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Photoionization of neutral atoms in a Bose–Einstein condensate

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Abstract. A process of photoionization of neutral atoms composing a Bose–Einstein condensate is examined theoretically. It is shown that the coherent nature of the initial atomic ensemble can lead to a relatively large phase-space density of the fermionic products (electrons and ions) and, hence, to photoionization rate suppression. This effect may serve as an experimental tool for investigation of collective interactions in a highly non-equilibrium coherent ensemble of charged fermions.

1. Introduction

The recent successes in reaching a Bose condensation of alkali atom vapours [1] have attracted much interest to new aspects of photon–matter interactions arising from the coherent nature of an atomic ensemble. Indeed, quantum collective effects give rise to a significant modification of the optical properties of atomic ensembles. For example, we can mention an increase of the radiative linewidth of an atomic transition caused by the large number of bosonic atoms in the final (ground) state [2]. Further accumulation of atoms in a Bose–Einstein condensate (BEC) may lead not only to enhancement of the excited state natural width by the corresponding phase-space density factor but also to non-exponential decay [3], revealing oscillatory behaviour characteristic of the case of very strong coupling [4].

In the present paper, we consider, in some sense, an opposite case—photoionization of a BEC by monochromatic laser light. Due to the coherent nature of the initial atomic ensemble and narrow spectral width of the light source, the occupation numbers of the final states can become close to unity. The products (electrons and ions) obey Fermi–Dirac statistics. Thus, further ionization is slowed down, and its rate is determined by a balance between laser light action on atoms and the rate of escape of the products from the system. In order to give some estimates, an analytically solvable model is worked out. We discuss possible applications of BEC photoionization as a tool for experimental investigation of charged fermion interactions in highly non-equilibrium electron–ion plasmas produced during this process.

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2. Basic equations

We consider a BEC of neutral atoms irradiated by monochromatic light with frequency ω_{L} exceeding the ionization threshold. This light is considered as a classical field coupling these atoms in the ground state to a continuum corresponding to infinite motion of electrons and ions produced in the ionization process. The corresponding Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{free}} + \hat{H}_{\mathrm{i}} + \hat{H}_{\mathrm{prod}} \tag{1}$$

consists of three terms. The first one,

$$\hat{H}_{\mathrm{free}} = -J\hat{b}_c^{\dagger}\hat{b}_c + \sum_p \omega_p \hat{a}_p^{\dagger}\hat{a}_p + \sum_q \Omega_q \hat{A}_q^{\dagger}\hat{A}_q \tag{2}$$

where $J > 0$ is the ionization potential, ω_p, Ω_q are the energies of the particular modes of translational motion of electrons and ions, respectively. We use the system of units where $\hbar = c = 1$. $\hat{b}_c$ is the annihilation operator of bosonic atoms in the condensate, $\hat{a}_p$ and $\hat{A}_q$ are the annihilation operators of fermions—electrons or, respectively, ions—in the corresponding modes.

Coupling of the bound state to the ionization continuum by means of the laser radiation is described by the second term,

$$\hat{H}_{\mathrm{i}} = \sum_p \left[\kappa_p \exp(-\mathrm{i}\omega_{\mathrm{L}}t)\hat{b}_c\hat{a}_p^{\dagger}\hat{A}_p^{\dagger} + \kappa_p^{*}\exp(\mathrm{i}\omega_{\mathrm{L}}t)\hat{b}_c^{\dagger}\hat{a}_p\hat{A}_p\right]. \tag{3}$$

Here the rotating-wave approximation is used. The coupling constants κ_p are proportional to the light wave amplitude.

A remark should be made with respect to momentum conservation. After an act of photoionization both the ion and electron acquire additional kinetic momenta (the sum of these two vectors is equal to the kinetic momentum of a photon). However, due to the large difference of the masses of the ion and electron, the electron carries almost all the kinetic energy released in this process. Thus the number of final states per unit energy is determined by the kinematics of the electron's motion. Being neutral atoms in a BEC does not modify the product state density, but changes dramatically, as we demonstrate in the present paper, the occupation numbers of these states in comparison with the case of a non-degenerate atomic ensemble.

The last term, $\hat{H}_{\mathrm{prod}}$, describes the interaction of photoionization products. There are processes of two types: (i) Coulomb interactions and (ii) recombination. Firstly, it is necessary to note that we need to consider interaction between ions and electrons just after an act of photoionization, i.e. when these fermions are very far from thermodynamical equilibrium with respect to both the energetic and angular distribution of particles. Hence, we cannot use the well known formulae for transport processes derived in a case of a plasma deviating slightly from the equilibrium state. Secondly, the wavepacket of a particular fermion is spread widely and overlaps significantly with wavepackets of many other fermions. This fact disallows us from using the standard binary-collision approach. It is necessary to turn to a quite complicated picture of collective excitations in such a plasma. In the present paper we assume that the action of $\hat{H}_{\mathrm{prod}}$ to the fermionic field operator can be treated within the framework of the relaxation constants model. Recombination is followed by photon emission, hence, it is quite natural to describe it in terms of irreversible processes. But even regular electrostatic interaction of particles composing a coherent quantum ensemble can lead to a wavefunction phase randomization and thus simulate a dephasing [5].

Thus, in Heisenberg's picture we find for the operators

$$\begin{aligned}\hat{b} &= \hat{b}_c \exp(-\mathrm{i}Jt)\\ \hat{f}_p &= \hat{a}_p \hat{A}_p \exp[\mathrm{i}(\omega_\mathrm{L} - J)t]\\ \hat{\nu}_p &= \hat{a}_p^\dagger \hat{a}_p + \hat{A}_p^\dagger \hat{A}_p\end{aligned} \tag{4}$$

the following set of equations:

$$\begin{aligned}\frac{\partial}{\partial t}\hat{b} &= -\mathrm{i}\sum_p \kappa_p^* \hat{f}_p\\ \frac{\partial}{\partial t}\hat{f}_p &= -(\gamma_2 + \mathrm{i}\delta_p)\hat{f}_p - \mathrm{i}\kappa_p \hat{b}(1 - \hat{\nu}_p)\\ \frac{\partial}{\partial t}\hat{\nu}_p &= -2\gamma_1 \hat{\nu}_p - 2\mathrm{i}\kappa_p \hat{b}\hat{f}_p^\dagger + 2\mathrm{i}\kappa_p^* \hat{b}^\dagger \hat{f}_p.\end{aligned} \tag{5}$$

Here $\delta_p = \omega_p + \Omega_p - \omega_\mathrm{L} + J$, and the relaxation constants γ_1, γ_2 arise from interaction of photoionization products and are the analogues of longitudinal and transversal damping rates in optical Bloch equations. Naturally, their ratio $\eta = \gamma_2/\gamma_1$ cannot be less than unity.

The initial conditions are the following. Firstly, at $t = 0$ there are no fermions in the system:

$$\hat{f}_p|_{t=0} = 0 \qquad \hat{\nu}_p|_{t=0} = 0.$$

Secondly, the initial number of atoms in the BEC is very large. This allows us to replace $\hat{b}$ with a c-number b (in particular, in the initial condition $\hat{b}|_{t=0} = b_0$).

3. Analytic solutions

For the sake of simplicity, we assume that the photoionization rate is the smallest rate in the system:

$$-\frac{\partial}{\partial t}\ln|b|^2 \ll \gamma_1. \tag{6}$$

It allows us to exclude the fermionic variables adiabatically:

$$\hat{f}_p = -\frac{\mathrm{i}\kappa_p b(\gamma_2 - \mathrm{i}\delta_p)}{\gamma_2^2 + 2|\kappa_p|^2|b|^2\eta + \delta_p^2} \tag{7}$$

$$\hat{\nu}_p = \frac{2|\kappa_p|^2|b|^2\eta}{\gamma_2^2 + 2|\kappa_p|^2|b|^2\eta + \delta_p^2}. \tag{8}$$

In the right-hand side of equations (7) and (8) the c-numbers appear. In particular, we can refer further, after we have already taken into account anticommutation relations when deriving the set of equations (5), to both the number ν_p of fermions in a given mode and the total number of fermions

$$\nu_\mathrm{tot} = \sum_p \nu_p \tag{9}$$

as to c-numbers, too. Using equation (7) it is easy to find the equation describing the decrease of the number of atoms in BEC due to photoionization:

$$\frac{\partial}{\partial t}|b|^2 = -\gamma_2 \sum_p \frac{2|\kappa_p|^2|b|^2}{\gamma_2^2 + 2|\kappa_p|^2|b|^2\eta + \delta_p^2} \tag{10}$$

or, equivalently,

$$\frac{\partial}{\partial t}|b|^2 = -\gamma_2 \int_{-\infty}^{\infty} \frac{2|\kappa(\epsilon)|^2|b|^2}{\gamma_2^2 + 2|\kappa(\epsilon)|^2|b|^2\eta + (\epsilon - \omega_L + J)^2} g(\epsilon)\,\mathrm{d}\epsilon. \tag{11}$$

In the latter expression we turn from summation over translational motion modes to integration over energy ϵ using the state number density $g(\epsilon)$. As usual in the Wigner–Weisskopf approach, the lower limit of integration is set as $-\infty$. Finally, we can write down the third form of equation (10):

$$\frac{\partial}{\partial t}|b|^2 = -\gamma_1 \nu_{\mathrm{tot}}. \tag{12}$$

Now one can easily see under which condition the collective quantum effects do not modify the photoionization rate. Namely, if

$$|\kappa(\epsilon_k)|^2|b|^2 \ll \gamma_1\gamma_2 \tag{13}$$

then equation (11) represents the standard Wigner–Weisskopf formula for the width of a decaying state, so photoionization proceeds with the usual rate

$$\Gamma_i = \pi|\kappa(\epsilon_k)|^2 g(\epsilon_k) \tag{14}$$

according to the exponential law $|b|^2 = |b_0|^2\exp(-2\Gamma_i t)$. Here $\epsilon_k = \omega_L - J$ is the excess of the photon energy over the ionization threshold. Due to the large difference in mass of the electron and ion, ϵ_k can be associated with the electron kinetic energy.

Obviously, for the single-particle case, $|b|^2 = 1$, equation (13) coincides with the condition of the coupling strength being weak enough for the exponential decay theory [4] to be applicable. If we consider a non-degenerate ensemble of atoms, an inhomogenous broadening usually prevents, if a medium is not too dense, the photoionization rate from being influenced by the Fermi statistics of the products.

Of course, this value of Γ_i can also be found without the assumption of equation (6), within the framework of the usual Wigner–Weisskopf theory. $\Gamma_i = \sigma_i I_L/\omega_L$, where σ_i is the photoionization cross section, I_L is the radiation intensity. However, equation (6) is essential for calculations in the case when equation (13) is violated. To analyse the consequences of violation of the condition given by equation (13) more concretely, we consider the three following models for the relaxation rates.

3.1. Constant relaxation rates

The first case implies constant values for γ_1 and γ_2. In this case, if equation (13) holds, the total number of fermions is $\nu_{\mathrm{tot}} = 2\Gamma_i|b|^2/\gamma_1$, and equation (13) takes the form

$$g(\epsilon_k)\gamma_1 \gg \nu_{\mathrm{tot}}. \tag{15}$$

This relation can be interpreted as follows. The lifetime of a fermion in the system is γ_1^{-1}, hence the product of the state number density with the width γ_1 is the number of states available for filling by fermions produced in the photoionization process. If this number is smaller than ν_{tot} then no suppression of the photoionization rate takes place. If, in contrast, the condition of equation (13) is violated, the photoionization rate is decreased down to the value for which the number of fermions in the system matches the number of configuration space cells available. In this case the decrease of the number of atoms in the BEC is not exponential:

$$|b|^2 = \left(|b_0| - \frac{\Gamma_i\gamma_2 t}{\sqrt{2\eta}|\kappa(\epsilon_k)|}\right)^2. \tag{16}$$

The corresponding number of fermions is approximately equal to $\pi\sqrt{2\eta}|\kappa(\epsilon_k)|g(\epsilon_k)|b|$. The self-consistency of the theory (in particular, satisfaction of equation (6)) requires $\nu_{\rm tot} \ll |b|^2$, or, in other words,

$$|b|^2 \gg \eta\, g(\epsilon_k)\, \Gamma_{\rm i}. \tag{17}$$

3.2. Fermion-number-dependent relaxation rates

The second model takes into account a possible dependence of the relaxation constants on the total number of fermions in the system, namely, we assume that

$$\gamma_\mu = \gamma'_\mu \nu_{\rm tot} \qquad \mu = 1, 2. \tag{18}$$

This model corresponds to the case where binary interactions between the fermions are predominant. The condition of the absence of a quantum statistics influence on the photoionization rate is given again by equation (15). Using equation (18), one can represent it as follows:

$$g(\epsilon_k)\, \gamma'_1 \gg 1. \tag{19}$$

In the opposite case, when $g(\epsilon_k)\, \gamma'_2 \ll 1$, the decrease of the number of atoms in the BEC is also exponential, $|b|^2 = |b_0|^2 \exp(-2\tilde{\Gamma}_{\rm i} t)$, but with the photoionization rate

$$\tilde{\Gamma}_{\rm i} = 2\pi g(\epsilon_k)\, \gamma'_2 \Gamma_{\rm i} \tag{20}$$

reduced due to Fermi statistics of the products. Again, equation (6), which is to be satisfied for a correct derivation of equation (20), can be equivalently transformed into equation (17).

3.3. Boson-number-dependent relaxation rates

The last of our three models of relaxation operates with the expressions

$$\gamma_\mu = \gamma_\mu^{(1)} |b|^2 \qquad \mu = 1, 2. \tag{21}$$

In this case, unlike the two previous ones, the modified decay law

$$|b|^2 = \frac{1}{\left(|b_0|^{-1} + \sqrt{\pi \gamma_1^{(1)} \gamma_2^{(1)} g(\epsilon_k) \Gamma_{\rm i}/2}\, t\right)^2}$$

takes place if the number of atoms in the BEC is not too great:

$$|b|^2 \ll \frac{\Gamma_{\rm i}}{g(\epsilon_k) \gamma_1^{(1)} \gamma_2^{(1)}}. \tag{22}$$

If the number of bosons in the condensate exceeds the right-hand side of equation (22) then the fermion escape rate is enhanced enough to eliminate the influence of the product statistics on the photoionization process.

The self-consistency requirement can again be expressed in the form of equation (17).

4. Discussion

Let us make some estimates of the BEC photoionization rate, taking into account recombination as a main mechanism of product losses. The state number density can be written as $g(\epsilon_k) \sim V m^{3/2} \epsilon_k^{1/2}$, where m is the electron mass, V is the quantization volume, which is arbitrary and appears in the final results only in expressions for particle densities ($|b|^2/V$ or $\nu_{\rm tot}/V$). The real volume of BEC in a trap is of the order of l^3 where

$l \sim 0.01$ cm [1]. For alkali metals $J \approx 4$ eV. We assume that $\epsilon_k \ll J$ (for the estimates we choose the kinetic energy of the electron to be of the order of 10^{-6} times its relativistic rest energy, i.e. $\epsilon_k \approx 0.5$ eV). Under these conditions, the photoionization cross section σ_i is of the order of a^2 (a is the atomic radius); the recombination cross section σ_r always obeys the relation $\sigma_r = \sigma_\mathrm{i}\omega_\mathrm{L}^2/(m\epsilon_k)$ [6].

Firstly, it is necessary to check whether the self-consistency condition equation (17) can be satisfied. Substitution of the system parameters gives the lower limit to the boson density of about $0.3 \times 10^8 \Gamma_\mathrm{i}$ cm^{-3} s. A realistic number of atoms in a BEC sample occupying the volume $l^3 \sim 10^{-6}$ cm^3 is of the order of 10^5 [1]. Thus our theory is reliable until the photoionization rate is smaller than 10^3 s^{-1}, which corresponds to $I_\mathrm{L} < 10$ W cm^{-2}.

Recombination of electrons and ions can proceed in two ways. The first one is the usual process that is described by the relaxation rate proportional to the total number of fermions, and $\gamma_1' = \gamma_2' \sim V^{-1}\sigma_r\sqrt{\epsilon_k/m}$. The second is specific for the case of the presence of a BEC of neutral atoms. Each electron–ion pair produced by laser radiation can undergo recombination with the atom reappearing in the BEC. A general picture in such a case is completely analogous to that considered by Javanainen with respect to radiative relaxation of an atomic excited state in the presence of BEC [2]. The large number of atoms in the BEC leads to an increase in the phase space corresponding to the final state and, hence, the large factor $|b|^2$ appears in the expression for the recombination rate. On the other hand, a photon is emitted during this process only into the very narrow body angle w_eff determined by the BEC sample size ($w_\mathrm{eff} \sim (\omega_\mathrm{L} l)^{-2}$ and under our assumption is as small as 10^{-6}). This body angle also appears as a factor in the relaxation rates given by equation (21). For any realistic conditions when the photoionization rate can differ from Γ_i the usual kind of recombination is predominant because $\nu_\mathrm{tot} \gg |b|^2 w_\mathrm{eff}$, and we should use the model of fermion-number-dependent relaxation rates. We find that $g(\epsilon_k)\gamma_1' \sim (\omega_\mathrm{L} a)^2 \sim 10^{-6}$, equation (19) is not satisfied obviously, hence, the photoionization rate is equal to $\tilde{\Gamma}_\mathrm{i} \ll \Gamma_\mathrm{i}$.

So we can conclude that photorecombination cannot effectively remove fermionic products from their final states, and the photoionization process would be almost stopped, if photorecombination were the only cause of the decrease of the population of these states. However, another factor, Coulomb interactions, remains. Calculation of its contribution to γ_1, γ_2 is beyond the scope of the present work. However, we should stress that experimental determination of the BEC photoionization rate can serve as a tool for investigation of electrostatic interactions of charged fermions in the coherent highly non-equilibrium ensemble. Checking the photoionization rate modification for different numbers of atoms accumulated in a BEC and using equation (13), one can give certain estimates or limits on relaxation rates originating providing data necessary for testing of different theoretical models, taking into account the collective nature of this effect.

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