Durability of Neutron Star Crust

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How long do neutron star mountains last? The durability of elastically deformed crust is important for neutron star physics including pulsar glitches, emission of gravitational waves from static mountains, and flares from star quakes. The durability is defined by the strength properties of the Yukawa crystals of ions, which make up the crust. In this paper we extend our previous results [Mon. Not. R. Astron. Soc. **407**, L54 (2010)] and accurately describe the dependence of the durability on crust composition (which can be reduced to the dependence on the screening length λ of the Yukawa potential). We perform several molecular dynamics simulations of crust breaking and describe their results with a phenomenological model based on the kinetic theory of strength. We provide an analytical expression for the durability of neutron star crust matter for different densities, temperatures, stresses, and compositions. This expression can also be applied to estimate durability of Yukawa crystals in other systems, such as dusty plasmas in the laboratory.

1 Introduction

The outer layers of neutron stars are composed of fully ionized nuclei (ions), degenerate electrons and, in the inner crust, unbound neutrons (see e.g. [1]). Ions interact via Coulomb potentials that are screened by electrons. In the simplest Thomas-Fermi model, the corresponding potentials have Yukawa form: $u(r) = Z^2e^2/r \exp(-r/\lambda_e)$. Here Ze is the charge of the ions (we assume all ions have equal charges) and r is the inter-ionic distance. Finally, λ_e is the electronic screening length. As a result, ions from a strongly coupled Yukawa system, which is know to crystalize if the temperature T is low enough. Such conditions are realized in not too young neutron stars and the crystalized part of the envelope is known as the neutron star crust. Its elastic and strength properties are defined by the Yukawa crystal of ions. As for all crystals, the crust will break if the applied stress is too large. Crust breaking may be important for a large set of observable phenomenona including pulsar glitches (see, e.g., [2–4]) and magnetar flares (see, e.g., [5]). The strength of the crust provides an upper limit for possible gravitational wave emission associated with elastically supported "mountains" (asymmetric mass distributions) see e.g. [6, 7]. Accurate microphysical studies of the material strength of the neutron star crust are needed for realistic neutron star models to compare with observations.

Until recently only the elastic constants of the neutron star crust were calculated with high accuracy [8, 9] (see also [10] for recent results including quantum effects). Strength estimations for the neutron star crust were based on analogies with terrestrial materials [3, 11] instead of on accurate calculations. Only in the last few years have large scale molecular dynamics (MD) simulations been performed in this field [12, 13]. In these papers the breaking stress (the maximum of the stress-strain curve) was determined by direct computer simulations with linearly increased strain. In ref. [13] we concentrated on how the breaking stress depends on the shear rate and on the temperature. We concluded that the durability of the crust agrees with the kinetic theory of strength (see [14], for example). However, all previous simulations were done only for one crust composition. In the present work we apply the same MD techniques as in ref. [13] and consider the dependence of the crust strength on composition. As a final result we provide an universal analytical approximation [Eqs. (4) and (6)] of durability of neutron star matter as functions of composition, temperature, density, and applied stress. This expression can also be applied to estimate the durability of dusty plasmas in the laboratory.

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2 Formalism

We describe electron screening by a Thomas-Fermi screening length

$$\lambda_{\mathrm{e}} = \frac{\sqrt{\pi}}{2}\left(\frac{\hbar c}{e^2}\right)^{1/2}\left(3\pi^2 n_{\mathrm{e}}\right)^{-1/3}, \tag{1}$$

where n_{e} is the electron number density and c is the speed of light. For simplicity, electrons are assumed to be ultrarelativistic. The state of the ion system can be characterized by the classical coupling parameter

$$\Gamma = \frac{Z^2 e^2}{aT}. \tag{2}$$

Here $a = [3/(4\pi n_{\mathrm{i}})]^{1/3}$ is the ion sphere radius.

The ion number density is $n_{\mathrm{i}} = n_{\mathrm{e}}/Z$ because the system is neutral. In the ultrarelativistic limit, the ratio of the electron screening length λ_{e} to the ion sphere radius a depends only on Z: $\lambda_{\mathrm{e}}/a \approx 5.41/Z^{1/3}$. As a result, we can model the dependence on composition by variation of the ratio λ_{e}/a.

A typical frequency of ion motion is the ion plasma frequency $\omega_{\mathrm{p}} = \sqrt{4\pi Z^2 e^2 n_{\mathrm{i}}/m_{\mathrm{i}}}$, where $m_{\mathrm{i}} = A m_{\mathrm{u}}$ is the ion mass. We measure time in units of ω_{p}^{-1}. At temperatures $T \lesssim T_{\mathrm{p}}/3 = \hbar\omega_p/3$ quantum effects can be important for ion motion. In our classical MD simulations we neglect quantum effects (assuming $\hbar \to 0$), but they can be important at the conditions of realistic neutron star crust. The role of quantum effects on thermodynamics of the crystal is accurately known [15, 16]. For the strength of a crust quantum effects can be included in an approximate way as developed in ref. [13].

2.1 Molecular dynamics simulations

To calculate the breaking stress we perform a large number of runs of the YUKAWAMD code (see [12, 13]) for matter composed of ions with $Z = 6$, $Z = 29.4$ and 100. In this code the ion system is strained by deforming periodic boundaries according to $x \to x + \epsilon y/2$, $y \to y + \epsilon x/2$, and $z \to z/(1 - \epsilon^2/4)$. Here the strain $\epsilon = vt$ increases with time at constant strain velocity v. We choose a time step $\sim 0.1/\omega_{\mathrm{p}}$ in the velocity Verlet algorithm [17]. We define stress as $\sigma = \partial\mathcal{E}/\partial\epsilon$ [18], where $\mathcal{E}$ is the internal energy at unit volume. The breaking stress σ_{b} is the maximum stress obtained during the shear simulation. The corresponding strain we refer to as the breaking strain ϵ_{b}. All simulations discussed here are done for 9826 ions in a periodic box. For more details about the numerical procedure see [12, 13].

2.2 Kinetic theory of strength

Following [13], we apply the kinetic theory of strength (see e.g. [14, 19–21]) to describe our MD simulations. In this theory, the breaking event occurs due to thermal fluctuations. To achieve breaking, the fluctuation should have energy U. This threshold energy is reduced by the applied stress σ to $U - \sigma V$, where V is the activation volume. Finally, the durability of matter τ under applied stress σ can be estimated as

$$\tau = \tau_0 \exp\left(\frac{U - \sigma V}{T}\right), \tag{3}$$

where τ_0 is a typical timescale for ion motion. As in ref. [13] we take $\tau_0 = \omega_{\mathrm{p}}^{-1}$. We fit the free parameters U and V to reproduce the results of our MD simulations. In dimensionless variables $\bar{U} = U\,a/(Z^2 e^2)$ and $\bar{\sigma} = \sigma/(n_{\mathrm{i}} Z^2 e^2/a)$, Eq. (3) can be rewritten in the form

$$\tau = \frac{1}{\omega_{\mathrm{p}}} \exp\left(\bar{U}\Gamma - \bar{\sigma}\bar{N}\Gamma\right), \tag{4}$$

where $\bar{N} = V n_{\mathrm{i}}$. Formulated in dimensionless form, the kinetic theory of strength can be applied to Yukawa crystals at any physical conditions from dusty plasmas to neutron star crust. We rewrite Eq. (4) as

$$\frac{v}{\omega_{\mathrm{p}}} = \frac{\epsilon_{\mathrm{b}}}{\bar{N}\Gamma\bar{\sigma}_{\mathrm{b}}} \exp\left(-\bar{U}\Gamma + \bar{\sigma}_{\mathrm{b}}\bar{N}\Gamma\right). \tag{5}$$

in order to directly apply to results of our MD simulations with linearly increasing strain (see ref. [13, 22], for example).

3 Numerical results

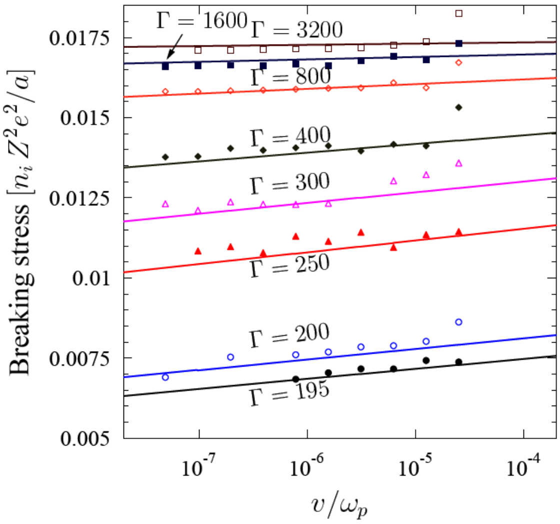

Fig. 1 The breaking stress vs shear rate v for $Z = 100$ ($\lambda/a = 1.16$). Symbols are results of MD simulations, solid lines correspond to the Eq. (5) with parameter set given by Eq. (6). See text for details.

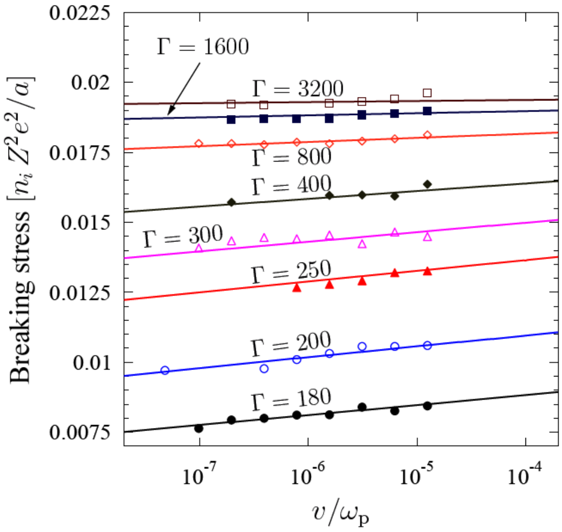

Fig. 2 Same as Fig. 1, but for $Z = 29.4$ ($\lambda/a = 1.75$).

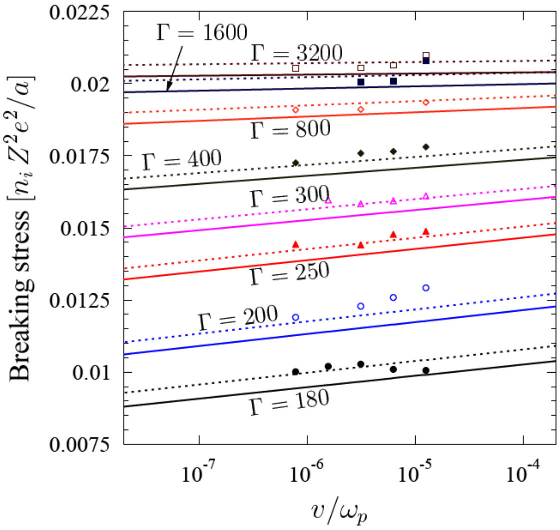

Fig. 3 Same as Fig. 1, but $Z = 6$ ($\lambda/a = 2.97$). An additional dotted lines correspond to parameters (6), but Γ_m is taken to be $\Gamma_\mathrm{m} = 175$.

Our MD simulation results of for the breaking stress of a system of 9826 ions for $Z = 100$ ($\lambda/a = 1.16$), $Z = 29.4$ ($\lambda/a = 1.75$) and $Z = 6$ ($\lambda/a = 2.97$) are shown by symbols in Figs. 1, 2 and 3, respectively. Most of the data for $Z = 29.4$ are taken from [13]. The data are fitted by Eq. (5) with the following parameters $\bar{U}$ and $\bar{N}(\Gamma)$:

$$\bar{U} = 68.6/\Gamma_\mathrm{m}, \quad \bar{N} = \frac{500}{\Gamma - 0.8\,\Gamma_\mathrm{m}} + 18.5. \tag{6}$$

Here $\Gamma_\mathrm{m} = \Gamma_\mathrm{m}(\lambda_\mathrm{e}/a)$ is the melting coupling parameter for Yukawa crystals at a given ratio λ_e/a. We apply the approximation from [23]

$$\Gamma_\mathrm{m} = \Gamma_\mathrm{m}^\mathrm{OCP} \frac{\exp(\kappa)}{1 + \kappa + \kappa^2/2}, \tag{7}$$

where $\kappa = \lambda_e n_{\rm i}^{1/3} = (3/4\pi)^{1/3}\ \lambda_e/a$ and $\Gamma_{\rm m}^{\rm OCP} \approx 175$ — the melting coupling parameter for the OCP [24]. This fit is shown by solid lines in Figs. 1–3 and one can see a very nice agreement with our MD data. For $Z = 6$ a slightly better fit can be achieved using Eq. (6) by adjusting of $\Gamma_{\rm m}$. This is shown by dotted lines in Fig. 1 and corresponds to using $\Gamma_{\rm m} = 175$, which is slightly lower than $\Gamma_{\rm m} = 178$ predicted by Eq. (7) for $\lambda/a = 2.97$. This difference in melting parameters is not very large and both fits are accurate enough for applications to neutron star crust.

4 Conclusions

We have performed extensive MD studies of the dense matter of neutron star crust with different composition. We describe our breaking stress results with the kinetic theory of strength. The corresponding parameters that we find are given by Eqs. (6) and (4) and can be applied to estimate the durability of the neutron star crust as a function of density, temperature, composition, and stress. Our results can be used as physical input for models of neutron star phenomena associated with crust breaking. They can also be applied to describe breaking events of Yukawa crystals for other physical conditions, such as dusty plasmas in the laboratory [25].

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