recently, Benhar et al. (2006) showed

that the combined knowledge of the frequency of emitted gravitational waves and the mass or radius of a compact object would allow one to discriminate between a strange star and a neutron star and set stringent bounds on the parameters of quark matter. Both groups of authors analysed global oscillations of strange stars.

In this paper, we focus on pressure oscillations (p and f modes) of strange star crusts. In our previous papers (Chugunov & Yakovlev 2005; Chugunov 2006), we have studied oscillations localized in a neutron star crust due to large multipolarity $l \gtrsim 100$. Here, we show that acoustic oscillations in a strange star crust are very specific even for low l because of a huge density jump at the interface between the normal crust and the quark core.

2 FORMALISM

Because a strange star crust is very thin (a few hundred metres for a strange star of mass $M \sim 1.4 M_{\odot}$), the approximation of plane-parallel layer can be used. Then, the space-time metric in the crust can be written as

$$ds^2 = c^2 dt^2 - dz^2 - R^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (1)$$

where the local time t and the local depth z are related to the Schwarzschild time \tilde{t} and the circumferential radius r by

$$t = \tilde{t} \sqrt{1 - R_G/R}, \quad z = (R - r)/\sqrt{1 - R_G/R}. \quad (2)$$

$r = R$ is the circumferential radius of the stellar surface, ϑ and φ are spherical angles, $R_G = 2GM/c^2$ is the gravitation radius and M is the gravitational mass of the star. The metric (1) is locally flat and allows us to use the Newtonian hydrodynamic equations for a thin envelope with the gravitational acceleration

$$g = \frac{GM}{R^2 \sqrt{1 - R_G/R}}. \quad (3)$$

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The pressure in the strange star crust is mostly determined by degenerate electrons and is almost independent of temperature T . Accordingly, we can use the same zero-temperature equation of state (EOS) for the equilibrium structure of the crust and for perturbations. We will neglect the buoyancy force and study pressure modes. We will also neglect elastic stresses which are unimportant for crust oscillations in the frequency range of interest. (They would be important for lower frequencies ~ 1 Hz.) The linearized hydrodynamic equations (for a non-rotating star) for the velocity potential ϕ (so that the oscillation velocity is $\mathbf{V} = -\nabla\phi$) can be rewritten as [see e.g. the monograph by Lamb (1975)]

$$\frac{\partial^2 \phi}{\partial t^2} = c_s^2 \Delta \phi + \mathbf{g} \cdot \nabla \phi, \quad (4)$$

where $c_s^2 \equiv \partial P_0 / \partial \rho_0$ is the squared sound speed. The potential ϕ can be presented in the form

$$\phi = e^{i\omega t} Y_{lm}(\vartheta, \varphi) F(z), \quad (5)$$

where ω is an oscillation frequency and $Y_{lm}(\vartheta, \varphi)$ is a spherical function (see e.g. Varshalovich, Moskalev & Khersonskii 1988). An unknown function $F(z)$ obeys the equation

$$\frac{d^2 F}{dz^2} + \frac{g}{c_s^2} \frac{dF}{dz} + \left(\frac{\omega^2}{c_s^2} - k^2 \right) F = 0, \quad (6)$$

where $k^2 = l(l+1)/R^2$. The requirement of vanishing Lagrange variation of the pressure at the surface implies $F(z)$ to be bounded as $z \rightarrow 0$. Because the strange quark matter at the interface between crust and the quark core is very dense and almost incompressible, we impose the condition $F'(h) = 0$, which means the vanishing radial displacement at the crust bottom $z = h$. In other words, we study oscillations of the crust alone.

Following standard prescription, we will call the modes without radial nodes of $F(z)$ by f modes, and the modes with nodes by p modes.

In analogy with the oscillation problem for neutron star crusts (Chugunov 2006), the oscillation problem for strange star crusts is self-similar. Taking the equilibrium pressure P as an independent variable in equation (6), we come to the equation for an eigenvalue $\lambda = \omega^2/g^2$; this equation contains the scaling parameter $\zeta = k/g$. Accordingly, the eigenfrequencies can be written as

$$\omega_k^2 = g^2 f_k(\zeta). \quad (7)$$

Here, $f_k(\zeta)$ are functions which can be calculated numerically. Note that for $l=0$ (radial oscillations), the eigenfrequency is proportional to the gravity g .

For the polytropic EOS, the squared sound speed is $c_s^2 = gz/n$, and equation (6) transforms to

$$z \frac{d^2 F}{dz^2} + n \frac{dF}{dz} + \left(\frac{n\omega^2}{g} - k^2 z \right) F = 0. \quad (8)$$

The EOS of a strange star crust can be well approximated by the polytropic EOS with the index $n=3$. Thus, equation (8) with $n=3$ will be used below for an analytical consideration of the oscillation problem.

The frequencies ω refer to the local crust reference frame (see equation 1). The frequencies $\tilde{\omega}$ of oscillations detected by a distant observer are

$$\tilde{\omega} = \omega \sqrt{1 - R_G/R}. \quad (9)$$

2.1 Radial oscillations

For radial oscillations ($l=0$), equation (8) is simplified as

$$z \frac{d^2 F}{dz^2} + n \frac{dF}{dz} + \frac{n\omega^2}{g} F = 0. \quad (10)$$

The solution bounded at the surface ($z=0$) is

$$F = A J_{n-1} \left(2\omega \sqrt{nz/g} \right) / z^{(n-1)/2}, \quad (11)$$

where A is a constant, and $J_n(x)$ is a Bessel function of the first kind (see e.g. Abramowitz & Stegun 1971). Then, the eigenfrequencies are given by the equation

$$J_n \left(2\omega \sqrt{nh/g} \right) = 0. \quad (12)$$

Formally, the solution contains the eigenfrequency $\omega=0$ which corresponds to $F(z) \equiv \text{const}$ (i.e. to a meaningless vanishing oscillation velocity). In addition, the equation gives an infinite number of eigenfrequencies,

$$\omega_i = \frac{j_{n,i}}{2} \sqrt{\frac{g}{nh}}, \quad (13)$$

where $j_{n,i}$ is an i th zero of $J_n(x)$. For a given n and a given maximum density in the crust, ρ^{max} , the crustal depth $h \propto g^{-1}$, and ω is proportional to g , in agreement with the general relation (7). Note that a mode labelled by i has i nodes of $F(z)$. At $n=3$, we have $j_{3,i} = 6.38, 9.76, 13.0, 16.2, 19.4$ for i from 1 to 5. For $i \geq 2$, the difference of neighbouring values is $j_{3,i+1} - j_{3,i} \approx 3.2$ (and it is slightly larger, ≈ 3.38 , for $i=1$). Accordingly, the eigenfrequencies are approximately equidistant. This feature can be used to distinguish such oscillations from other ones (see Section 3.1).

2.2 Non-radial oscillations

The problem of non-radial oscillations of a thin polytropic layer was studied by Lamb (1911, 1975). Following those studies, let us introduce $f(z) = \exp(kz)F(z)$ and $a = n(1 - \omega^2/gk)/2$ in equation (8). Then, equation (8) reduces to

$$z \frac{\partial^2 f}{\partial z^2} + (n - 2kz) \frac{\partial f}{\partial z} - a f = 0. \quad (14)$$

The solution to this equation bounded at $z \rightarrow 0$ is $f(z) = AM(a, n, 2kz)$, where A is a constant and $M(a, b, x)$ is the Kummer function (see e.g. Abramowitz & Stegun 1971). The boundary condition $F'(h) = 0$ leads to the equation

$$\frac{2a}{n} M(a+1, n+1, 2kh) = M(a, n, 2kh). \quad (15)$$

For a not very large multipolarity $l \ll R/h \sim 50$, the value of kh is small, and the long-wavelength approximation applies which simplifies the problem. The eigenfrequency of the fundamental mode can be determined using asymptotic series expansions, which give

$$\omega_f^2 = k^2 gh / (n+1) = l(l+1)gh / [R^2(n+1)]. \quad (16)$$

The respective eigenfunction is $F_f = A [1 + k^2(z^2/2 - hz)/(n+1)]$ (Lamb 1911, 1975). In this case matter, elements move nearly horizontally; the ratio of the radial to the horizontal velocity components can be estimated as $kh/(n+1) \ll 1$.

The p modes can be studied numerically. However, they have relatively high frequencies, so that $k^2 \ll \omega^2/c_s^2$ in equation (6). Neglecting k^2 , we come to the same equation as for radial oscillations. Thus, higher modes of non-radial oscillations have almost the same frequencies (13) and radial parts (11) of the velocity potential $F(z)$,

as radial oscillations (see Section 2.1). These results can also be obtained using the asymptotes of the Kummer function. In these cases, the velocity is mostly radial; the ratio of the radial to the horizontal velocity components can be estimated from equation (8) as $1/kh$.

2.3 Oscillation damping

We define the oscillation damping time (in the local stellar reference frame) as

$$\tau = E/|dE/dt|, \quad (17)$$

where the oscillation energy is

$$E = \frac{R^2}{2} \int_0^h \rho [(F')^2 + k^2 F^2] dz \quad (18)$$

and

$$\frac{dE}{dt} = \frac{dE_{\text{el}}}{dt} + \frac{dE_{\text{grav}}}{dt} + \frac{dE_{\text{visc}}}{dt}. \quad (19)$$

Here, dE_{el}/dt , dE_{grav}/dt and dE_{visc}/dt are oscillation energy loss rates owing to the emission of electromagnetic and gravitation waves and owing to the viscous dissipation.

To estimate the gravitational radiation rate (for $l \geq 2$), we employ the multipole expansion formula (Balbinski & Schultz 1982)

$$\frac{dE_{\text{grav}}}{dt} = -2\pi \frac{l(l+1)(l+2)}{(l-1)[(2l+1)!!]^2} \frac{GR}{\omega} \left(\frac{\omega R}{c}\right)^{2l+1} I_{\text{grav}}^2, \quad (20)$$

where

$$I_{\text{grav}} = \int_0^h \rho [F'(z) + (l+1)F(z)/R] dz. \quad (21)$$

The electromagnetic damping rate (for $l \geq 1$) calculated in the model of a frozen-in dipolar magnetic field disturbed by oscillations, with the vacuum boundary conditions at the stellar surface (McDermott et al. 1984; Muslimov & Tsygan 1986), is given by

$$l=1 \quad \frac{dE_{\text{el}}}{dt} = -\frac{c}{720\pi} B^2 R^2 \left(\frac{\omega R}{c}\right)^6 \left[\frac{2F(0)/R + F'(0)}{R\omega} \right]^2; \quad (22)$$

$$l \geq 2 \quad \frac{dE_{\text{el}}}{dt} = -\frac{c}{32\pi} B^2 R^2 \left(\frac{\omega R}{c}\right)^{2l} \times \frac{l(l-1)}{(2l+1)(2l-1)[(2l-3)!!]^2} \times \left[\frac{2(l+1)F(0)/R - F'(0)}{R\omega} \right]^2, \quad (23)$$

where B is the magnetic field strength at the magnetic pole. The presence of the magnetosphere can change the emission power (Timokhin, Bisnovatyi-Kogan & Spruit 2000), but we neglect this effect.

The viscous damping of oscillations was studied by Chugunov & Yakovlev (2005) who showed that

$$\frac{dE_{\text{visc}}}{dt} = -\frac{1}{4} \int_0^R r^2 \eta \left(I_1 - \frac{4}{3} I_2 \right) dr, \quad (24)$$

where

$$I_1 = 4 \left[(F'')^2 + 2 \frac{1+l(l+1)}{r^2} (F')^2 - 6 \frac{l(l+1)}{r^3} F'F + l(l+1) \frac{1+l(l+1)}{r^4} F^2 \right],$$

$$I_2 = \left[F'' + \frac{2F'}{r} - \frac{l(l+1)}{r^2} F \right]^2,$$

and η is the shear viscosity. Note that the viscous damping can be additionally enhanced by thin viscous layers near weak first-order phase transitions associated with the transformation of nuclides in dense matter. The viscosity in these layers can be diffusive or turbulent; we neglect this additional dissipation in our calculations.

Taking the equilibrium pressure P as the integration variable in equation (18), we obtain

$$E = \frac{g R^2}{2} \int_0^{P_{\text{max}}} (\rho^2 F_p'^2 + \zeta^2 F^2) dP, \quad (25)$$

where P_{max} is the pressure at the crust bottom, and $F_p' = dF/dP$. Note that for given EOS, ρ^{max} , ζ and the rms radial displacement at the surface, the energy scales as gR^2 .

As will be shown in Section 3, the viscous damping dominates at $l=0$ and $l \gtrsim 30$. Dipole modes decay usually owing to electromagnetic radiation. Depending on the magnetic field strength, modes with $2 \leq l \lesssim 30$ damp mainly under the action of either electromagnetic or gravitational radiation.

Let us consider the oscillation damping for a polytropic EOS for which the equilibrium density is $\rho = \rho^{\text{max}}(z/h)^n$. In this case, the damping rate via gravitational radiation is evaluated analytically and the oscillation energy (for a given rms amplitude of radial surface displacements) is independent of l . The results are outlined below.

2.4 The damping of f modes

The oscillation damping of a fundamental mode comes mainly from horizontal motions of the matter. Accordingly, it is sufficient to take $F_f = A[1 + k^2(z^2/2 - hz)/(n+1)] \approx A$. Then,

$$I_{\text{grav}} = A \rho^{\text{max}} \frac{h}{R(n+1)} \quad (26)$$

and

$$E = \frac{l(l+1)}{2} \rho^{\text{max}} A^2 h. \quad (27)$$

Note that for a given rms radial displacement a at the surface, the normalization constant is $A \propto a\omega/k^2 \propto a/k$ (as follows from equation 16). Accordingly, the energy is independent of l .

2.5 The damping of p modes

p modes (with at least one radial node) of not very large multipolarity $l \ll R/h$ are accompanied by nearly radial velocities of the matter. Their velocity potential F and oscillation frequencies are almost the same as for radial oscillations. Thus, their energy can be estimated as

$$E \approx \frac{R^2}{2} \int_0^h \rho F'^2 dz. \quad (28)$$

For the same rms radial displacement on the surface, the energy is independent of l . One has

$$I_{\text{grav}} \approx \int_0^h \rho F'(z) dz = -A^2 h^{(1-n)/2} \rho_{\text{max}} J_{n+1}(2\omega\sqrt{hn/g}); \quad (29)$$

the last equality is obtained using equation (11) for the function F .

3 NUMERICAL RESULTS

All numerical results are presented for a strange star of the gravitational mass $M = 1.4 M_{\odot}$, the circumferential radius $R = 10$ km and the crust depth $h = 250$ m. The results can easily be rescaled to

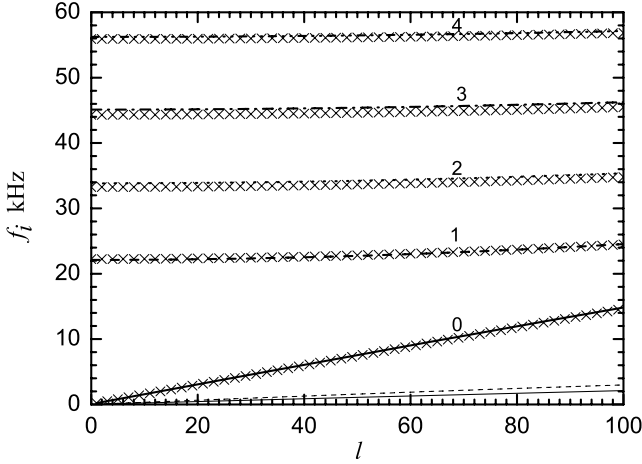


Figure 1. Frequencies of oscillations localized in the crust of a strange star ($M = 1.4 M_{\odot}$, $R = 10$ km and $h = 250$ m) as detected by a distant observer. The numbers next to the curves indicate the number of radial nodes of the function F . Crosses are plotted for the EOS of the accreted crust (Haensel & Zdunik 1990); lines are for the polytropic EOS with $n = 3$. The two oscillation branches of the accreted crust (shown by thinner lines) with lowest frequencies are density discontinuous g modes.

other strange star models [with the same EOS and the same maximum density of the normal matter] using the scaling relation (7).

Eigenfrequencies have been determined from equation (6) (with the boundary conditions $F'(h) = 0$ and F bounded at $z \rightarrow 0$) by the Runge–Kutta method.

Fig. 1 shows oscillation eigenfrequencies calculated for a distant observer as a function of multipolarity l . Crosses are plotted for the EOS of the accreted matter (Haensel & Zdunik 1990) with the accurate treatment of phase transitions (see e.g. Chugunov 2006). Lines are for the polytropic EOS ($n = 3$), which describes the normal matter composed of ^{56}Fe nuclei and ultrarelativistic electrons. The density at the crust bottom is $\rho^{\text{max}} = 7.65 \times 10^{10} \text{ g cm}^{-3}$ for the accreted crust and $3.9 \times 10^{10} \text{ g cm}^{-3}$ for the polytropic one. Note that the polytropic EOS accurately describes most of eigenfrequencies, but oscillations of the accreted crust have two specific additional branches (shown by thinner lines in Fig. 1) known as density discontinuity g modes (see e.g. McDermott 1990). These oscillations are caused by buoyancy forces associated with phase transitions; they are present in neutron stars and in strange stars with crust. For a vanishing shear modulus of the matter in the vicinity of a phase transition, the frequencies of such modes can be estimated as (McDermott 1990)

$$f \approx 1.8 \left\{ l(l+1) \frac{1 - \exp(-n)}{n} \left(1 - \frac{R_G}{R} \right) \right\}^{1/2} \left(\frac{10 \text{ km}}{R} \right)^{3/2} \times \left(\frac{M}{M_{\odot}} \frac{\Delta \rho}{\rho_-} \frac{z_{\text{ph}}}{R} \right)^{1/2} \text{ kHz}, \quad (30)$$

where z_{ph} is the depth of the phase transition, $\Delta \rho$ is the density jump and ρ_- is the density just after the jump. The parameters of such modes are sensitive to the model of accreted matter and to the shear modulus of this matter, etc. Note that the distinctness of phase transitions in the accreted crust is still not clear – they could be smoothed (which would affect the respective g mode oscillation frequencies). Therefore, we will not include density discontinuity g modes in our analysis and will study other oscillation branches in the polytropic approximation.

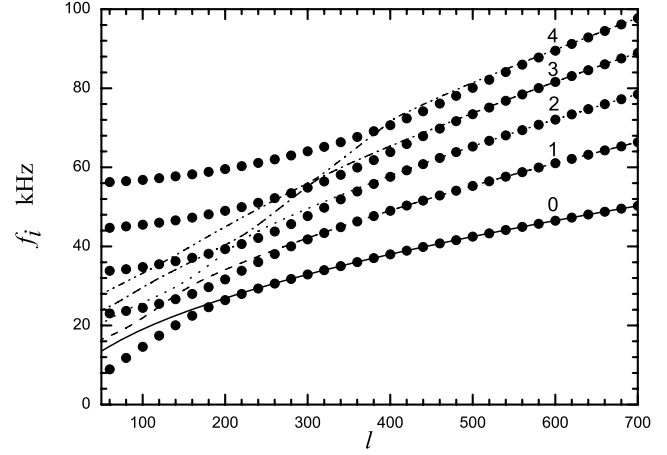


Figure 2. Frequencies of oscillations (as detected by a distant observer) localized in crusts of strange stars (filled dots) and neutron stars (lines) of the same mass $M = 1.4 M_{\odot}$ and radius $R = 10$ km. For the strange star, the crust depth is chosen to be $h = 250$ m. The density discontinuous g modes are not shown. Numbers near curves show the number of radial nodes of the function F .

Fig. 2 shows the frequencies of oscillations of a strange star and a neutron star of the same mass $M = 1.4 M_{\odot}$ and radius $R = 10$ km. Both stars are assumed to have an accreted crust. For the strange star, the crust depth is chosen to be $h = 250$ m. Details of calculations of neutron star oscillations are described by Chugunov (2006). If $l \gtrsim 300$, oscillations are localized in a thin surface layer of the depth $\lesssim h$; then both the strange star and the neutron star have the same oscillation spectrum. For lower l , the oscillations can penetrate into the layers of the depth $\gtrsim h$, and the oscillation spectra of the strange star and the neutron star are seen to become different.

Let us focus on oscillations of the crust of strange stars. For not very large $l \lesssim 150$, the frequencies of fundamental modes depend linearly on l , but for p modes (with at least one radial node) the frequencies are approximately constant and equidistant. This feature is in good agreement with analytical results of Sections 2.1 and 2.2. For higher l , the l -dependence of the frequencies becomes more complicated because of the violation of the long-wave approximation (which requires $kh \ll 1$). Finally, for $l \gtrsim 300$, the oscillations are localized in the outer layers of the crust, and their frequencies become the same as for neutron stars.

Oscillations of strange stars with crust have several specific features which can be used to distinguish them from oscillations of neutron stars.

- (i) The frequencies of fundamental modes have linear dependence on l .
- (ii) The frequencies of p modes (with at least one radial node) are approximately equidistant and have very weak dependence on l .

Fig. 3 shows the damping time of oscillations, rescaled for a distant observer [$\tilde{\tau} = \tau (1 - R_G/R)^{-1/2}$]. The crust temperature (important for the viscous damping) is assumed to be $T = 10^7$ K and the magnetic field strength at the magnetic poles is 10^{12} G. For higher T , the viscous damping time becomes larger.

The damping time is seen to vary by many orders of magnitude, from $\tilde{\tau} \approx 10^3 \text{ yr}$ for the fundamental mode with $l = 5$ to $\tilde{\tau} \lesssim 10^{-3} \text{ s}$ for the mode with at least two radial nodes of $F(z)$ and $l \sim 10$. This huge difference is produced by several damping mechanisms. In particular, radial oscillations ($l = 0$) decay exclusively through the viscous dissipation (generating neither gravitational nor

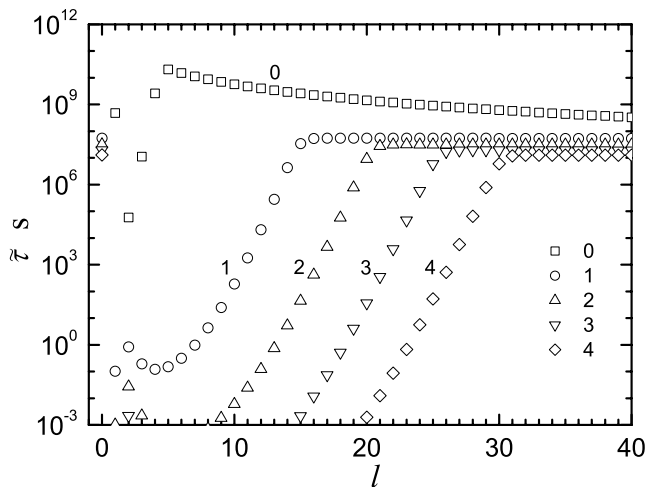


Figure 3. Damping times of oscillations (for a distant observer) localized in the crust of a strange star ($M = 1.4 M_{\odot}$, $R = 10$ km and $h = 250$ m). The crust temperature is $T = 10^7$ K and the magnetic field at the magnetic poles is $B = 10^{12}$ G. Numbers indicate the number of radial nodes of the function F .

electromagnetic radiation). They damp slowly, with $\tilde{\tau} \sim 1$ year. The modes with $l = 1$ undergo powerful electromagnetic damping which greatly decreases the damping time (Fig. 3).

Fundamental modes have low frequencies. Accordingly, they mainly undergo the viscous damping and do not damp efficiently via the emission of electromagnetic and gravitation waves. Their damping times are relatively large, up to 10^3 yr. The exclusion is provided by the modes with $l = 1, 2, 3$ and 4 , which decay quicker via the emission of electromagnetic waves.

For p modes (with at least one radial node), the picture is different. The modes with $1 \leq l \lesssim 30$ decay primarily through the electromagnetic channel. Because neither oscillation frequency nor $F(z)$ depends on l (see Section 2.2), the l -dependence of the dissipation time is fully determined by the electromagnetic energy losses (see equation 23). For higher multipolarity $l \gtrsim 30$, the electromagnetic and gravitational emissions are strongly suppressed by large l (see equations 20 and 23), and the viscous damping prevails. As a result, the damping times become approximately constant due to a weak l -dependence of oscillation frequencies and radial parts $F(z)$ of the velocity potential. Note that for lower $B \sim 10^9$ G, the main damping mechanism of f modes with $2 \leq l \lesssim 20$ consists in the emission of gravitational waves, while the electromagnetic channel is important only for dipole modes.

To conclude, the fundamental modes and all radial oscillations have damping times exceeding 1 yr and have better chances to be detected. The modes with $l \gtrsim 30$ have also relatively large damping times $\gtrsim 1$ yr, but it could be difficult to excite and detect them owing to large multipolarity.

3.1 Inferring strange star parameters

Let us assume that some oscillation frequencies are detected and corresponding modes are identified. The frequencies of pressure modes (in the local stellar reference frame) are given by equation (13). Then the measured frequencies $\tilde{\omega}_i$ would give us the value of

$$\alpha = \frac{g}{h} (1 - R_G/R) = 3 \left(\frac{2\tilde{\omega}_i}{j_{3,i}} \right)^2. \quad (31)$$

The frequencies of fundamental models are given by equation (16). If $\tilde{\omega}_f$ were measured, it would provide us with the value of

$$\beta = \frac{g h}{R^2} (1 - R_G/R) = 4 \frac{\tilde{\omega}_f^2}{l(l+1)}. \quad (32)$$

Formally, we need to detect only one mode of each type to infer the values of α or β , but detecting several modes would give more confidence to the results.

Having α and β , we could obtain the value of $\gamma = \sqrt{\alpha\beta} = g(1 - R_G/R)/R$ without any additional assumption on the value of ρ^{\max} . Note that $g\sqrt{1 - R_G/R}/R \propto \bar{\rho}$, the mean density of the star.

Let us assume that we can deduce the value of ρ^{\max} from the theory or from interpretation of some observations. Then, the crust depth h could be calculated for a given EOS. For the polytropic EOS, it can be estimated as

$$h \approx 8.31 \times 10^4 (\rho_{11}^{\max})^{1/3} / g_{14} \text{ cm}, \quad (33)$$

where $g_{14} = g/10^{14} \text{ cm s}^{-2}$ and $\rho_{11}^{\max} = \rho^{\max}/10^{11} \text{ g cm}^{-3}$. In that case, we would obtain two equations (for α and β) for two important unknown stellar parameters, the mass M and the radius R , and could determine these parameters. If we do not know ρ^{\max} , we would be unable to determine M and R , but could constrain them taking into account that ρ^{\max} cannot exceed the neutron drip density $\rho_d \approx 6 \times 10^{11} \text{ g cm}^{-3}$ (for the accreted envelope). Substituting $\rho^{\max} = \rho_d$ into (33) and h into (32), we would obtain the upper limit on R . With this upper limit, we could easily derive the upper limit on M from the value of γ .

If, on the other hand, the mass M is known from independent observations (e.g. the star enters to a binary system), then the equations for α and β would allow one to determine R and h .

Note that we do not include density discontinuous g modes into our analysis. The frequencies of these modes depend on many factors (such as the densities of phase transitions, associated density jumps, etc.). The uncertainties of such factors would complicate the theoretical interpretation of measured g mode frequencies. However, if observed, g modes could provide some useful additional information. For instance, the number of such modes for a fixed $l > 0$ is equal to the number of phase transitions N . (This statement is strict only in the absence of other sources of buoyancy.) This number could bound the maximum crust density ρ^{\max} in the region between theoretically predicted N th and $N + 1$ phase transitions and impose then better bounds on the inferred values of M and R .

4 CONCLUSIONS

We have studied pressure oscillations of strange star crusts. Our main conclusions are as follows.

(i) The oscillations are almost insensitive to the various modifications of the EOS in the normal crust. The polytropic EOS provides approximately the same eigenfrequencies as the EOS of the accreted crust, except for density discontinuous g modes, which are absent for the polytropic EOS (see Section 3 and Fig. 1).

(ii) The oscillation problem for acoustic modes is self-similar (in the plane-parallel approximation). Once the problem is solved for one stellar model, it can easily be rescaled to strange star models with any mass and radius (but the same EOS and the maximum crust density; see Section 2).

(iii) For a thin polytropic crust, the oscillation problem is solved analytically (Sections 2.2 and 2.1).

(iv) The oscillation spectrum of a strange star crust is *specific*. The frequencies of fundamental modes depend linearly on l ; the

frequencies of p modes are almost independent of l . These features are *unmistakable seismic signatures of strange stars with crust* (Section 3).

(v) A detection and identification of one fundamental mode and one p mode would enable one, in principle, to infer the mass and radius of a strange star (if the maximum crust density is known) or at least to obtain corresponding upper limits (Section 3.1).

Therefore, oscillation modes of strange stars with crust are potentially good tools to distinguish these strange stars from neutron stars and to determine their masses and radii. The oscillation frequencies could be detected very precisely by radio-astronomical methods.

A search for these oscillation modes could be useful. Some of them do not damp quickly and can survive for a long time (Section 3). Pressure modes are robust because they are relatively independent of the thermal state of the crust, and they should not be strongly affected by the crustal magnetic field.

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